Computer-aided analysis of engine noise

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Abstract: The overall process of low-noise engine design is outlined. Computational techniques for the analysis of engine noise are considered: the finite element method for the vibratory analysis of the engine structure, the boundary element method for the acoustic analysis of the bare engine and the boundary and shell element method for the prediction of the effect of noise shields. Results from the application of the techniques to an engine-like structure or to test problems are given.

Reference to this article should be made as follows: Kirkup, S.M. and Tyrrell, R.J. (1992) 'Computer-aided analysis of engine noise', *Int. J. of Vehicle Design*, vol. 13, no. 4, pp. 388–402.

Keywords: acoustics, boundary and shell element method (BSEM), boundary element method (BEM), computer-aided design, crankcase simulation rigs (CSR), engine, finite element method (FEM), noise, pre- and post-prototypes, Rayleigh integral method (RIM), shell integral equation, shields, sound power integral method (SPIM), vibration.

1 Introduction

Over recent decades the volume of traffic in developed countries has increased at a dramatic rate. The potential for a corresponding increase in traffic noise pollution has been countered, in most countries, by legislation which restricts the noise output of new vehicle designs (Waters, 1980). The criterion for measuring the noise produced by a vehicle is the *drive past test* (Hassall and Zaveri, 1979; Waters, 1980). In brief, this measures the A-weighted total sound pressure at a point a particular distance from the path of the vehicle. The test site is regulated and the vehicle is operating under a specific set of conditions for the duration of the test. Legislation generally places an upper limit on the measured noise for each different category of vehicle. With the clear aim of keeping traffic noise to a reasonable level, over the years it has been the tendency for legislators to systematically reduce these noise limits.

The pressure of keeping the noise from vehicles to a certain level and the need to satisfy the customers' personal demands for less noisy vehicles has meant that noise has become an important factor for automotive engineers to consider. The noise arises as a result of the vibration of the vehicle. Close inspection of any vehicle reveals that there are several discernible sources of vehicle vibration. Of these, the engine is found to be a major source under most operating conditions. Hence a large part of the burden of designing low-noise vehicles has fallen to the engine designer.

The relationship between the noise produced by engines, the vibration of the engine, the design of the engine structure and the excitation forces has interested engineers for several decades. Equipment exists for the measurement of each of these properties. Usually, the relationships between these properties are analysed only after they are resolved into their frequency components, although recent advances in the analysis of non-linear effects of joints and interaction through oil films have led to some work being performed in the time domain. The results of such experimental work help the engineer to understand how the noise arises and perhaps to decide how the design may be modified to reduce the noise. However, development in this way is costly and hence a computer-aided approach to the design of low-noise engines has been pursued over recent years.

Once the design of the basic powertrain structure itself has been finalized, shielding may be used to effect a further reduction in noise. Shielding may take the form of complete or substantial enclosure of the engine block or it may consist of one or more shields which together only partially enclose the engine. Both methods add bulk to the engine block and hence neither method is an easy option for designers. However, whereas complete or substantial enclosure is generally obstructive to routine maintenance operations, partial shielding need not be. For discussions on the practicalities of shielding, see Waters, 1980 and Thien, 1982. In this paper, the partial enclosure of the engine using a set of close-fitting shields is considered.

The steady improvement in both numerical techniques and computer equipment over recent decades has meant that it is possible to make computational predictions of both the vibratory and acoustic properties of engines. The object of this paper is to describe the advanced computational techniques that may be employed: the use of the *finite element method* (FEM) for the prediction of the vibratory properties, the *boundary element method* (BEM) for the prediction of the sound field around a vibrating engine and the use of the *boundary and shell element method* (BSEM) for the prediction of the effect of an engine noise shield. Results from the application of the FEM and BEM to engine-like structures are given. The BSEM is applied to a suitable test problem and the results from this are presented.

The ultimate aim of this work is to provide a range of software tools to aid the design development process. In other words, the aim is to develop an integrated *computer-aided design* approach to low-noise engine design. The design development of any system may ultimately be regarded as a repetitive process, each stage consisting of an analysis of the existing design followed by a decision on the modifications for the new design. In this paper, we are concerned with computational techniques for the analysis stage of the design process. Such techniques are considered alongside standard measurement techniques and within the *computer-aided design* context.

2 The process of designing low-noise engines

The function of the engine is to convert fuel into the mechanical forces which power the vehicle. The vibration of the engine structure arises from the mechanical events within the engine and the direct excitation from the fuel combustion process. The noise observed by the occupant or bystander is the perceived acoustic response of the air to the various radiating surfaces. These surfaces may be those of the powertrain itself or of the vehicle structure. The vehicle structure may be excited either from the air (in turn excited by the powertrain surfaces), or directly, for example via the engine mounts. Background work on this can be found, for example in Grover and Lalor, 1973; Challen, 1982; and Morrison 1985.

