Estimating the probability of large negative stock market returns: the case of food retailing and processing firms.

By
Philip Kostov[a] and Seamus McErlean[b]

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[a] Agricultural and Food Economics, School of Agriculture and Food Science, Queen's University Belfast, Newforge Lane, Belfast, BT9 5PX. Tel: +44 (0)28 9025 5237, Fax: +44 (0)28 9025 5327
e-mail: P.Kostov@qub.ac.uk

[b] Agricultural and Food Economics, School of Agriculture and Food Science, Queen's University Belfast, Newforge Lane, Belfast, BT9 5PX. Tel: +44 (0)28 9025 5621, Fax: +44 (0)28 9025 5327
e-mail: Seamus.McErlean@dardni.gov.uk

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Abstract

Correct assessment of the risks associated with likely economic outcomes is vital for effective decision making. The objective of investment in the stock market is to obtain positive market returns. The risk, however, is the danger of suffering large negative market returns. A variety of parametric models can be used in assessing this type of risk. A major disadvantage of these techniques is that they require a specific assumption to be made about the nature of the statistical distribution. Projections based on this method are conditional on the validity of this underlying assumption, which itself is not testable. An alternative approach is to use a non-parametric methodology, based on the statistical extreme value theory, which provides a means for evaluating the unconditional distribution (or at least the tails of this distribution) beyond the historically observed values. The methodology involves the calculation of the tail index, which is used to estimate the relevant exceedence probabilities (for different critical levels of loss) for a selection of food industry companies. Information about these downside risks is critically important for investment decision making. In addition, the tail index estimates permit examination of the stable Paretian hypothesis.

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1. Introduction

The volatility of stock market prices can be considerable and there is a common perception that volatility levels are increasing over time. Certainly, the recent series of stock market crashes has brought the issue of downside risk into sharp focus. This is particularly so for those investors who regularly have to liquidate their assets at short notice. Danielsson and de Vries (1997) cite the example of a pension fund that must be able to pay out every period. Investors in this situation often favour the 'safety first' rule, which explicitly takes into account the probability of a high negative return (Roy, 1952; Bernstein, 1992).

Downside risk is perceived to vary according to share type. For example, one might expect the average level of downside risk associated with holding the shares of well established food processing and retailing to be lower than that for companies operating in emerging markets or within the new technologies sector. On the other hand it is likely that these downside risk levels vary considerably across food firms.

The potential for capital gains is one of the main attractions of holding shares or other financial assets. However, as recent stock market crashes have demonstrated, all share prices are subject to substantial falls. A large reduction in share prices represents a capital loss to an investor, which may trigger serious financial problems. In this paper, a loss (gain) is defined as a drop (rise) in share value over the course of a single day's trading, calculated by comparing the opening and closing prices quoted for the share in question at the beginning and end of the same day's trading. The probability of a loss exceeding a given magnitude, such as for example, a 30% drop in share price, can be estimated. However, it is preferable to estimate the probabilities over a range of loss levels. Various techniques can be used to estimate these exceedence probabilities. Some of these techniques attempt to fit the entire probability distribution, while others concentrate only on the (left) tail, given that this is the area of most relevance.

The simplest method of calculating exceedence probabilities is to make use of sample equivalents, as in the case of non-parametric Value-at-Risk methods[1]. In these methods, a long historical run of daily data on share price movements is used to determine the frequency with which a capital loss of a given size occurs. Thus, the frequency of losses greater than 10 per cent occurring in a sample of say 1000 could be used to calculate the probability of a loss greater than 10 per cent. While this method has the advantage of requiring no assumptions to be made about the underlying distribution, it suffers from two serious problems. Firstly, these sample equivalents will be inefficient because of their high variance, which means they provide unreliable estimates of the true distribution. Secondly, since the probability distribution is calculated directly from a sample of historical share price movements, there is no way to obtain probabilities for out-of-sample losses.

