# AN INVESTIGATION INTO THE DESIGN OF A SELF ADAPTIVE DIGITAL TEMPERATURE CONTROLLER FOR INDUSTRIAL FURNACE CONTROL.

Thesis submitted to the Council for National Academic Awards in partial fulfilment of the requirements for the degree of

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The work presented in this thesis was carried out in the School of Electrical and Electronic Engineering at Preston Polytechnic,

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## SUMMARY

The work presented in this thesis is directed towards the investigation of the possibilities of using adaptive or learning processes in order to achieve a greater degree of temperature control in a range of furnaces which are normally used for the heat treatment of metals. In this investigation the author has limited his work to two furnaces, a medium sized furnace and a small sized furnace. It is argued that the methods employed in controlling these two furnaces may be readily applied to other furnaces of the same general type.

The first objective of the work presented in this thesis was to produce a satisfactory mathematical model for the medium size furnace, check its validity by the use of analogue simulation techniques and finally to use the parameters elucidated from the work on the mathematical model to close the adaptive loop.

Next a review of the various methods of system identification was carried out and particular attention was given to the problems associated with the long (several hours) time constants involved in the work on heat treatment furnaces. The difficulties involved when working with long time constants were resolved by making use of a digital controller and by the use of Z-transforms as applied to the furnace mathematical model.

The closing of the adaptive loop was achieved by the use of the digital computer as the controller. The identification of the furnace parameters was achieved by a model adjustment strategy and by use of a continually changing index of performance dictated

by the monitoring of the apparent changes in the furnace parameters.

Finally the results obtained by controlling the furnace are given, which show that good temperature control has been achieved, but it appears that further work will need to be carried out before a universally acceptable control strategy can be developed.

ACKNOWLEDGEMENTS

The author wishes to express his deepest gratitude to Dr. T.J. Terrell who supervised this research, for his guidance and encouragement throughout the course of its progress. The author is also indebted to Cooperheat Ltd., and C.W. Control Ltd., for the use of their equipment and for guidance on furnace control problems, and also to Preston Polytechnic for providing equipment and financial backing for the project. Thanks are also due to my colleagues, especially Mr. N. Owen for his suggestions during the operation of the mini-computer and to Mrs. T. Smith for her patience and skillful typing of this thesis. I thank my wife, Pat, for her support and understanding during the course of the project.

LIST OF CONTENTS

Page No.

## FIGURES AND TABLES

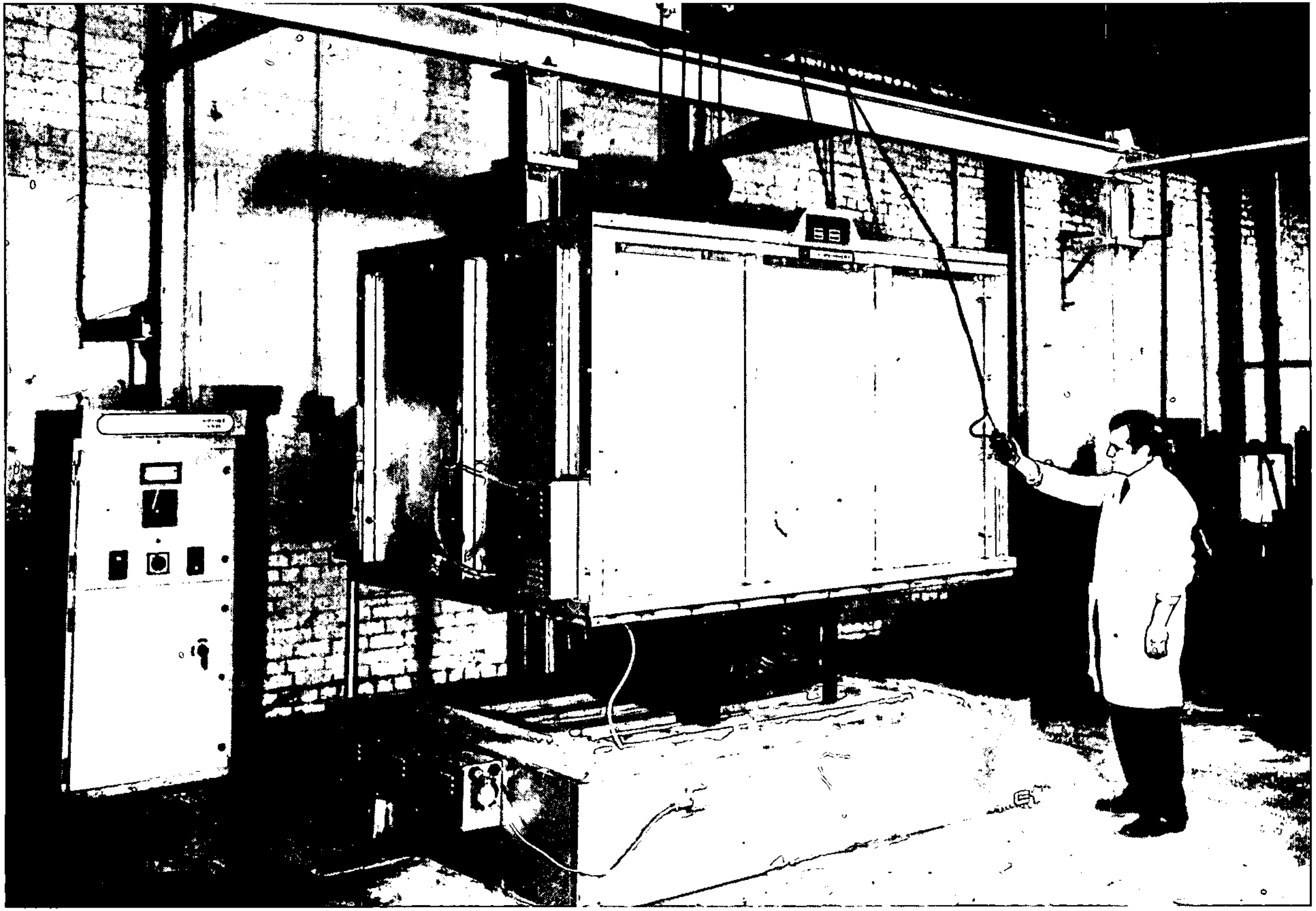
### FIGURES

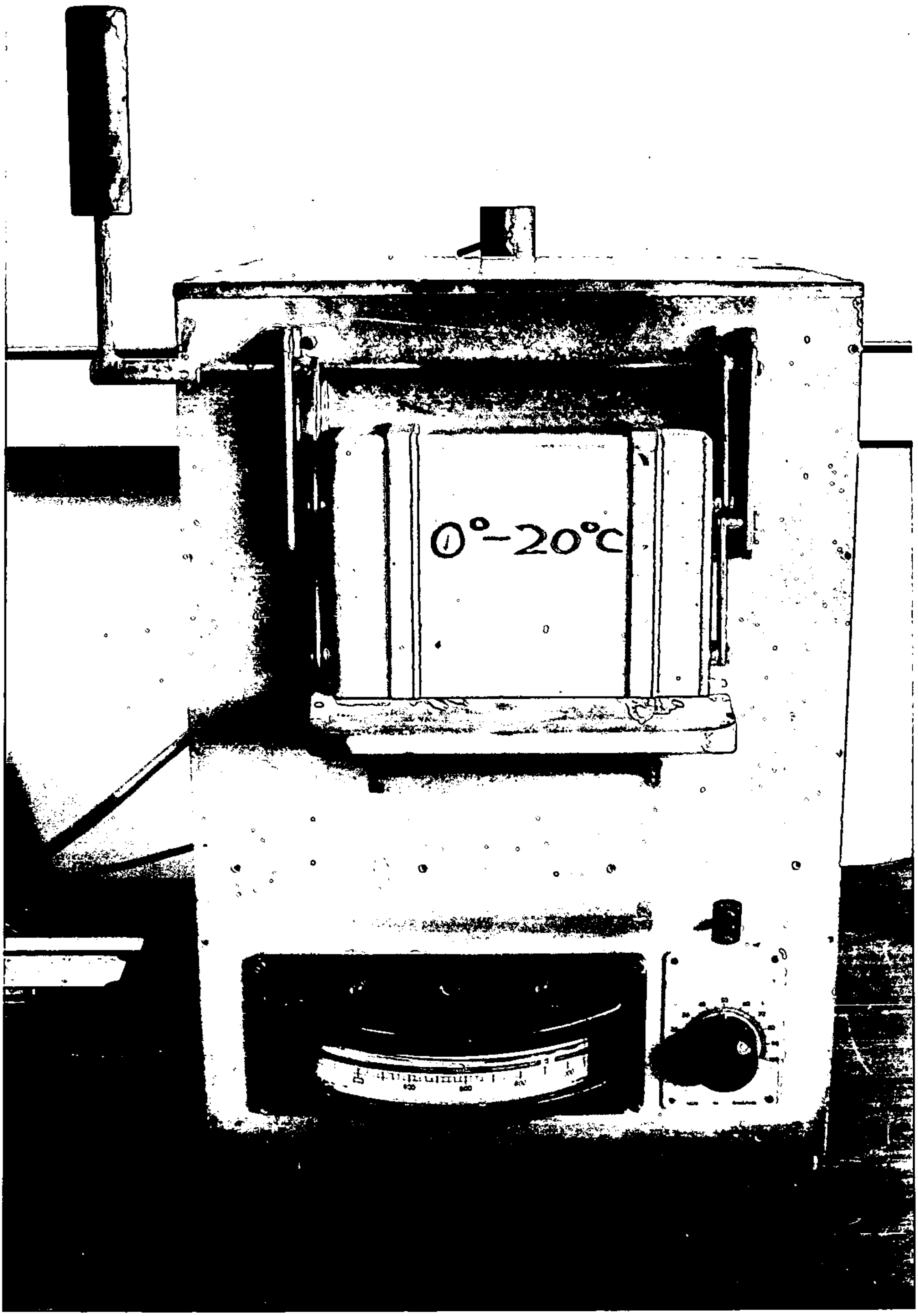


*Fig. 2.1 is contained in a pocket at the end of this thesis.

Page No.

TABLES

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# CHAPTER 1

# INTRODUCTION

## 1.1 Preliminary Remarks

The research reported in this thesis is directed towards solving some of the problems associated with the adaptive control[1,2,3] of an industrial stress-relieving-furnace. In the early stages of the investigation a medium sized furnace was studied, and the knowledge gained was subsequently used on a second smaller furnace, thereby testing, to some degree, the validity of the derived furnace model. The medium size furnace, rated at 21kW (71500 Btu/hr) is shown in figure 1.1, and the smaller furnace, rated at 3.6 kW (12,500 Btu/hr), is shown in figure 1.2.

Prior to the commencement of this research project, the type of furnace control,[4] used by some industrial organisations, was largely dependent upon the basis that the step-response of the furnace was assumed to be linear. That is, the furnace temperature was assumed to increase, or decrease, at a fixed rate over the operating cycle of the process involved. This assumption was found to be true only if relatively short time periods were taken, for example times of the order of several minutes. If however, a time scale of the order of several hours is used, the step-response of the furnace is non-linear.

In practice the step-response of the furnace depends upon many random and uncontrollable factors. Some of the most important factors are, for example:

(i) external temperature variation,

(ii) heat storage in the load,

(iii) the mass of the load,

(iv) heat storage in the furnace walls, and

(v) heat storage in the hearth,

This occurrence of unknown and uncontrollable factors naturally suggests that an adaptive approach may lead to a much better understanding of the furnace performance and hence more accurate control of the furnace. Improved control of the furnace may result in better stress relieving and possibly be more efficient as far as the utilisation of energy is concerned. For the two furnaces used in this investigation the results in economic terms, i.e. in energy savings, will not be large. However, the possibility that better stress relieving, together with the hope that the techniques presented in this thesis may apply to much larger industrial furnaces justifies this investigation into adaptive control of industrial furnaces.

In order to gain a better understanding of operating conditions a mathematical model of the furnace was derived. In the initial investigations the theoretical temperature profile of this model was found to be slightly in error when compared with the actual temperature profile for the furnace. The model was then empirically changed, which resulted in improved correlation between the measured and theoretical furnace data, see figure 2.3. The furnace mathematical model and its detailed derivation is discussed in chapter 2.

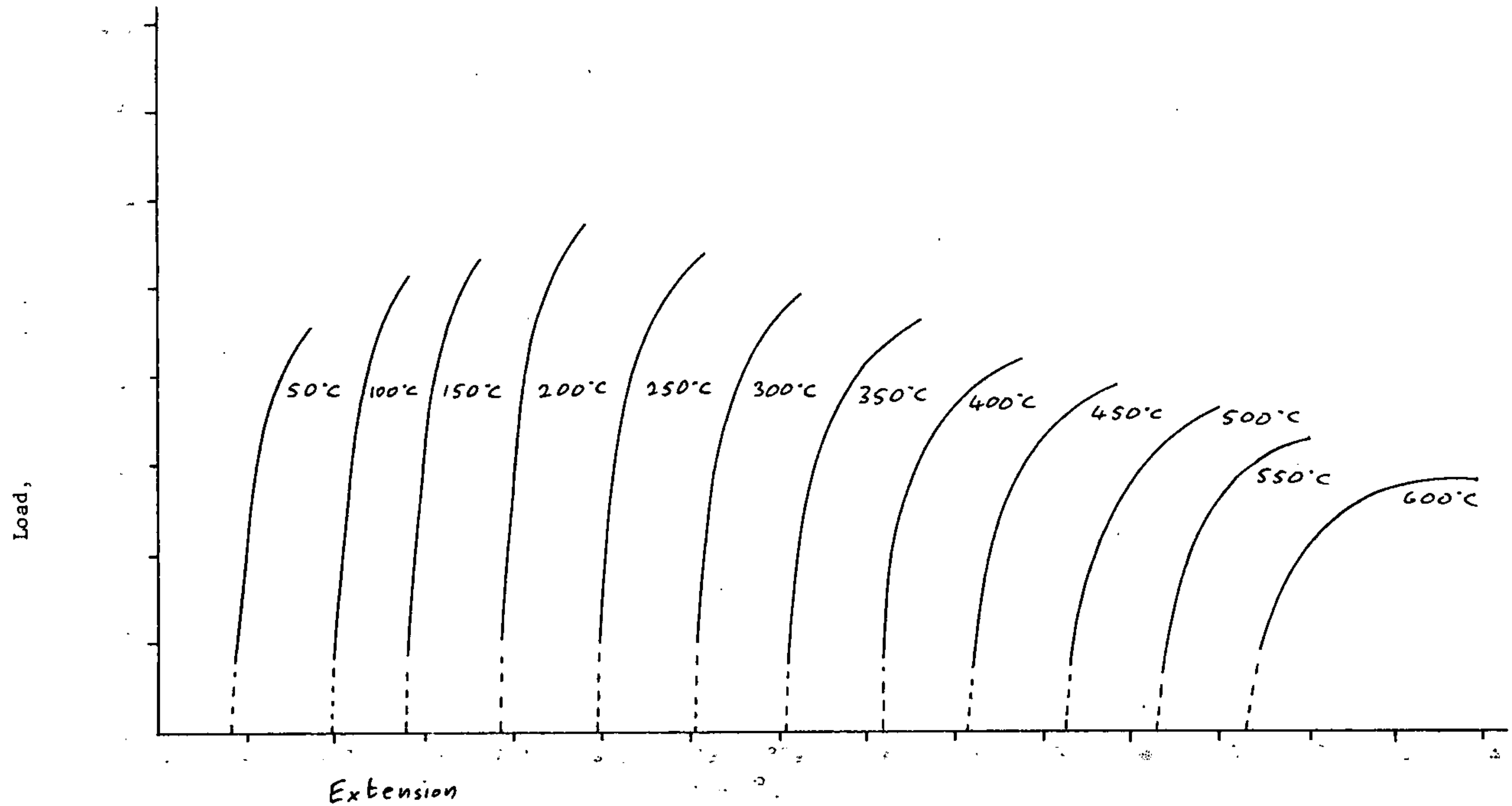



Fig. 1.3 Load-extension diagrams of high-carbon wire, cold drawn and then reheated at indicated temperatures (from Greaves).

## 1.2 Heat Treatment of Metals (Stress Relieving)

The main purpose of the type of furnace dealt with in this thesis is to relieve the internal stresses which are produced in, for example, steel objects which have been 'worked' when cold. The stress relieving is achieved by subjecting the metal to a controlled heating process, this type of heating is sometimes called a "stress relief anneal".[5]

If a 'cold worked' material is reheated the material undergoes a cycle of changes in strength and toughness, and this process is governed by the temperature of the reheating cycle.

Typical strength changes in high carbon steel composition[6] (carbon 0.74%. Manganese 0.74% and Silicon 0.14%) are shown in figure 1.3. After being cold drawn in wire form the steel was heated to the temperatures shown, then cooled to normal room temperature and tested in tension up to fracture. It can be seen that mild reheating, providing the temperature changes are sufficiently slow, causes both the elastic limit and the tensile strength to increase, up to a certain temperature limit.

When reheating occurs at too fast a rate, other undesirable effects become observable. For this reason the heat-treatment-furnace needs to be controlled in some measure. If the heating of the steel is too fast, or not uniform in some way, then regardless of its composition, changes in its structure will take place.

Steel expands on heating. This expansion progresses at a uniform rate until a critical range is reached. In this critical range a marked reduction in volume occurs. This reduction in volume can and does cause large stresses to be set up in the steel, in particular this can happen when steel structures are welded.

These internal stresses, if sufficiently large, can produce warping or even cracking of the steel. At the very least these internal stresses will weaken the steel and could lead to premature failure of a component. Also, the large temperature gradients produced will result in the grains, in the areas near to the weld, being non-uniform in size. Slow, careful, controlled reheating will tend to relieve any stresses produced in the steel and also reduce the non-uniformity in the grain size.

Some industrial organisations (e.g. Cooperheat Ltd), produce furnaces which are used to relieve stresses in the steel structures of oil rigs in the North Sea. In an environment like the North Sea, premature failure of any structural component could have disastrous consequences, and hence heat-treatment is an essential and very important process.

## 1.3 Adaptive Control

This thesis is primarily concerned with the investigation of adaptive control and its application to the control of an industrial heat-treatment furnace. However, as a prelude to the discussion of the control problem in detail, it is worth while to briefly consider some of the more interesting and useful historical developments of control in general, and adaptive control in particular.

The art of control is very old indeed, in fact probably as old as life itself. Man is a very good example of an adaptive control system, that is, a system which can adjust and adapt to its environment.

The control of automatic systems, in the first instance, developed towards empirical solutions for primarily engineering problems. The principle and application of feedback control, in its mathematical form developed as engineers attempted to understand the operation of automatic systems.

### 1.3.1 Deterministic Processes

The first attempts to analyse automatic systems used such classical methods as the Nyquist plot, Bode diagrams, Nichol's chart and the root locus method. This type of analysis was,in the early stages, often no more than trial and error, and the period which embraced this development of control is now often referred to as the 'deterministic-period'.

It is, however, important to realise that deterministic methods may only be used when control systems can be predicted with complete certainty. Bellman and others[7,8] delineate properties of a deterministic process as follows:

(i) The state of the system is known at each stage of the process prior to the decision which has to be made at that stage.

(ii) The set of possible decisions is known at each stage.

(iii) The effect of a choice of any member of this set is known at each stage.

(iv) The duration of the process is known in advance.

(v) The criterion function is prescribed in advance.

The five statements above define that which is generally known as a deterministic process.

### 1.3.2 Stochastic Process

The next stage in the continuing development of control was the period when engineers attempted to control processes in which one or more of the parameters was statistical or random in its nature[8]. That is, the parameter could be described in mathematical form, and its mean and standard deviation were either known or could be measured. Bellman[7] superimposes various elements of uncertainty using the following criterion, for the properties of a stochastic process[9].

(i) An unknown initial state with a given probability distribution.

(ii) A distribution of allowable sets of decisions at each stage of the process.

(iii) A distribution for the outcomes of any particular decision.

(iv) A distribution for the duration of the process, or equivalently, a probability of the termination of the process at any stage, dependent upon the state and the decision made.

(v) A distribution for the criterion function to be used to evaluate the sequence of decisions and states.

Processes of the stochastic type generally present a more difficult control problem than those of a deterministic nature, but with time, the problems were analysed, and some solutions obtained.

### 1.3.3 Adaptive Processes

The stochastic process, whilst interesting and demanding in its self, does not present the most difficult challenge from a control point of view. The processes which present a very demanding challenge for a control engineer are the type of process in which the parameters are varying, and possibly, as in the stochastic control system, some are statistical in their nature[10,11,12]. But unlike the stochastic system the exact distribution of this statistical variation is either unknown or at best only partially known.

This type of process cannot easily be investigated using the techniques of deterministic or stochastic processes. The controller of this type of process will need to 'adapt' to any changes in the process, and hence the controller will need to be investigated from an adaptive view point[13,14,15].

The problem of designing a controller which is capable of adjusting its parameters in order to stabilize the feedback control systems' characteristics, when the characteristics, or the parameters of the controlled plant are changing, was the origin of the self adaptive system[16].

The first such reference to a self adaptive system was in 1958 by Whitaker, Yarman and Kezer[17]. Since 1958 the number of papers and research publications dealing with adaptive control topics has probably grown at an exponential rate, and will probably continue to do so in the immediate future. First, however, it is necessary to specify more closely what is meant by an adaptive system.

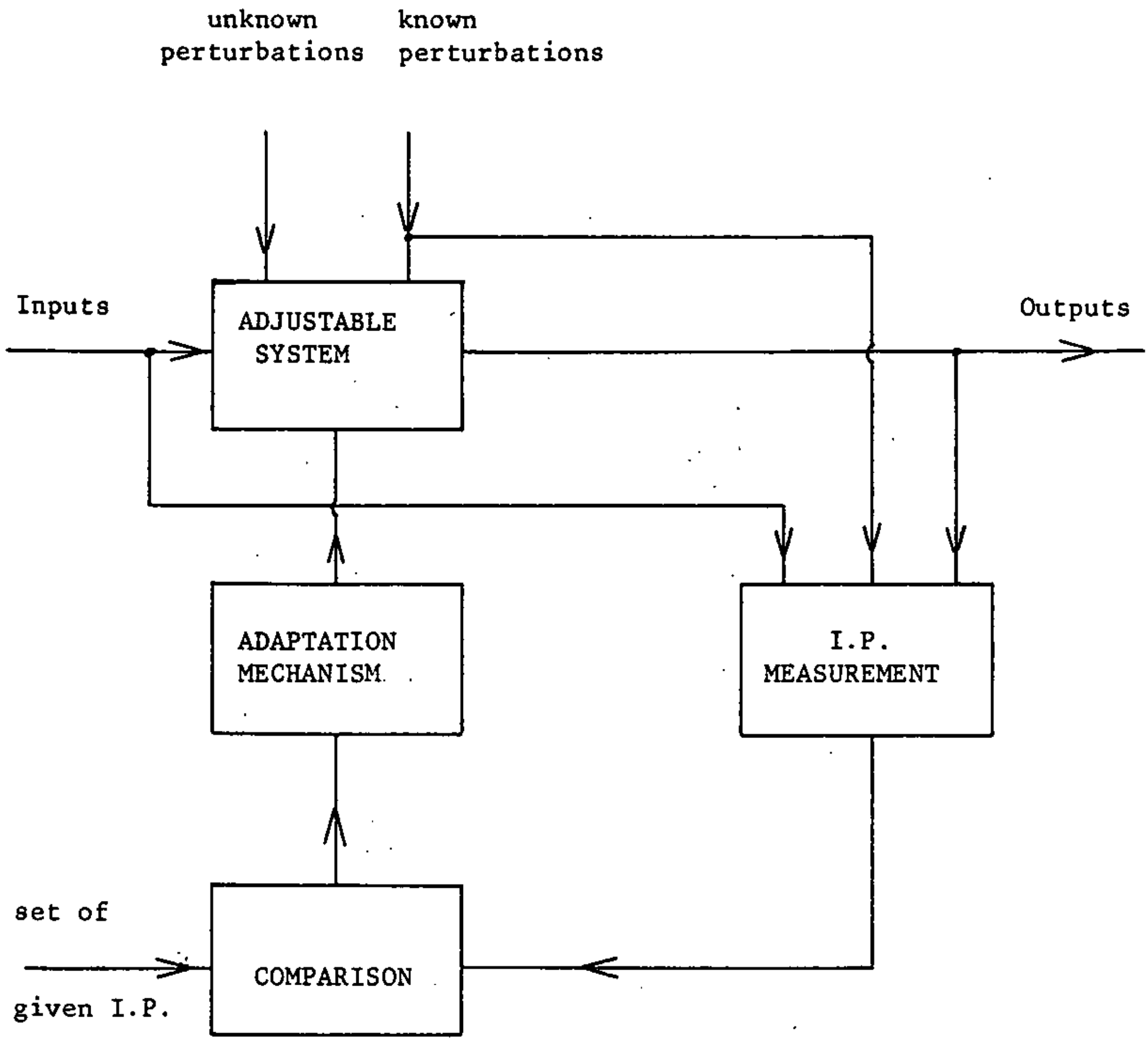



Fig. 1.4  Basic configuration of an adaptive system.

At this time, it appears that there is no generally accepted definition of what is meant by an adaptive system. It will, however, be useful as a guide line, to quote definitions of adaptive systems from other publications.

According to Tsypkin[18] adaption is described as follows: "By adaption we mean the process of changing the parameters, structure and possibly the controls of a system on the basis of information obtained during the control period so as to optimise, from one point of view or another, the state of a system when the operation conditions are either uncompletely defined initially or are changing". A more specific definition has been proposed by Landau[13,19] which is: "An adaptive system measures a certain index of performance using the inputs, the states, and the outputs of the adjustable system. From the comparison of the measured index of performance and a set of given ones, the adaption mechanism modifies the parameters of the adjustable system or generates an auxiliary input in order to maintain the index of performance close to the set of given ones", see figure 1.4. This type of adaptive control is often referred to as a "non-scanning" or "searchless" adaptive system.

It would seem that each of the many definitions of an adaptive process has its own particular advantages and disadvantages when looked at from a general point of view. It is also probable that there are nearly as many definitions of adaptive processes as there are researchers in this area[20]. Bellman[7] calls processes of an adaptive nature "learning processes" and as such each individual process will have features of interest to each researcher, and consequently a definition to cover all types of adaptive or learning processes will be very difficult to produce. The definition which will be used in this thesis is that any system which is encountered in which a learning[21] or adaptive process takes place will be called an "adaptive control process".

It is clear from the definition above that at least one parameter of the system will vary, sometimes appreciably, with changes in the 'environment'. Hence it is possible to further classify adaptive systems according to the mechanism of adaption which the particular system employs[22]. Some of these categories are as follows:

(i) *Passive Adaptive Systems:* these are systems which through their design give a satisfactory performance despite possible wide variations in environmental conditions and without changes of the system parameter or parameters by the controller.

(ii) *Adaption of input signal:* This type of system monitors the input signal and bases its control strategy on the changes it detects. This system is essentially open loop because the output of the system is not monitored.

(iii) *System Adaptive control:* In this type of system an adjustment is made in its own parameters in order to compensate for changes in the transfer function of the controlled plant. In this case either one or more of the time dependent variables could be changed or the transfer function as a whole could be determined and then changed by the controller.

(iv) *Parameter Adaptive control :* This type of control is achieved by adjustment of parameters such as time constants, system gain or other loop parameters.

(v) *Adaptive shaping of input signal:* In this type of system the controller makes changes to the shape of the input signal in order to achieve the desired response which will be necessitated by system parameter changes. Adaptive shaping of the required input signal is sometimes computer controlled, where the input waveform will be based on the computed dynamic response of the controlled system.

(vi) *Optimum adaption:* Sometimes called extremum adaptation. This system is adjusted in order to give a minimum or a maximum in one or more of the variables.

It can be seen from the above classification that there are several different types of adaptive system, and it will be useful to consider which components it is generally thought must be present in an adaptive system.

It is apparent from the literature which has been published in the area of adaptive control that there are always at least two parts to an adaptive system. There may in fact be more than two parts to a practical adaptive control system, but from a theoretical point of view there must be at least two.

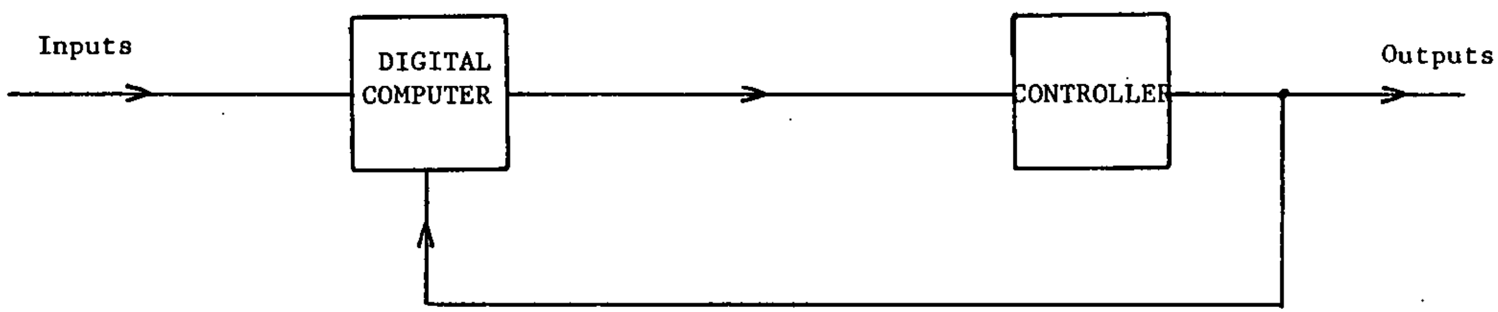



Fig. 1.5 Digital computer controlled adaptive system.

The first part of any adaptive system must be identification[23]. This refers to the determination of the dynamic response of the process to be controlled. The second part is the actuation of the appropriate controlling signal which will in its turn modify the response of the system under control.

The actuation part of the control process can be subdivided into two further classes or parts. The first part is the decision function, in which the adaptive controller must decide what type, or of what value of control signal it must generate, in order to satisfy the second part of the adaption process, namely, the control signal modification function.

The two parts of an adaptive system as discussed above may, in a practical adaptive system, be inseperable, but it follows that in the design of an adaptive control system, these parts are combined with techniques developed for applications of the deterministic and/or stochastic type of control, thereby producing the logic and configuration of the desired adaptive control system[24,25], see Fig. 1.5.

When an adaptive controller is designed there are a number of problems which need to be considered in order that a reasonable system performance is obtained. The first problem which arises with an adaptive system is that with the application of an adaptive loop, which will probably be parameter varying, the system will become non-linear even if the system was originally linear. This non-linearity will undoubtedly increase as the system complexity increases.

The next difficulty arises with constraints of the available measuring equipment. This problem may mean that the measuring equipment may not be able (or it may not be permissible) to measure all the parameters which it is desired to measure. As a result the identification of the system may only be partially completed, and hence the system controller may have to function with incomplete information.

A further problem which may arise is concerned with the time needed for the identification phase[26]. The adaptive controller will perform better if complete identification is possible. However, this is not always possible for a number of reasons, some of which have been briefly outlined above, and others which depend upon the identification period being as small as possible. A working rule-of-thumb, suggested by Davies,[27] is that the identification time should be comparable with the time constants of the system. Ideally the identification time should be much less than this.

Identification of the plant should normally be achieved during the normal operation of the plant, and as a result any test disturbances should not affect the plant. Hence the plant should not be subjected to large test disturbances and neither should it be necessary for the plant to be removed from its normal operation.

Another problem which needs to be considered, is the determination of the optimum operating condition[28] for the plant. This optimum condition for the plant is very important, and is usually characterised by a single number, which is often referred to as the 'figure of merit'. It is obviously difficult to specify the performance of a complex system by one number, and hence the selection of the performance index (figure of merit) must be made with great care. The figure of merit should be as simple as possible and should also include, if possible, a long term objective as well as a short term one.

System stability is yet another problem with which the designer has to contend. Often, unstable systems result from poor design or bad decisions in the selection of the figure of merit, the identification method or the adaptive loops. When systems are in the process of being designed it may be advantageous to consider the system stability by utilising the stability theory of Liapunov,[29,30] by which the designer will be able to obtain estimates of the type of configurations allowable.

The reason for the use of Liapunov's stability criterion is that as a result of using an essentially non-linear feedback system, the use of non-linear feedback theory often does not give adequate information for a complete analysis of a complex adaptive system. As a result, stability cannot be assumed unless Liapunov's criterion is obeyed.

### 1.3.4 Adaptive Furnace Control

In the previous section various adaptive systems have been discussed, and now it is appropriate to state why adaptive control[31,32] has been used in the case of a heat treatment furnace[33,34].