Radiated noise levels must be considered at every stage in the development of the design of modern engines. At concept design, only broad guides to the overall engine structure will be given. During the initial design phase, however, accurate predictions of engine vibrations may be made; these may form the basis of noise prediction systems. Such analysis work may be used to help generate improved designs, prior to manufacture of prototypes.

The overall design process may be divided into the pre-prototype and post-prototype phases. Each of these phases is likely to be partially iterative in nature. At any given point during the post-prototype phase, the engineer may be satisfied that the noise is sufficiently low and that no further deliberate action is necessary. On the other hand, the engineer may be already planning further measures to reduce the noise. One such measure is that of partial enclosure of the engine block using noise shields.

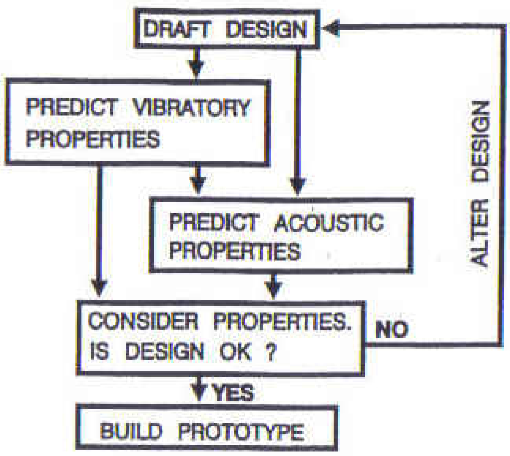
In this Section, a simplified and schematic description of each design stage is given. It is shown how computational techniques may be usefully employed in the design process.

2.1 The pre-prototype phase

In the process of designing an engine, methods for predicting the vibratory and acoustic properties from a draft of the design would be useful. Such methods would show whether or not the engine would be too noisy when built, and also give indications as to how the design may be improved. Once the design of the engine is such that the predicted noise is satisfactory, a prototype of the engine may then be built.

Figure 1 illustrates this part of the design process. The reason for carrying out this pre-prototype phase, rather than directly building a prototype, is that it makes the overall design process more economical. The efficiency and even the success of this process is dependent on the accuracy of the prediction of the vibratory and acoustic properties, the interpretation of these properties and the ability to make successful modifications to the design of the engine based on this interpretation.

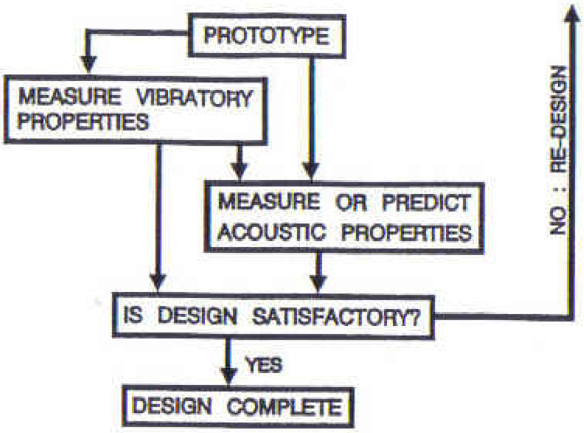
Figure 1 Pre-prototype design process.



2.2 The post-prototype phase

Once the prototype is built, the vibratory and acoustic properties can be measured. However, some of the desired acoustic information may be difficult or costly to measure and it may therefore be more feasible to predict the acoustic properties from the surface vibration. This part of the design process is illustrated in Figure 2.

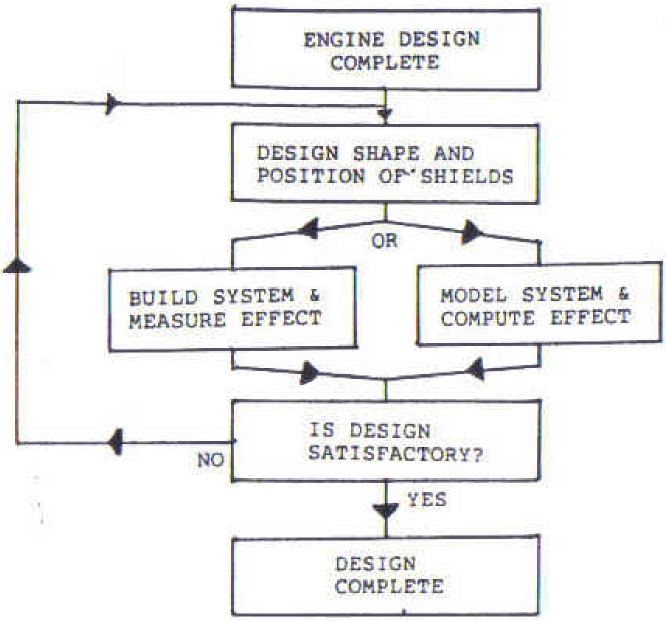
Figure 2 Post-prototype design process



2.3 Noise shields

On fitting one or more noise shields around the engine block, the change in noise will depend on the shape of the engine block and the nature of its vibration, the size, shape and position of each shield and the vibratory properties of the shield itself. Diagnostic information about the vibratory and acoustic properties of the isolated engine block will help the design engineer decide where the shields should be placed. Without the aid of computational tools for predicting the effect of the shield on the noise, the shield may only be developed through generating a physical model and measuring the effects. The overall process of designing an engine-shield system is illustrated in Figure 3.