An alternative approach, which is parametric, makes use of the 'true' underlying distribution for the market returns under consideration. The parameters of the 'true' distribution are estimated from available data and used to construct a negative returns-probability table. The problem, however, with this approach is that the true underlying distribution is typically unknown. Thus, to proceed with this approach one has to make an assumption about the nature of the underlying distribution. For example, McCulloch (1981) fits the Cauchy distribution in calculating the bankruptcy probabilities of commercial banks. However, such an assumption is unlikely to be validated, with the result that inferences based on the estimated parameters loose validity. Clearly, the choice of the underlying distribution affects the results obtained, particularly when it is the tail of the distribution that is of most interest. Another major problem with this and other parametric approaches stems from the fact that they attempt to fit the entire probability distribution. In estimating the parameters of the entire probability distribution from all the data available these approaches give low weighting to extremely large negative (or positive) returns simply because of the rarity of these returns. Therefore, paradoxically, the most important information is belittled.

A third approach was suggested in the early 1990s and is based on a proposal by Du Mouchel (1983) to concentrate on the behaviour of the tails and thereby avoid the need to make any assumptions about the centre of the distribution. The theoretical background of this approach is provided by the statistical theory of extreme values (see, for example Leadbetter *et al*., 1983; Beirlant *et al*., 1996). The main result of this theory is that it shows the limiting

[1] Value-at-Risk analysis is normally used to assess downside risks associated with a portfolio of shares, whereas we are concerned with analysing single shares. However, the principles are the same.

distribution of extremes (maxima or minima) can be characterised regardless of the underlying statistical distribution that generates the data. Thus, instead of considering the entire distribution, one can fit the asymptotic limiting distribution to the tail for the purposes of inference. This tool is made all the more useful by the fact that the tail can be characterised by single statistic, namely the ‘tail index’. This statistic can be thought of as a measure of tail thickness. Importantly, exceedence probabilities can be calculated from the tail index estimates. Essentially, this amounts to calculating quantiles probabilities from the limiting distribution of the extreme values. Under this approach the probable occurrence of extreme returns falling outside the range of the empirical data set can also be estimated.

An added advantage of the tail index is that it can be used to provide information about the underlying distribution of the returns data. Thus, for example, semi-parametric estimates of the tail index can be used to test the stable Paretian hypothesis.

The aim of this paper is to use tail index estimates to calculate the relevant exceedence probabilities (for different critical levels of loss) for a selection of food industry companies. Information about these downside risks is critically important for investment decision making. The exceedence (or crash) probabilities estimated are compared with each other. It is concluded that the companies in this sample can be broadly classified into either high risk or low risk types. We also use the tail index estimates to examine the stable Paretian hypothesis. The next section describes the concept of the tail index, the stable Paretian hypothesis and other relevant statistical concepts. Sections 3 and 4 describe the methodology and data sets, respectively. Results are presented in section 5 and conclusions are drawn in section 6.

2. The Theoretical Concepts

The statistical properties of financial data have long been the subject of learned discussion and academic research. Over a hundred years ago Bachelier, in his Ph.D. thesis entitled ‘Théorie de la Spéculation’ (reprinted in Cootner (1964)), formulated one of the first testable hypotheses on the statistical behaviour of financial data by proposing the use of the Normal distribution as a model for share price movements. While most empirical studies reject the Normal hypothesis, many theorists are of the view that share price movements should follow a distribution from the stable distribution family (also called Paretian distributions), of which the Normal (Gaussian) is a member. Stable distributions possess the appealing property of stability-under-addition. Given that daily, weekly, etc., returns from financial assets can be considered sums of independent, identically distributed (IID) share price changes it is expected that the limiting distribution will come from the stable distribution family. There is intuitive appeal also in that the stable distributions, other than the normal, share the features of fat tails and high peaks (leptokurtosis) observed in financial data.

The Normal is the most familiar stable distribution with a characteristic component, α_s, equal to 2. However, in empirical studies the Normal is frequently rejected as the limiting distribution for share price movements because of the leptokurtic nature of the observed data. This led Mandelbrot (1963) and Fama (1963) to propose stable Paretian distributions as the limiting distributions for financial market returns. These non-normal stable distributions have a characteristic component $0 < \alpha_s < 2$. Under this hypothesis relative price changes would have no finite second and higher moments and would, therefore, only obey the generalised central limit law under time aggregation.