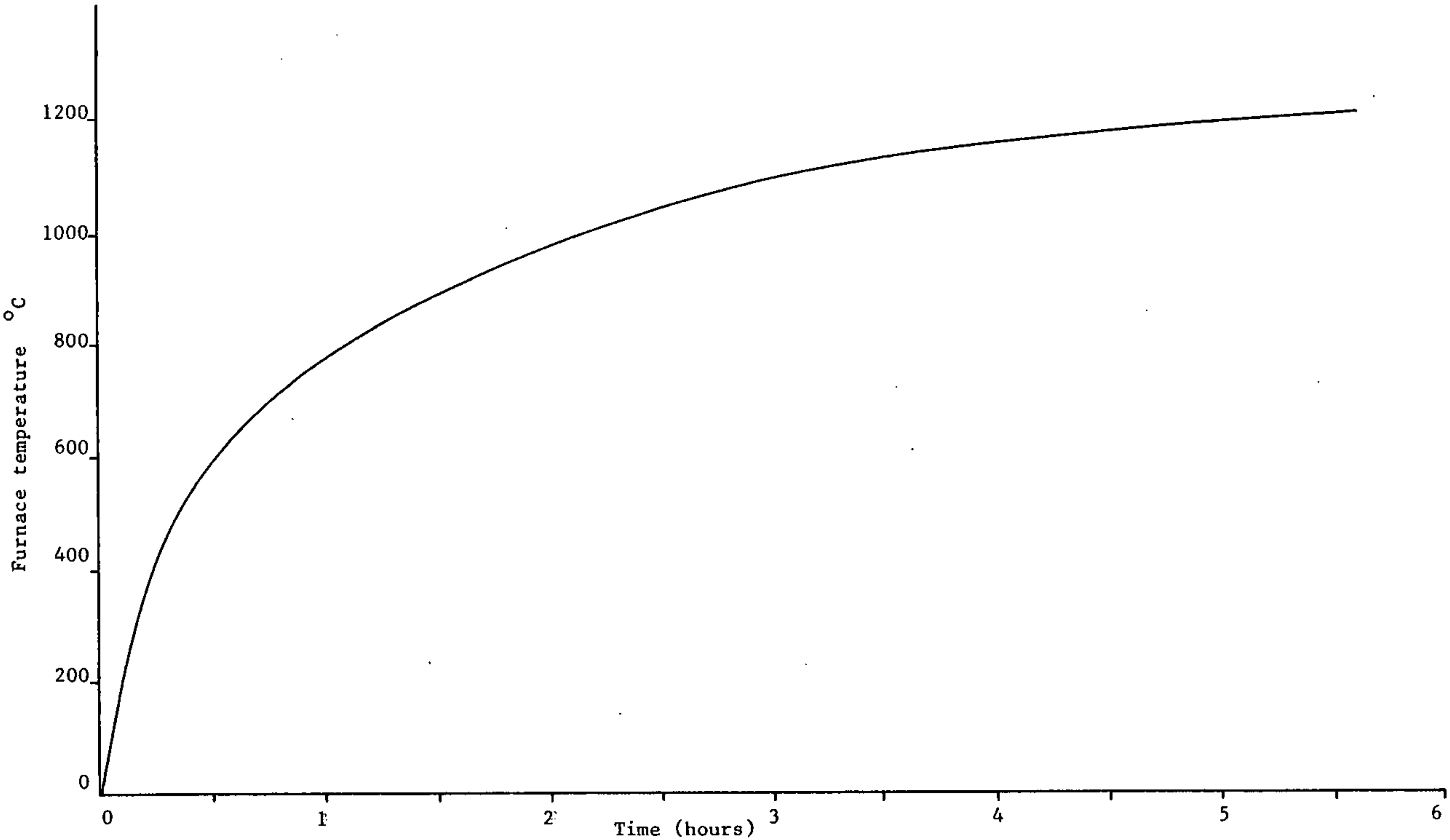



Fig. 1.6 Step response of the Cooperheat Ltd. furnace

The basic step-response of the medium sized furnace (Cooperheat Ltd) is shown in figure 1.6. This characteristic is non-linear and in order to describe its variation with time, a variable time constant needs to be involved. Also, an unknown number of other disturbances will affect the furnace[35,36]. Some of these disturbances can be enumerated, but their effect on the system performance cannot usually be predicted in advance with any degree of certainty[27,38,39]. For example, the mass of the load and its surface area will have an effect on the furnace temperature profile, and their influence cannot be estimated accurately. Similarly other changes will affect the furnace performance, such as ambient temperature changes - generally increasing during the day time with furnace use, but usually decreasing in the evening and at night. Further more, voltage fluctuations, due to problems experienced by the Generating Boards, may also affect the furnace performance.

Clearly, from the above considerations, the furnace may be described with a specified accuracy only over a small range of its operation. However in order to achieve accurate control over a relatively large time period some method of adaptive compensation is required. This is especially true if the furnace is to conform to a specific temperature-profile and not to just maintain a relatively high temperature, which has to be reached in the shortest possible time.

## 1.4 The project discussed in this thesis

The initial part of this investigation is concerned with the general area of adaptive control, and a brief review of the subject has been presented in this chapter.

Chapter 2 describes the layout of a 'top hat' furnace, and then considers the detailed derivation of the furnace mathematical model. This model is then modified to take into account various changes of parameters. The model is then compared with the actual known furnace performance, and finally analogue computer simulation is used to verify the mathematical model.

Chapter 3 describes the results of investigations into various methods of identifying the plant (furnace). It includes results obtained on the analogue computer model of the furnace, and also includes various methods of overcoming the problems of the long time-constant of the type of plant under investigation.

Chapter 4 describes the investigation of a suitable self-adaptive controller. This chapter also includes a detailed description of the type of control system which was eventually considered the most appropriate for control of the furnace.

Chapter 5 describes the implementation of the control system chosen in Chapter 4. Also included in this chapter are the results obtained both during the setting up of the adaptive loop as well as results of the furnace operation when controlled by the adaptive controller. The furnace used in this part of study is a different type from that used in the initial study. This second furnace was used in order to demonstrate that the type of model produced, can, with only slight modification, be used on various types of heat treatment furnace.

The final chapter deals with the conclusions which have been drawn from the investigation, and suggestions for further work, which may be carried out in the same area, are included.

# CHAPTER 2

# PLANT MODELLING

## 2.1 The Furnace Layout

Figure 2.1* shows the detailed drawing of the medium sized furnace (courtesy of Cooperheat Ltd) which was used in the initial part of this investigation. The furnace is of a conventional design and is usually referred to as a 'top hat' type. The designation 'top hat' refers to the fact that the whole of the top of the furnace is removable. That is, the top and the four sides (forming the top hat) lift off in one piece, usually by the use of a small pulley system.

Once the top has been removed the interior layout and load-base are easily accessible. The outer case, or top hat, is constructed from cast steel and between the internal and external walls there is a composite layer of insulation material. The exact composition will vary from furnace to furnace, and is determined by the normal working temperature of the furnace.

The load-base of the furnace is constructed from fire bricks, these being surrounded by a cast steel outer case. On the top of the fire bricks there is a layer of sand which, when the furnace is in operation, forms a seal between the upper and lower parts of the furnace. Detailed information regarding the construction materials of the furnace can be obtained from reference 42, and this information is used in the evaluation of the furnace mathematical model in section 2.2 of this chapter.

*Fig. 2.1 is contained in a pocket at the end of this thesis.

This type of furnace usually uses electrical energy for its operation. The heating elements,covered with porcelain,are located in the base of the furnace, and they are set below the general level of the furnace base to protect them from accidential damage during loading and unloading operations.

The furnace is rated at 75,000 Btu/hour (or 21kW) and is supplied by a normal 3-phase system. The control of the furnace, prior to this investigation, was achieved by interruption of the source of energy. That is, the furnace control was of the bang-bang type, however, as the variation of temperature with time will normally be non-linear in its nature over the operating period of the furnace (several hours), this type of control has only limited use.

The measured internal furnace temperature will depend upon a number of variables, for example, load mass and density, external temperature, supply voltage and furnace and/or load heat storage. These variables together with the inevitables furnace 'lags', will result in a temperature/time curve which will be non-linear. As a result, simple methods of furnace control will in general, produce poor temperature control.

When the furnace is allowed to function without any control, with the normal maximum heat input, the typical variation of temperature against time is shown in figure 1.6. In the time taken for this type of operation (no-load data) the external air temperature was found to have changed by $20^{\circ}F$ ($11^{\circ}C$) and the furnace walls and top were found to have increased in temperature by an average of $140^{\circ}F$ in a time period of between two and three hours.

The effect of the subsequent heat storage in the hearth and walls, and the change in air temperature can be allowed for in the derivation of the furnace mathematical model as shown in the following section.

## 2.2 Plant Modelling

Figure 2.1 shows the furnace upon which the mathematical model[40,41] derived in this section is based. This mathematical model, which describes the furnace performance, is based on the detailed drawing shown in figure 2.1, supplied by Cooperheat Ltd, and also the information supplied by G.R-STEIN Refactories Ltd.[42] The constants and variables used in this theoretical derivation are listed in Appendix 1.

The derivation of the furnace mathematical model is as follows: Referring to Appendix 1 we have

Heat loss from the 'top hat' = $A_c U_c \ (\theta_f - \theta_a)$ ......2.1

Heat loss from the hearth = $A_h U_h \ (\theta_f - \theta_a)$ ......2.2

The heat storage in the hearth can be conveniently expressed as a fraction of $(h_c + h_h)$, that is:

Heat storage $h_s = x(h_c + h_h)$ where $0 \leqslant x \leqslant 1$ .....2.3

The total heat loss $h_T = h_c + h_h + h_s$

$$h_T = (x + 1)(h_c + h_h) \qquad .....2.4$$

Substituting equations 2.1 and 2.2 in equation 2.4 yields:

$$h_T = y(x + 1)(\theta_f - \theta_a) \qquad .....2.5$$

where

$$y = \left[A_c U_c + A_h U_h\right]$$

Hence the total heat supplied, h, is

$$h = \left[Q_m + (1 + x)Un\right] \frac{d\theta_f}{dt} + h_T \qquad .....2.6$$

Substituting equation 2.5 in equation 2.6 and rearranging yields:

$$\frac{h}{y(x + 1)} + \theta_a = \theta_f + \frac{(Qm) + (1 + x)(Un) \frac{d\theta_f}{dt}}{y(x + 1)} \qquad .....2.7$$

now letting $\tau = \frac{(Qm) + (1 + x)(Un)}{y\,(x + 1)}$

The heat input h will normally be a step function applied at t = 0 and $\theta_a$ would be of any form but initially is assumed to be a non-delayed ramp function, and by taking Laplace transforms, equation 2.7 becomes:

$$\frac{h}{s(x + 1)y} + \frac{\overline{\theta}_a}{s^2} = \overline{\theta}_f + \tau\,(s\,\overline{\theta}_f - \theta_{fo}) \qquad .....2.8$$

assuming, for convenience at this stage, that

$\theta_{fo} = 0$ at $t = 0$, then:

$$\overline{\theta}_f = \frac{\overline{\theta}_a}{(1 + s)s^2} + \frac{h}{s(1 + \tau s)(x + 1)y} \qquad .....2.9$$

taking inverse Laplace transforms yields:

$$\theta_f = \overline{\theta}_a\,\tau\,(e^{-t/\tau} - 1) + \overline{\theta}_a t + \frac{h}{y(x + 1)}\left[1 - e^{-t/\tau}\right] .....2.10$$

The initial assumption that $\theta_{fo} = 0$ at $t = 0$ is a particular solution of equation 2.9 for an ambient temperature of $0^oF$. In general the ambient temperature will be non-zero, and a general solution of equation 2.9 must contain a term, say $\theta_o$, to take into account the initial ambient temperature.

Hence $$\theta_f = \overline{\theta}_a\,\tau\left[e^{-t/\tau} - 1\right] + \overline{\theta}_a t + \frac{h}{y(x + 1)}\left[1 - e^{-t/\tau}\right] + \theta_o \qquad ..... 2.11$$

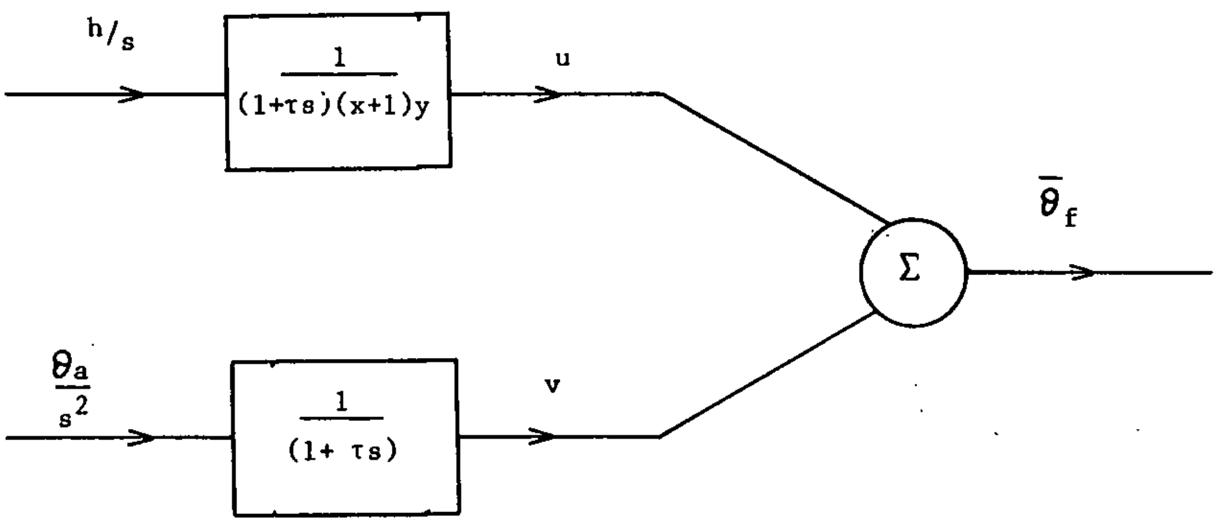



Fig. 2.2 Block diagram for furnace mathematical model

The block diagram shown in Fig. 2.2 represents equation 2. 9, and this equation forms the basis of the mathematical model for the furnace. It is possible to implement this mathematical model on an analogue computer. However, before analogue simulation is carried out, it is necessary to estimate the values of various parameters used in equation 2. 9, and then they must be verified mathematically.

## 2.3 Determination of Furnace-hearth heat capacity

### 2.3.1 Method 1

The heat capacity of the furnace hearth is given by the following expression:

$$\text{Heat Capacity}/_{m^2} = \left[\frac{(t_1 + t_2)}{2} - t_o\right] \text{x (specific heat)x(Density)x(d)} \quad .....2.12.$$

where $t_1$ and $t_2$ are the inner and outer hearth wall temperatures ($^oC$)

$t_o$ is the mean air temperature ($^oC$)

d is the hearth depth.

In order to calculate the heat capacity of the hearth the specific heat was assumed to be 1.05 kJ/kg$^oC$ (Btu/lb $^oF$) and the density of the hearth material was found using the following method in conjunction with figure 2.1 and the table of densities, reference 42.

| | | |
|---|---|---|
| Sand volume | = 0.0731 $m^3$ | |
| weight | = 0.0731 x 2720 | = 198 kg |
| Bricks 'A' volume | = 0.045 $m^3$ | |
| weight | = 0.045 x 2120 | = 97.2 kg |
| Bricks 'B' volume | = 0.0218 $m^3$ | |
| weight | = 0.0218 x 2120 | = 46.2 kg |

Bricks 'C' volume = $0.343 \text{ m}^3$

weight = $0.343 \times 630$ = 216.2 kg

Total weight of hearth = 558.4 kg

$$\text{Average density} = \frac{558.4}{1.22} = 455 \text{ kg/m}^3$$

Hence the average density after allowing a reduction of 10% for air spaces is 409 $\text{kg/m}^3$.

$$\text{Heat capacity}/_{m^2} = \left[\frac{(68 + 600)}{2} - 20\right] (1.05)(409)(0.76)$$

$$= 102.10^3 \text{ kJ/m}^2$$

Total Heat capacity of furnace hearth at 600°C

$$= 102.10^3.\text{hearth area}$$

$$= 102.10^3.\ 1.61$$

$$= 165.10^3 \text{ kJ or } (45.8 \text{ kW - hr})$$

Similarly at 900°C heat storage is

$$= 246.10^3 \text{ kJ or } (68 \text{ kW - hr})$$

and at 1000°C heat storage is

$$= 274.10^3 \text{ kJ or } (76 \text{ kW - hr})$$

### 2.3.2 Method 2

Using the data listed in Appendix 3, supplied by Cooperheat Ltd, the heat capacity of a furnace hearth may be determined as follows:

Total heat storage of the hearth at

600°C is $96.10^3$ kJ/$\text{m}^2$

900°C is $147.10^3$ kJ/$\text{m}^2$

1000°C is $179.10^3$ kJ/$\text{m}^2$

Hence total heat capacity of the furnace hearth is:

at 600°C ; $96.10^3.1.61 = 154.10^3$ kJ or 43 kW - hr at $900^{o}C$ ; $147.10^{3}.1.61 = 236.10^{3}$ kJ or 66 kW - hr

at $1000^{o}C$ ; $179.10^{3}.1.61 = 288.10^{3}$ kJ or 80 kW - hr

It may be seen from methods 1 and 2 in section 2.3.1 and 2.3.2 respectively, that there is a reasonable agreement between the heat capacity values. Hence an average value for Un, in equation 2.7, may be arrived at by the following:

$$\text{at } 600^{o}C \quad Un = \frac{165.10^{6}}{600-68} = 310 \text{ kJ or } 86 \text{ W - hr}$$

$$\text{at } 900^{o}C \quad Un = \frac{246.10^{6}}{900-77} = 298 \text{ kJ or } 83 \text{ W - hr}$$

$$\text{at } 1000^{O}C \quad Un = \frac{274.10^{6}}{1000-82} = 298 \text{ kJ or } 83 \text{ W - hr}$$

Hence the average value for Un = 302 kJ or 84 W - hr.

Now that a value has been arrived at for Un it is possible to determine a value for the time constant of the furnace. From equation 2.7 the time constant is given by:

$$\tau = \left[\frac{Qm + (1 + x)(Un)}{y(x + 1)}\right] \qquad .....2.13$$

Initially it must be assumed that:

(i) The mass m of the load is zero. This variable will, of necessity, be allowed for when the adaptive loop is closed.

(ii) A value must be assigned for the variable x. This value is determined by reference to the data supplied by Cooperheat Ltd, see Appendix 2, and the value of x will be 0.1.

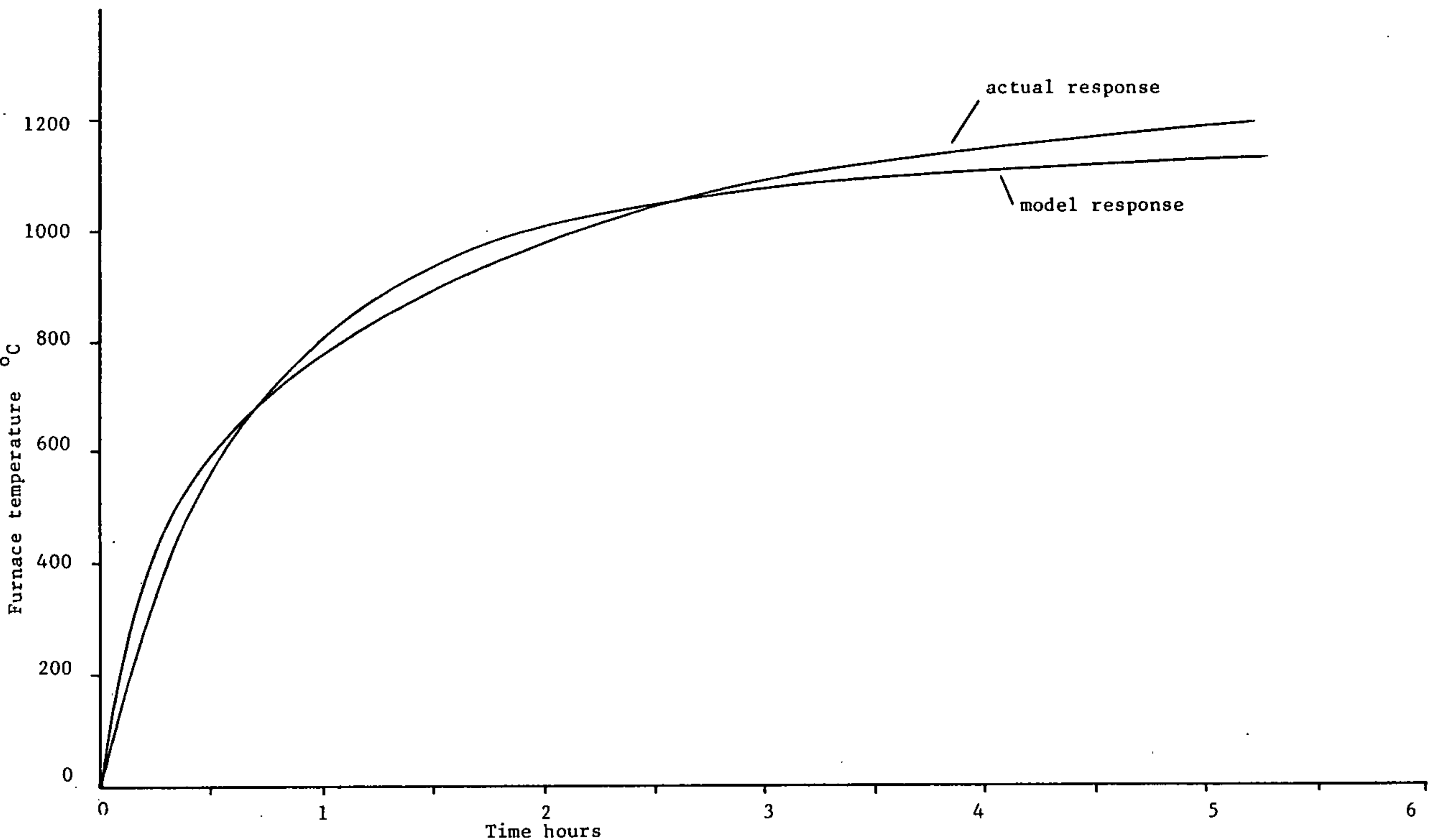



Fig. 2.3 Comparison of model and actual furnace response.

Hence $\tau = \dfrac{[1 + 0.1]\,302\ 10^3}{1.1\left[\dfrac{4.36.2.12.10^6}{(68-20)} + \dfrac{1.61 \times 2.12.10^6}{(68-20)}\right]}$ hours

$= 1.15$ hours

This value of the time constant compares well with the value of time constant estimated using the actual response of the furnace. The time constant, however, does vary with time and the value produced above is only valid after an operating period of about one hour. The estimated value of time constant is smaller for operation times of less than one hour, and larger for operation times which are greater than one hour.

## 2.4 Furnace Model Verification

### 2.4.1 Comparison of the Mathematical model with the furnace response

In order that the mathematical model produced in the preceeding sections can be used in an adaptive control strategy, it is necessary to demonstrate that the mathematical model produced is a reasonable approximation to the actual furnace response.

Figure 2.3 shows a comparison between the actual furnace response and the predicted response from the mathematical model of equation 2. 9. The time constant used for this predicted response is 0.8 hours, which is smaller than the theoretical value. However this smaller value of time constant gives a much better agreement with the actual furnace response at the start of the furnace operation.

It should be noted that with this type of furnace there is no appreciable time lag at the start of the furnace operation.

It is thought that this is due to the placement of the heating elements in the heating chamber of the furnace, resulting in a very rapid transfer of heat energy to the furnace chamber.

The small furnace used in the adaptive control section of this project, does contain a small time lag, and this time lag may be allowed for in the direct operation of the adaptive controller or by modification of the mathematical model to allow for this time shift.

### 2.4.2 Modification of mathematical model to take into account a time delay

If it is required to replace the transfer function of the furnace by a second function which is delayed by a time $T_1$,[43] a function is required which is zero from $-\infty$ to a time $(T_1)$ and equal to $f(t - T_1)$ when t is greater than $T_1$. If $\mathcal{L}\left[f(t)\right] = F(s)$, then the transform of the function delayed by a time $T_1$ is given by the following:

$$\mathcal{L}\left[f(t - T_1)\ u\ (t - T_1)\right] = \int_{T_1}^{\infty} e^{-st} f\ (t - T_1)dt$$

Introducing a new dummy variable $T = t - T_1$ we have

$$\mathcal{L}\left[f(T)\ u\ (T)\right] = \int_{0}^{\infty} e^{-s(T + T_1)}\ f(T)dT$$

$$= e^{-T_1 s} \int_{0}^{\infty} f(T)\ e^{-sT} dT$$

$$= e^{-T_1 s}\ F(s) \qquad \ldots\ldots 2.14$$

Hence the transform of a function delayed by a time $T_1$ is:

$$\mathcal{L}\left[f(t - T_1)\ u\ (t - T_1)\right] = e^{-T_1 s}\ F(s) \qquad \ldots\ldots 2.15$$

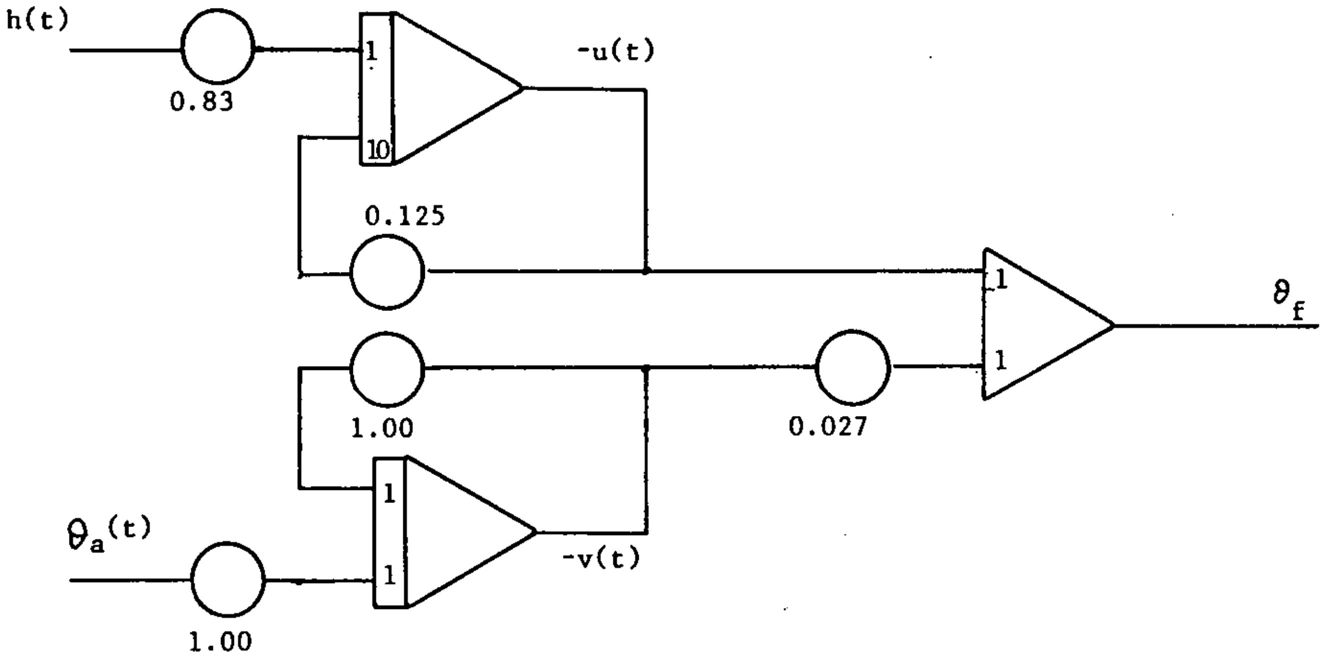



Fig. 2.4 Analogue computer model of Cooperheat Ltd. furnace

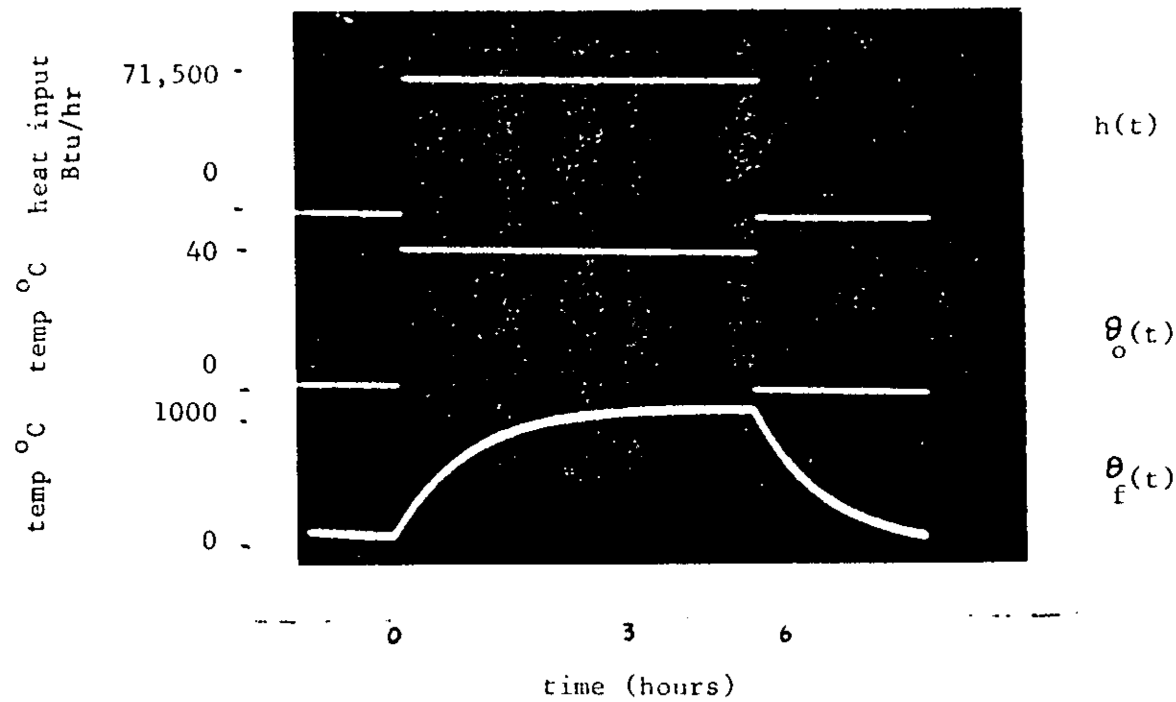
Hence in equation 2.15 $f(t - T_1)\ u\ (t - T_1)$ is the delayed or time shifted function, and the term $e^{-T_1 s}$ is a time delay operator.

## 2.5 Analogue Computer Simulation

In order to subject the derived furnace mathematical to further tests in order to verify its validity it was thought that a simulation of the model, using an analogue computer, would give a good indication regarding the validity of the model.

It is also useful to simulate the mathematical model on an analogue computer because this analogue simulation may in the future be used as the basis for closing the adaptive loop if a solution to the problem of accurate analogue integration, over a period of several hours, is produced.

The analogue simulation, as shown in figure 2.4 was produced by reference to the basic mathematical model as derived in section 2.2. The potentiometer settings and amplifier gain settings are determined by (i) the parameters defined in the basic equation and with reference to the particular furnace (ii) the ratio between the amplifier gain settings and potentiometer setting for the inputs h(t) and $\theta_a(t)$ were dictated by the maximum amplitudes of their respective inputs. (iii) Time scaling was used in order to make the process observable over a relatively short time period and to make the output consistent with the operation period of the analogue computer. (iv) The values of the variables x and y in equations 2.11 were given a value of 0.1. This value is consistant with the data given in Appendix 2, which was supplied by Cooperheat Ltd., for a range of heat treatment furnaces. The value of 0.1 for x and y may need to be changed later as more work and information is gained.