Figure 3 Noise shield design process



2.4 *Discussion*

Noise has been an important factor in engine design for several decades. Over this period, engineers have become more experienced in designing low-noise engines, using simple relationships between design parameters and noise (e.g. see Grover and Lalor, 1973). However, this approach is severely limited as the relationship between the design parameters and the noise produced by the engine is not trivial. With the increasing capabilities of computational tools, the more valid approach of basing the noise prediction methods on the principles that govern the properties of the engine structure and the air which surrounds it has been considered over recent years (e.g. see Dowling, 1991).

In the design process, illustrated in Figures 1–3, the design engineer has to be able to interpret the vibratory and acoustic information and judge whether corrective treatment in the design is required. Clearly, the more reliable and comprehensive this information is, the better placed the engineer is to make these judgements. Hence the aim of developers of both computer software and measuring equipment is to provide engineers with as much reliable information as they can usefully handle.

3 Evaluation of the vibratory properties of the engine

The response of the structure to the internal forces, the structural mode shapes and the resonant frequencies are some of the vibratory properties that are useful to the design engineer. The finite element method is now generally regarded as a reliable tool for modelling the vibratory behaviour of elastic structures (see Zienkiewicz, 1977) and there is continued development of methods for the calculation of the excitation and response. The application of such methods to engine blocks is considered in Croker, Lalor and Petyt, 1979; Lalor and Petyt, 1982; and Tyrrell and Croker, 1987.

4 Evaluation of the acoustic properties of the engine

A sound power spectrum, surface intensity patterns, the sound pressure at points in space and the radiation ratio curves for the engine's structural mode shapes are some of the acoustic properties that are useful to the design engineer. Methods for predicting the acoustic properties of the isolated engine from the surface vibration can be based on the *Rayleigh integral method* (RIM) or the *boundary element method*. There is also a simple method for estimating the sound power termed the *sound power integral method* (SPIM). These methods are reviewed in this Section and their suitability for predicting the acoustic properties of the air surrounding the engine block is studied.

4.1 *The sound power integral method*

Assuming that the sum of the sound powers from each point on the surface radiating independently is equal to the total sound power allows us to derive the *sound power integral method*. The method is very crude and it is inappropriate for the estimation of the other acoustic properties. It is equivalent to assuming that the radiation ratio is equal to one at all wavenumbers. Loosely speaking, for engines the radiation ratio is roughly unity for frequencies greater than approximately 1 kHz and the method can therefore yield a rough estimate of the sound power at these higher frequencies. An assumed radiation efficiency curve may be used to modify the predicted frequency spectrum below 1 kHz, such that better correlation is obtained with experience. The method has the benefit of being computationally cheap and it has been widely used in this application. An example of its use is given in Croker, 1987a,b.

4.2 The Rayleigh integral method

Assuming that the engine is made up of a set of flat plates which radiate independently allows us to apply the Rayleigh integral method. Applying the method to the faces where the vibration is strongest can give satisfactory approximations to the local acoustic properties. The sound power and radiation ratios calculated via this method are usually reasonably accurate for in-line engines. There are several reported applications of this method to the engine noise problem, e.g. see Yorke, 1975; and Croker, 1987a,b.

4.3 The boundary element method

The *boundary element method* is the general term given to numerical methods for the solution of partial differential equations where the method is derived from a boundary integral equation formulation. The method is applicable to a wide range of physical or engineering problems (Brebbia, 1978; Banerjee and Butterfield, 1981). For the problem considered in this Section, the equation we need to solve is the Helmholtz equation or reduced wave equation in a region exterior to an arbitrary surface with a velocity boundary condition at the surface. The derivation of a reliable BEM for the solution of this problem has interested researchers for several decades. The method introduced in Schenck, 1968 is now a popular BEM for the computation of the properties of the acoustic field surrounding a vibrating body. However, though this method is reasonably easy to implement, it is difficult to automate for general surfaces. On the other hand, methods based on the integral equations introduced in Kussmaul, 1969 and Burton and Miller, 1971 have been found much easier to automate but more difficult to implement. For reviews on these methods see Burton, 1976 and Kleinmann and Roach, 1974.

From the theoretical point of view, the BEM is clearly well-suited to the problem of predicting the acoustic properties of the air surrounding a vibrating engine. It is able to closely represent the physical situation of the engine in free-space or in an anechoic chamber. The main difference between the physical problem and the model is that the engine surface must be idealized as a simple closed surface so that many details will need to be omitted.