Conventional estimation of the characteristic exponent explicitly relies on the assumption that the underlying distribution is stable. Thus using this statistic for inference about the prevalence of a stable distribution is certainly open to criticism (DuMouchel, 1983). The characteristic component could be used to test the null of Gaussian against all other Paretian distributions. However, to test the IID hypothesis itself, the assumption of stability must be abandoned. As DuMouchel (1983) suggests, this can be achieved by using semi-parametric techniques to estimate the tail index, which can be used to test between the stable and other candidate distributions. The tail index is derived from extreme value theory (see Longin, 2001 for an application of this theory) and in simple terms is a measure of the thickness of the tail(s) of a statistical distribution.

It is known that in the case of the non-Gaussian stable Paretian distributions the tail index, α, coincides with the characteristic exponent, α_s. Therefore, an estimate of a tail index outside the interval (0, 2] (in the statistically significant sense) would indicate rejection of the stable distribution hypothesis. In other words, if the tail index $\alpha > 2$, then the stable Paretian hypothesis is rejected[2]. Statistically the significance of the tail index is that it denotes the

[2] Another avenue for testing the stable distribution hypothesis is to directly estimate the characteristic exponent for different frequencies (e.g. daily, weekly, monthly) and to exploit the stability-under-addition property of stable distributions, which implies that the characteristic exponents calculated at these different frequencies should be the same.

largest finite moment (in the sense that a distribution with a tail index of 5.6 will have finite values for its first five moments, for proof see Gumbel, 1958: 266).

Another important use of the 'tail index' estimates is that they can be used to calculate exceedence probabilities. The concept of the 'tail index' derives from the statistical theory of the extremes and before proceeding to the tail index calculations, it is useful to briefly review this theory (for more detail see Beirlant *et al*., 1996; Leadbetter *et al*., 1983, Embrechts *et al*. 1999; Embrechts, 2000). One of the basic results of the theory of extremes is to show that under quite general conditions the limiting behaviour of the tails of a distribution of IID variables follows one of only three max-stable distributions (Leadbetter et al., 1983)[3]. In the brief exposition that follows the concepts are explained in terms of the maxima, although generalisation to the case of the minima is possible simply by changing the sign of the market returns.

Define the order statistic $M_n = \max(x_1, x_2, ..., x_n)$ as the maximum values from a sample of observations $\{x_i\}$. It can be shown that with appropriate scaling the limiting distribution of M_n belongs to one of only three classes of distribution. Expressed more formally, the limiting distribution of the tail, $\Pr[a_n M + b_n \leq x]$, with a_n and b_n denoting normalising constants, converges to one of the following types of generalised extreme value (GEV) distributions:

$$G_{1,\alpha}(x) = \begin{cases} 0 & x \leq 0 \\ \exp(-x^{-\alpha}) & x > 0 \end{cases}, \qquad (1.1)$$

$$G_{2,\alpha}(x) = \begin{cases} \exp(-(-x)^{\alpha}) & x \leq 0 \\ 1 & x > 0 \end{cases}, \qquad (1.2)$$

$$G_3(x) = \exp(-e^{-x}) \qquad x \in \Re \qquad (1.3)$$

where α denotes the tail index. These three limiting distributions are actually the Fréchet (1.1), the Weibull (1.2) and the Gumbel (1.3) distributions. Alternatively the above three

[3] This result is known as the Fisher-Tippett theorem (Fisher and Tippett, 1928), although the formal mathematical proof was provided much later by Gnedenko (1943).

equations [(1.1) to (1.3)] can be concisely written in the so-called Cramer - von Mises representation of the GEV distributions as follows:

$$G_\gamma(x) = \exp(-(1+\gamma x/\sigma)^{-1/\gamma}) . \tag{2}$$

In this formalisation, $\sigma > 0$ is a scale parameter and the three elementary types of extremal behaviour are characterised by $\gamma > 0$, $\gamma < 0$, and the limit in the case $\gamma \to 0$. For the first two types above, the shape parameters of both representations (1.1) and (1.2) are related to the parameters of the von Mises representation by: $\gamma = \pm 1/\alpha$ (+ for (3.1) and – for (3.2)).