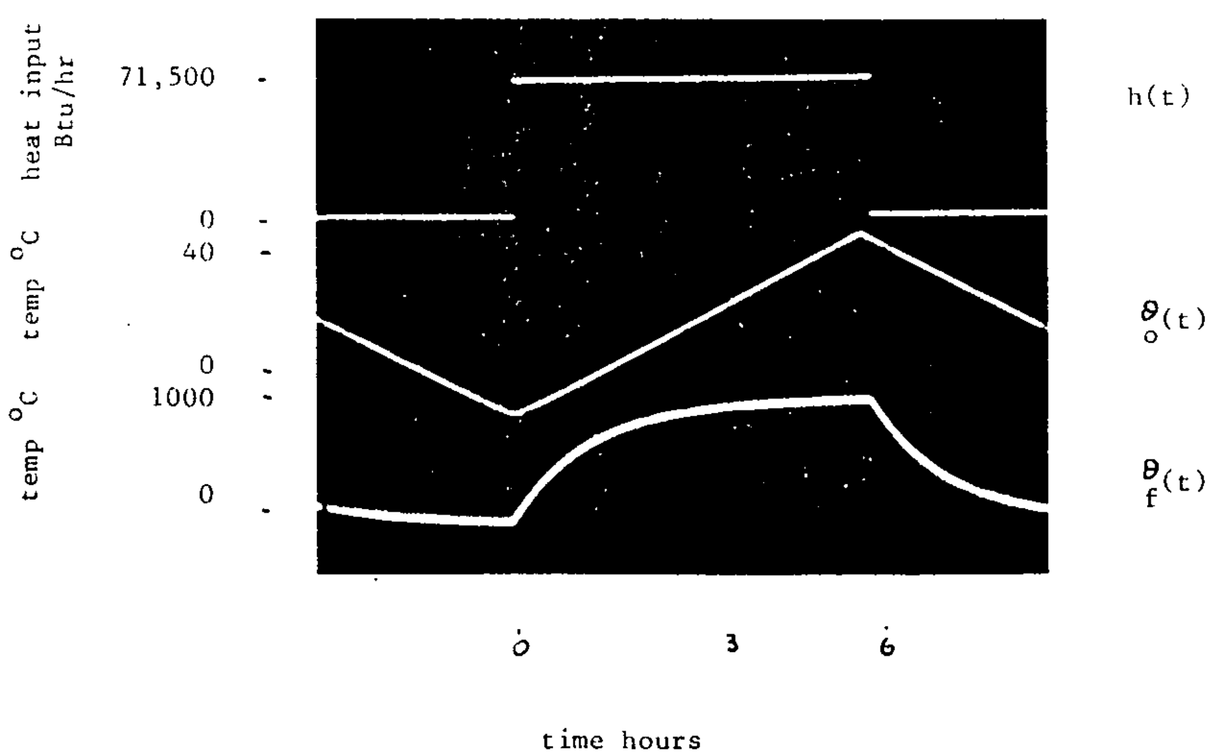




Fig. 2.5 Results of the analogue simulation of the furnace mathematical model for various inputs of $h(t)$ and $\theta_o(t)$

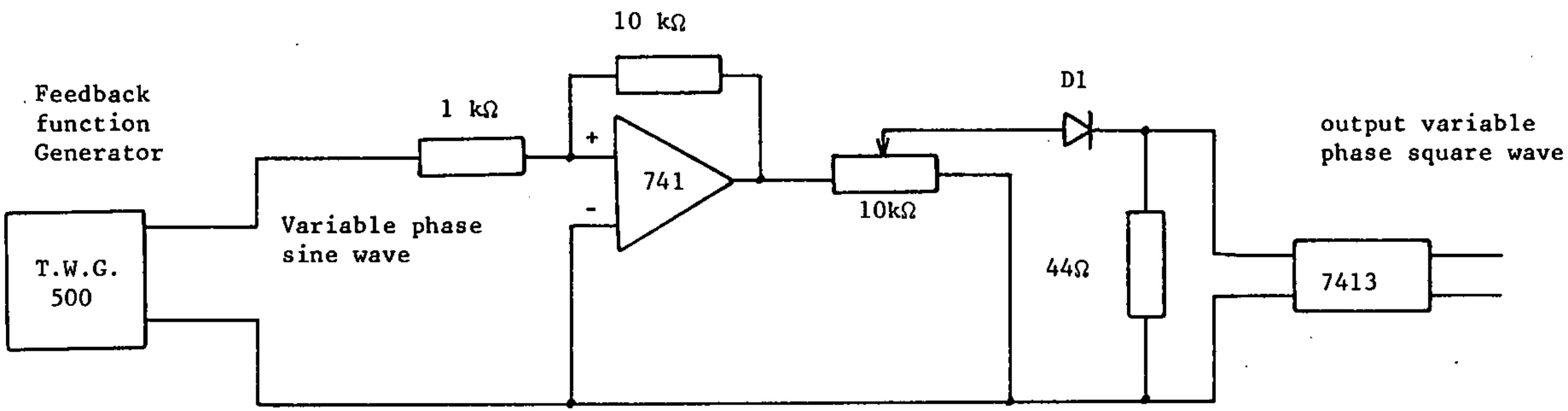



Fig. 2.6 Circuit for the production of a variable phase square wave.

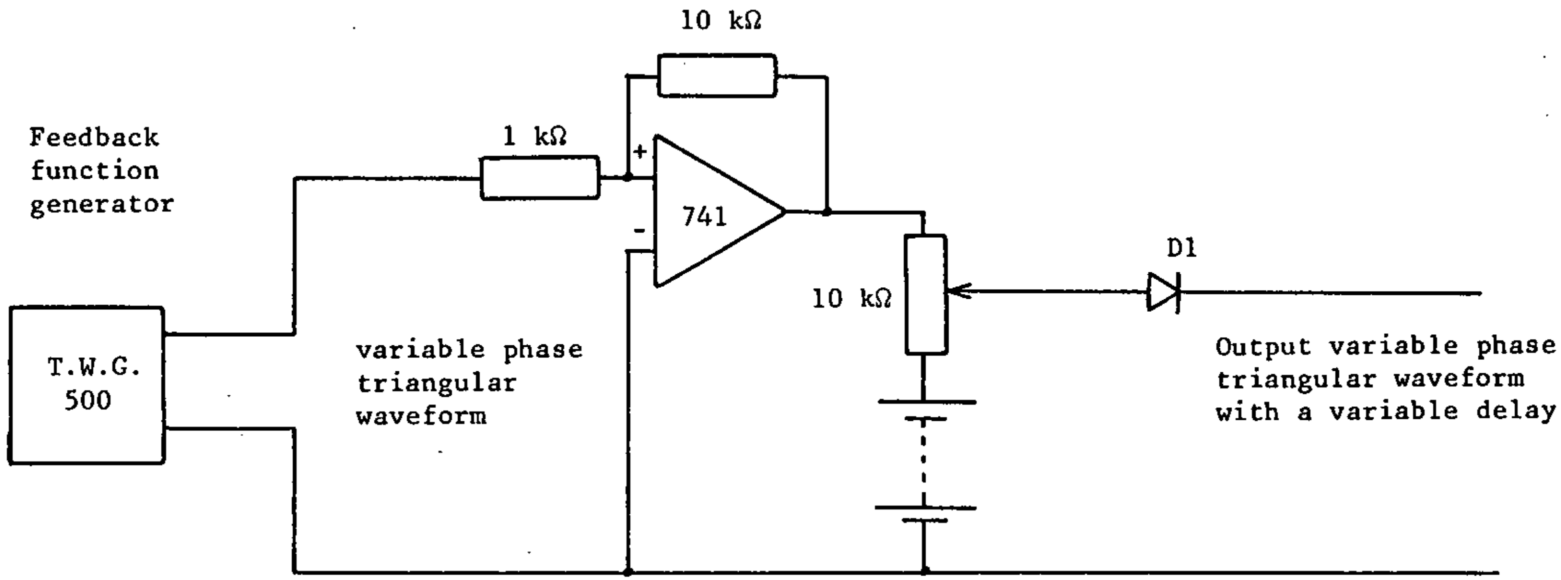



Fig. 2.7 Circuit for the production of a variable phase triangular waveform with a variable delay factor.

Figure 2.5 shows the results obtained for various inputs of h(t) and $\theta_a(t)$. The detailed information regarding the various time delayed and triangular waveforms, is given in section 2.6.

## 2.6 Analogue Computer test waveforms

In order to adequately test the validity of the analogue computer simulation of the furnace mathematical model, it was necessary to produce a series of variable phase and time delayed waveforms of suitable amplitude and frequency.

The most sophisticated function generator available which was compatible with the analogue computer was a Test Waveform Generator, variable phase function generator. However, in order to produce variable phase square waves and triangular wave forms, it was necessary to use the circuits shown in figures 2.6 and 2.7.

It should be noted that the output waveform for the circuit in figure 2.7 is an amplitude and time delay variable waveform, with respect to the square wave output from the circuit shown in figure 2.6.

## 2.7 Deductions from Model tests

From the results reported in sections 2.4.1 and 2.5, it can be seen, in the opinion of the author, that the mathematical model of the furnace derived in section 2.2, is a valid one, and with a certain amount of confidence it can be used in a model-adjusting adaptive control strategy as described in Chapter 4.

It must be remembered, however, that the model will need to undergo a degree of change to take into account the variations of the operating conditions of the furnace and also this will be especially true if a different furnace is used.

It is the opinion of the author that the results presented in this chapter may with a degree of confidence, be extended to other similar types of heat treatment furnace as demonstrated in Chapter 4.

# CHAPTER 3

## IDENTIFICATION

### 3.1 Introduction

Identification[44,45,46] is the procedure by which the plant, or system under consideration, is tested by some specific method, in order to determine a reliable relationship between the various input and output quantities, both controllable and random. This relationship may take several forms, but it is often produced in the s-domain or z-domain. In the case of the furnace(s) investigated in this thesis, the identification procedure is carried out in the time-domain, although it relies on the z-domain model, which in turn is based on the transfer function produced in the s-domain (chapter 2).

It is often necessary to identify a system in order that the identifier and/or process controller can establish a reliable mathematical model for the plant or system,[47,48] so that a control law may be determined from the latest information available from the system under test.

Once the system or plant has undergone an identification process it may then be possible to formulate a reliable strategy for the control of the plant.[49] As a result of this identification procedure the parameter(s) of the mathematical model, which has been identified,may be used as a basis for the control of the plant or system. This identification procedure may take one or other of several different types[50,51] which will be dependent upon the type of system under consideration.

## 3.2 Random Signal Testing

In order to design an adaptive control system, it is necessary to evaluate the characteristics of the system. It may be possible to determine the system characteristics by simulation techniques or by testing off-line. However, in most cases the off-line testing means that the lost production involved would normally be probibitively expensive. Thus identification should normally take place when the plant or furnace is on-line and also in its normal environment.

A further problem is that some identification methods require the introduction of disturbance signals at the inputs.[52] These signals should be small in their amplitude so that the test signals do not disturb the operation of the plant, and also to ensure that the characteristics obtained describe a linear system, and not a system which has been made non-linear by the use of these disturbance signals.

The use of random signal testing usually gives an estimate of the system's impulse response function and from this it is fairly simple to locate the main components of the system's transfer function.

In order to show how this type of random signal testing may be applied to the problem of identifying the major components of the furnace under study, it is necessary to use the following mathematical relationships to justify the theoretical basis of this method.

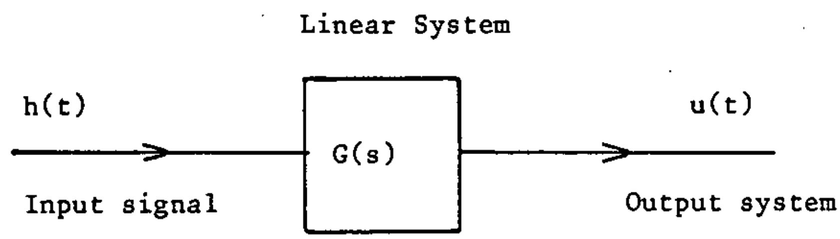



Fig. 3.1 Linear System

## 3.3 The Superposition or Convolution Integral

Referring to figure 3.1 and using Laplace transforms we may define the following:

$$\mathcal{L}\left[h(t)\right] = H(s)$$

$$\mathcal{L}\left[u(t)\right] = U(s)$$

and $$\frac{\mathcal{L}\left[h(t)\right]}{\mathcal{L}\left[u(t)\right]} = \frac{H(s)}{U(s)} = G(s) \qquad \text{.....3.1}$$

The function g(t) is the inverse Laplace transform of G(s) and is the system's impulse response.

Hence $$g(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} G(s)\, e^{st} ds \qquad \text{.....3.2}$$

and using the relationships shown in equation 3.1 we find that

$$U(s) = \int_0^{\infty} e^{sw}\, g(w)\, dw \int_0^{\infty} e^{-sx}\, h(x)\, dx$$

where w and x are dummy variables

$$U(s) = \iint_a e^{-s(w+x)}\, g(w)\, h(x)\, dwdx$$

where $a$ is the area of integration

let $w + x = t$ and $w = \tau$

$$U(s) = \iint_{a'} e^{-st}\, g(\tau)\, h(t-\tau)\, ds'$$

$$= \int_0^{\infty} e^{-st}\left[\int_0^{t} g(\tau)\, h(t-\tau)\, d\tau\right] dt$$

However $$H(s) = \int_0^{\infty} e^{-st}\, h(t)\, dt$$

$$u(t) = \int_0^{t} g(\tau)\, h(t-\tau) d\tau \qquad \text{.....3.3}$$

Hence it can be seen from equation 3.3 that the output signal u(t) may be expressed in terms of the input signal and also the systems impulse response, that is, via the convolution integral.

## 3.4 Impulse Response Identification using the Cross Correlation function

The cross-correlation function may be defined when it is possible for one variable to influence the future value of a second variable. If a quantitative measure of these signals (time varying) is required, the relationship is denoted by the following cross correlation

$$\phi_{hu} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} h(t)\, u(t+\tau)dt \qquad \ldots..3.4$$

It should be noted that in the particular case when h(t) = u(t) then the cross-correlation function reduces to the auto-correlation function thus auto-correlation

$$\phi_{hh} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} h(t)\, h(t+\tau)dt \qquad \ldots..3.5$$

In order to identify the furnace's impulse response using the cross-correlation function defined above it is necessary to extend the limits of integration used in equation 3.3 to $\pm\infty$. This is because the impulse response must be zero for $t < 0$ i.e. before any signal is applied, and as the whole of the impulse response is of interest, the response will only decay to zero as t tends to infinity.

Hence the convolution integral from equation 3.3 may be rewritten in the form

$$u(t) = \int_{-\infty}^{+\infty} g(\lambda)\, h(t-\lambda)d\lambda \qquad \ldots..3.6$$

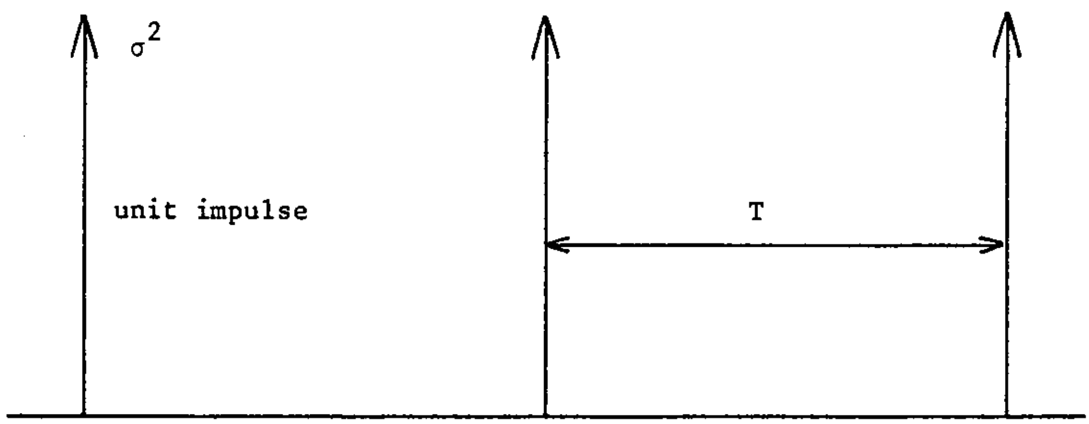



Fig. 3.2 $\emptyset_{hh}(\tau)$ for a Pseudo-random noise input.

Also the cross-correlation function of equation 3.4 can now be expressed as

$$\phi_{hu}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} h(t) \int_{-\infty}^{+\infty} g(s)\, h(t+\tau-s)\,ds\,dt \quad .....3.7$$

and by changing the order of integration

$$\phi_{hu}(\tau) = \int_{-\infty}^{+\infty} g(s) \left[ \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} h(t)\, h(t+\tau-s)\,dt \right] ds \quad .....3.8$$

$$\phi_{hu}(\tau) = \int_{-\infty}^{+\infty} g(s)\, \phi_{hh}(\tau - s)\, ds \quad .....3.9$$

Hence it can be seen from equation 3.9 that if a signal with a known auto-correlation function has a measurable cross-correlation function it is possible, by deconvoluting equation 3.9 to obtain the required impulse response g(t).

## 3.5 Identification using Pseudo-Random Noise Input

It is possible to identify a system by using a white noise input. However the main disadvantage of the white noise identification method is that of the length of time taken to identify the system. This identification time may be reduced considerably by using a pseudo-random noise input.

The pseudo-random noise input would have the same type of auto correlation function as a white noise input, but the signal would be repeated with a time period of T. That is, the function $\phi_{hh}(\tau)$ would have a value $\sigma^2$, the mean squared value, at $\tau = 0, T, 2T, 3T, ...... nT$ and zero for any other value of $\tau$, see figure 3.2.

$$\phi_{hh}(\tau) = \frac{1}{T} \int_{0}^{T} h(t)\, h(t+\tau)\,dt \quad .....3.10$$

The limits of integration for equation 3.10 must be 0 to T because these are the limits over which the function $\phi_{hh}$ is evaluated. Hence at a time $(\tau - s)$ later

$$\phi_{hh}(\tau - s) = \frac{1}{T}\int_0^T h(t)\, h(t - \tau - s)dt \qquad .....3.11$$

$$\phi_{hu}(\tau) = \int_{-\infty}^{+\infty} g(s)\left[\frac{1}{T}\int_0^T h(t)\, h(t + \tau - s)\, dt\right] ds$$

and by changing the order of integration,

$$\phi_{hu}(\tau) = \frac{1}{T}\int_0^T h(t)\left[\int_{-\infty}^{+\infty} g(s)\, h(t + \tau - s)\right] dt$$

$$= \frac{1}{T}\int_0^T h(t)\, u(t + \tau)dt \qquad .....3.12$$

Hence by using pseudo-random noise, the cross-correlation function may be obtained to full accuracy by integration over one period of the noise only.

## 3.6 Identification Using Pseudo-Random Binary Sequences

The problems of using a white noise signal as a method of identification are well known and in practice only approximations to white noise may be generated in a practical situation. The elaborate gain control and sampling/filtering requirements to meet the computer specification are usually too costly or too complicated, and as a result, suffer from poor reliability.

It is, however, possible to extend the concepts of the auto-correlation and cross-correlation function and apply them to a form of binary signal. The type of binary signal which may be used for identification of a system is called a pseudo-random binary sequence, or a chain code. These are linear recurring

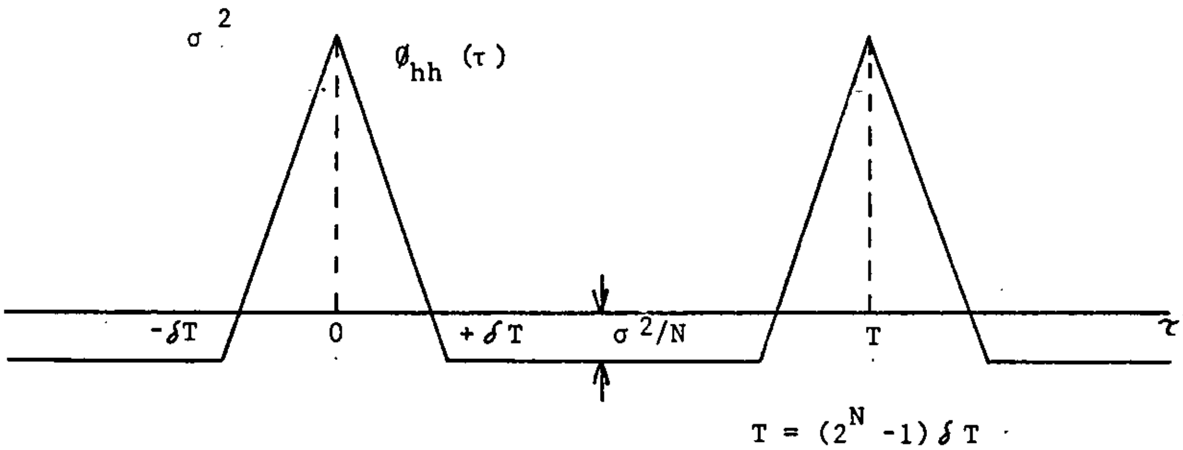



Fig. 3.3 Auto-correlation function for a binary maximum length sequence

sequences, the generation of which is discussed in great detail in other texts.[53,54]

The auto-correlation function of a signal h(t) for a delay $\tau$ is shown in figure 3.3.

If the waveform, whose auto-correlation function is as shown in figure 3.3, then the equation of this function may be written as:

$$\phi_{hh}(\tau) = \frac{1}{T}\int_0^T h(t)\,h(t+\tau)\,dt \qquad \ldots\ldots 3.13$$

## 3.7 Furnace model test using a chain code correlator

In order to determine if impulse response identification could be carried out using the furnace mathematical model, the chain code correlator[55] (type CCC352 manufactured by Feedback Ltd., Sussex England) was used in conjunction with the analogue computer, the initial'ising of which was discussed in chapter 2. The analogue computer model of the furnace is shown in figure 2.4.

The test on the analogue model was time scaled in order to allow the tests to be carried out in a reasonably short length of time. In contrast, the identification on the furnace, when used for the adaptive control of the plant, would have to be carried out in real time, and hence the identification time would be much greater.

The tests using the chain code correlator were performed with the re-set function on the VIDAC analogue computer suppressed, and the rep-op switch in the non-operate position. The correlation function curves for both h(t) and $\theta_a(t)$ were obtained separately,

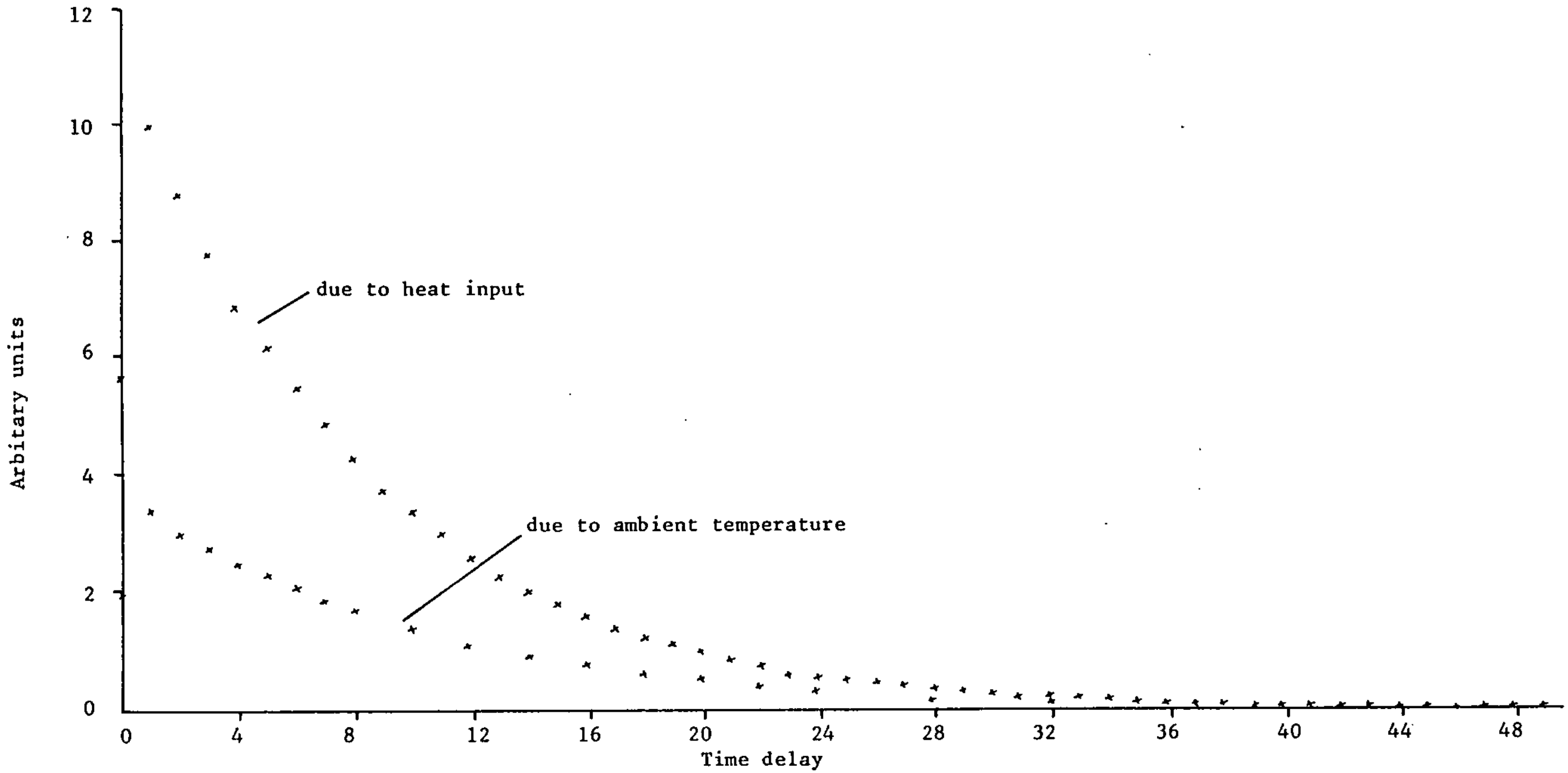



Fig. 3.4 Impulse response for both parts of the furnace mathematical model.

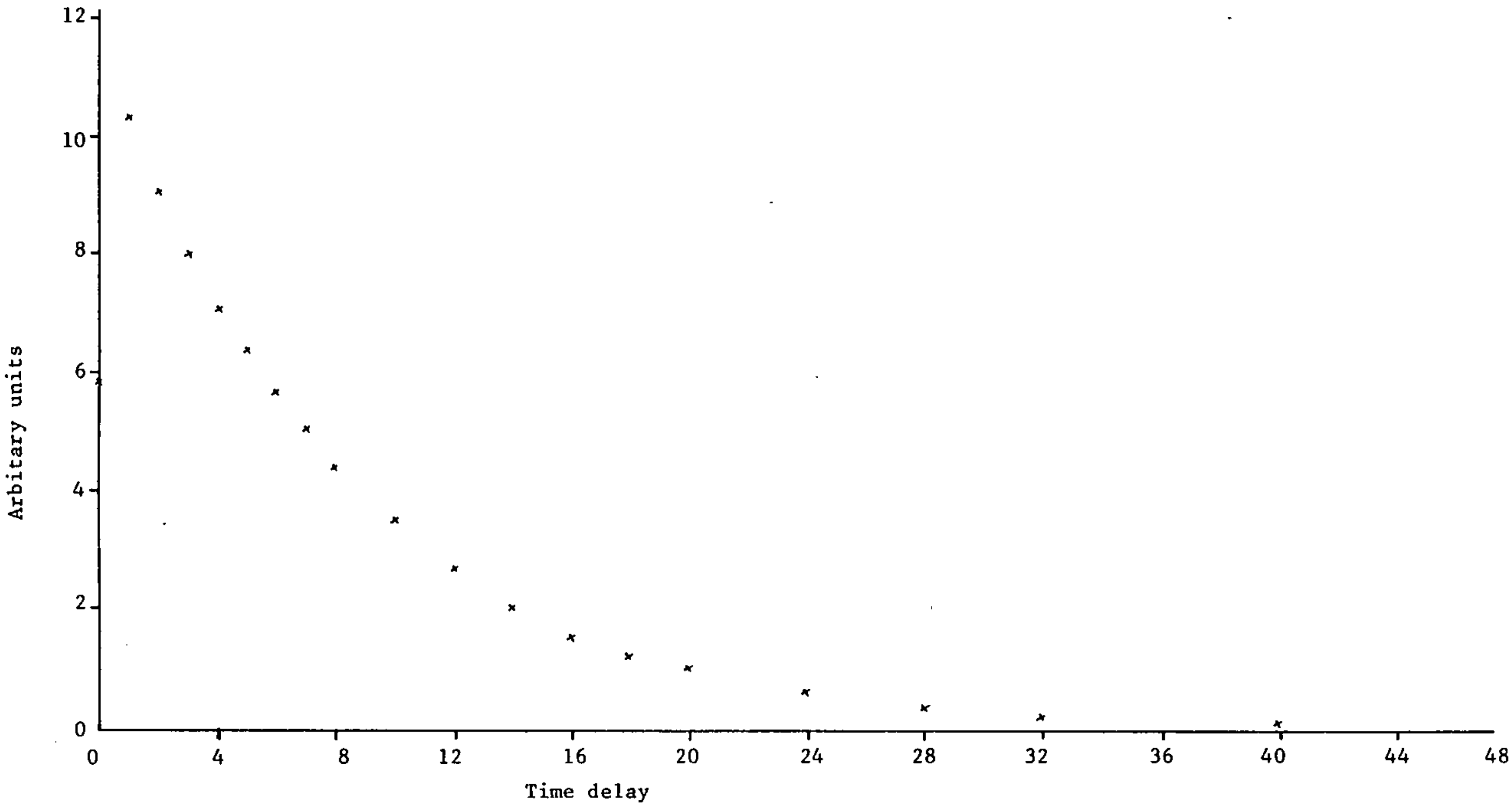
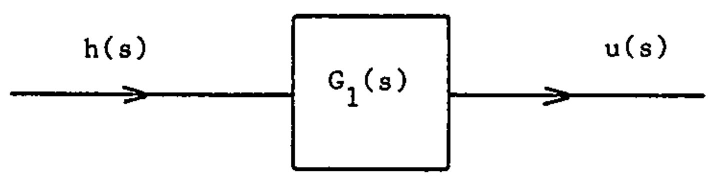
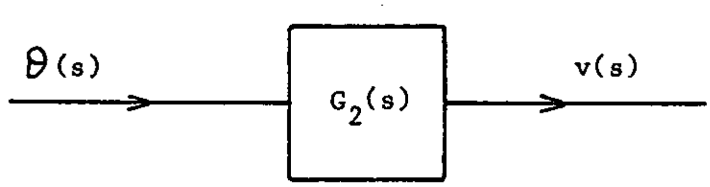



Fig. 3.5 Combined impulse response for the furnace mathematical model using chain code correlator.




$$G_1(s) = \frac{1}{(1 + \tau s)(x + 1)y}$$

Fig. 3.6 Part one of the furnace mathematical model.




$$G_2(s) = \frac{1}{(1 + \tau s)}$$

Fig. 3.7 Part two of the furnace mathematical model.

the output being taken at $\theta_f$ (see figure 2.4) in each case.

The results so obtained are not amplitude scaled and in order to produce a combined output for $\theta_f$ it is necessary to perform some amplitude scaling. The results obtained are shown in figure 3.4, and 3.5.

## 3.8 Derivation of Furnace impulse response from the mathematical model

In order to verify the results from the chain code correlator, it is necessary to investigate the form of the impulse response using the mathematical model derived in chapter 2. Referring to figure 3.6 we may write:

$$u(s) = \frac{h(s)}{(1+\tau s)(x+1)y} \qquad \ldots\ldots 3.14$$

and the impulse response is given by:

$$u(t) = \mathcal{L}^{-1}\left[\frac{1}{(1+\tau s)(x+1)y}\right] \qquad \ldots\ldots 3.15$$

For the second part of the furnace model refer to figure 3.7.