Figure 4 A comparison of methods of engine noise estimation

	SPIM	RIM	BEM	Measuring
sound pressure	×	?	✓	✓
surface sound intensity	×	?	✓	✓
sound power	?	? or ✓	✓	? or ✓
radiation ratio	×	? or ✓	✓	?

There have been a number of reports on the application of the BEM to engine and machine noise problems: Hall, 1982; Koopman, 1982; Planchard, Guisnel and Smadja, 1985, Sas and VandePonseele, 1985; Seybert and Holt, 1985; Soenarko and Seybert, 1985; Seybert, Wu and Li, 1989; and Riehle, Allen and Branch, 1990. These reports consider the applicability of the method and give some calculated results. However, previous research generally discusses the potential of the BEM and generally presents results at only one or two frequencies. The method is generally regarded as being computationally expensive.

4.4 Summary

The potential for success or otherwise in the use of the different methods for evaluating the acoustic properties of the air surrounding a vibrating engine for noise purposes is illustrated in the Table in Figure 4. It must be realized that for engine noise purposes, the most important region of the frequency range is generally approximately 0.5 kHz to 2.5 kHz and that the acoustic properties need not be evaluated to high numerical accuracy.

5 Analysis of the engine-shield system

The computational prediction of the effect of noise shields on engine noise has received the least attention of the problems considered in this paper. In order that the noise shields do not add significantly to the weight of the vehicle, they will generally be made of light, flimsy materials and hence they may also vibrate significantly. The vibration of the noise shields may arise through mechanical coupling with the vehicle vibration or because of acoustic coupling between the engine and shields.

The frequencies that largely determine the noise produced by the engine-shield system are the frequencies at which the engine vibration is strong; the structural resonant frequencies of the shield; and the acoustic resonant frequencies that may occur in the gap between the engine and the shield. Moreover, should any of the resonant frequencies of each of these components coincide in the frequency spectrum, then the shield may well increase rather than decrease the noise. Clearly, therefore, much useful information about the system can be derived by considering the properties of the engine, the shield and the gap in isolation. However, in order to model the full system, an advanced computational method termed the *boundary and shell element method* is required. It is this method that we consider applying in this paper.

6 The simulation of engine structures

In order to gain an understanding of the effect of different designs of engine on the vibratory and acoustic properties, the Noise Refinement Centre at Ricardo embarked on building and testing engine-like structures which are known as *crankcase simulation rigs* (CSRs). These structures had all the main features of a heavy duty diesel engine crankcase/cylinder block/cylinder head assembly, but with the minimum of irregular details which characterize a real engine design. The CSRs are slab-sided in-line engine structures. Two different forms were constructed — the symmetric CSR (CSRSYM) and the asymmetric CSR (CSRASYM). The structures were tested and measurements of the excitation forces and acoustic and vibratory properties were taken. These measurements are useful in the analysis of the relationships between these properties. The data from these experiments are also useful for the verification of results from the application of computational methods. For further details on the CSR project, see Croker, 1987a,b and Croker and Tyrrell, 1989.

In order to take measurements of the acoustic and vibratory properties, the CSRs were suspended on elastic cords in a semi-anechoic test cell. Surface vibration response to internal excitation was measured at some 250 points using a miniature accelerometer. The sound pressure was measured at 10 microphone points on an imaginary hemisphere of radius 1.4 m and at 8 microphone points on an imaginary cylinder of the same radius which joins onto the hemisphere. The sound power was determined by calculating a numerical approximation based on an integral over the imaginary surface. The surface acceleration, the sound pressures, the sound powers, and the cylinder pressure were converted into 10 Hz narrow band frequency spectra. For more details on this, see Croker, 1987a,b.

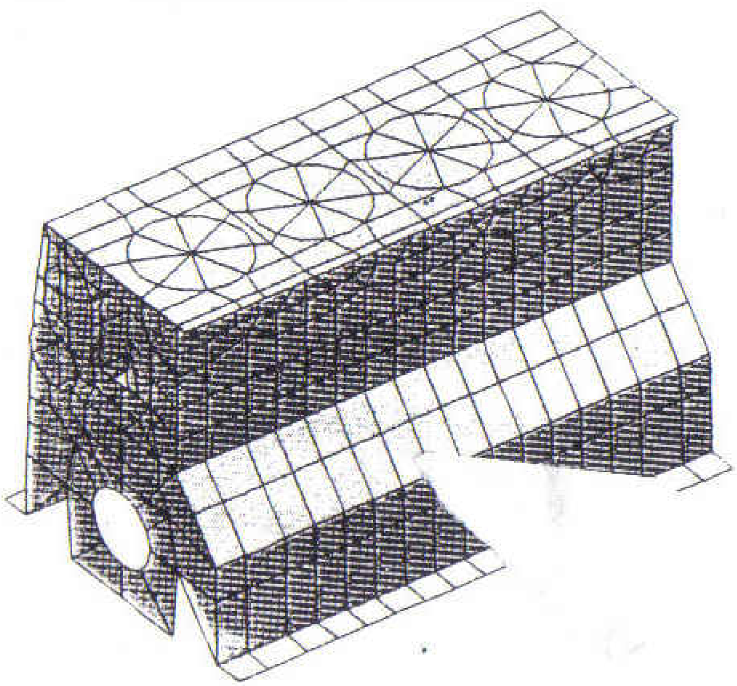
7 Vibratory analysis via the finite element method

In this Section, we consider the application of the PAFEC finite element package (PAFEC Ltd) to the vibratory analysis of the CSRs. For the CSRs, large PAFEC finite element models containing up to 4000 high order elements with up to 50 000 degrees of freedom were used. The finite element model of CSRASYM is illustrated in Figure 5.