From this classification of the extreme values, one can derive a similar classification of the behaviour in the outer parts of the statistical distribution. More specifically, by denoting $W \equiv \text{Prob}[X_i \le x]$, it follows directly from the classification of extremes in (1.1) to (1.3) that if the maximum of a distribution follows a GEV of type i (i = 1,2,3) then the upper tail of the distribution is close to

$$W_{1,\alpha} = 1 - x^{-\alpha}, \qquad x \ge 1, \tag{3.1}$$

$$W_{2,\alpha} = 1 - (-x)^{\alpha}, \qquad -1 \le x \le 0, \tag{3.2}$$

$$W_3 = 1 - \exp(-x), \qquad x \ge 0, \tag{3.3}$$

More formally, the above is usually referred to as the so-called Generalised Pareto Distribution (GPD) with the same tail shape parameter α as the GEV distribution. The Cramer - von Mises representation of the GPD is given by:

$$W_\gamma = 1 - (1+\gamma x/\sigma)^{-1/\gamma} \tag{4}$$

The three elementary types of tail behaviour can be described as hyperbolic decline (3.1), distributions with finite end-points (3.2) and exponential decline (3.3). As in the GEV case they are respectively characterised by $\gamma > 0$, $\gamma < 0$, and the limit in the $\gamma \rightarrow 0$ case (where γ is the inverse of the tail index α). An advantage of estimating this measure of tail behaviour is that allows a number of candidate 'limiting distributions' to be excluded from the outset. For example, an estimate of γ significantly different from zero would imply rejection of the normal distribution, distributions that involve mixtures including the normal and diffusion-jump processes.

A number of estimators have been developed for the 'tail index' α or its reciprocal γ. However, two non-parametric estimators for the tail index stand out because of their widespread use. The first of these is the Pickland estimator (Pickland, 1975), $\hat{\gamma}_P$, which in the case of maximum values reads:

$$\hat{\gamma}_P = (\hat{\alpha}_P)^{-1} = \frac{1}{\ln 2}\ln\left(\frac{x_{n-m+1} - x_{n-2m+1}}{x_{n-2m+1} - x_{n-4m+1}}\right). \tag{5}$$

where (x_i), i=1,…,n are the returns ordered in ascending order and *m* (the number of tail observations to be considered) depends on the total number of observations in the sample.

The second estimator is the Hill estimator, $\hat{\gamma}_H$, which is obtained by maximising the likelihood of the relevant tail function conditional on the chosen size of the 'tail' (Hill, 1975). It is computed as follows:

$$\hat{\gamma}_H = (\hat{\alpha}_H)^{-1} = \frac{1}{m}\sum_{i=1}^{k}[\log x_{(n-i+1)} - \log x_{(n-m)}]. \tag{6}$$

In equation (6), the sample elements are put in descending order: $x(n) \geq x(n-1) \geq ... \geq x(n-m) \geq ... \geq x(1)$ where *m* is the number of 'tail' observations considered.

The Hill estimator, unlike the Pickland estimator, is appropriate only where the Fréchet is the limiting distribution of the extreme values. However, the Hill estimator is more efficient than the Pickland in these circumstances. Some previous empirical studies have used the normal distribution, mixed diffusion jump processes (Press, 1967) and discrete mixtures of normal distributions (Kon, 1984), which lead to the choice of the Gumbel distribution as the limiting distribution for extreme values. However, the reasons behind these choices were linked to convenience rather than solid theoretical support. The Paretian (fat-tailed) distributions all lead to the Fréchet as the choice of the limit, as do ARCH type processes. The Fréchet distribution has almost unanimous support as the choice for the limiting distribution of the extremes in the case of financial market returns where fat tails prevail. Longin (1996) shows empirically that the Fréchet is the correct choice for market return data. Consequently, recent empirical research on financial market returns has concentrated on the Fréchet case.