$\therefore$ the impulse response is given by:

$$v(t) = \mathcal{L}^{-1}\left[\frac{1}{(1+\tau s)}\right] \qquad \ldots\ldots 3.16$$

Hence the total impulse response is given by:

$$u(t) + v(t) = \mathcal{L}^{-1}\left[\frac{1}{(1+\tau s)(x+1)y} + \frac{1}{(1+\tau s)}\right] \qquad \ldots\ldots 3.17$$

Table of comparison between impulse model and the response obtained from the chain code correlator

| t (hours) | impulse response function | | chain code correlator output | (Error) |
|---|---|---|---|---|
| | $\theta_f$ | $\theta_f$ (scaled) | | |
| 0 | 1.27 | 11.70 | 5.85 | 6.12 |
| 0.1 | 1.12 | 10.34 | 10.35 | 0.01 |
| 0.2 | 0.98 | 9.13 | 9.10 | 0.03 |
| 0.3 | 0.87 | 8.06 | 8.05 | 0.01 |
| 0.4 | 0.77 | 7.11 | 7.15 | 0.04 |
| 0.5 | 0.68 | 6.28 | 6.40 | 0.12 |
| 0.6 | 0.60 | 5.54 | 5.70 | 0.16 |
| 0.7 | 0.53 | 4.89 | 5.10 | 0.21 |
| 0.8 | 0.47 | 4.31 | 4.40 | 0.09 |
| 0.9 | 0.41 | 3.80 | | |
| 1.0 | 0.36 | 3.36 | 3.50 | 0.14 |
| 1.2 | 0.28 | 2.61 | 2.70 | 0.11 |
| 1.4 | 0.22 | 2.04 | 2.05 | 0.01 |
| 1.6 | 0.17 | 1.58 | 1.55 | 0.03 |
| 1.8 | 0.13 | 1.23 | 1.25 | 0.02 |
| 2.0 | 0.10 | 0.96 | 1.05 | 0.09 |
| 2.2 | 0.08 | 0.75 | | |
| 2.4 | 0.06 | 0.58 | 0.65 | 0.07 |
| 2.6 | 0.05 | 0.45 | | |
| 2.8 | 0.04 | 0.35 | 0.35 | 0.00 |
| 3.0 | 0.03 | 0.28 | | |
| 4.0 | 0.01 | 0.074 | 0.10 | 0.03 |
| 5.0 | 0.00 | 0.018 | | |
| 6.0 | 0.00 | 0.00 | | |

Table 3.1

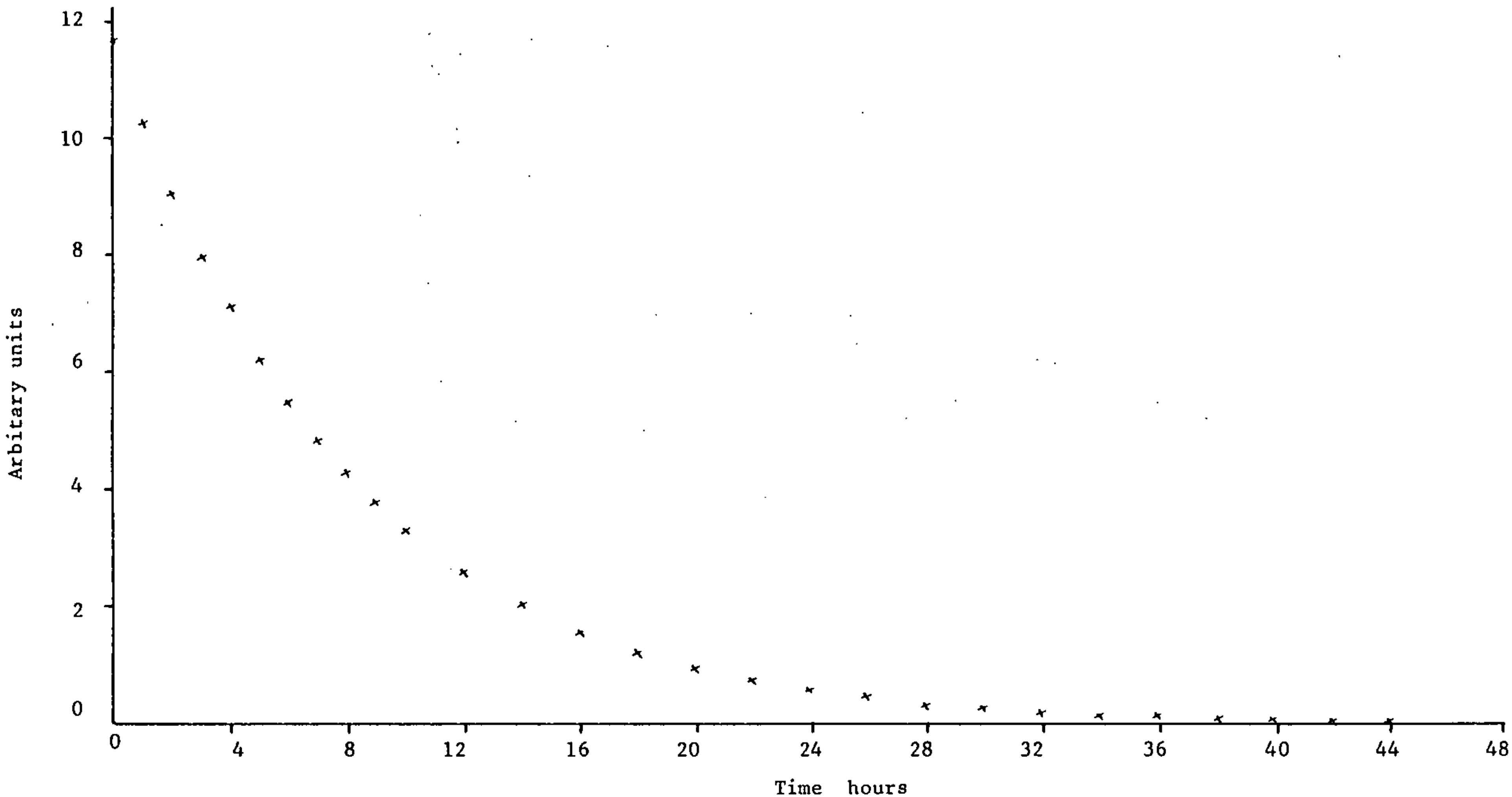



Fig. 3.8 Combined impulse response for furnace mathematical model using $\theta_f(t) = \frac{1}{m} e^{-t/\tau}$.

$$= \mathcal{L}^{-1}\left[\frac{[1 + (x+1)y]}{(1+\tau s)(x+1)y}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{[1 + (x+1)y]}{(x+1)y\ \tau(s+\frac{1}{\tau})}\right]$$

Taking inverse Laplace transforms we get:

$$\theta_f = \frac{[(x+1)\ y + 1]}{(x + 1)y\ \tau}\ e^{-t/\tau}$$

$$\theta_f = \frac{1}{m}\ e^{-t/\tau} \qquad .....3.18$$

Now letting x = 0.1 (from Cooperheat Ltd, data Appendix 3 )

and letting y = 65 (determined from the actual furnace response)

we obtain $\tau$ = 0.8 hours, as an average value.

$$\frac{1}{m} = 1.27$$

and $$\theta_f = 1.27\ e^{-t/0.8} \qquad .....3.19$$

As can be seen from table 3.1, the impulse response function and the output from the chain code correlator are in good agreement, apart from the first value,see Fig. 3.8. This is due to the form of the correlation function, and is a well known error.

## 3.9 Plant (Furnace) Identification problems

In the previous section it has been established that the plant has an average time constant of 0.8 hours, but variable from a minimum of 0.2 hours up to a maximum of over 1.5 hours. This means that in order to identify the plant using binary maximum length sequences,[56] then the minimum time required to gain even partial identification will normally be between 4 and 6 hours. If complete identification is needed then the time taken will be very long indeed.

In order to minimise this problem, some method of time scaling would be required,but this assumes that some knowledge of the plant has already been obtained.[57,58]

To obtain the impulse response of the plant we can assume that

$T = 10^{-3}$ sec. is an appropriate value,

and $T = \dfrac{\tau_o}{m - 1}$ .....3.20

where m is the number of shift registers in chain-code hardware, and $\tau_o$ is the effective vanishing point of the function.

hence $2^m$ = 4 hours (settling time for the furnace)

$$2^m = 4.3600.1000 \text{ ms}$$

$$m = \frac{\log_e (4.3600.1000)}{\log_e (2)}$$

$$m = 24 \text{ or greater}$$

We must now assume that time scaling is possible and that the identification time may be reduced by some factor, say 60. This would mean that:

$$\tau_o = 4.60 = 240 \text{ s.}$$

Further assume that m = 64, which is the same as for the Feedback Ltd, chain code correlator, then from equation 3.20

$$64 \geqslant \frac{240}{T}$$

$$T \geqslant 4s$$

This still means that identification will take up to16s, even by time scaling the plant by a factor of 60.

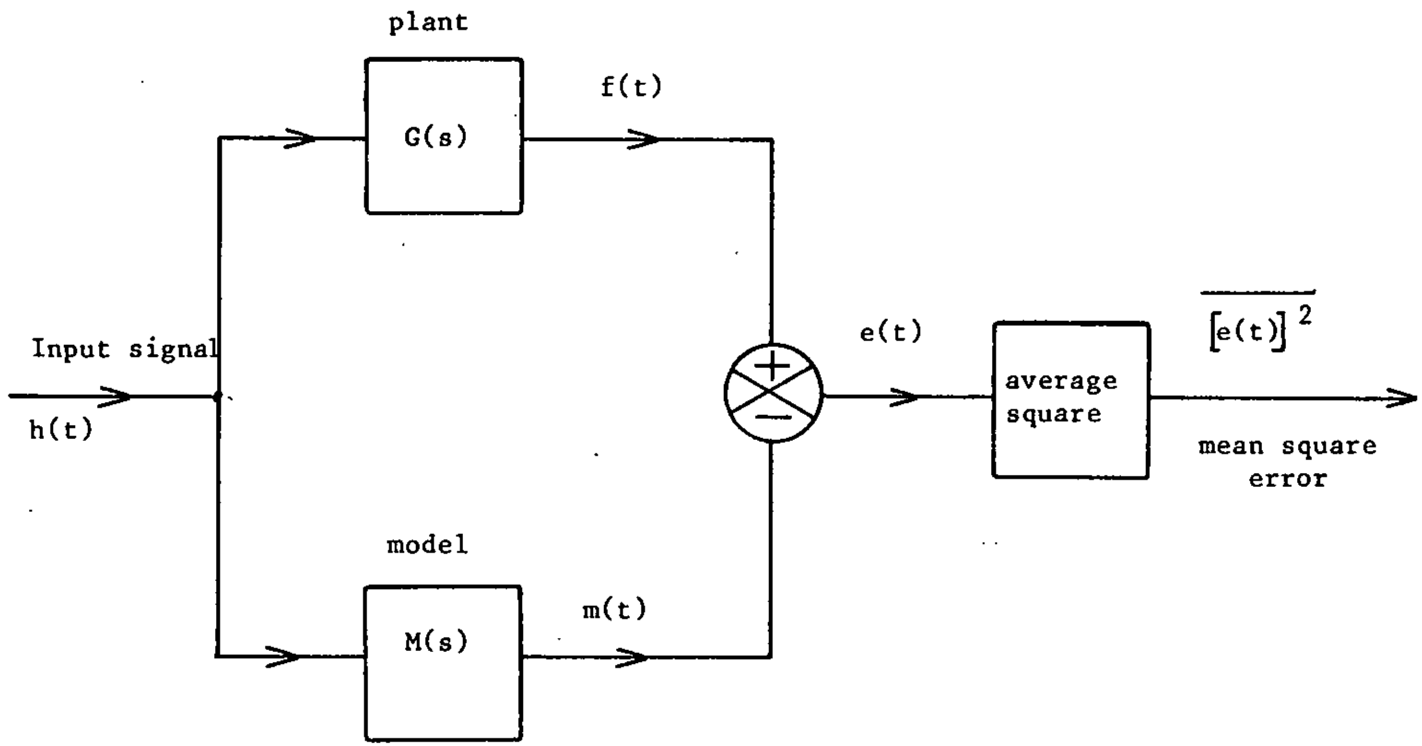



Fig. 3.9 Block diagram of an on line parameter adjustment identification system.

The problems associated with time scaling the plant by some arbitrary amount make it rather difficult undertaking. That is, in order to time scale the plant a mathematical model of the plant must be used.[59]

Hence it would seem that the most appropriate method of identification would be not to work directly on the plant, but to work on the mathematical model of the plant, and to identify the plant by adjusting a parameter or parameters of the mathematical model. The error between the mathematical model and the plant could be monitored and a mathematical model produced whose parameters matched those of the plant.

The advantages of 'identifying' via the mathematical model, in this manner, are that time scaling can be introduced into the model, and also the adaptive computer can change the parameters of the mathematical model as many times as it is considered necessary. Hence if a digital method of producing the mathematical model is implemented, then the problems associated with the long time constant of the plant will be minimised.[60]

Once a model adjustment method has been chosen it only remains to choose an adjustment approach which is suitable for the plant being investigated in this research project.

## 3.10 On line or Parameter Adjustment System Identification

### 3.10.1 General Approach

The mean square error for the system used in figure 3.9 is given by:

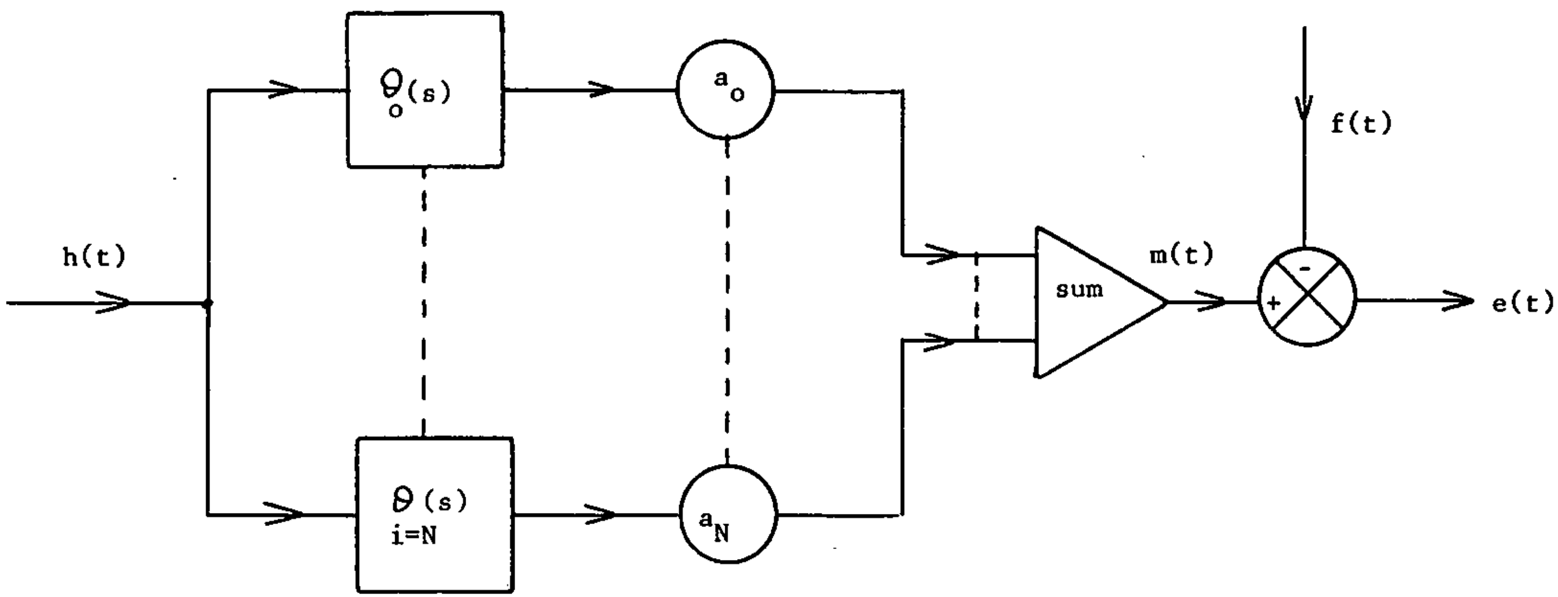



Fig. 3.10 Block diagram of system model.

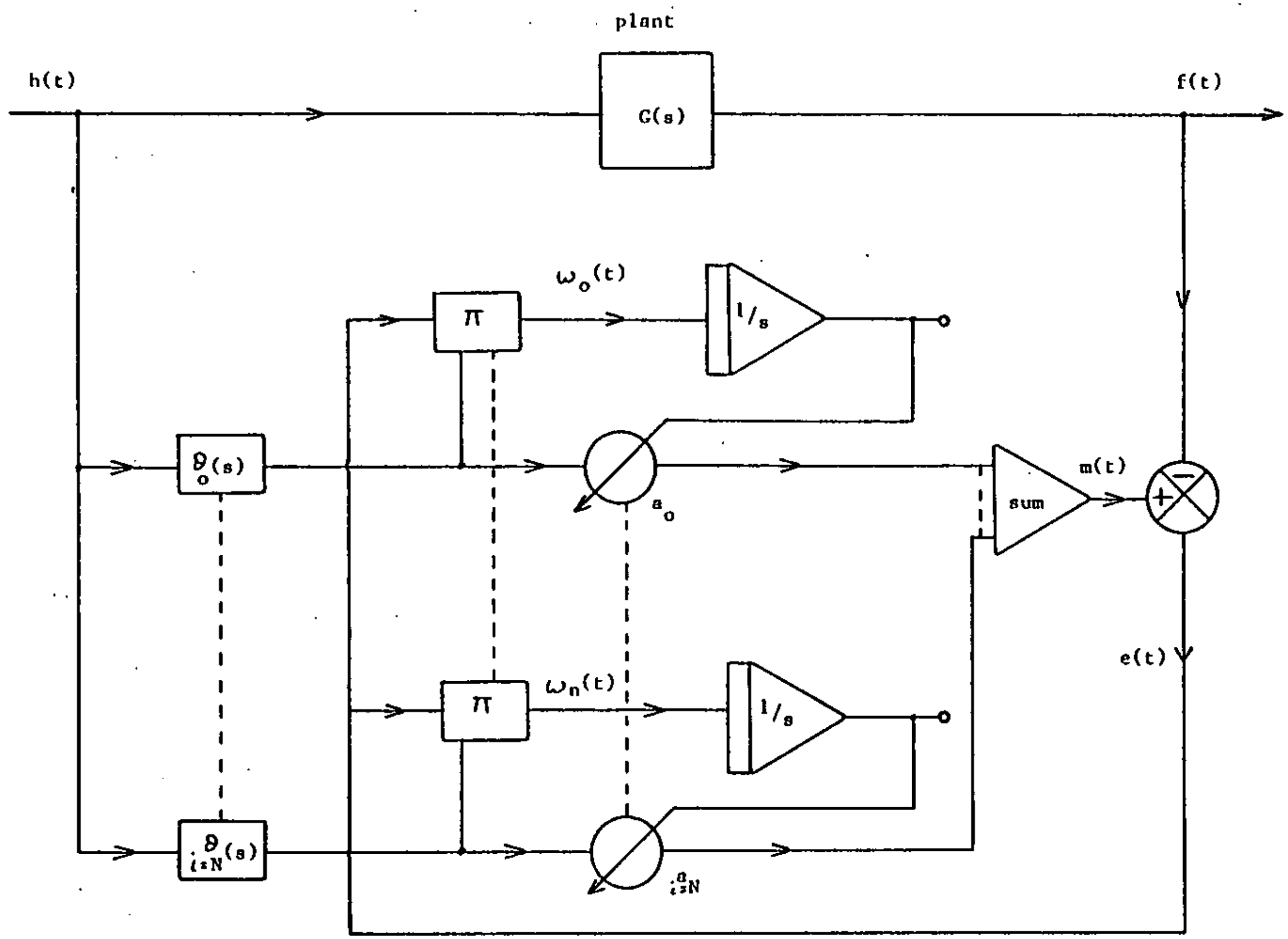



Fig. 3.11 Block diagram of a practical on line identification method.

$$\overline{e^2(t)} = \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} \left| G(s) - M(s) \right| \phi_{hh} (s)ds \qquad .....3.21$$

where $\phi_{hh}(s)$ is the spectral density of the function h(t)

and $$\overline{e^2(t)} = \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} \left| F(s) - M(s) \right|^2 ds \qquad .....3.22$$

where F(s) and M(s) are the Laplace transforms of f(t) and m(t) respectively.

The function M(s), which is the approximate model of the plant, may be represented by a linear combination of functions.

$$H(s) = \sum_{i=0}^{i=N} a_i \, \theta_i(s) \qquad .....3.23$$

Hence H(s) may be varied by suitable adjustment of $a_i$.

By use of the above theory, there are at least two methods by which identification may be achieved, and these are discussed below.

### 3.10.2 Method 1

By direct implementation of equation 3.23, the model M(s) may be shown as in figure 3.10. This is the general approach using equation 3.23. However if this is to be included in a particular application it must be modified to include some method of adjusting the parameters of $\theta_i$. This is usually done by using some method of adjusting $a_i$ until the desired output is reached. Figure 3.11 shows a standard approach.

By analysis of the method used in figure 3.11 it can be shown that there is a minimum value for the rate of change of each

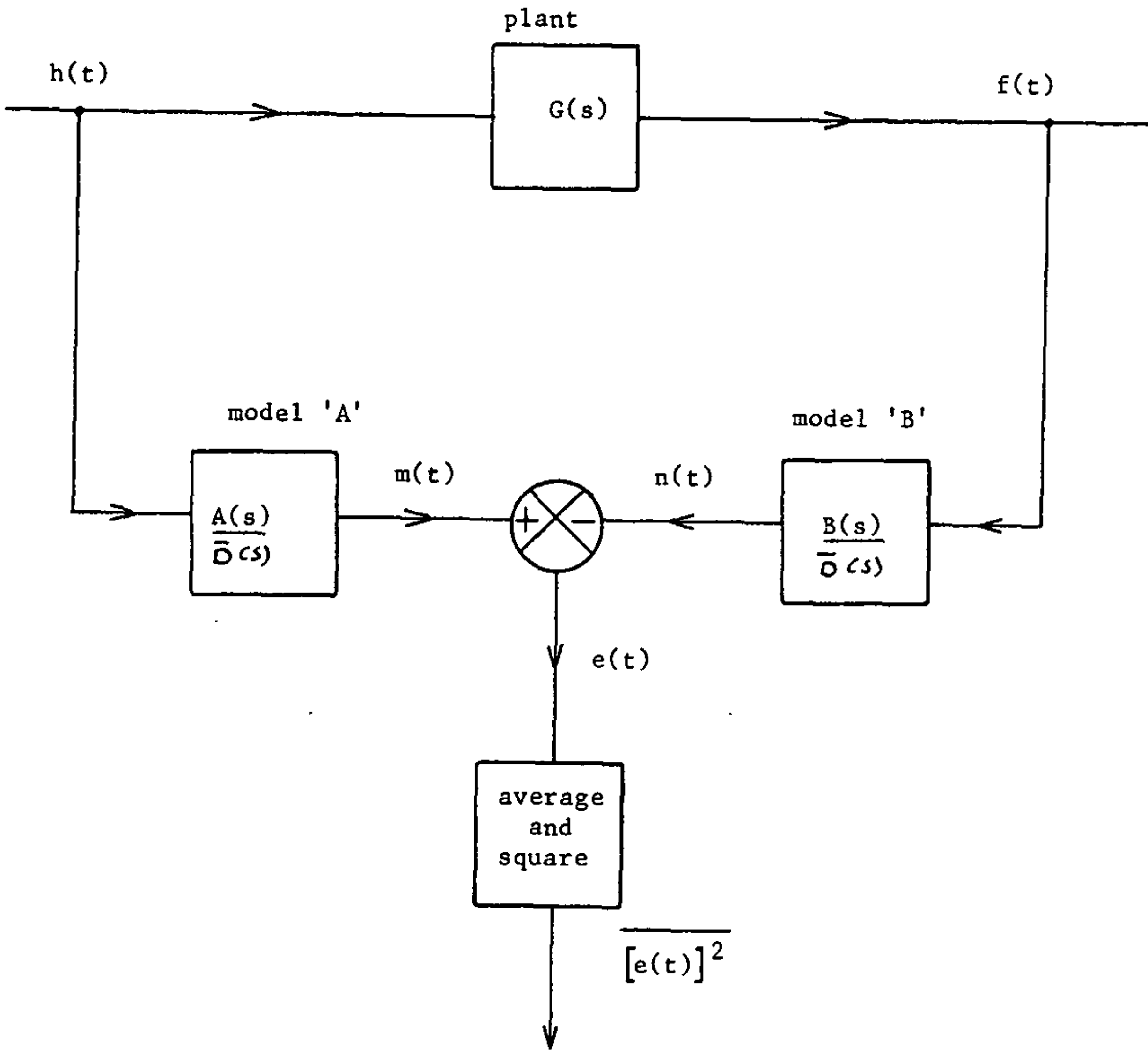



Fig. 3.12 Block diagram of system model.

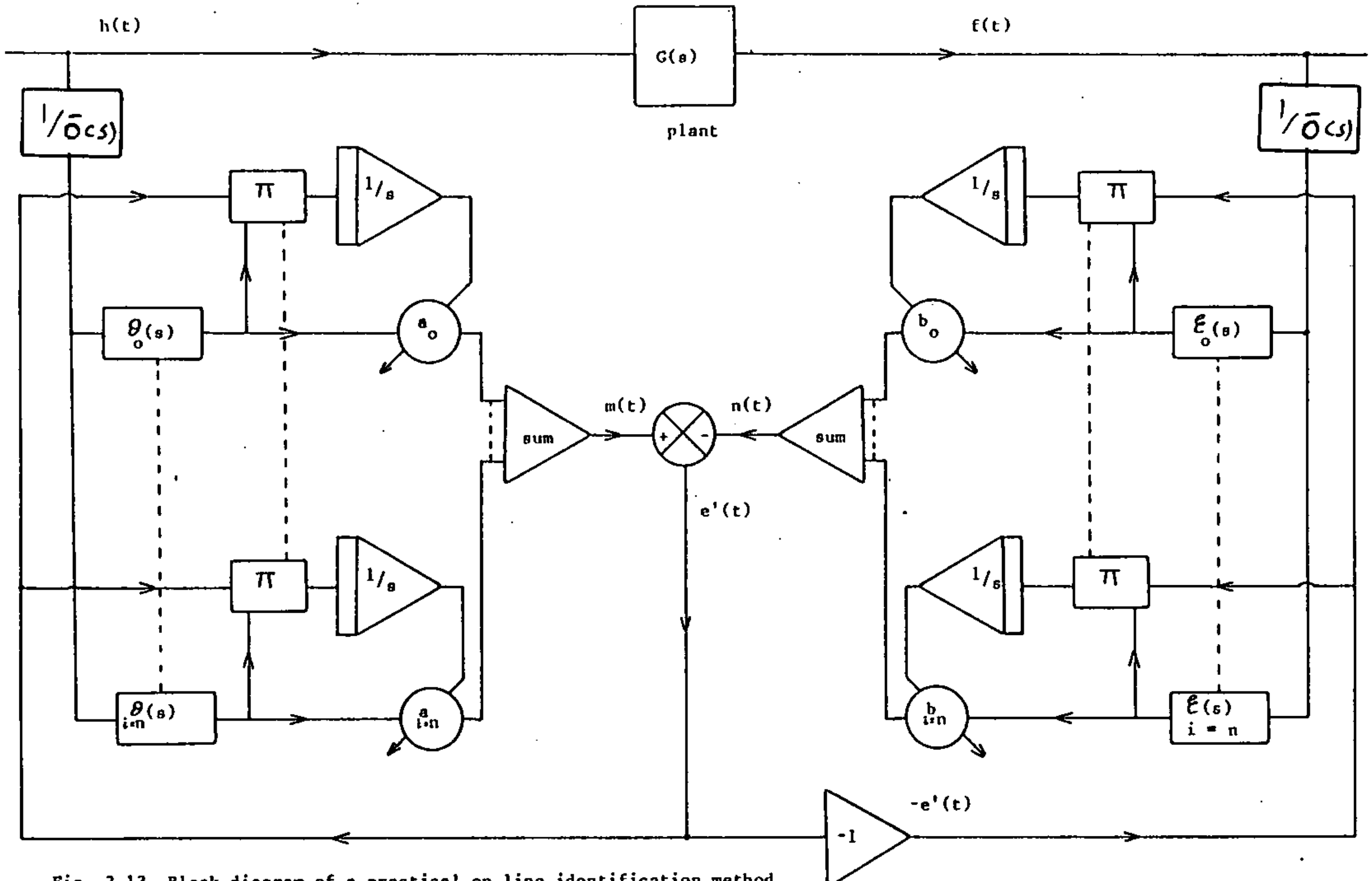



Fig. 3.13 Block diagram of a practical on line identification method.

parameter.[27] Hence there is a minimum value for the integral squared error and so an optimum condition or setting for the mathematical model may be found. The accuracy associated with this mathematical model will depend upon the time available for its estimation.

### 3.10.3 Method 2

From equation 3.23 a second method of identification also suggests itself. This basic method is shown in figure 3.12. For this system the mathematical models A(s), B(s) and $\bar{D}$(s) are polynomials of order N, which are governed by the plant characteristics.

The implementation of the method suggested by figure 3.12 is shown in figure 3.13. By analysis it can be shown that e'(t) may become zero either when $a_i$ or $b_i$, the adjustable coefficients, have been adjusted to their correct values or when both $a_i$ and $b_i$ are zero.

In this type of model $a_i$ and $b_i$ are adjusted in an opposite sense to the signs of their respective gradients and at a rate which is proportional to those gradients. By this method it is possible to change the values of $a_i$ and $b_i$ along the path of steepest decent to their optimum values at the point of the minimum mean square error.

Both these methods could be applied to the control of the furnace(s) under investigation in this research project, however the first of the two methods has the advantage of being less complex and also being better suited to the type of mathematical model developed in Chapter 2. Hence the identification scheme used in the first method is the scheme which will be used in order to identify the furnace mathematical model, and for use in the adaptive control system.

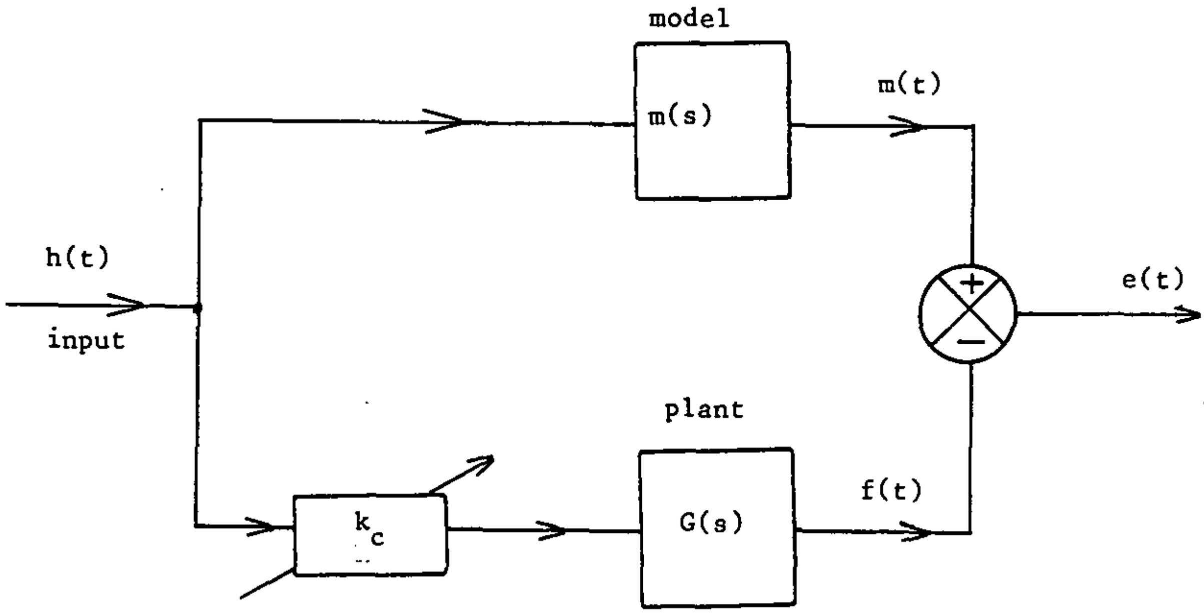



Fig. 4.1 A basic M.R.A.C. gain adjustment system.

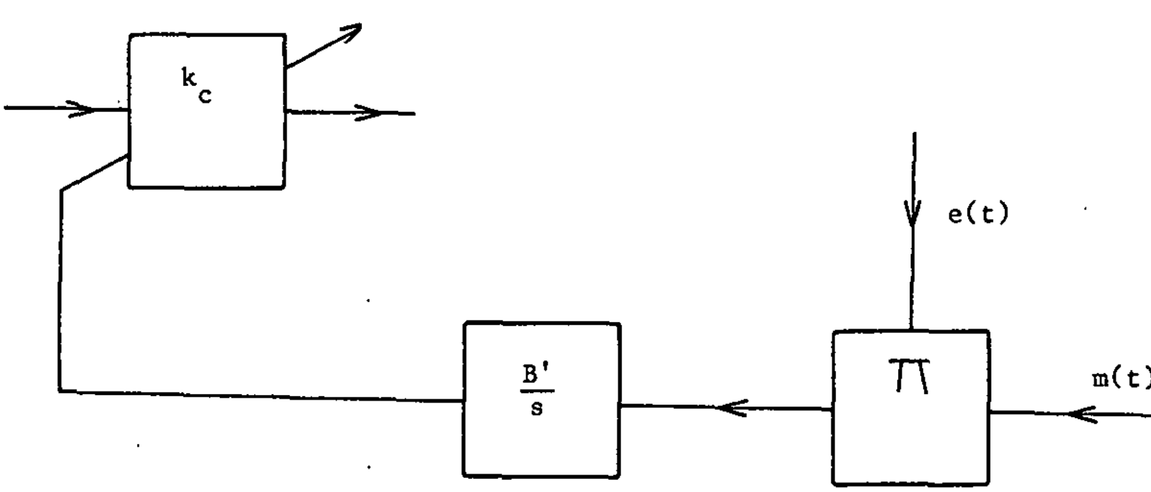



Fig. 4.2 M.I.T. adaptive loop

# CHAPTER 4

## SELF ADAPTIVE FURNACE CONTROL SYSTEM

### 4.1 Introduction

Many ingenious design rules have been extensively reported in the literature, especially in the areas of the design of continuous model reference, parameter adaptive control systems, see Fig. 4.1. There are basically two main approaches to the synthesis of this class of adaptive control system. One type is centred upon the production of some form of performance index [P.I.] and the other on a Liapunov function; the exact form of these classes of adaptive control systems will be discussed in detail in section 4.2. Both of these methods of adaptive control have their own particular advantages and limitations. Hang[61,62] has carried out a critical comparison of the merits of the various design rules which come under the above headings. These comparisons will be reviewed in the next section.