The comparison of computed and measured vibratory properties for structures of this complexity is a severe test on the FEM. In order to predict the forced response, a knowledge of the acting internal forces is required. These forces may be specified by measurement or analysis, though usually it is not feasible to obtain a detailed description of them.

For the CSRs, calculations are based on the concept of a unit magnitude flat in-cylinder pressure spectrum. The use of this excitation results in forced vibration data in the form

Figure 5 Finite element mesh of the CSRASYM



of transfer functions which are readily comparable with experimentally derived results. Pressure forces are applied directly to model cylinder flame faces. Forces on the main bearings are calculated by means of calculated transfer paths through the piston/connecting rod/crankshaft assembly.

Mode shapes, resonant frequencies and forced response of the CSRs for frequencies up to 2500 Hz were computed via the model illustrated in Figure 5. The resonant frequencies were generally predicted to within 5%. Mode shape prediction and forced vibration analyses yielded results which compared well with those measured on CSRs. Further details on this work are given in Tyrrell and Croker, 1987.

8 Acoustic analysis via the boundary element method

In order to apply the boundary element method to the rig, the surface is idealized into a simple closed surface and approximated by around 550 planar triangular elements. Generally, the positions of the elements are chosen so that their vertices are at the accelerometer points. The boundary element mesh of the CSRASYM is shown in Figure 6. On each boundary element, the surface vibration is specified by interpolating the values at the vertices of the components of the velocity normal to the element, derived from the accelerometer readings. At vertices where there was no accelerometer reading, the velocity is prescribed zero values.

The particular BEM used was based on the integral equation of Kussmaul, 1969. Further detail on the method employed is given in Kirkup, 1989, and Kirkup and Henwood, 1992. Figure 7 presents a comparison of the measured and computed sound power transfer function (with respect to cylinder pressure) over the range of 400 Hz to 2400 Hz.