There is a significant practical problem in applying the Hill (or the Pickland) estimator, namely the method of determining the number of observations in the tail. The problem manifests itself in the choice of the tail length, m. Resolution of this problem necessarily involves an implicit assumption about the true tail index. Given that the exact distribution generating financial market returns data is unknown and the limiting distribution used is only an approximation, it remains the case that estimates obtained will be biased (in general). If optimal estimates are to be obtained (using the MSE criteria) then this bias and the variance must diminish at the same rate as the sample size tends to infinity (Danielsson and de Vries, 1997). However, the bias tends to be non-linear in terms of the sample size and tail length. Thus, the standard procedures (such as a bootstrap) can not be applied. The sub-sample bootstraps proposed by Hall(1990) and developed by Danielson et al. (1996) and Danielson and de Vries (1997) could be used to calculate an optimal length of the tail. However, this complicated procedure depends critically on a large sample size.

3. Methodology

The first step is to estimate point estimates of the tail index for both positive and negative returns for each of the six share price time series using the Hill estimator (equation 6). The Hill estimator was chosen because it is more efficient than the Pickland estimator when the Fréchet is the limiting distribution of the tail and the Fréchet most is likely to be the limiting distribution of the tail given the fat-tailed nature of financial market returns data.

One of the issues to be resolved in applying the Hill estimator is that of choosing the optimal value for m. Although it would be possible to use the sub-sample bootstraps proposed by

Hall (1990) to calculate an optimal value for m, this procedure is not employed in this study given insufficient size in the sample used. Instead, the Hill tail index estimator is calculated over a range (grid) of possible values for m. Following Lux (1996) the values for m range from 5% to 15%. Point estimates of the tail index for each of the six share price data series are estimated using the Hill estimator for values of m equal to 15%, 12.5%, 10%, 7.5% and 5%. These tail index estimates can be used to test the stable Paretian hypothesis, which is rejected if $\alpha>2$. However, while the estimated tail index may be greater than two, the question is whether the statistic is significantly greater than two. The point estimates of the tail index will be unreliable indicators of the true value for the tail index if they have a very large variance. Consequently, we construct confidence intervals for these point estimates for the tail index for both negative and positive returns.

Denoting $\gamma = 1/\alpha$, where α is the tail index and γ_H as the Hill estimate it can be shown that $(\gamma_H-\gamma)m^{1/2}$ is asymptotically normal with zero mean and a variance of γ^2 (Goldie and Smith, 1987). This property can be exploited in constructing asymptotic confidence intervals for the estimated tail index. It follows directly from above that since $(\gamma_H-\gamma)m^{1/2}/\gamma = (\alpha-\alpha_H)_* m^{1/2}/\alpha_H$ is asymptotically distributed as N(0,1), then if we denote by *f* the relevant confidence point from N(0,1) (normal distribution with zero mean and unity variance) then the confidence intervals will be $\alpha_H \pm f * \alpha_H/m^{1/2}$. Upper and lower confidence intervals are calculated at the 95% level for each tail index point estimate. Thus, if $\alpha>2$, but the confidence limits fall either side of 2 then the stable Paretian hypothesis could not be rejected.

Another important issue is whether the distribution is symmetric. Do the tail indexes of the positive and the negative extreme returns coincide, as is expected to be the case for Paretian distributions? If they do coincide, the positive and negative returns can be pooled together to allow for an improved estimate for the tail index. In order to construct an appropriate test statistic to examine symmetry, the asymptotic normality of the Hill estimator can be exploited once again. Since $(\gamma_H-\gamma)m^{1/2} \sim N(0, \gamma^2)$ and therefore $(\gamma_H-\gamma)m^{1/2}/\gamma \sim N(0, 1)$. If we use + and – superscripts to denote Hill estimates for the tail index of the positive and negative returns respectively, then if the true tail index is the same for both these cases the sum (*q*, as indicated in equation 7) represents (asymptotically) a sum of two squared normal random variables. If it is assumed that these variables are independent[4], the sum (*q*) should follow chi square distribution with 2 degrees of freedom.