### 4.2 Comparison of Adaptive Design Rules

In the synthesis of model reference adaptive control systems the following design rules are probably the most well known:

(i) the M.I.T. design rule[63]
(ii) the Liapunov synthesis[64]
(iii) Dressler's gradient rule[65]
(iv) Price's gradient rule[66]
(v) Monopoli's design rule[67]

There are several other design rules which are less well known, the limitations of these methods are discussed in section 4.2.6.

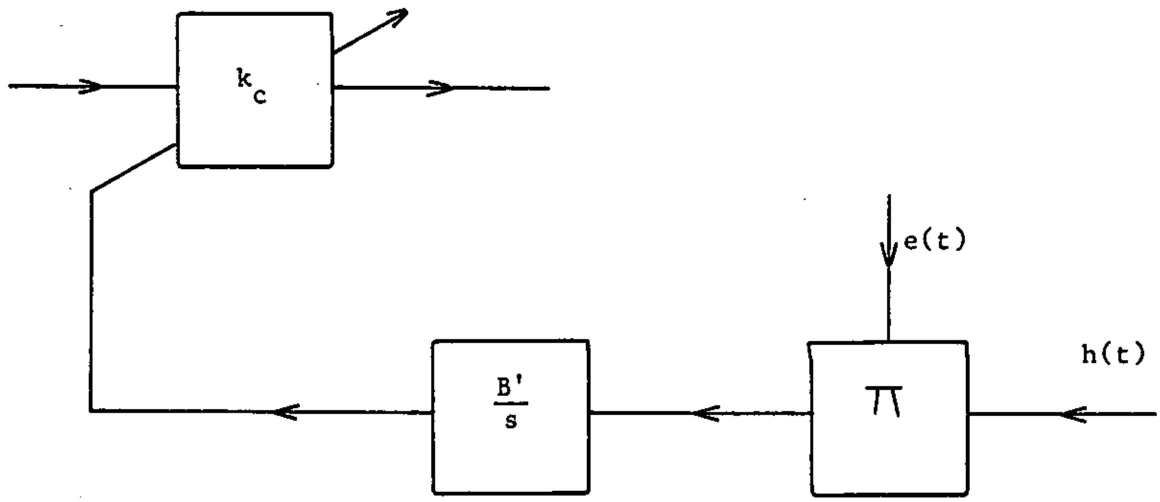



Fig. 4.3 Dressler's adaptive loop.

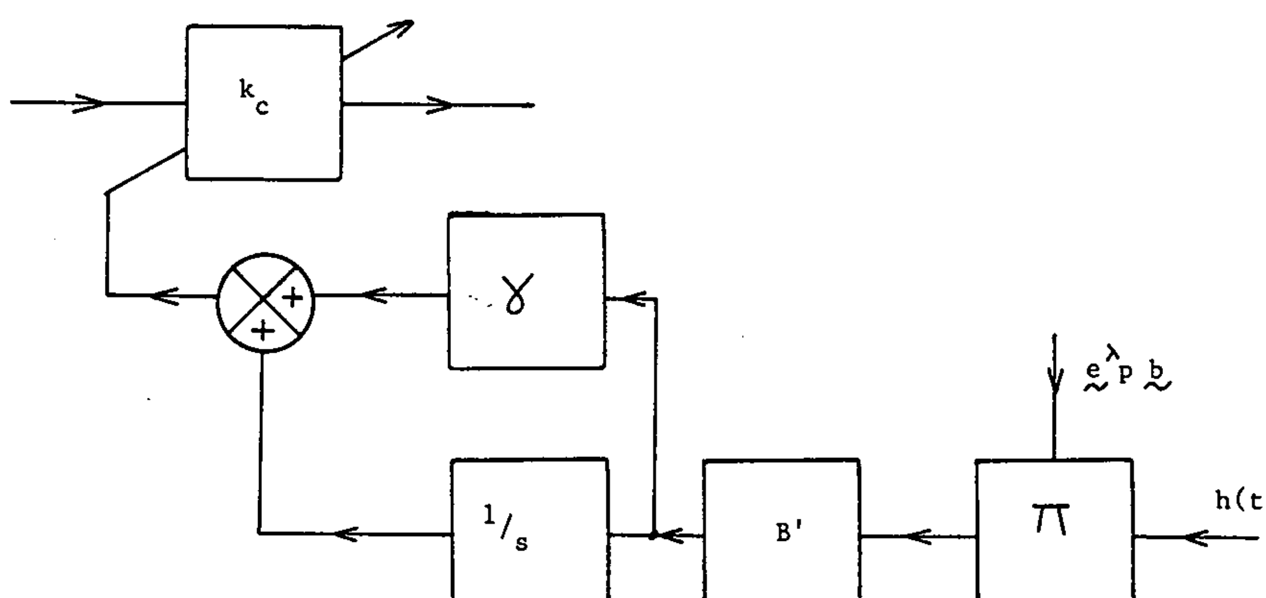



Fig. 4.4 Liapunov's adaptive loop

### 4.2.1 The M.I.T. Design rule

The basic configuration of the M.I.T. design rule is shown in Fig. 4.2. The performance index and parameter adjustment technique is found as follows:

$$\text{Performance Index } [\text{P.I.}] = \int e^2 \, dt \qquad \ldots\ldots 4.1$$

and using the parameter adjustment law of steepest descent minimization technique we have:

$$\frac{\partial k_c}{\partial t} = B \, e \, \frac{\partial m(t)}{\partial k_c} \qquad \ldots\ldots 4.2$$

and $\dfrac{m(t)}{k_c}$ is proportional to f(t)

$$\frac{k_c}{t} = B' \, e \, f(t) \qquad \ldots\ldots 4.3$$

where B' is the adaptive gain.

The advantage of this method is that equation 4.2 can be easily implemented. However doubts about the stability of this method under certain conditions have been expressed by Parks[64] and James[68].

### 4.2.2 Liapunov Synthesis

A successful form of the Liapunov function is that proposed by Gilbert et al[67] and has the form of:

$$V = \hat{e}^T P \hat{e} + (X + r \, k_v \, m)^2 \qquad \ldots\ldots 4.4$$

where $m = B' \, \hat{e}^T P \hat{b} \, r$ and $X = k - k_c \, k_v$

$$\frac{\partial v}{\partial t} = -\hat{e}^T Q \hat{e} - 2 \lambda r \, k_v^2 \, m^2 \qquad \ldots\ldots 4.5$$

These result in a stable adjustment law of the form:

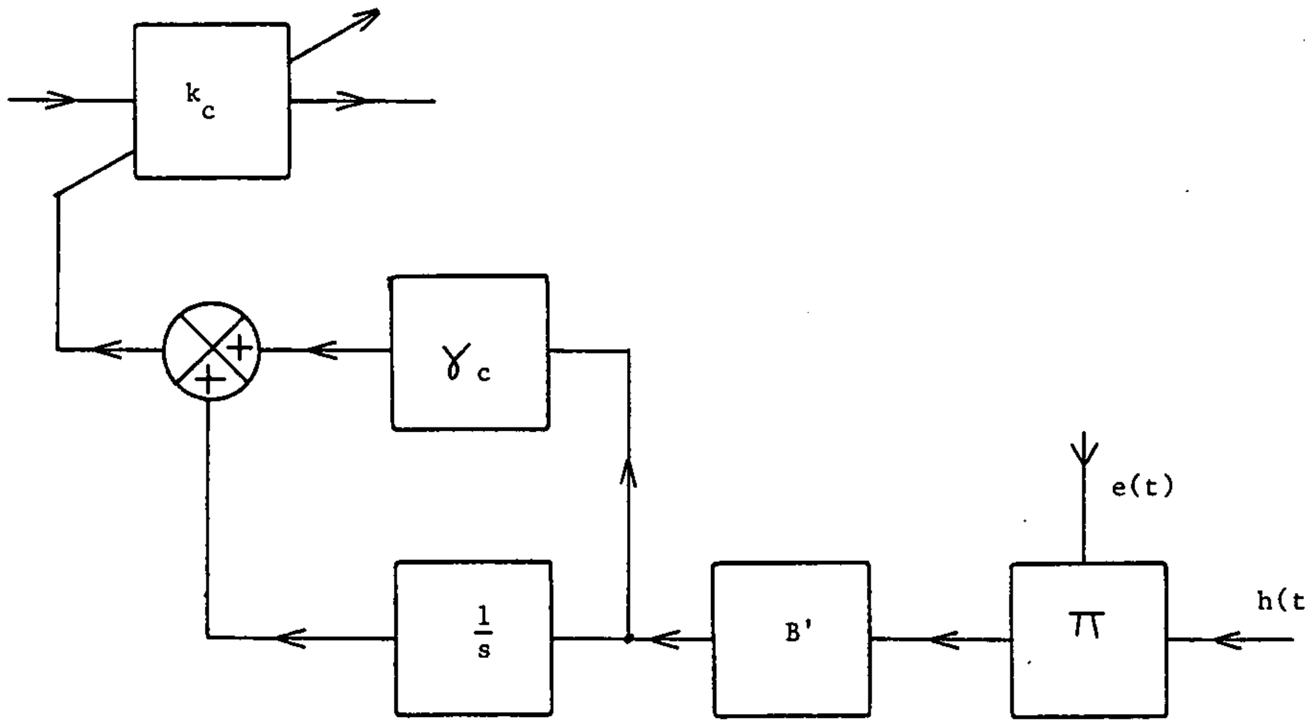



Fig. 4.5 Price's adaptive loop.

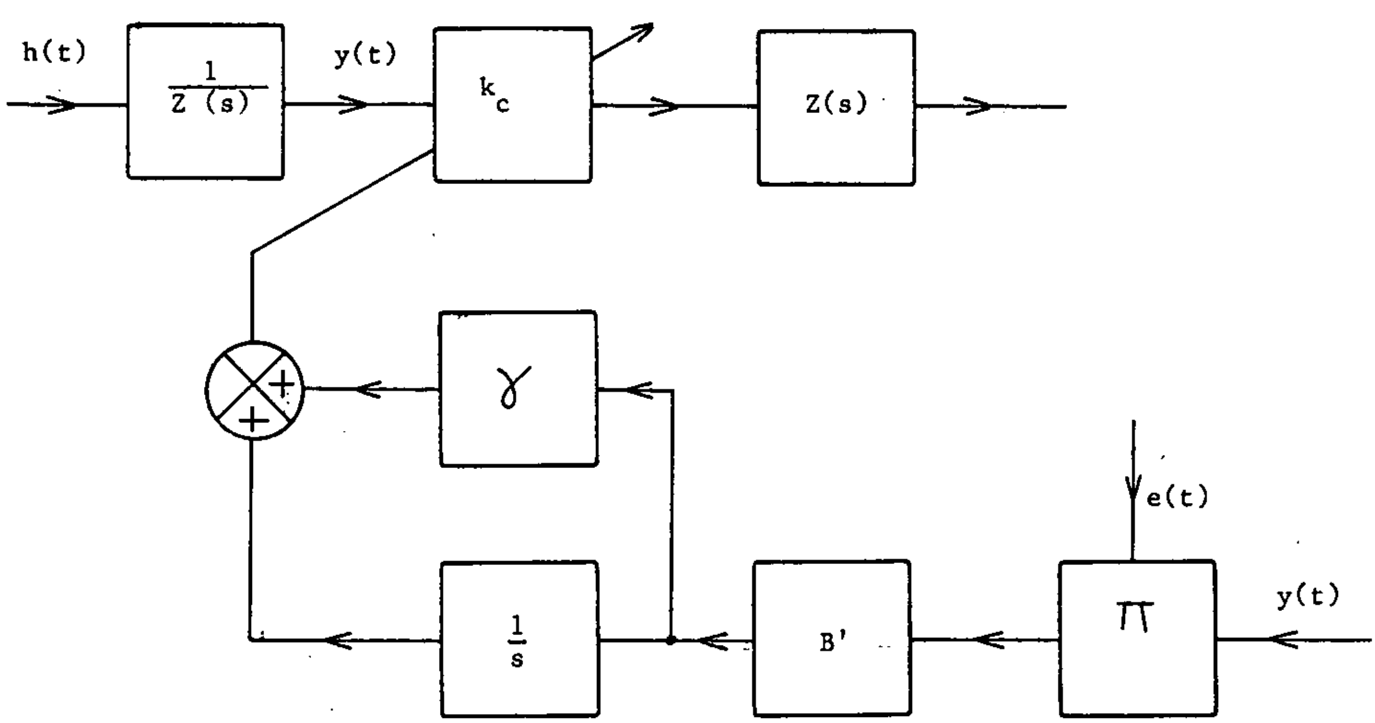



Fig. 4.6 Monopoli's adaptive loop.

$$\frac{k_c}{t} = m + \gamma m \qquad .....4.6$$

It is interesting to note that by letting $\gamma = 0$ we have the design rule used by Parks[64]. Equation 4.6 for the Liapunov method has been reported in the literature[62,69] as being predictably stable, but the equation requires the estimation of the complete state vector which is not often available. This may often necessitate the use of differential networks with associated noise problems.

### 4.2.3 Dressler's Gradient Rule

The parameter adjustment law for the Dressler's gradient rule[65] is:

$$\frac{\partial k_c}{\partial t} = B' \; e \; h(t) \qquad .....4.7$$

This type of adaptive controller is easily implemented. It does, however, suffer from the disadvantage that at larger loop gains its damping suffers and it has stability problems similar to that of the MIT rule[62].

### 4.2.4 Price's Gradient Rule

This is sometimes referred to as the accelerated gradient method[66]. The parameter adjustment law is given by:

$$k_c = B' \; e \; h\,(t) + c \frac{d}{dt} \left(B' \; e \; h\,(t)\right) \qquad .....4.8$$

where c = const.

This type of control law is similar to the control law in the Dressler case, however there is the addition of a feed forward term which has the effect of improving the damping and stability of the system. The disadvantage of this method is that stability cannot always be guaranteed.

### 4.2.5 Monopoli Design Rule

This is based on a modified Liapunov scheme[67]. The function Z(s) is used to produce a modified plant transfer function such that Z(s) G(s) is real and positive, and the Kalman-Meyer theorem is used to eliminate any error derivatives[63,65]. Hence the parameter adjustment law may be written as:

$$\frac{\partial k_c}{\partial t} = B' \, e \, y(t) + h(t) \frac{d}{dt} (B' \, e \, y(t)) \qquad .....4.9$$

where y(t) is the input signal (modified) to the plant, which may be obtained by passing the original input signal h(t) through a filter (1/Z(s)). It is also possible[62] to extend this technique to the case of a general time varying gain.

### 4.2.6 Other Design Rules

The design rules considered in section 4.2.1 to 4.2.5 are probably the most well known adaptive design rules. There are, however, several other design rules, such as those of Kokatovic,[70] Nikiforuk[71] and Choe[72]. These designs are not considered in detail here as they have been well documented in the past, but most importantly they are thought to possess basic weaknesses. These weaknesses are well documented by Hang[62] who points out their shortcomings.

## 4.3 Furnace Adaptive Loop

The choice of the method of closing the furnace adaptive loop[73.74.75] is probably the most critical decision which has to be made, in order that precise temperature control may be achieved using an adaptive control system.

The choice of the method of identification which was discussed in detail in section 3.10.2 inevitably restricts the options available.[76] In order to close the adaptive loop with the type of identification system chosen, the adaptive control law initially used, was that of the Dressler's gradient rule, modified to suit the particular requirements and restrictions[77,78] of the furnace control system. It was however, thought that the MIT rule may also be appropriate, and that when the adaptive loop was in the process of being closed it was envisaged that the MIT rule may be implemented at some later date.

The technique used for the parameter adjustment law was of steepest descent minimization, in order to change the parameters of the mathematical model in a time scale (maximum allowable time of 19.96 sec) which was compatible with the control law and also the PDP8 digital computer which is used as the process controller.

The modification to the Dressler's Gradient rule in order to make it compatible with the furnace adaptive control system is as follows:

Dressler's Gradient rule for adaptive control is repeated here again for convenience.

$$\frac{\partial k_c}{\partial t} = B' \, e \, h(t)$$

where $e = \Delta\tau$ .....4.10

$$\frac{\partial k_c}{\partial t} = B' \Delta \tau h(t)$$

Hence $\Delta I_c = F \dfrac{\partial k_c}{\partial t} = B \Delta \tau h(t)$ .....4.11

*In equation 4.10 $\Delta\tau$ is the difference between $\tau$ for the model and $\tau$ for the plant.*

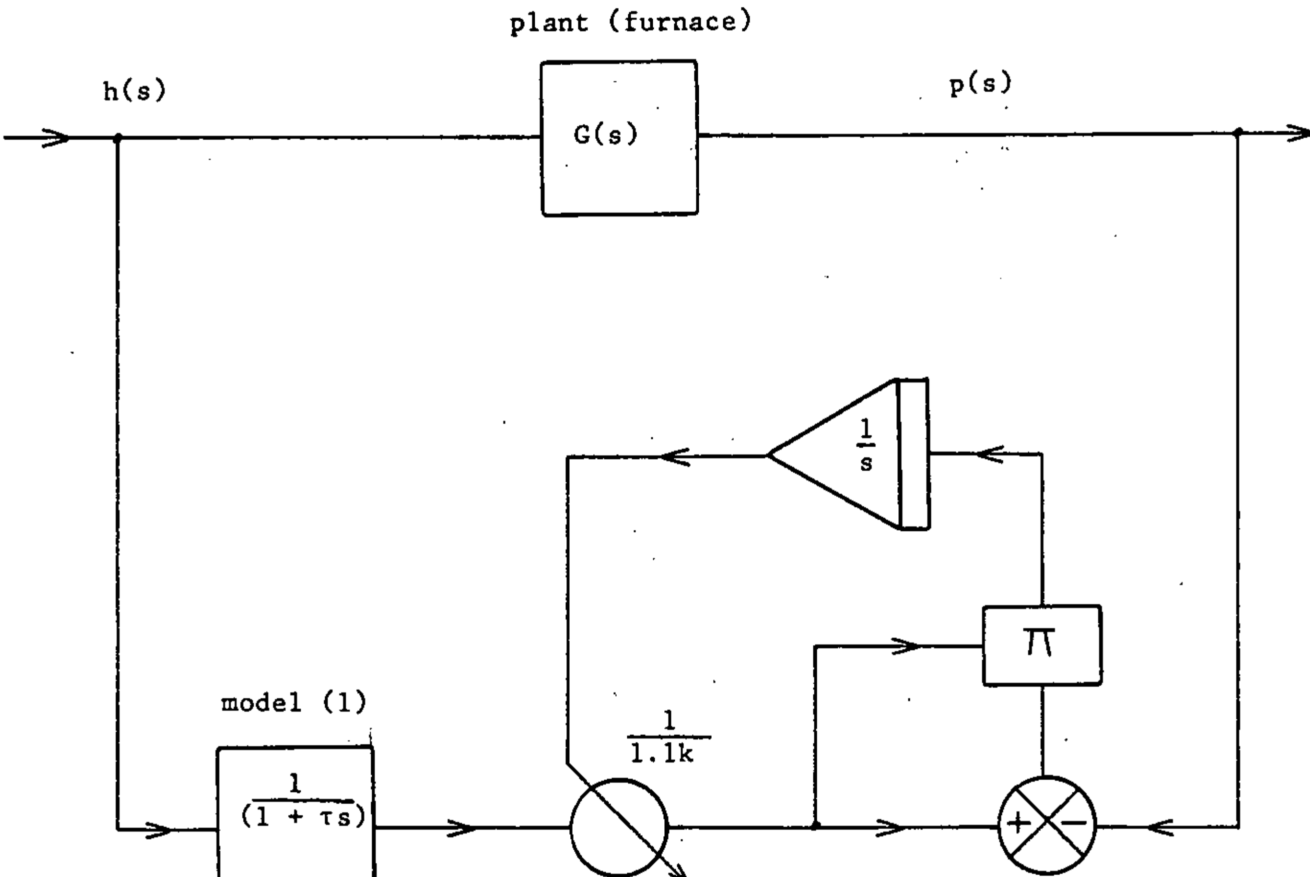



Fig. 4.7 Part of the furnace mathematical model identification scheme.

This adaptive control law is the arrangement which is used to produce the required change in the furnace current ($\Delta I_c$), if the measured furnace temperature is either lower or higher than that which is required by the pre-determined temperature/time profile. This pre-determined temperature/time profile is characterised by the parameter $\Delta\tau$. The exact method of the implementation of this adaptive control law is discussed in the following sections.

## 4.4 Furnace Control System

### 4.4.1 Analogue System

The implementation of the furnace identification and adaptive control system may be either analogue or digital in its concept. If an analogue system can be implemented, it may be possible to use relatively cheap and easily obtainable circuit elements. In contrast, a digital implementation, would require a computer controlled system together with more expensive digital control and possibly interface circuits.

In order to demonstrate the feasibility of an analogue adaptive control and identification system, only part of the furnace identification system will be initially implemented, that is the identification of the first part of the furnace mathematical model, as shown in figure 4.7. If this part of furnace mathematical model identification scheme can be implemented successfully, then from the experience gained during its implementation, the second part of the mathematical model may be included and hence the whole of the identification procedure may be tested. Finally the adaptive loop can be closed.

The circuit details for the analogue implementation of the

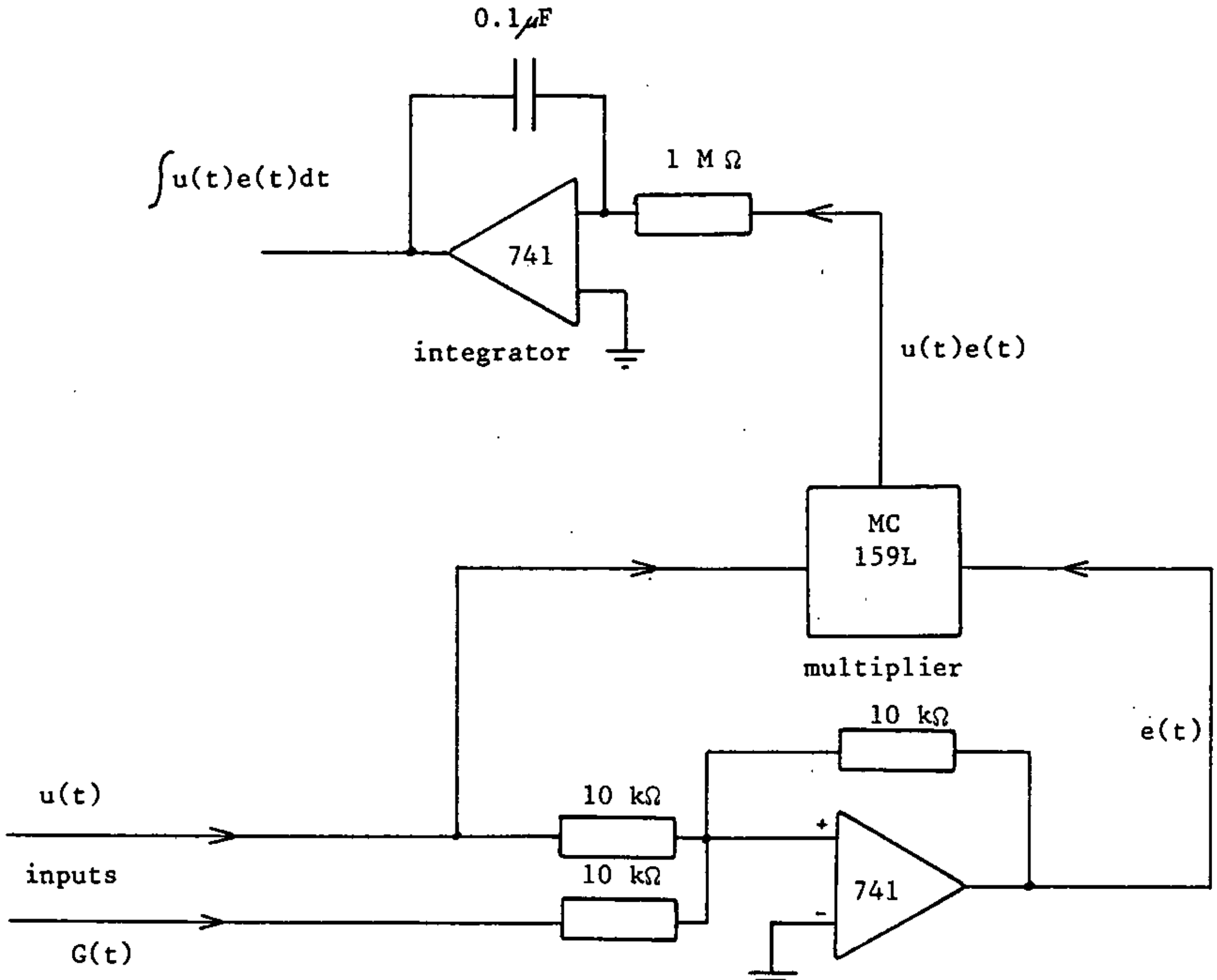



Fig. 4.8 Analogue implementation of the furnace mathematical model identification scheme.

identification scheme, for the first part of the mathematical model, is shown in figure 4.8.

The relative phases of the inputs u(t) and G(t) are arranged such that the output from the comparator gives u(t) - G(t) which is the error signal e(t). The components used for the integrator time period, are determined by reference to the time scale used for the analogue model discussed in section 2.4. It is however, not necessary to use 0.1$\mu$F and 1 M$\Omega$ only, as these values may be changed if it is found to be desirable, and providing the new integrator time period is within the operating limits of the identification procedure.

The testing of the analogue identification scheme was carried out in the following manner. The inputs u(t) and G(t) need first to be simulated. These inputs, in the adaptive control situation envisaged would be continuously varying and hence initially it was thought that a sine or a cosine waveform, offset by various dc voltages would give realistic simulated inputs. In practice, this type of input was found to be unrealistic as the phase shift introduced by the active circuit elements produced an inherently unstable system as soon as completion of the parameter adjustment loop was attempted[79,80,81].

A second attempt to simulate the inputs u(t) and G(t) was to use variable dc voltages. This method solved the phase shift problems associated with the sine and cosine inputs and the integrator produced a measurable output signal, which was proportional to the product of u(t) and e(t), over the time period involved. This second method did, however, have a serious drawback, namely that the integral function, performed by the use of a 741 operational amplifier, suffered from a serious output drift problem. This meant that if the functions u(t) and G(t) were the same, then the error signal e(t) should be, and was, zero. Hence the input to the integrator was obviously also zero, and the integrator output should remain constant. In practice the integrator output voltage either slowly increased or decreased under any zero or slowly changing inputs, and was only limited by the output reaching the $\pm$ 15V supply voltage. This was obviously most unsuitable for the type of identification system envisaged.

The problem of integrator drift could be eased by using more stable, but also considerably more expensive, operational amplifiers. However the problem of output drift would probably still be apparent over the operating period of the furnace (up to several hours).

The problem of integrator output voltage drift is a very serious drawback to the implementation of an analogue identification and adaptive control system, but the scheme still could be used if an integrator was available with a stable output over the operating period of the furnace[82,83].

### 4.4.2 Implementation of an Analogue Parameter Adjustment System

If it is assumed that the problem of integrator output drift may be controlled by the use of a more stable type of operational amplifier, it becomes necessary to demonstrate that completion of the parameter adjustment loop is viable, using an analogue

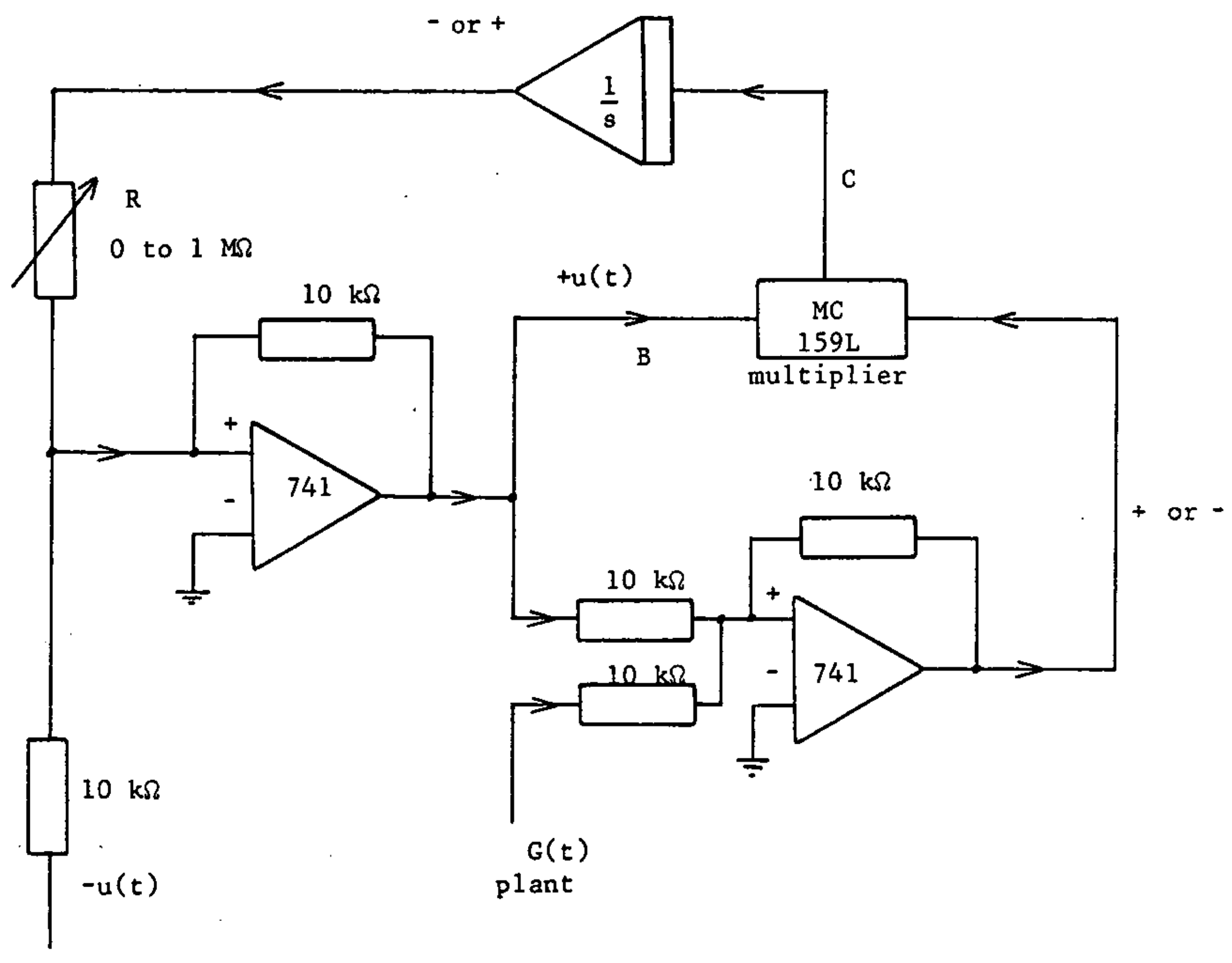



Fig. 4.9 Analogue parameter adjustment loop.

identification system. Completion of the parameter adjustment loop may be achieved in several different ways[84]. Two methods are discussed below.

Method 1

The output of the integrator must be used to effect an adjustment to the parameter(s) of the furnace mathematical model. It was thought that a voltage controlled resistor (MOSFET in this case) operated over its limited linear range, would present a suitable solution. The disadvantage of this method is that the characteristics of a MOSFET are far from linear and when this type of device was tested in the circuit shown in Figure 4.9 far too little control over the resistance R, could be achieved for satisfactory model adjustment performance.

Method 2

The parameter adjustment loop may be completed by use of a light dependent resistor, operated by the level of the integrator output voltage. This method would have the advantage of isolating the feedback path, in an electrical sense, but the stability of the circuit could still not be guaranteed unconditionally.

In order to establish the validity of using the method of resistance adjustment in the parameter adjustment loop, the circuit of Fig. 4.9 was used, with variable dc inputs for G(t) and u(t), and also with an initial manual adjustment to the resistor R.When tested under the foregoing conditions, the circuit was found to be basically unstable and oscillations at a frequency of approximately 500 kHz were observed. The commencement of oscillation was dependant upon the value of the resistor R, and a measure of controllability of the error signal e(t), was achieved before oscillations occurred. These oscillations however, commenced before an acceptable low level of error signal was achieved.

In conclusion, the problems associated with an analogue implementation of the identification scheme for the furnace mathematical model, would seem to be very great. Hence it was considered, at this stage of the investigation that an analogue mathematical model identification scheme is not a practical proposition for this type of heat treatment furnace. In the next section a digital solution to the problem will be discussed.

### 4.4.3 Digital Implementation of a parameter adjustment system

As a result of the very serious problems encountered during the implementation of the parameter adjustment system using the analogue techniques detailed in the previous section, it was thought that a practical solution may be obtained by using a digital system of mathematical model parameter adjustment, in conjunction with a digital minicomputer, namely the PDP8[85,86,87].