Figure 6 Boundary element mesh of the CSRASYM

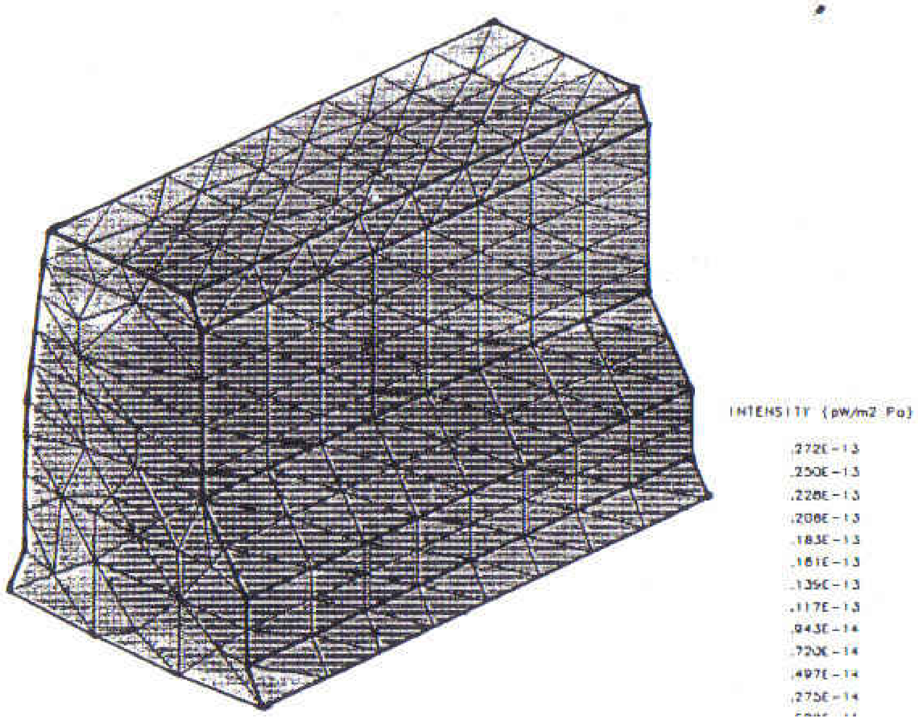
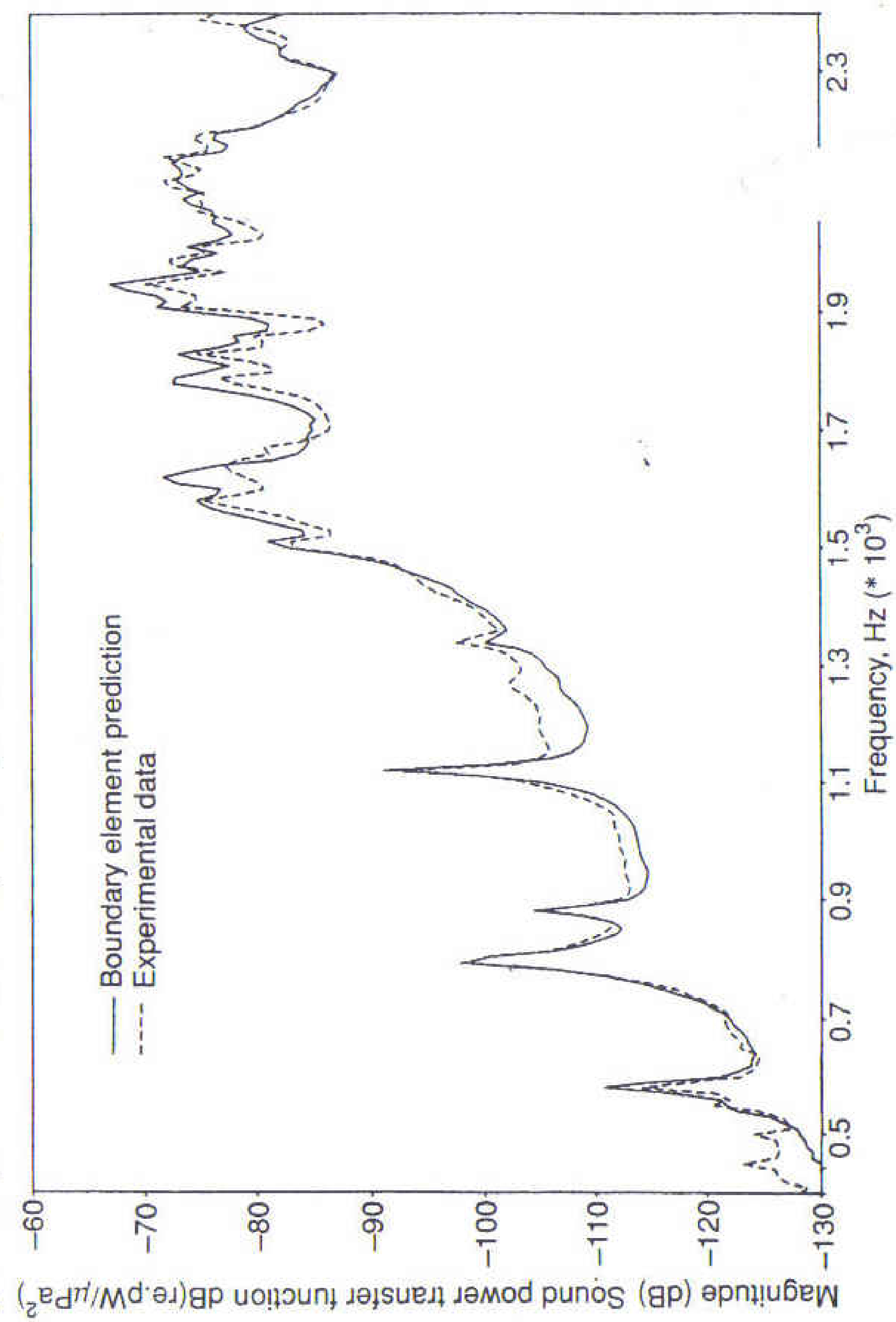


Figure 7 Comparison of the measured and computed sound power transfer function spectra



In Figure 7, the graphs of the sound power spectrum have peaks at 800 Hz and 1120 Hz. The peaks occur at these frequencies because the rig has structural resonances at 795 Hz and 1121 Hz. Figure 8 illustrates the surface sound intensity with respect to cylinder pressure at 800 Hz and 1120 Hz. The shading scheme in Figure 8 indicates the average intensity on each boundary element.