[4] Unfortunately, the assumption of independence is not testable.

$$q = [(\gamma^{+}-\gamma)(m^{+})^{1/2}/\gamma)]^{2} + [(\gamma^{-}-\gamma)(m^{-})^{1/2}/\gamma]^{2} = (\alpha/\alpha^{+}-1)^{2}\,m^{+} + (\alpha/\alpha^{-}-1)^{2}\,m^{-} \qquad (7)$$

The chi-square test statistic calculated from equation (7) is used to test if the right and left tails of the returns distribution can be treated as symmetrical. If the tails are not symmetric then the stable Paretian hypothesis can be rejected.

One of the most important features of the tail index is that estimates for this statistic can be used to calculate crash probabilities, by taking advantage of the fact that a low value for the tail index indicates a high probability of the occurrence of an extreme event. In calculating these crash probabilities the concept of the quantile function is employed. The usual definition of the s^{th} quantile x_s of a continuous distribution with distribution function F is:

$$x_s = F^{-1}(s) \qquad (8)$$

where F^{-1} is the inverse of the distribution function. In other words the quantile function is the inverse of the distribution function. It is worth noting that the widespread Value at Risk (VaR) measure is usually calculated as the $(1\text{-}s)^{th}$ quantile , as follows:

$$VaR_{1-s} = F^{-1}(1-s) \qquad (9)$$

In this paper an alternative quantile concept is used namely, the so-called return level. Suppose we define:

$$\text{Prob}(X_k > b) = 1/k \qquad (10)$$

where $k>0$, for given b. The above states that the level b will be exceeded once every k periods. If these periods are defined as years, then one would expect this level to be exceeded only once per k years. It is straightforward to show that this alternative definition is equivalent to the one based on the probability distribution. The quantile given by equation (9) is identical to the s^{th} quantile calculated using equation (8) if $k=1/(1-s)$. Equation (9) is used because it can be directly related to the Extreme Value Theory results summarised in equation (4) and thus enables the estimated tail index to be used in calculating quantile-probability pairs.

Equation (4), or more specifically equation (3.1) as the most relevant specification of equation (4) in our case, seems to provide a suitable expression for quantile estimation.

However, the asymptotic nature of the underlying theory requires some additional manipulation to account for the small sample bias of potential estimators, which are directly based on equation (3.1). Furthermore, a direct application of equation (3.1) would also require the estimation of the normalising constants. Essentially, this means fitting the whole asymptotic distribution to the available data, which is a considerable computational burden (see Longin, 1996 for an overview on the distribution fitting methods). In addition, the fact that various estimated values (tail index and two normalising constants) are combined in this approach to calculating the quantile function can lead to serious bias due to the errors arising from the original estimation of these values. The main reason for such potential bias is the fact that the quantile functions are highly non-linear in the tail index and owing to this the standard approximations, which employ the Taylor series, may perform poorly. There are several alternative estimators proposed in the literature that can be used for calculating exceedence probabilities. Danielson and de Vries (1997) suggested a quantile estimator that involves a first order approximation and crucially, depends on the optimal choice of the tail length. Given the approached used in this study, an estimator is required that accounts for the bias arising from the choice of tail length. Consequently, we use the consistent large quantiles estimator due to an earlier suggestion by Dekkers and de Haan (1989)[5]. This estimator is specified as follows:

$$x_p = \{[(km/2pn)\gamma_H - 1]/(1 – 2\ \gamma_H)\}(X_{m/2} - X_m) + X_{m/2} \tag{11}$$

where k is the number of observations (in this case k = 250 which approximates the number of trading days per year) and n and m are the sample size and the number of observations in the tail region, respectively. x_p is the quantile, while X_i , i=1,2, m are decreasing order statistics (i.e. the market returns situated in the tail, ordered in descending order). Since eq. (11) provides us with an expression for the quantile function, the relevant expression for the probabilities associated with given quantile (i.e. any predetermined negative market returns values) can be obtained by simply inverting the latter with regard to the exceedence probability p.