This digital system, which will help minimise the problems associated with the integration function over the long furnace time constants, and some of the stability problems encountered with the analogue parameter adjustment system, involves for its implementation, the transformation of each part of the analogue system into a digital or digitized system by use of the

'Z transformation' method[88].

In essence the use of Z-transforms necessitates the sampling of the incoming signal or signals, and then performing the required mathematical operation on each individual digitized part of the signal. Hence the function of integration may be performed without any inherent output drift occurring, and it is the system designer who may decide how long the process computer needs to hold the output of the integrator function at a particular value. In practice this 'holding time', could be of any length, up to, but not including infinity.

The Z-transformation method is used to change a particular function expressed in terms of the Laplace operator, s, from the 's-domain' to the 'Z-domain'. The following notation will be used exclusively in this analysis of the furnace adaptive system.

$$Z = e^{sT} \qquad \text{.....4.12}$$

and $$Z^{-1} = e^{-sT} \qquad \text{.....4.13}$$

The function $Z^{-1}$ means that a delay of one complete sampling period T has occurred. That is, the function or operation occurs one time period later, thus it is delayed by a time T.

The Z-transformation of the integral function is given by:

$$Z\left[G(s)\right] = Z\left[\frac{V_o(s)}{V_{in}(s)}\right] = Z\left[\frac{1}{s}\right] \qquad \text{.....4.14}$$

and therefore

$$G(Z) = \frac{Z}{Z - 1} = \frac{V_o(Z)}{V_{in}(Z)} \qquad .....4.15$$

$$G(Z) = \frac{1}{1 - Z^{-1}}$$

$$V_{in}(Z) = V_o(Z)\left[1 - Z^{-1}\right]$$

$$V_{in}(Z) = V_o(Z) - V_o(Z)\, Z^{-1}$$

$$V_o(Z) = V_{in}(Z) + V_o(Z)\, Z^{-1} \qquad .....4.16$$

Now letting $V_{in}(k)T$ be the kth sample value of the input signal, where the sampling interval is T, and similarly letting $V_o(k)T$ be the kth sample value of the output signal, then from equation 4.16 it follows that

$$V_o(k)T = V_{in}(k)T + V_o(k - 1)T \qquad .....4.17$$

Hence for a unit-step input the output $V_o(k)T$ is a ramp function, and the function $G(Z) = Z/(Z - 1)$ gives the function of an integrator.

A second method of showing that $G(Z) = Z/(Z - 1)$ performs the function of integration is as follows

$$V_o(Z) = V_{in}(Z) \times \frac{Z}{Z-1} \qquad .....4.18$$

It can be shown that for a unit-step input

$$V_{in}(Z) = \frac{Z}{Z - 1} \qquad .....4.19$$

Hence to perform the function of integration on a step input substitute equation 4.19 in equation 4.18 above.

$$V_o(Z) = \frac{Z}{Z-1} \times \frac{Z}{Z-1} = \frac{Z^2}{(Z-1)^2} \qquad .....4.20$$

Dividing $Z^2$ by $(Z-1)^2$ gives:

$$V_o(Z) = 1 + 2Z^{-1} + 3Z^{-2} + 4Z^{-3} + \ldots\ nZ^{(n-1)} \qquad .....4.21$$

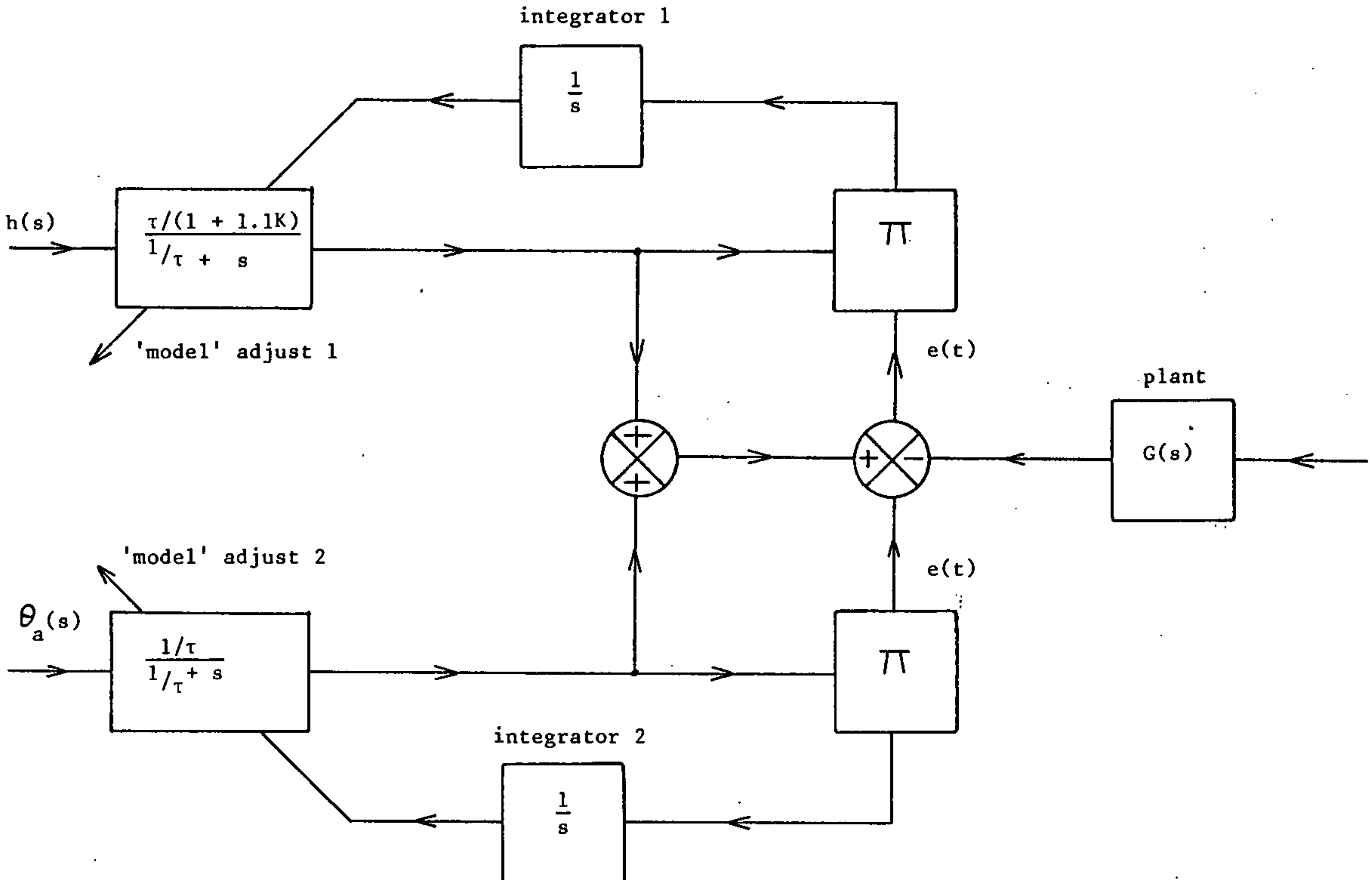



Fig. 4.10 Parameter adjustment scheme for the furnace mathematical model.

Using the notation of equation 4.17, equation 4.21 becomes:

$$V_o(k)T = 1 + 2\ (k-1)T + 3(k-2)T + 4(k-3)T+\ldots \quad \ldots\ldots 4.22$$

Hence it is seen that for a unit-step input the function $V_o(k)T$ is a ramp function, and the Z transform $Z/(Z-1)$ has performed the operation of integration.

### 4.4.4 Z-transformation of the furnace mathematical model to perform the parameter adjustment function

The furnace model in the 's-domain' together with the parameter adjustment scheme is shown in Fig. 4.10. In order to transform this scheme from the 's-domain' to the 'Z-domain' the standard Z transformation technique is used.

let $\beta = \dfrac{\tau}{(1 + 1.1K)}$ and $\alpha = \dfrac{1}{\tau}$

$$M_1(s) = \frac{\beta}{\alpha + s} \quad \ldots\ldots 4.23$$

and

$$M_1(Z) = \beta\left[\frac{Z}{Z - e^{-\alpha T}}\right] = \frac{0(Z)}{h(Z)} \quad \ldots\ldots 4.24$$

$$\frac{0(Z)}{h(Z)} = \frac{\beta}{1 - Z^{-1}\ e^{-\alpha T}} \quad \ldots\ldots 4.25$$

$$\beta\ h(Z) = 0(Z) - 0(Z)\ Z^{-1}\ e^{-\alpha T} \quad \ldots\ldots 4.26$$

$$0(Z) = \beta\ h(Z) + 0(Z)\ Z^{-1}\ e^{-\alpha T} \quad \ldots\ldots 4.27$$

$$0(k)T = \beta\ h(k)T + 0(k-1)T\ e^{-\alpha T} \quad \ldots\ldots 4.28$$

Also

$$M_2(s) = \frac{\alpha}{\alpha + s} \quad \ldots\ldots 4.29$$

$$M_2(Z) = \frac{\gamma\ (Z)}{\theta_a\ (Z)} = \frac{\alpha\ Z}{Z - e^{-\alpha T}}$$

$$= \frac{\alpha}{1 - \bar{Z}^{-1} e^{-\alpha T}} \qquad \ldots\ldots 4.30$$

$$\gamma(Z) - \gamma(Z)\, Z^{-1} e^{-\alpha T} = \alpha\, \theta_a(Z)$$

$$\gamma(Z) = \alpha\, \theta_a(Z) + \gamma(Z)\, Z^{-1} e^{-\alpha T} \qquad \ldots\ldots 4.31$$

Hence $\gamma(k)T = \alpha\, \theta_a(k)T + \gamma(k-1)Te^{-\alpha T}$ .....4.32

The whole mathematical model may be found by considering the sum of equation 4.32 and 4.28

$$\omega(k)T = 0(k)T + \gamma(k)T \qquad \ldots\ldots 4.33$$

The error signal, which is the difference between the signals from the model, $\omega(k)T$ and the plant $G(k)T$, is $e(k)T$ which is found by:

$$\omega(k)T - G(k)T = e(k)T \qquad \ldots\ldots 4.34$$

Hence the input to the integrator is given by:

$0(k)T \times e(k)T$ = parameter adjustment signal for part 1 of the model .....4.35

and $\gamma(k)T \times e(k)T$ = parameter adjustment signal for part 2 of the model .....4.36

4.4.5 Computer Implementation of the Z transformed furnace parameter adjustment scheme[89,90,91]

In order to test the parameter adjustment scheme it is necessary to show that the Z-transformed furnace model gives a representative response when compared with the response given from the actual furnace under consideration. Initially the furnace used was the large Cooperheat Ltd, furnace.

A computer programme was developed in order to test the validity of the Z-transformation model and to establish representative

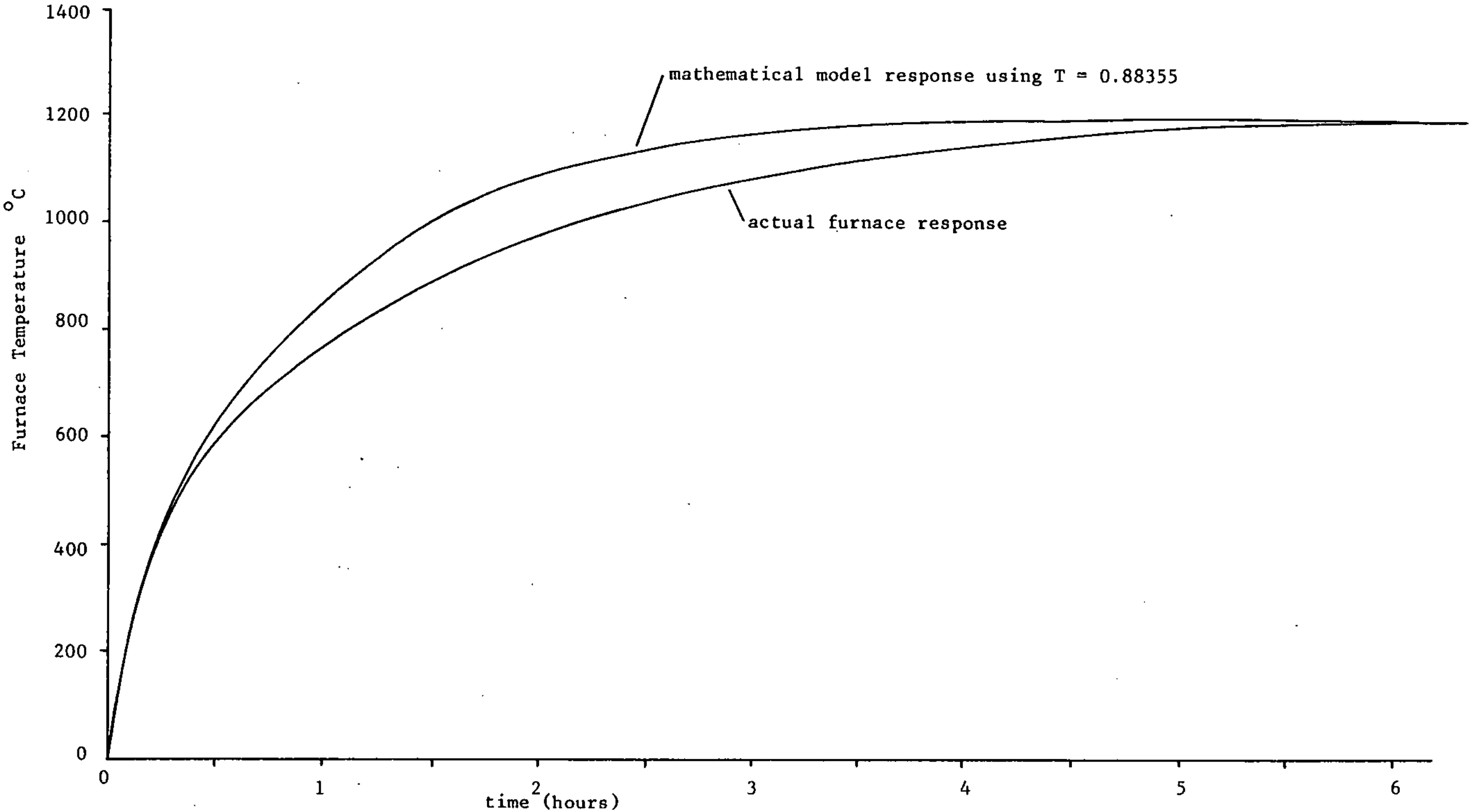
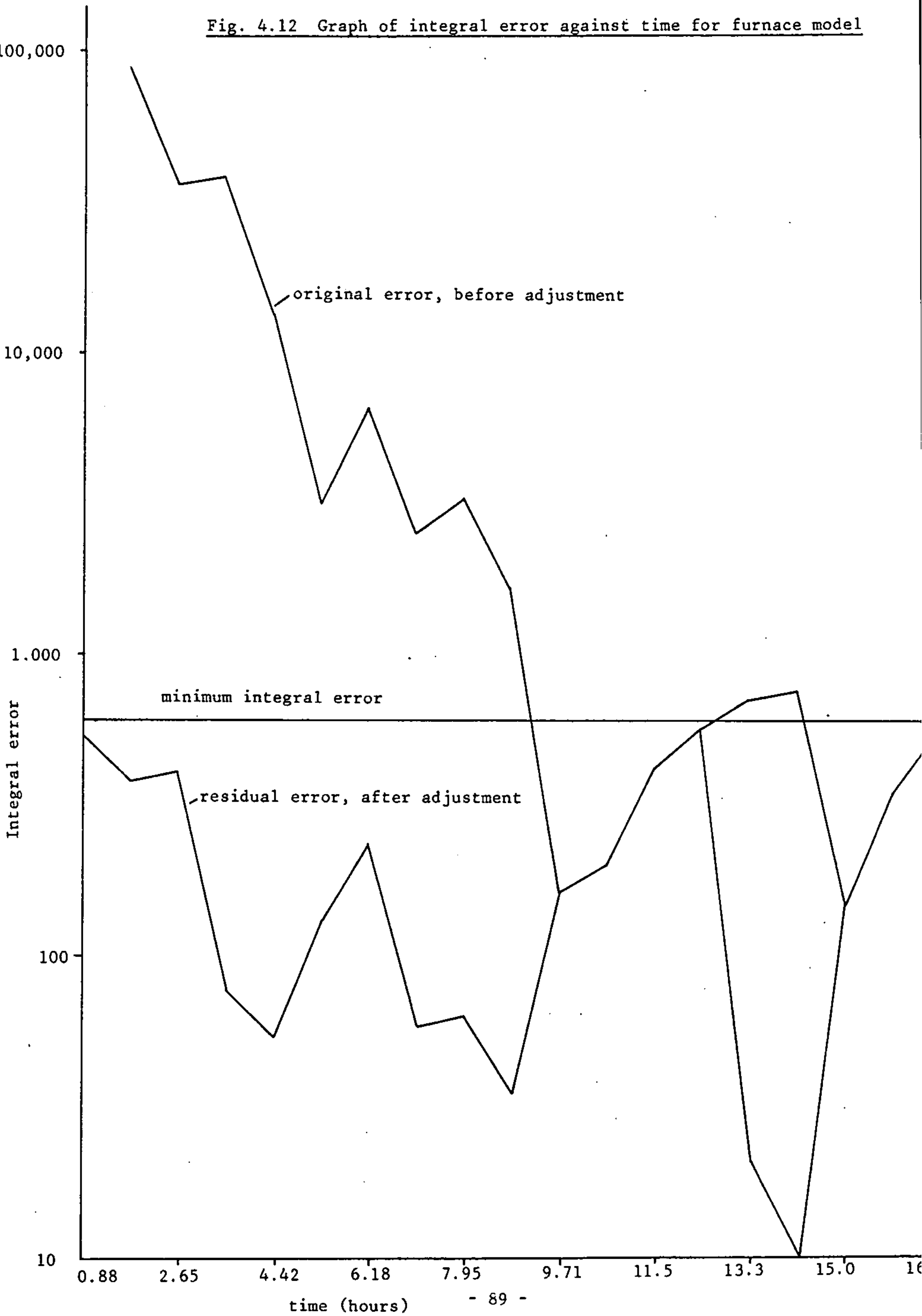



Fig. 4.11 Step response of the Cooperheat furnace compared with the Z-transformed mathematical model.

Fig. 4.12 Graph of integral error against time for furnace model



values for the sampling period, T, (see equation 4.28). The computer programme developed in the first instance is listed in Appendix 4, and the results obtained using T = 0.88355 hours are shown in Fig. 4.11. It should be noted that the mathematical model compares well with the actual furnace response at the extreme ends of the range, the largest error occurring at about 2 hours after switch on.

This error may be minimized by using a variable parameter within the mathematical model. This development to the programme is also shown in the programme listing in Appendix 4.

The result of this adjustment scheme is that the programme may now be made to follow a preselected temperature/time profile, up to the maximum realisable profile of that given in Fig. 4.11. It is of no practical interest in producing temperature/time profiles outside the range indicated by Fig. 4.11, because the furnace could not operate beyond these limits.

The results of trial runs of the programme given in Appendix 4 are shown in Figs. 4.11 and 4.12. It can be seen that integral error produced by the programme is at its maximum at the commencement of the programme, and in order to keep the running time of the programme within reasonable limits an arbitrary, but limiting,minimum integral error value was selected,as can be seen in Fig. 4.12. It should be noted that as the programme runs the initial error for each iteration becomes smaller. This is due to the 'learning nature' of the type of parameter adjustment used.

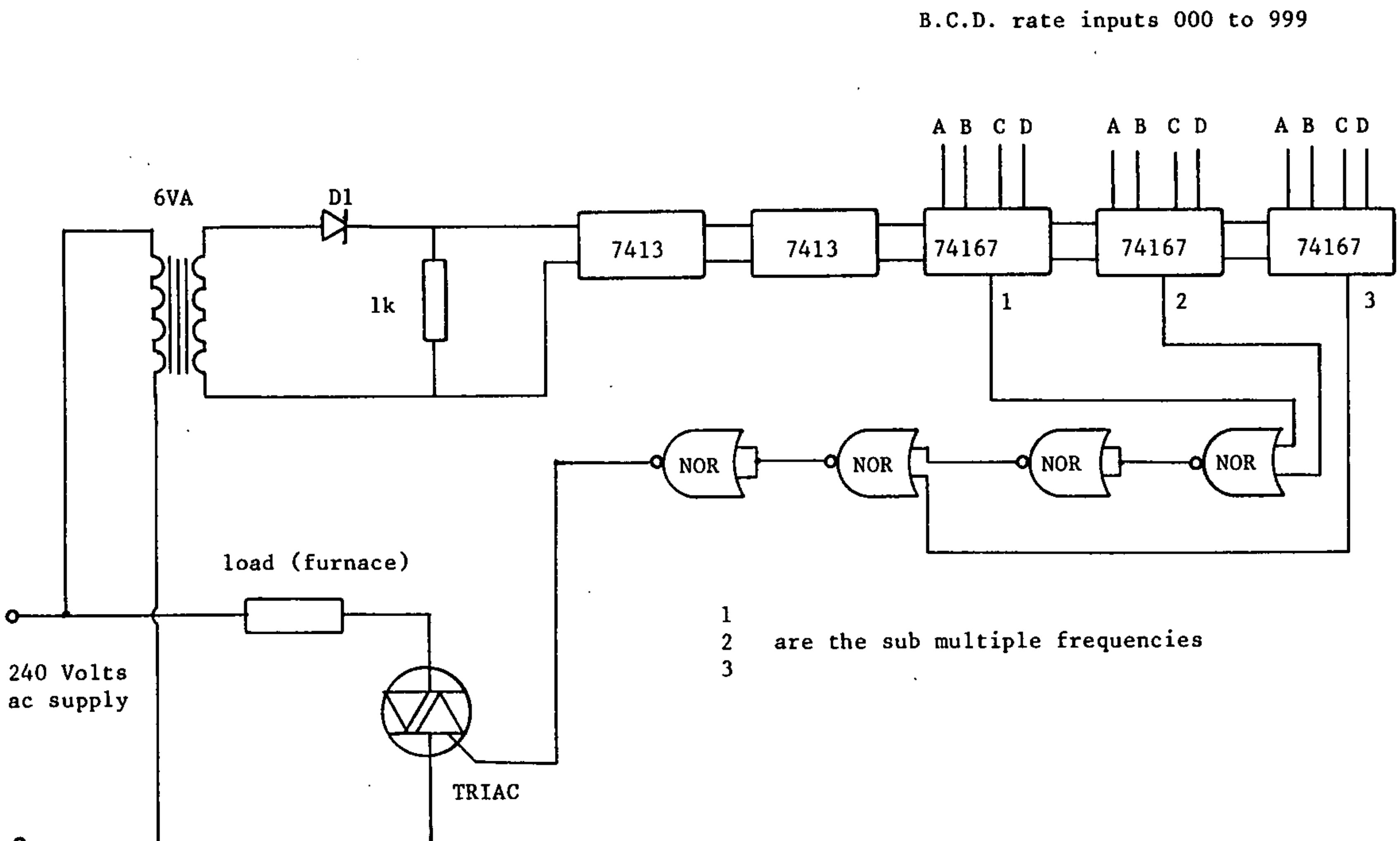



Fig. 4.13 Circuit for the pulse control of a TRIAC using B.C.D. rate inputs.

The next problem after the parameter adjustment scheme has been found to function in a satisfactory manner is to close the adaptive loop by using a suitable method of controlling the furnace current.

### 4.4.6 Furnace Current Control

The method of controlling the furnace current is by means of a TRIAC. The TRIAC used in this particular application is rated at 15A maximum. Two methods of triggering the TRIAC, and hence controlling the furnace current, were considered. These methods were (i) phase control and (ii) pulse control.

It was considered that of the two methods available, the method of pulse control offered the greatest range and precision in control of the current flowing through it, and could also be made compatible with the output interface requirements associated with the process computer. The method of phase control on the other hand did not offer as good a range of current control, and because it was basically an analogue system it could not be easily interfaced with the process computer.

Initially two methods of controlling the furnace current, using the pulse control method of triggering the TRIAC, were developed. The first method, shown in Fig. 4.13, was based on the control parameter being a three digit binary coded decimal number. By changing this control parameter, the rate at which pulses arrived at the gate of the TRIAC could be varied from zero out of 1000 pulses to 999 out of a 1000 pulses. The pulses used are synchronised to the a.c. mains waveform, which is derived from a small 6VA, 240V a.c. transformer.

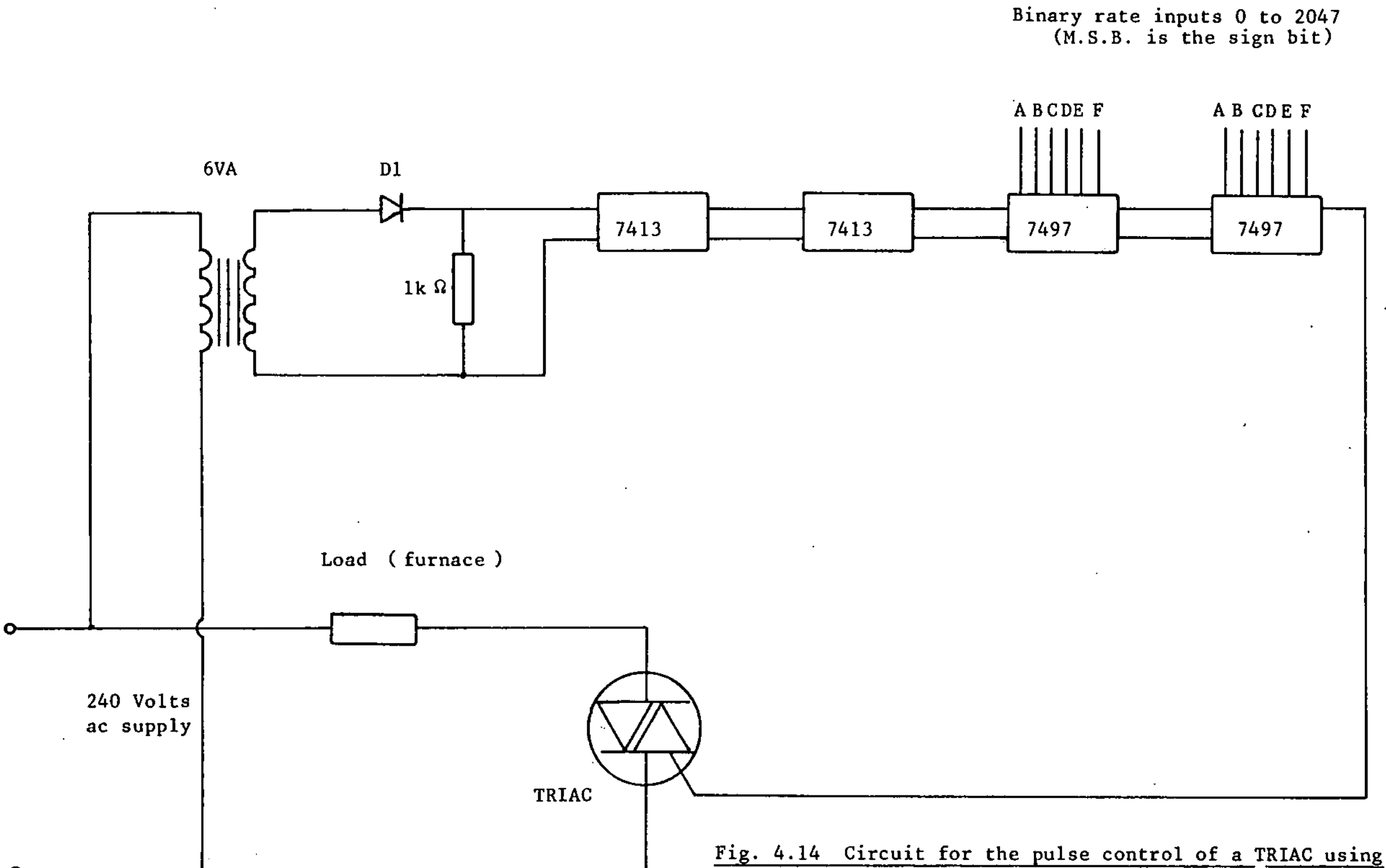



Fig. 4.14 Circuit for the pulse control of a TRIAC using Binary rate inputs.

The output (low voltage) of the transformer is then rectified and passed through a Schmitt triggered NAND gate in order to produce a square wave output with a frequency of 50 Hz. The second Schmitt triggered NAND is used (on the same chip) to produce a square wave in the required phase relationship, with the 240V a.c. supply, in order that the TRIAC may be triggered correctly.

The 50Hz square wave is then passed to the rate multipliers (three connected in cascade), and conditions established at the three submultiple frequency output terminals 1, 2, and 3 in figure 4.13, are determined by the conditions at the input terminals ABCD. These inputs can vary from binary 0000 to binary 1001 which corresponds to denary 0 to 9. The submultiple frequency outputs are then combined by the four NOR gates in order to produce correctly synchronised pulses which trigger the TRIAC.

An alternative arrangement for this type of circuit is shown in Figure 4.14. In this arrangement the binary coded decimal inputs are replaced by a 11-bit binary rate input. This circuit functions in a similar manner to the circuit shown in Figure 4.13, but the rate input can be varied from denary 0 up to denary 2047, there being no submultiple frequency outputs and hence there is no requirement for additional gate circuits. Both the circuits described above may be made compatible with the process computer interface output, the binary rate multiplier giving a greater precision control of the TRIAC current.