Figure 8(a) Surface intensity pattern at 800 Hz

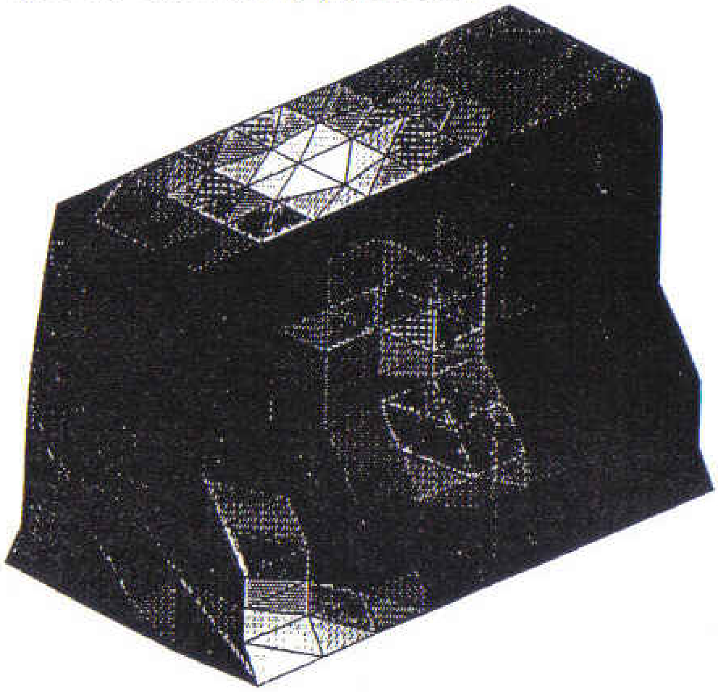
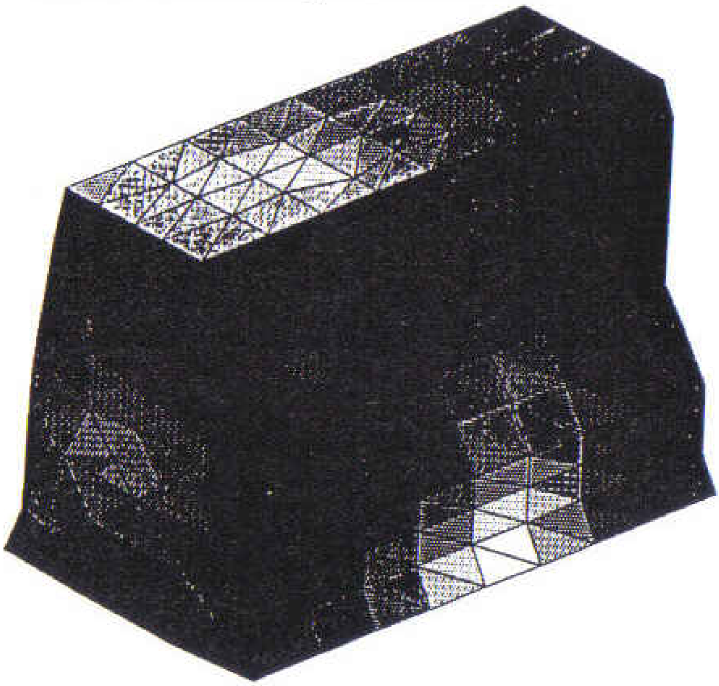


Figure 8(b) Surface intensity pattern at 1120 Hz



9 Potential use of the boundary and shell element method

The Helmholtz equation, which governs the acoustic field exterior to an isolated shield or shell, can be reformulated as an integral equation in a way similar to that for a closed surface (see Ben Mariem and Hamdi, 1987 or Warham, 1988). The resulting equation is termed a *shell integral equation*.

The situation we consider here consists of a closed boundary (the engine block) and at least one thin shield. This can also be reformulated as an integral equation termed a *boundary and shell integral equation* (BSIE) which is a hybrid of a *boundary integral equation* and a *shell integral equation*. Discretization of this integral equation allows us to derive the *boundary and shell element method*. The BSEM is clearly a generalization of the BEM and hence the background to the BEM in Section 4.3 is also relevant to the BSEM.

For the particular BSEM applied in this paper, the BSIE was based on the generalization of the integral equation of Burton and Miller, 1971. In order to derive the BSEM, the surfaces are approximated by a set of triangles and the surface functions are approximated by a constant on each triangle. Further detail on the method is given in Kirkup, 1991.

The test problems that are considered all have a 10 cm cube of which one face is vibrating uniformly, and the vibration of the other faces is zero. A square plate of side 10 cm is placed 10 cm from the vibrating face. Air is the acoustic medium and the shield and cube are assumed to be perfectly reflecting. The test problem is illustrated in Figure 9.

The radiation ratios of the system with or without the shield were computed at 10 Hz, 20 Hz, . . . , 5000 Hz. The radiation ratio curve for the system is given in Figure 10. The graph shows that the shield has a major effect on the radiation ratio of the vibrating cube. The radiation ratios for the shielded and unshielded cube are similar when the frequency is less than around 1300 Hz. However, the shield makes a significant reduction in radiation ratio at around 1800 Hz, 3000 Hz and 4500 Hz and significantly increases the radiation ratio at around 2200 Hz and 3700 Hz. Further results are given in Kirkup, Henwood and Tyrrell, 1991, and Kirkup, 1991.