[5] De Haan et al. (1994) proposed an alternative estimator. Our choice of estimator has been determined by practical considerations such as desirable statistical properties and ease of implementation.

4. The Data Set

The data set is made up of daily stock prices for six food-related companies traded on North American stock exchanges. Several important factors were considered in selecting these companies. First, due to the asymptotic nature of the underlying theory, preference was given to those stocks for which reasonably long time series was available. Better known companies were preferred mainly because of better information availability. Portfolio based stocks such as investment funds were omitted in favour of operating firms. The following food processing companies were chosen: Afton Food Group Limited (AFF.TO), Del Monte Foods Company (DLM), Dole Food Company Inc. (DOL), Kraft Foods Inc. (KFT), Vita Food Products Inc. (VSF), and Safeway Inc. (SWY). The symbols in the brackets are the so-called ticker symbols for these stocks. Afton is traded on the Toronto Stock Exchange (TSE), Vita quotations are from AMEX, while all the others are from the New York Stock Exchange (NYSE). The data used is historical daily data from Commodity Systems, Inc. (CSI). The last observation in each data set was the 12 August 2002. The earliest observation varied for each firm providing a range of different sample sizes, ranging from several thousand to several hundreds observations. The larger data sets were those of Dole (beginning 31 January 1985), Safeway (from 26 April 1990). In the case of Afton the data series began on 31 May 1994, for Vita the data set began on 9 May 1997. The smaller samples were those of Del Monte (beginning 5 February 1999,) and Kraft (from 13 June 2001). The available data contains opening, closing, high and low quotations as well as volume traded. In order to rule out phenomena resulting from thin trading, all the stocks selected are traded in significant volumes.

The return variable used in this analysis is the logarithmic return based on the difference between the opening and the closing values on the same day. An alternative approach is to calculate returns using the difference between the closing values of two consecutive days. The latter includes a period of non-trading that may have distorting effects, which can be avoided if returns are calculated on the basis of a single continuous day of trading, so as to focus entirely on market phenomena. It is well known that weekend effects (e.g. markets closing on Friday and reopening on Monday compared to the other weekdays) can create distortions, which may lead to the incorrect rejection of the IID hypothesis tested in this paper. During periods of continuous trading the continual arrival of new information is reflected in market prices. However, during a period of non-trading (say overnight) there is an accumulation of information, which may lead to a sudden shift in market prices when the market re-opens (see Sullivan *et al*. 1998). Note that it is possible in principle that these two different types of information (continuous over the trading period, and accumulated over the period of non-trading) may lead to different inherent price dynamics. Since the stable distributions are characterised by the property of stability-under aggregation, the mixing of two potentially different processes could violate this property even if the stable hypothesis is a valid one for both the continuous and the accumulation cases, because of the mentioned weekend effects. It is therefore appropriate to rule out such a possibility from the very outset of the study. Moreover such shifts may create a spurious fat-tailed character in the data. This would impact on the estimated tail index and ultimately on the estimates of the exceedence probabilities. It may also lead to spurious rejection of the stable Paretian hypotheses.

Most studies calculating tail indices concentrate on exchange rates and stock market indices where zero returns are relatively rare. In the case of individual company shares considered here, there is significant number of zero daily market returns even when the shares are traded in considerable volumes. Consequently, some adjustment to the data set is necessary. This adjustment consists of conflating the zero observations into a single observation, which reduces the total sample to a smaller 'effective' sample to which the conventional criteria for determination of the appropriate tail size can be applied. Note, however, that this additional manipulation is only used in estimating the tail index, because it corrects the distortionary effect of numerous zero returns on the relative measures of tail size (as percentage of the estimation sample). When estimating the quantile-probability pairs (i.e the exceedence probabilities), the full available sample is used.