The circuits of Figures 4.13 and 4.14 produce a variation in the furnace (TRIAC) load current, but they do suffer from a serious drawback, that is, because the trigger pulses are passed to the gate

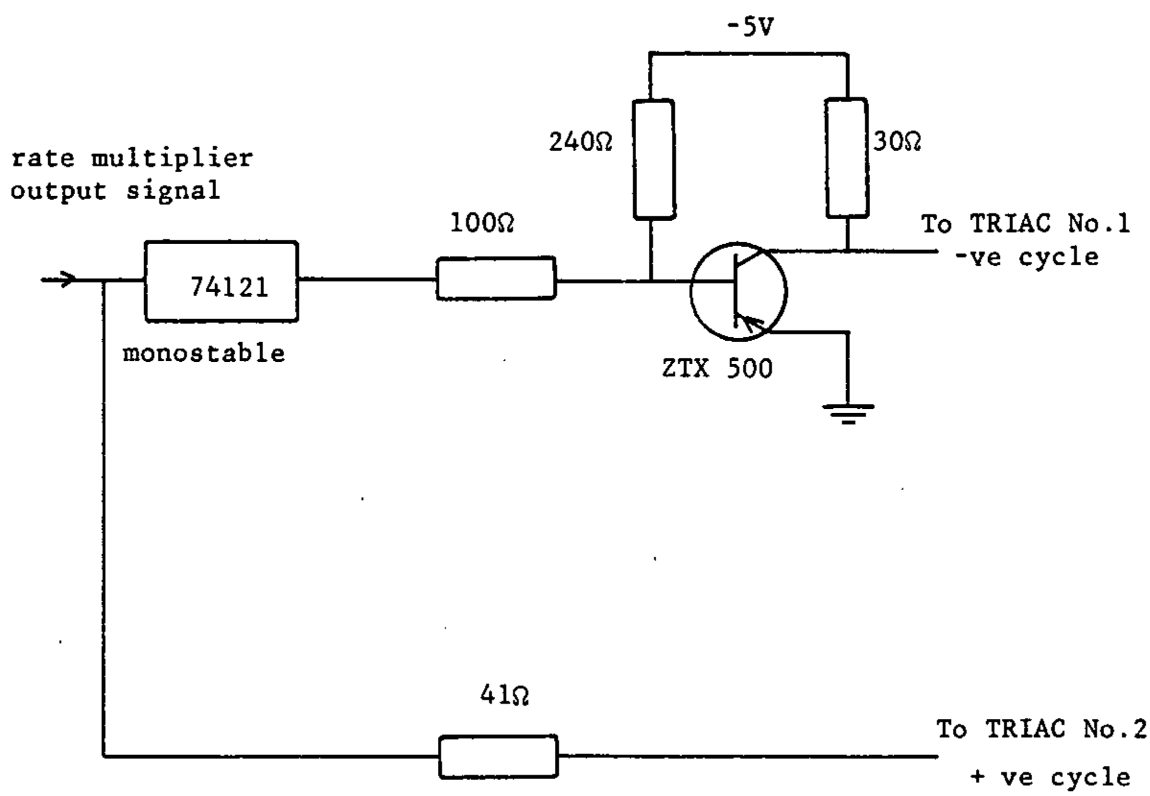


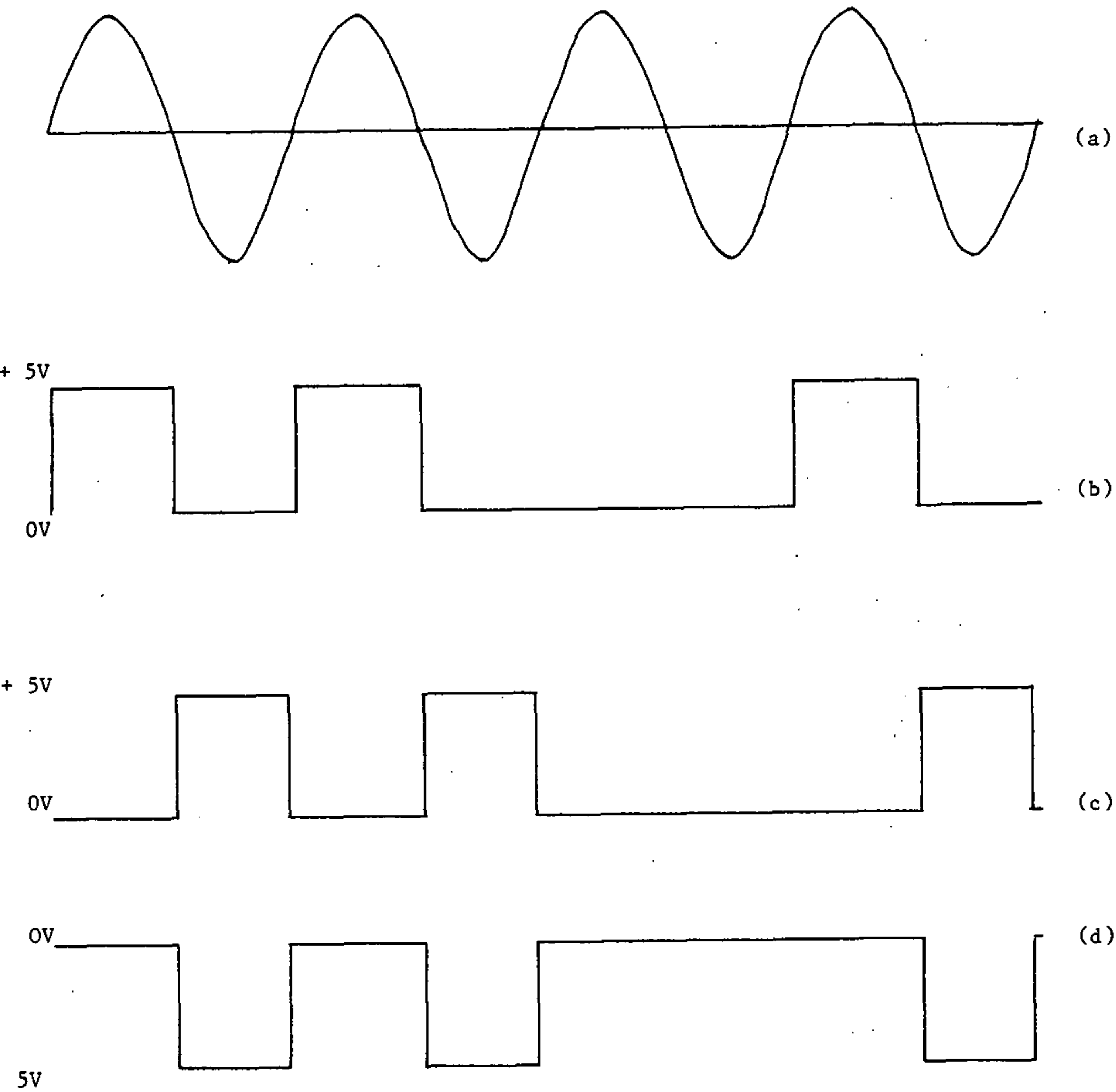

Fig. 4.15 Trigger circuits for both TRIACS

of the TRIAC at the start of a positive going a.c. input waveform, they can only control the furnace (TRIAC) current over this positive half cycle. Current can never flow through the load over the negative half cycle.

A refinement to the circuits of Figures 4.13 and 4.14 is required to enable negative going pulses to be applied to the gate of the TRIAC at the start of the negative going half-cycle, and hence give control over the furnace (TRIAC) current during the negative half-cycle, as well as during the positive half-cycle. The circuit arrangement to enable control to be also obtained over the negative half-cycle is shown in Figure 4.15.

The monostable, which must reset in less than half of one time period of the 50 Hz square wave, is used to enable the output from the rate multiplier to be phase shifted by $90^{\circ}$. This phase shifted waveform is then used to trigger a switching transistor. The output from the switching transistor gives a negative going square wave which can be used to trigger the TRIAC on its negative going cycle, and the transistor circuit will produce a sufficiently high gate current to trigger the TRIAC.

The main problem with this type of arrangement is that the output pulses to the gate of the TRIAC from the positive and negative triggering circuits are mutually incompatible, and hence two TRIAC's are needed if the full furnace current is to be controlled, i.e. one TRIAC to carry the positive half cycle load current, and the other to carry the negative half cycle of the load current. The phase relationships of the




(a) - 50Hz mains supply.

(b) - Output of the rate multiplier.

(c) - Output of monostable.

(d) - Output of ZTX 500 Transistor.

Fig. 4.16 Various control circuit waveforms.

various waveforms are shown in Figure 4.16.

The theoretical analysis of the current flow through the furnace load and hence through the TRIAC's is given below:

The setting on the rate multipliers input terminals will determine the exact number of pulses reaching the TRIAC gate and hence the average current flowing through the furnace will be controllable to a high degree of precision. The average current flowing will be given by the following expression.

$$I_{ave} = \left[ I \right]_{\substack{\text{max scale}\\\text{reading}}} \frac{n}{n + m} \qquad .....4.36$$

where $n$ = number of cycles/1000 or 2048 'on'

and $m$ = number of cycles/1000 or 2048 'off'

The reading on moving coil instrument of $I_{ave}$ was unsatisfactory for low values of n, and hence is not considered further.

For the furnace current it is the r.m.s. value of current which will be of interest, and hence the following analysis needs to be considered.

let $$i = I_m \sin \omega t \qquad .....4.37$$

and $$i^2 = I_m^{\,2} \sin^2 \omega t \qquad .....4.37a$$

The period for n 'on-cycles' is $2\pi n$

and the period for m 'off-cycles' is $2\pi m$

$$\bar{i}^2 = \frac{1}{(m+n)\pi 2} \left[ \int_0^{2\pi n} I_m^{\,2} \sin^2 \omega t \, d\omega t + \int_{2\pi n}^{2\pi m} I_m^{\,2} \sin^2 \omega t \, d\omega t \right] \qquad .....4.38$$

The second term is obviously zero, therefore

$$\bar{i}^2 = \frac{I_m^{\,2}}{(m+n)2\pi} \int_0^{2\pi n} \tfrac{1}{2} (1 - \cos 2 \omega t) \, d\omega t$$

Table of B.C.D. rate multiplier setting and r.m.s. current.

| rate output pulses/1000 | rms current (Amps) |
|---|---|
| 0 | 0 |
| 50 | 3.2 |
| 100 | 4.5 |
| 150 | 5.5 |
| 200 | 6.4 |
| 250 | 7.1 |
| 300 | 7.8 |
| 350 | 8.4 |
| 400 | 8.9 |
| 450 | 9.5 |
| 500 | 9.9 |
| 550 | 10.5 |
| 600 | 10.9 |
| 650 | 11.4 |
| 700 | 11.8 |
| 750 | 12.2 |
| 800 | 12.6 |
| 850 | 13.0 |
| 900 | 13.4 |
| 950 | 13.7 |
| 999 | 14.1 |

Table 4.1

Table of binary rate multiplier setting and r.m.s. current.

| rate output pulses/2048 | rms current (Amps) |
|---|---|
| 0 | 0 |
| 32 | 1.75 |
| 64 | 2.50 |
| 128 | 3.6 |
| 192 | 4.4 |
| 256 | 5.1 |
| 320 | 5.5 |
| 512 | 7.1 |
| 704 | 8.3 |
| 960 | 9.8 |
| 1024 | 10.0 |
| 1088 | 10.3 |
| 1152 | 10.5 |
| 1280 | 11.2 |
| 1536 | 12.2 |
| 1792 | 13.2 |
| 1920 | 13.6 |
| 1984 | 13.8 |
| 2047 | 14.1 |

Table 4.2

$$\bar{i}^2 = \frac{I_m^2 n}{2(m+n)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}\sqrt{\frac{n}{(m+n)}} \qquad .....4.39$$

Equation 4.39 gives the r.m.s. value of the current flowing through the furnace load for an input n to the rate multiplier. This number n can be in the range 0-999 or 0 - 2047 depending upon the type of control used. Readings taken during tests on both types of circuit are given in tables 4.1 and 4.2. The instrument used for the tests was an a.c. ammeter of the moving iron type.

It should be noted that the readings listed in tables 4.1 and 4.2 respectively are the r.m.s. values of the furnace load current, and do not produce a linear relationship between the current and pulse rate. If a moving coil instrument was used, the average current against pulse rate could be plotted, but as the readings taken on this type of instrument proved unreliable they are not reproduced here.

Trial readings of the furnace current were also made using a variety of different types of instruments, for example a true r.m.s. voltmeter and a thermal ammeter. These instruments proved most unsatisfactory at all low and intermediate values of rate inputs because the response is relatively fast for these types of instrument. That is, the indication on the instrument changes very rapidly at first, attempting to reach the maximum instantaneous indication and then returning to zero as soon as the pulse or pulses have passed. An instrument which was heavily damped may have helped, but since the readings taken

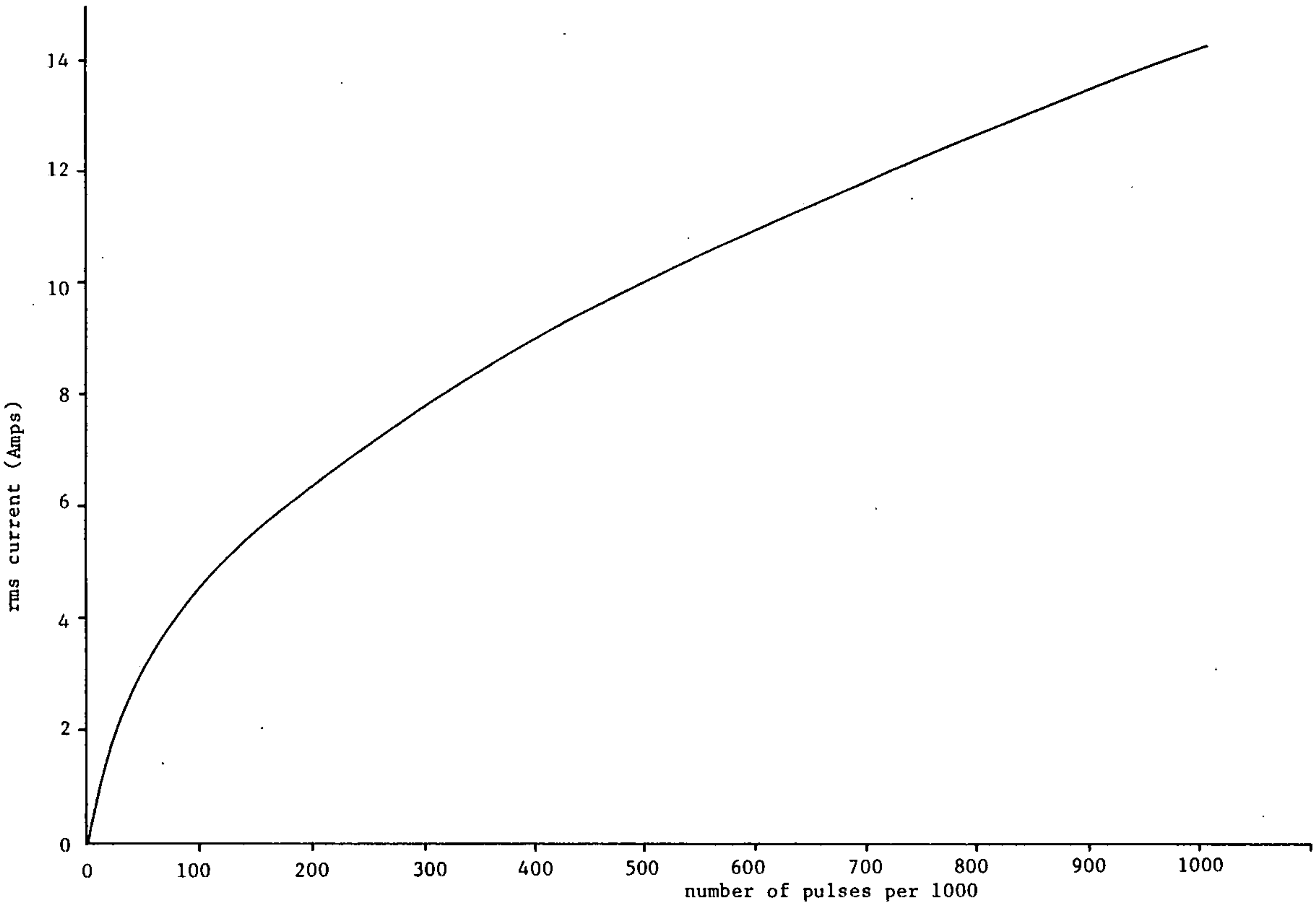



Fig. 4.17  Graph of rms current against the number of pulses per 1000.

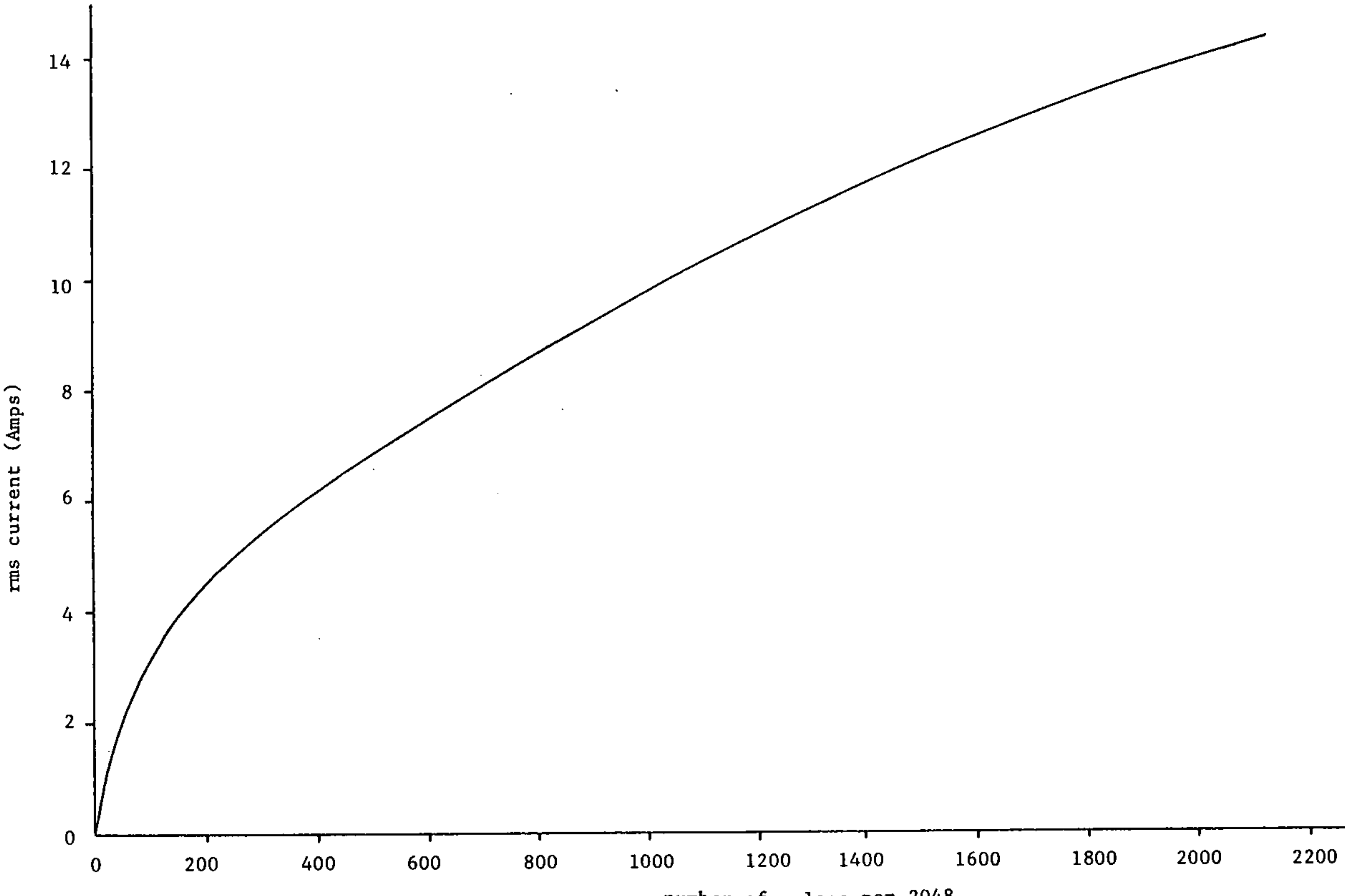



Fig. 4.18 Graph of rms current against the number of pulses per 2048.

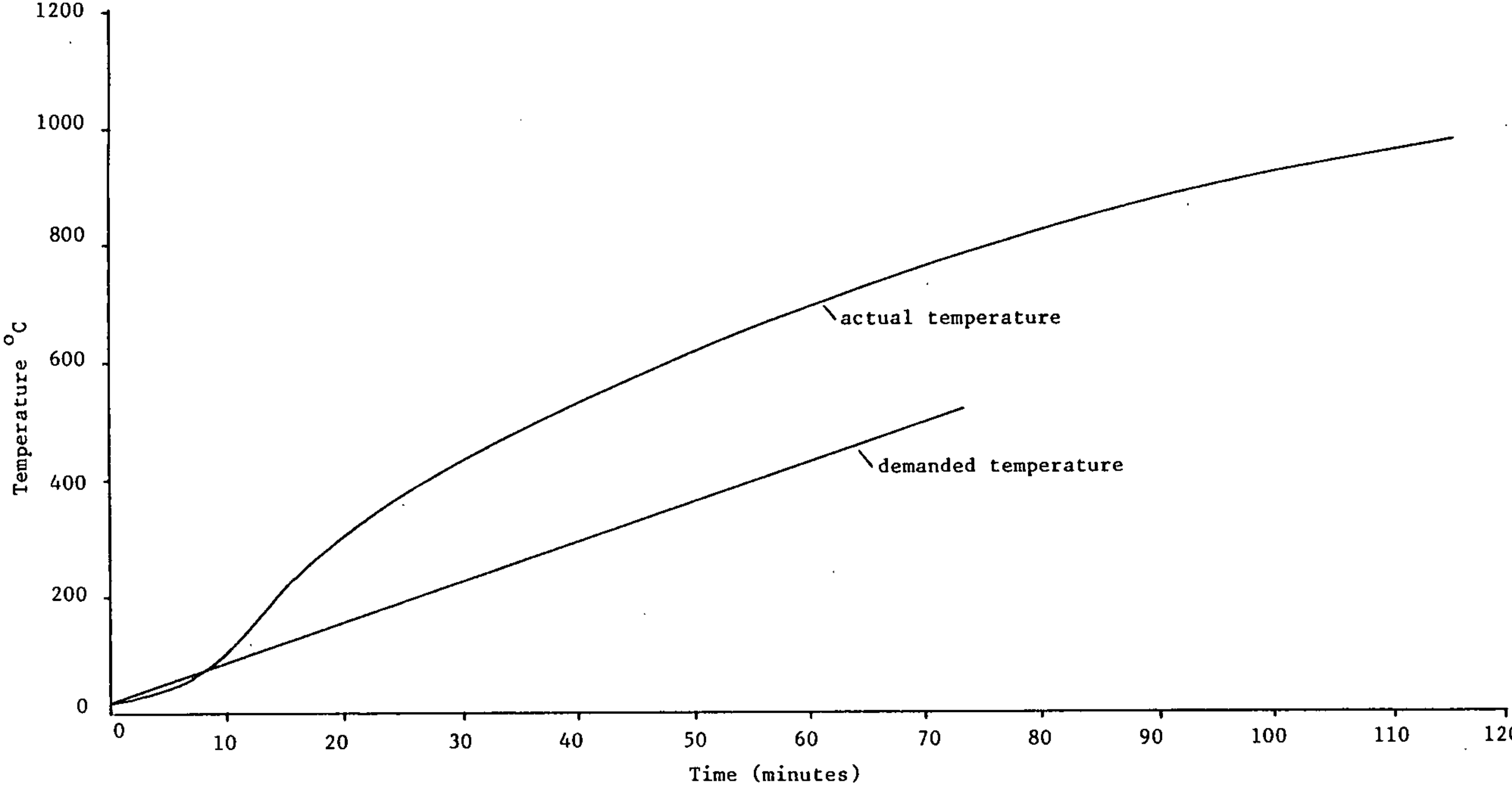



Fig. 4.19  Step response of the small furnace with no load.

from the moving iron type of instrument proved more reliable than any of the others obtained, it was considered that the readings on the moving iron instrument would be of the most value. Figs. 4.17 and 4.18 respectively show the results obtained by varying the rate inputs.

### 4.4.7 Adaptive Control Computer Programme

The temperature against time no-load profile for the small (3.6 kW) heat treatment furnace, which was used to investigate the adaptive control strategy discussed in section 4.3, is shown in Figure 4.19. The demanded temperature curve is also shown in Figure 4.19, and it will be noted that this demanded temperature function is of the form of a simple ramp, having for convenience a maximum demanded temperature of 520$^{\circ}$C. This demanded temperature curve can, of course, be changed or modified as necessary to suit the individual requirements of the furnace load.

One of the problems encountered when controlling the temperature of a furnace of this type, is that an inevitable time delay occurs in respect of the furnace response to any changes in the load current. This research project attempts to minimise this problem in two ways:

(i) by applying the basic properties of the model reference adaptive control strategy to the control of the furnace and using the furnace mathematical model as the reference model, and

(ii) by careful selection of the performance index for the furnace in the adaptive control programme.

The basis of the choice of the index of performance is to use one of the furnace mathematical model parameters, which will give

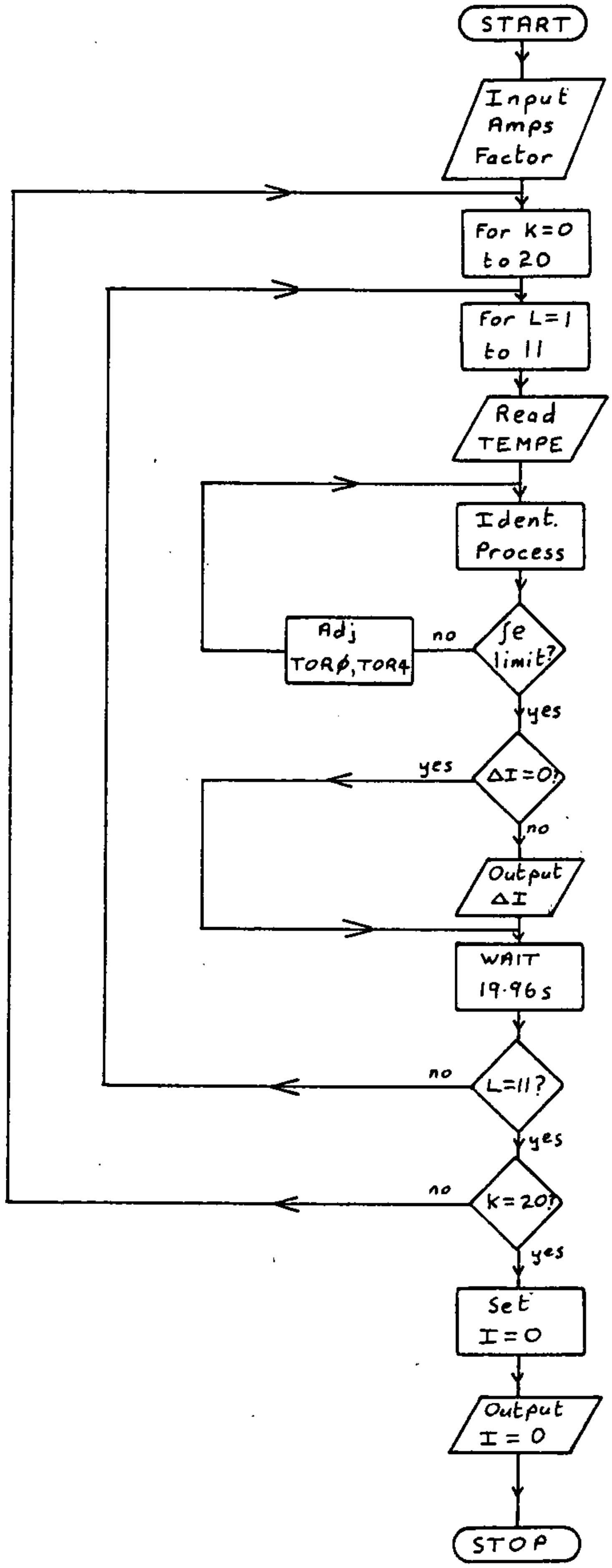



Fig. 4.20 Flow diagram for complete furnace control programme

a measure of prediction of the ultimate temperature which the furnace will reach, and thus enable the control strategy to be based on a parameter which will give the controller an indication of the value of the furnace temperature at some time in the future.

The mathematical model parameter which it was thought most appropriate to the above criterion, is the main, but variable, furnace time constant. This time constant will enable the process controller to have some knowledge of the temperature which the furnace will achieve with a particular set of conditions. Hence the controller will be able to adjust the load current to an extent which will take present factors into account.

The FORTRAN programme which has been developed to achieve the adaptive control strategy described above is listed in Appendix 5, and the associated flow diagram is shown in Figure 4.20.

The control programme functions as described below.

The furnace temperature, measured using a thermocouple with an output of the order of several milli-volts, is first amplified using a 741 operational amplifier, and then this output is fed into the PDP8 computer via an analogue-to-digital conversion channel. The sampled input is then compared with the temperature value produced by the mathematical model of the furnace, the initial parameters having been deposited in the computer at the start of the programme, see Appendix 5.

If an error exists between the input temperature and the temperature value produced by the mathematical model, the programme, being self-adjusting, will correct the model parameters until the resultant error is reduced to an acceptable small value see section 4.4.5. The furnace mathematical model parameter (main time constant) so determined, is then compared with a pre-programmed parameter for this particular demanded temperature. This difference of parameter values then produces an error quantity which is then used to determine any change in the furnace load current which is necessary.

The output from the PDP8 computer is in the form of a 12-bit binary word, which is connected to the rate-multiplier inputs, via a buffered digital input/output, which is compatible with both the TTL logic of the binary rate multipliers and the PDP8 computer output.

Once the model adjustment identification procedure, and the change of furnace current, has been completed, the programme then encounters a timing function: WAIT (see Appendix 5). This WAIT function is determined by the limitation on this timing function set by the PDP8 computer, and is completed after 19.96 sec. The maximum time allowable in this type of FORTRAN programme is 20 seconds. It is, however, required to operate the programme over a period (T) of 3.66 minutes during which time $k$ will be zero (see equation 4.33). This time period of 3.66 minutes is obtained by introducing a programming loop, and allowing the timing function, SET CLOCK and WAIT, to be set to 19.96 seconds and then repeating the timing loop 11 times (see Appendix 5). Hence the programme will

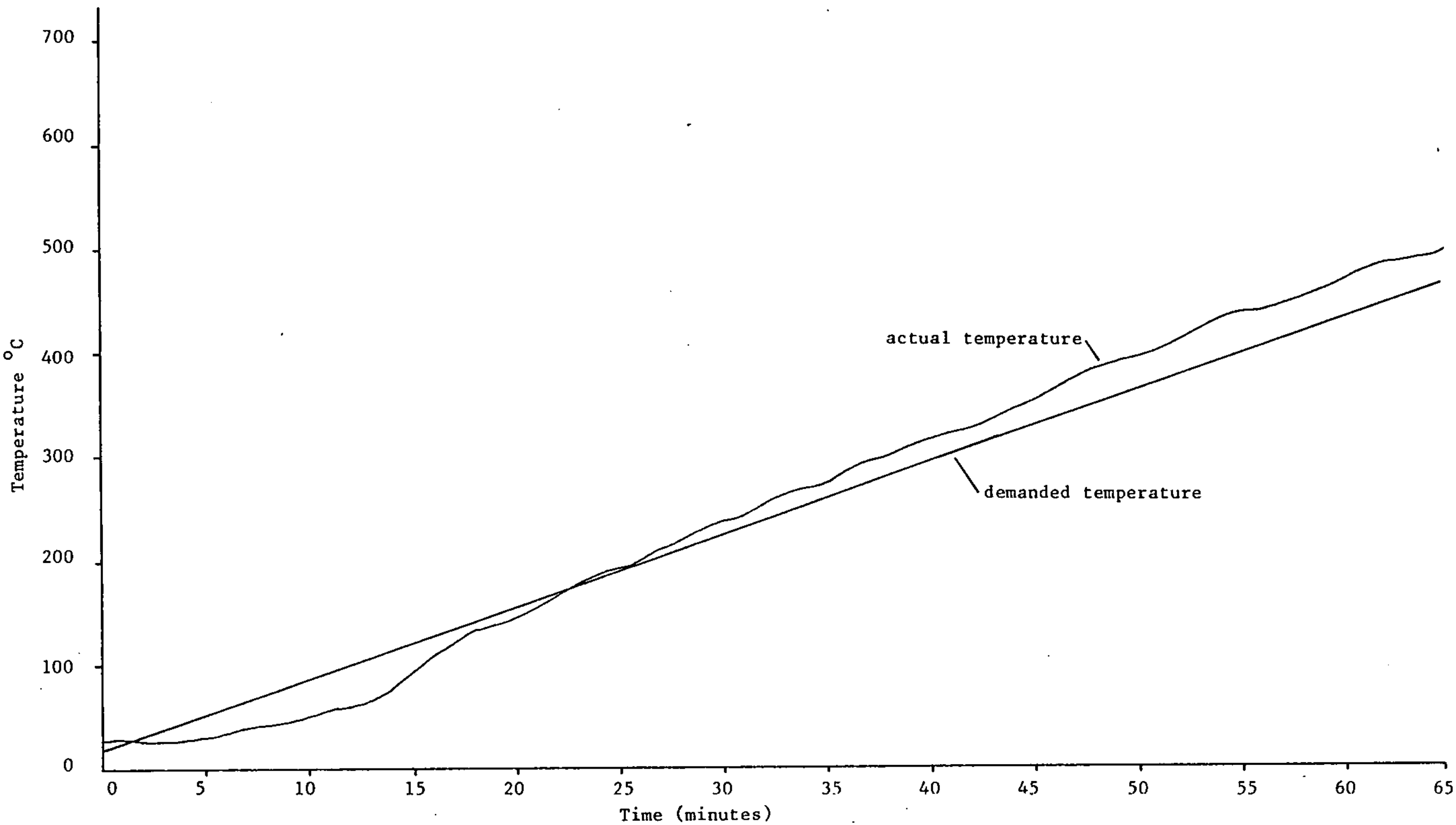



Fig. 4.21 Graph temperature against time for furnace when being adaptively controlled.

take 19.96 times 11 seconds for each discrete identification phase, during which time the current may be adjusted up to eleven times. The full programme is completed after 21 discrete identification steps, after which the total elapsed time will be 76.8 minutes.

### 4.4.8 On-Line Test Results

In order to test the operation and performance of the adaptive control programme listed in Appendix 5, it is necessary to select a suitable number of desired discrete temperature values, which the furnace should reach at particular times in the control cycle. These various temperature/timing points are pre-programmed into the adaptive control programme, and this information is required to be in terms of the time constant values required for the furnace at these temperatures. For the test results detailed in this section, the demanded temperature curve is that of a ramp function (see Figure 4.21). The various time constants demanded by this type of function have been determined by the programme listed in Appendix 4, and may be altered as required by the demands of the pre-programmed temperature profile.

The results of the operation of this basic furnace control system is shown in Fig. 4.21. It should, however, be noted that the adaptive control system described herein, does not automatically select the initial current, and this should thus be selected at the commencement of the programme. The initial current used was selected to be much less than the maximum furnace load current, thereby preventing a large increase in the furnace temperature while the adaptive programme identifies the furnace mathematical model over the first two identification phases. It can be seen from the results presented in Figure 4.21 that the adaptive control system does produce an approximately linear temperature/time profile, but the deviation from the demanded temperature curve is significant for temperatures below 150°C and above 250°C. It is evident from these results that a modification of the control strategy will be required in order to achieve better and more precise control of the furnace temperature.

It was thought that better control of the furnace temperature could be achieved if two changes are introduced into the initial control programme. That is, it was thought that the error in the control programme strategy consists of two distinct parts:

(i) The initial current is set too low, and hence produces unacceptably low initial temperatures. This problem may be solved be increasing the initial current.

(ii) The adaptive controller is over compensating for the low temperatures at the start of the programme and this may result in temperatures which are too large in the later stages of the control programme. This problem may be minimised by reducing the temperatures towards which the controller is aiming. In practice this will mean that the target time constant and hence the target temperature in each part of the control programme will be reduced or time shifted. The effect of this will be the same as introducing a time delay into the identification procedure (see section 2.4.2).