Figure 9 Illustration of the test problem

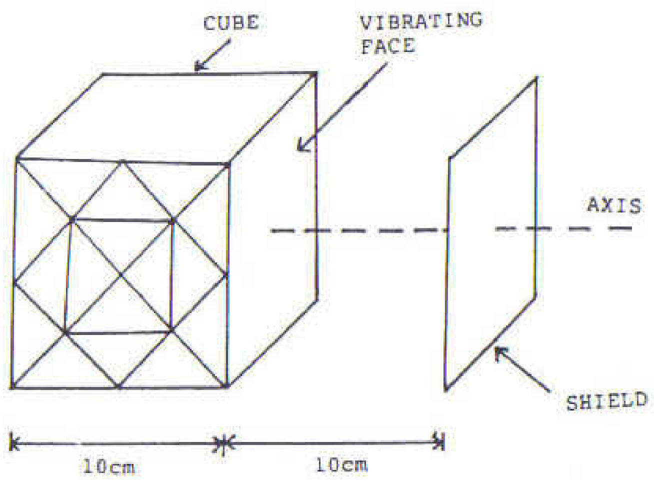
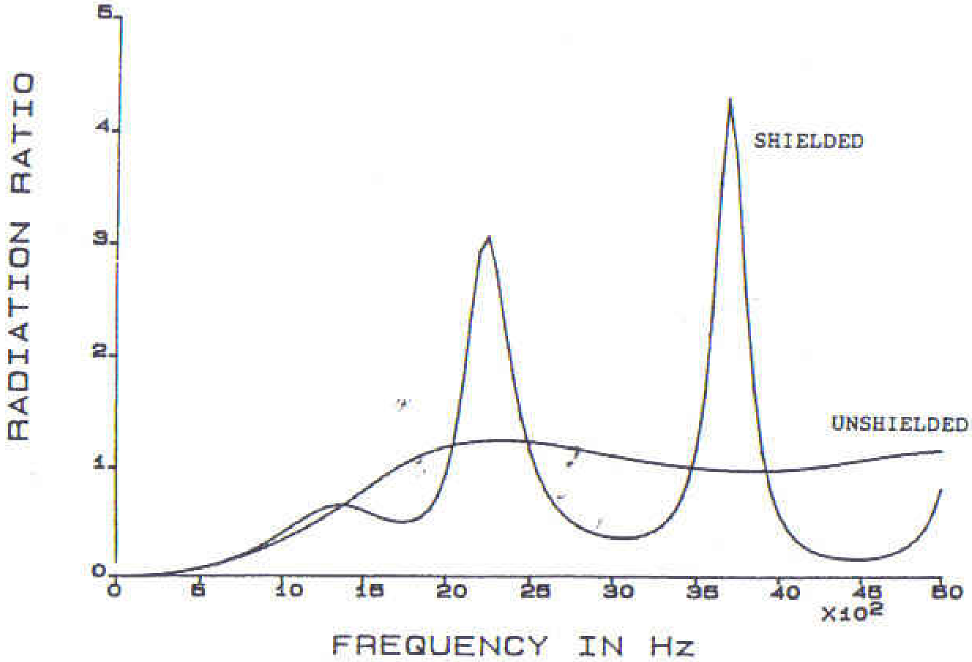


Figure 10 Radiation ratio curves for the shielded and unshielded cubes



10 Conclusion

In this paper, it has been shown that computational methods can be used to calculate the vibratory and acoustic properties of engine blocks and engine-shield systems. Attention in the vibratory calculation is now concentrated on improving the efficiency of model generation and computational efficiency. There is also work on the improvement of methods for estimating the excitation forces involving advanced non-linear calculations of such phenomena as the resonant coupling of vibrating crankshafts and cylinder blocks which are separated by oil-films.

For the prediction of the acoustic properties, the SPIM and the RIM are computationally much cheaper than the BEM. However, the potential for accuracy and adaptability in the BEM is much greater than that for the other methods. Hence, with steady improvements in computer equipment and in the method itself, it is likely that the BEM will prevail in the acoustic analysis of engines.

In this application, the BEM for the acoustic analysis of the bare engine block is at a much earlier development stage than is the FEM. The BSEM for the acoustic analysis of a shielded engine is at an even more immature stage of development. However, it has been demonstrated in this paper how the BSEM may be employed. The obvious next step is to validate the method through the comparison of computed with measured results on a real-world problem.

A culture of understanding can only evolve around any design development technique when there is confidence in its computational reliability and it is reasonably efficient. Equipment for the measurement of the acoustic and vibratory properties has been available for some decades and although improvements in the scope, reliability and efficiency of such equipment are being made, such a culture of understanding has developed over this time. Computational techniques such as the FEM and BEM have become practical in the engine noise area only over the last decade. Such a culture of understanding for the application of the methods and the interpretation of results will take time to develop fully. It is the opinion of the authors that the FEM, BEM and BSEM, applied as d ~ibed in this paper, will become increasingly important in the analysis of engine n

Acknowledgements

The authors would like to thank Dr D.J. Henwood of Brighton Polytech ..id Mr M.D. Croker of Ricardo for their advice and support in this work. The authors are also indebted to many of their former colleagues at Ricardo who have contributed to this work.

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