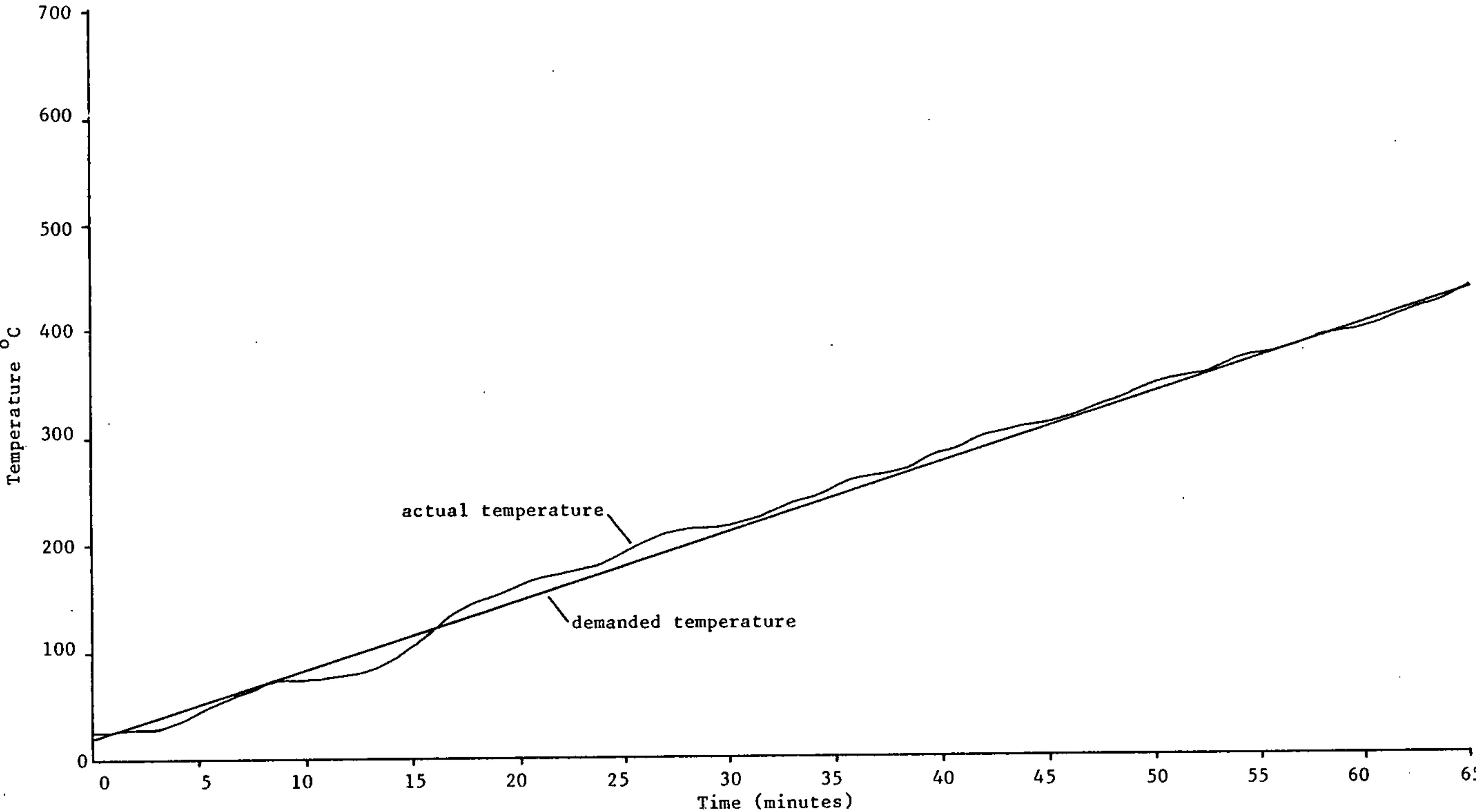



Fig. 4.22 Graph of temperature against time for furnace when being adaptively controlled.

The results shown in Fig. 4.22 are the outcome of shifting the time constants (or reducing the temperature) of the pre-programmed temperatures by up to two factors of 3.66 minutes, and also increasing the initial current by a factor of two, when compared with Fig. 4.21. It can be seen from Fig. 4.22 that an acceptable control of temperature was obtained with the exception of the first few minutes of operation. It is envisaged that this initial error may be minimised by further increasing the initial current by a suitable factor.

Once a satisfactory degree of temperature control has been achieved for normal operation, i.e. with 'noise' in the form of environmental temperature changes, it still remains to investigate the reaction of the furnace adaptive control systems when an extreme (high level disturbance), which is also random in its nature, is introduced into the system. For the furnace system, the factor which is potentially the most disruptive, is that of an operator opening the furnace door for an unspecified period. It is thought that in the extreme, the furnace door would not remain open for more than 1 minute, but probably a more realistic time period is 15 seconds or less.

As a consequence of the above arguments, tests were carried out in order to determine the effects on the furnace control of;

(i) first opening the furnace door for 1 minute (see Figure 4.24 and

(ii) for 15 secs (see Figure 4.23).

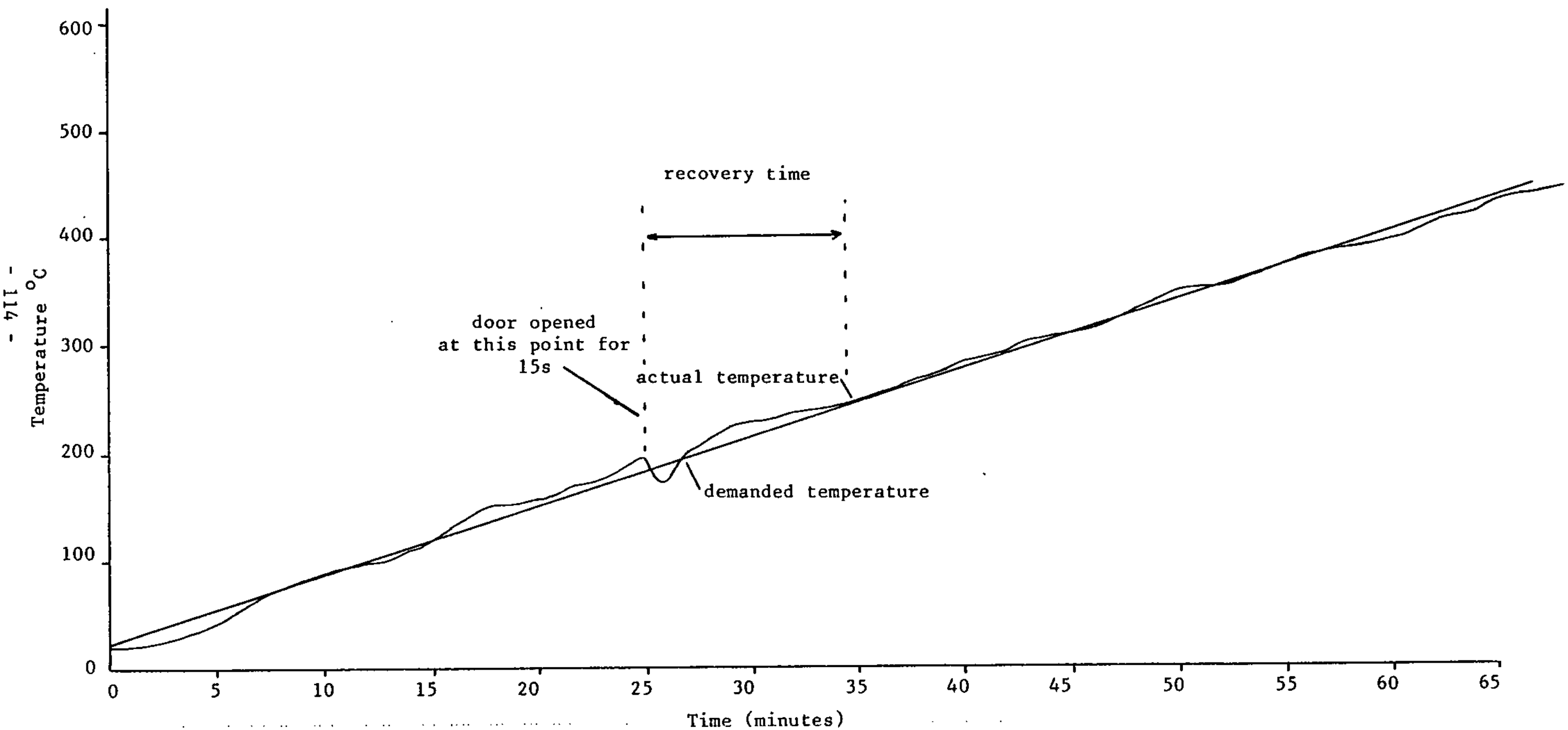



Fig. 4.23 Graph of temperature against time for furnace when being adaptively controlled and subject to a high level disturbance.

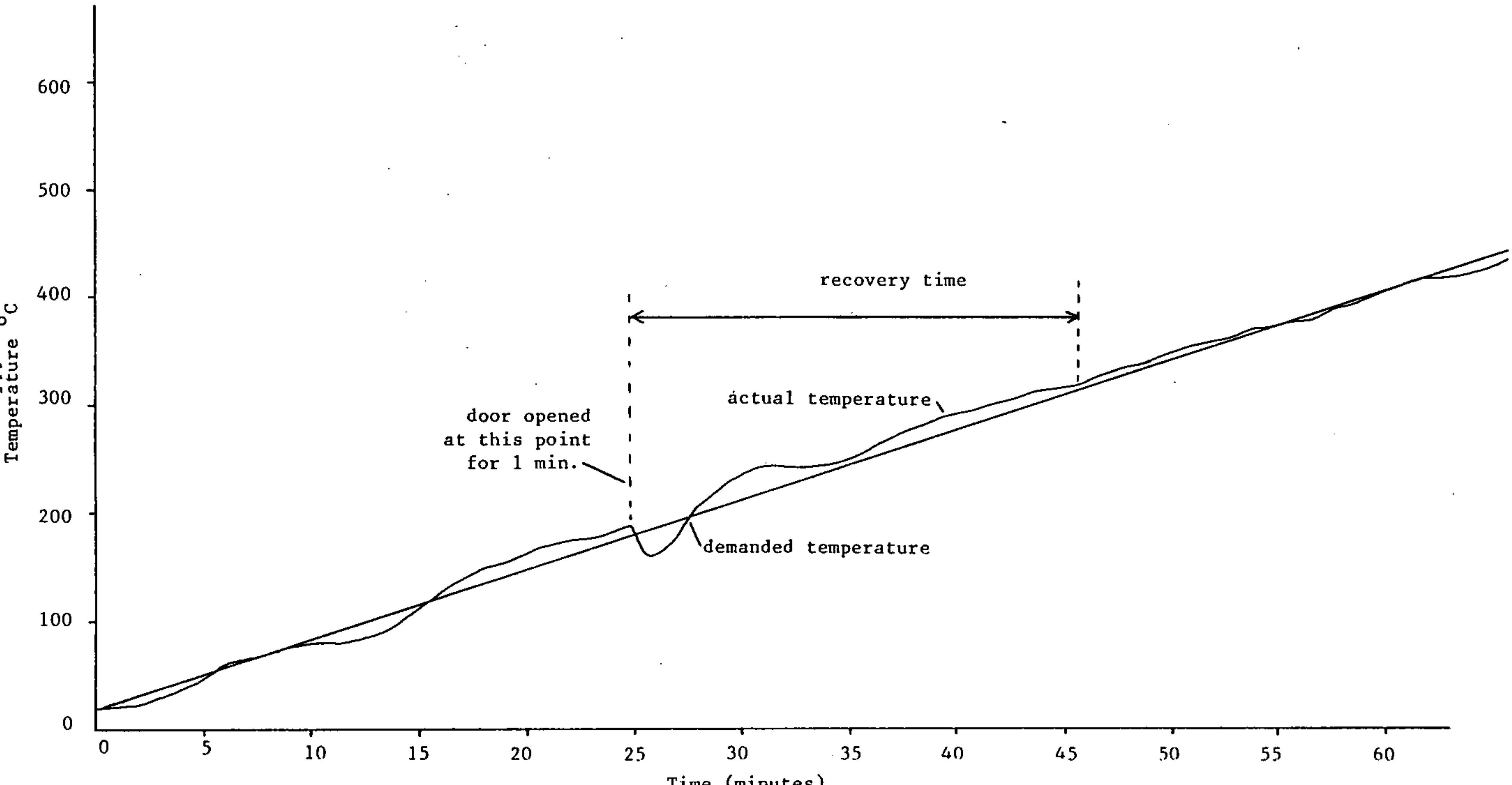



Fig. 4.24 Graph of temperature against time for furnace when being adaptively controlled and subject to a high level disturbance.

Predictably the recovery time for the furnace temperature in the case when the furnace door was open for 1 minute is longer than when the door is only left open for 15 seconds. In both cases, however, recovery of the furnace temperature to its pre-programmed level, takes no more than 20 minutes in that of the worst possible case.

# CHAPTER 5

## CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

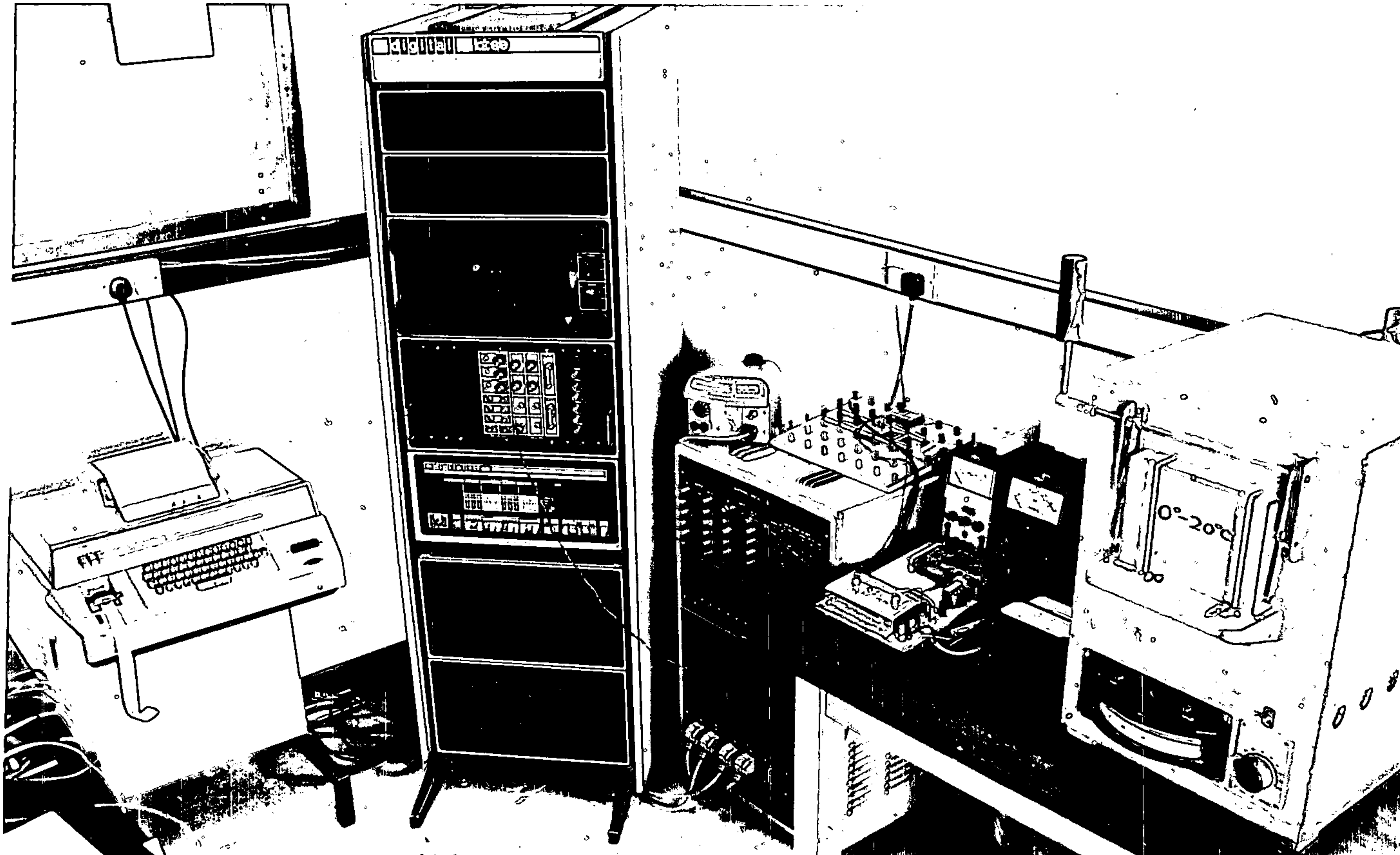
### 5.1 Conclusions

The performance of the small heat treatment furnace after closing the adaptive loop was found to be close to that demanded by the control programme. It has also been demonstrated that when an extreme disturbance is introduced to the system, (i.e. that of opening the furnace door), the adaptive nature of the control system ensures that normal operation of the furnace is restored relatively quickly. Hence it is concluded that the adaptive control system investigated in this research project gives good temperature control for low-level disturbance inputs (changes in environmental temperature) when used in conjunction with a small heat treatment furnace.

### 5.2 Suggestions for further work

It has been demonstrated in Chapter 4 that the control of a small heat treatment furnace using a model reference adaptive control strategy, is both viable and produces results which are acceptable from a heat treatment point of view. It is possible that the adaptive control system described herein may be applied to a variety of industrial furnaces, however, it is not assumed that the techniques presented in this thesis could be extended to all types of heat treatment furnaces operating over a wide range of temperatures.

In order to extend the techniques to a wide variety of furnaces and a wide variety of temperature ranges, a more general


digital
0°-20°C

control algorithm would have to be developed, which in itself would have to be adaptable to accommodate different types of furnaces and different temperature ranges. The type of control algorithm envisaged would probably be developed and implemented using a state-space approach, which has been specifically excluded from this study in favour of a classical approach.

The classical approach, used in this investigation of an adaptive system, was adopted to enable the techniques presented in this thesis to be readily extendable to low cost digital controllers once the initial work has been concluded, in contrast to the bulky mini-computer adaptive control system shown in Fig. 5.1.

Low cost micro processors (microcomputers) are now available, and the control algorithm developed in this thesis may be readily implemented. The major expense in this type of project is the software development costs, and it would now be a relatively simple operation to convert the programme in Appendix 5 to an equivalent machine code listing for a microprocessor controller. For example, the M6800 microprocessor, two M6820 peripheral interface adaptors (PIA's), an A/D converter, RAM and ROM memories and an arithmetic processor (AM 9511) can be programmed to perform the adaptive control operation. The cost of the system hardware is currently about £200, and bearing in mind that the cost of hardware is likely to continue being reduced and that the chip count is likely to be reduced, then quite clearly the digital adaptive controller described in this thesis is worth developing further for subsequent implementation using a microcomputer system.

Recent work has suggested that providing that the time constant of the furnace is relatively short, other methods of identification, of the furnace mathematical model may be used in order to control the furnace temperature[92]. It is also evident from the work discussed in Chapter 4 that further work may be undertaken in the various methods of closing the furnace adaptive loop. It is the opinion of the author that several of the other methods discussed in Chapter 4 would also be worth further investigation, and may thereby give adequate control of the furnace temperature.

It is clear that the work on the adaptive control system described in this thesis may be extended as suggested above. Surely any resulting improvement in the control of furnace temperature will be worthwhile, and to this end the work described in this thesis is dedicated.

APPENDIX 1

| Symbol | Definition | Units |
|---|---|---|
| Q | specific heat of load | Btu/lb$^{o}$F(kJ/kg$^{o}$C) |
| U | specific heat of hearth | Btu/lb$^{o}$F(kJ/kg$^{o}$C) |
| m | mass of load | lbs (kg) |
| n | mass of hearth | lbs (kg) |
| $\theta a$ | ambient temperature | $^{o}$F ($^{o}$C) |
| $\theta f$ | furnace temperature | $^{o}$F ($^{o}$C) |
| $\overset{o}{\theta}$ | rate of change of temperature | $^{o}$F/hr ($^{o}$C/hr) |
| $A_c$ | total area of 'top hat' | ft$^2$ (m$^2$) |
| $A_h$ | total area of hearth | ft$^2$ (m$^2$) |
| $h_c$ | heat loss from 'top hat' | Btu/hr (kW) |
| $h_h$ | heat loss from hearth | Btu/hr (kW) |
| $h_s$ | heat loss due to sand seal | Btu/hr (kW) |
| $U_c$ | heat loss coefficient of top hat | Btu/ft$^2$ $^{o}$F hr (kW/m$^2$ $^{o}$C) |
| $U_h$ | heat loss coefficient of hearth | Btu/ft$^2$ $^{o}$F hr (kW/m$^2$ $^{o}$C) |

APPENDIX 2

Useful data for electric furnaces courtesy of Cooperheat Ltd.

Heat losses from walls and hearths:

| | Casing temperature | Heat loss/hour |
|---|---|---|
| at 600/650$^{\circ}$C (1112/1202$^{\circ}$F) | 68$^{\circ}$C (155$^{\circ}$F) | 0.59 kW/m$^2$ (190 Btu/ft$^2$) |
| at 1000/1050$^{\circ}$C (1830/1922$^{\circ}$F) | 82$^{\circ}$C (180$^{\circ}$F) | 0.81 kW/m$^2$ (260 Btu/ft$^2$) |

The above figures should always be multiplied by the number of hours to temperature. These figures are assuming the furnace temperature has reached its steady state.

---

Standby losses with casing temperatures as above

| | |
|---|---|
| at 600/650$^{\circ}$C | 0.86 kW per sq metre per hour |
| or (1112/1202$^{\circ}$F) | 275 Btu per sq foot per hour |
| at 1000/1050$^{\circ}$C | 1.18 kW per sq metre per hour |
| or (1830/1922$^{\circ}$F) | 370 Btu per sq foot per hour |

Add 10% to the above for sand seal losses.

APPENDIX 3

Heat Storage:

Depending on heating time: Hearth

units are kW per sq metre

| Temperature | 4hr | 6hr | 8hr | 10hr | 12hr | 16hr |
|---|---|---|---|---|---|---|
| 600°C | 15 | 18 | 21 | 23 | 24 | 25 |
| 900°C | 23 | 28 | 32 | 34 | 36 | 39 |
| 1000°C | 29 | 34 | 40 | 43 | 45 | 48 |

Walls

For heat storage in the walls take 10% of heat stored in the hearth.

Heat storage in the load.

at 600°C (1112°F) 0.11 kW per kg

at 900°C (1652°F) 0.18 kW per kg

at 1050°C (1922°F) 0.21 kW per kg

APPENDIX 4

BASIC Computer Programme.

```
LIST
10 REM MATHEMATICAL MODEL OF A COOPERHEAT FURNACE USING
20 REM Z TRANSFORMS WITH SCALING AND MODEL ADJUSTMENT
30 REM OF THE TIME CONSTANT
35 REM THE TEMP INPUTS AND OUTPUTS ARE IN DEG C BUT THE PROG.
36 REM MODEL USES DEG F FOR COMPUTATION PURPOSES
40 PRINT "THE VALUES OF THE TIME CONSTANT TIME INCREMENT"
50 PRINT "HEAT INPUT AND THE FURNACE CONSTANT USED"
60 PRINT "INITIALLY IN THIS PROGRAMME ARE"
70 PRINT "0.8,0.88355,71500,65"
80 PRINT "ALSO THE VALUE OF THETA A T(2) IS 20 DEG F"
90 DIM T(4),I(4)
100 DIM O(20),G(20)
110 DIM W(20),E(20)
120 DIM A(20),C(20)
130 DIM D(20),B(20)
140 LET G(0)=0
150 LET C(0)=0
160 LET D(0)=0
170 LET O(0)=0
175 LET W(0)=0
180 READ T(0),T(4),T(1)
190 READ H,C
200 READ N
205 LET T(2)=N*9/5
207 READ X,Y
210 LET R1=1
220 LET R2=.6325
230 FOR K=1 TO 20 STEP 1
240 READ L
250 LET G=T(0)/(1+1.1*C)
260 LET P=H*G
270 LET Q=EXP(-T(1)/T(0))
280 LET O(K)=(P+O(K-1)*Q)*R1
285 LET G(0)=0
290 LET G(K)=(T(2)/T(4)+G(K-1)*EXP(-T(1)/T(4)))+R2
300 LET M=G(K)+O(K)
305 LET W(K)=M*5/9+20
310 LET E(K)=L-W(K)
320 LET A(K)=E(K)*O(K)
330 LET B(K)=E(K)*G(K)
340 LET C(K)=A(K)+C(K-1)
350 LET D(K)=B(K)+D(K-1)
360 LET A=INT(W(K)+.5)
370 LET B=INT(E(K)+.5)
380 LET I(1)=INT(C(K)+.5)
390 LET I(2)=INT(D(K)+.5)
400 IF I(I)>650GO TO 550
410 IF I(I)<-650GO TO 550
415 LET S=INT(T(4)*100+.5)/100
```

```
416 LET R=INT(T(Ø)*1ØØ+.5)/1ØØ
42Ø PRINT K,A,B,I(I),I(2)T(Ø)T(4)
43Ø NEXT K
44Ø DATA.8,.8
45Ø DATA.88355
46Ø DATA 715ØØ
47Ø DATA 38
48Ø DATA 11.1
482 REM THE FOLLOWING DATA IS FOR THE MODEL ADJUSTMENT
483 REM OF THE FIRST PART OF THE MODEL
485 DATA 5.5ØØØØØE-Ø7
486 REM THE FOLLOWING DATA IS FOR THE MODEL ADJUSTMENT
487 REM OF THE SECOND PART OF THE MODEL
488 DATA 3.55ØØØØE-Ø6
49Ø REM THE FOLLOWING DATA IS FOR THE PLANT G(S) ON NO LOAD
5ØØ DATA 73Ø,935,1Ø5Ø,113Ø,117Ø
51Ø DATA 118Ø,119Ø,1195,12ØØ,12ØØ
52Ø DATA 12ØØ,12ØØ,12ØØ,12ØØ,12ØØ
53Ø DATA 12ØØ,12ØØ,12ØØ,12ØØ,12ØØ
54Ø STOP
55Ø REM THIS PART OF THE PROGRAMME IS TO ADJUST THE MODEL
56Ø REM FOR C(K) AND D(K) BOTH POSITIVE AND NEGATIVE BY
57Ø REM ADJUSTING THE TIME CONSTANT
575 LET S=INT(T(4)*1ØØ+.5)/1ØØ
576 LET R=INT(T(Ø)*1ØØ+.5)/1ØØ
58Ø PRINT A,B,I(I),I(2)RS
59Ø LET T(Ø)=T(Ø)+I(I)*X
6ØØ LET T(4)=T(4)+I(2)*Y
61Ø GO TO 25Ø
62Ø END
```

APPENDIX 5

FORTRAN Computer Programme.

```
L
C      THIS PROGRAMME IDENTIFIES A MATHEMATICAL MODEL
C      OF A 3.6 KW HEAT TREATMENT FURNACE USING Z TRANSFORMS
C      WITH SCALING AND MODEL ADJUSTMENT OF THE TIME
C      CONSTANTS. THE TEMPERATURE INPUTS AND OUTPUTS
C      ARE IN DEG C, BUT THE PROGRAMME MODEL USES DEG
C      F FOR COMPUTATION PURPOSES. THE INITIAL VALUES OF
C      THE CONSTANTS USED IN THIS PROGRAMME ARE:
C      TORØ AND TOR4 Ø.75 HOURS, TIME (INC) 3.66 (MIN),
C      HEAT INPUT 12,3ØØ BTU/H,FURNACE CONSTANT 49,
C      TOR3 2Ø DEG OR 11.1 DEG C (TOR2)
C      T(K) IS THE TIME CONSTANT OF THE REQUIRED TEMPERATURE
C      THAT IS THE DEMANDED TEMPERATURE
       DIMENSION O(2Ø),G(2Ø),W(2Ø)
       DIMENSION E(2Ø),A(2Ø),C(2Ø)
       DIMENSION F(2Ø)
       DIMENSION T(2Ø)
       O(Ø)=Ø
       G(Ø)=Ø
       W(Ø)=Ø
       E(Ø)=Ø
       A(Ø)=Ø
       C(Ø)=Ø
       D(Ø)=Ø
       F(Ø)=Ø
       TORØ=Ø.Ø1274
       TOR3=2Ø.Ø
       TOR4=Ø.75
       TIME=-Ø.Ø61
       HEAT=123ØØ.Ø
       CONST=49.Ø
       X=1.6E-Ø5
       Y=7.ØE-Ø7
       B(Ø)=Ø
       R1=1.Ø
       R2=Ø.6325
       READ(1,14)AMPS
14     FORMAT('AMPS='E8.2)
       READ(1,15)FACTOR
15     FORMAT(1'FACTOR='E8.2)
       T(Ø)=Ø.Ø1274
       T(1)=Ø,Ø1274
       T(2)=Ø.Ø1274
       T(3)=Ø.2176
       T(4)=Ø.2176
       T(5)=Ø.2299
       T(6)=Ø.2674
       T(7)=Ø.2674
       T(8)=Ø.3Ø29
       T(9)=Ø.3327
       T(1Ø)=Ø.3582
       T(11)=Ø.3582
       T(12)=Ø.3819
       T(13)=Ø.4Ø35
       T(14)=Ø 4Ø35
```

```
      T(15)=Ø.4231
      T(16)=Ø.4418
      T(17)=Ø.4598
      T(18)=Ø.4762
      T(19)=Ø.4925
      T(2Ø)=Ø.5Ø84
      TOR2=TOR3*9.Ø/5.Ø
      WRITE(1,1Ø)
 1Ø   FORMAT('PROGRAMME STARTS')
      CALL SETCL(2,1996)
      DO 621 K=Ø,2Ø
 2Ø   AK=FLOAT(K)
      DO 62Ø L=1,11
      CALL ADINIT
       INPUT=IADC(2)
      TEMPA=FLOAT(INPUT)
      TEMPE=TEMPA*Ø.978+2Ø.Ø
      AL=FLOAT(L)
      AMINS=AK*3.66+(AL-1.Ø)*19.96/6Ø.Ø
 25   IF(K)89,89,85
 85   G=TORØ/(1.Ø+1.1*CONST)
      P=HEAT*G
      Q=EXP(TIME/TORØ)
      O(K)=(P+O(K-1)*Q)*R1
      S=EXP(TIME/TOR4)
      G(K)=(TOR2/TOR4+G(K-1)*S)*R2
      TEMPF=G(K)+O(K)+68.Ø
      TEMPO=TEMPE*9.Ø/5.Ø+32.Ø
      E(K)=TEMPO-TEMPF
      A(K)=E(K)*O(K)
      B(K)=E(K)*G(K)
      C(K)=A(K)*C(K-1)
      D(K)=B(K)+D(K-1)
      TEMPC=(TEMPF-32.Ø)*5.Ø/9.Ø
      ITEMP=TEMPC+Ø.5
      F(K)=E(K)*5.Ø/9.Ø
      IE=F(K)+Ø.5
29Ø   IF(C(K)-65.Ø)3ØØ,4ØØ,22Ø
3ØØ   IF(C(K)+65.Ø)22Ø,4ØØ,4ØØ
C     THIS PART OF THE PROGRAMME IS TO ADJUST THE MODEL
C     FOR C(K) AND D(K) BOTH POSITIVE AND NEGATIVE
C     BY ADJUSTING THE TIME CONSTANTS
22Ø   IF(K-1)26Ø,26Ø,28Ø
26Ø   X=8.ØE-Ø6
28Ø   IF(K-8)25Ø,24Ø,24Ø
24Ø   X=3.5E-Ø6
25Ø   TORØ=TORØ+C(K)*X
      TOR4=TOR4+D(K)*Y
      GOTO 85
4ØØ   IF(T(K)-TORØ)4Ø1,2ØØ,4Ø2
4Ø1   DELT=T(K)-TORØ
      AMPS=AMPS+(DEL*FACTOR*TEMPE)
      GO TO 89
4Ø2   DELT=TOK-T(K)
      AMPS=AMPS-(DELT*FACTOR*TEMPE)
89    IF(AMPS-2047.Ø)2Ø1,9Ø,9Ø
9Ø    AMPS=2Ø47.Ø
2Ø1   IF(AMPS)95,95,2ØØ
95    AMPS=Ø.Ø
```

```
200     IAMPS=IFIX(AMPS)
590     CALL DIOUT(IAMPS)
        WRITE(1,24)AMINS,TEMPE
24      FORMAT(IF8.2'M'IF8.2'C')
610     CALL WAIT
620     CONTINUE
621     CONTINUE
210     WRITE(1,215)
215     FORMAT('PROGRAMME ENDS')
        IAMPS=0
        CALL DIOUT(IAMPS)
230     STOP
        END
```

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