5. Results

The point estimates for the tail index are presented in table 1. The symbols, + and –, are used to denote the positive and negative returns (i.e. the right and the left tail). A feature of the estimates of the tail index, presented in Table 1, is the degree of uniformity, which suggests that the estimates are not highly sensitive to the choice of the size of the tail. Exceptions to this uniformity are the tail index estimates for companies such as Kraft and Del Monte estimated at the lower (5%) tail sizes. However, sample size for these firms is smaller than for the other firms and at the 5% tail size the sample size is further reduced. Therefore, it is to be expected that with a relatively low number of tail observations the estimation of the tail index may be inefficient. At the other end of the scale, tail estimates based on larger

(15%) tail size may include to many observations from the centre of the statistical distribution. Thus, to some extent, the 10% and 7.5% tail sizes may provide more representative, and therefore, more reliable estimates of the tail index for this specific sample of company shares.

With only a single exception (the negative returns for Afton at the 5% tail size), the point estimates of the tail index for the eight firms are greater than 2. However, before rejecting the IID hypothesis, it is important to examine the 95% confidence intervals for the point estimates of the tail index. The 95% confidence intervals for the point estimates of the tail index are presented in Table 2. Again, with only a single exception, the results in Table 2 indicate that the lower limits of the 95% confidence intervals for the tail index estimates exceed two. Therefore, the IID hypothesis is rejected at the 95% level of significance. This result suggests that the IID hypothesis is inadequate for characterising these market prices.

Using equation (7) the hypothesis that tail indices are identical for the upper and the lower tails of the statistical distributions of the market price returns can be tested. The results from these tests are presented in table 3. The q values represent the calculated chi-square, while the Prob(q) values represent the corresponding probability for this value from the Chi square distribution with 2 degrees of freedom. The chi-square test results presented in Table 3 indicate that, with exception of the cases of the 5% tail size for Afton and Del Monte, the null cannot be rejected at 95% level of confidence. However, the discriminative power of the chi square test used here may be low[6]. Note that the confidence intervals for the tail index estimate for the lower and upper tails (presented in table 2) rarely overlap. Furthermore, with only a few exceptions (likely due to small samples), the estimates for the left and the right tail index differ in the same direction when estimated over a range of tail sizes (see Table 1). This systematic deviation probably indicates that the assumption of independence used in constructing the q statistic does not hold. Consequently, the left and right tail indices are treated separately despite the chi-square test results.

The primary aim of the paper is to demonstrate the use of the tail index in calculating exceedence probabilities for the selected market asset returns. These probabilities can be calculated by inverting equation (11) with regard to the probability p (i.e. solving for p). The exceedence probability is then calculated for predetermined loss levels. These exceedence

[6] This means that while rejecting the null means violation of the assumption for identical tail indices, failure to reject it may be due to other unspecified reasons. In other words if the tail indices are identical, the null must to hold. The fact that it holds however does not necessarily mean that the indices are identical.

probabilities are presented in table 4 for different tail sizes and negative returns ranging from 0.15 to 0.40. The probabilities in table 4 are annual probabilities which denote the number of times a negative return exceeding the level indicated is likely to occur within the period of one year. Thus, for example, the exceedence probability at 10% tail size for negative return of 40% for Del Monte is approximately 0.25. Basically, this means that for Del Monte shares (asymptotically, i.e. in the long run) negative daily market returns are expected to exceed 40% on average 0.25 times per year or once every 4 years (=1/0.25). Clearly, the result changes when the exceedence probability calculated from the tail index estimated at a different tail size is used. Therefore, there is uncertainty about which exceedence probability to use when making financial decisions. Nevertheless, these calculations do provide useful information. Note that when only the 12.5%, 10% and 7.5% tail sizes are considered the values for the exceedence probabilities are more alike. As indicated earlier, it is likely that the other tail sizes produce rather biased results. In a situation where the consequence of a loss is high, a more conservative approach may be favoured and investors may prefer to rely on estimates based on larger tail sizes.

The range of the exceedence probability estimates across the firms in the sample is considerable. When only the 12.5%, 10% and 7.5% tail sizes are considered the exceedence probabilities for a 40% loss range from 0.033 (Kraft) to 0.6 (Afton). This means the expected frequency of a single occurrence of a 40% fall in share price for these firms is between 1.5 and 30 years. Averaging the highest and lowest exceedence probabilities for each firm estimated from the 12.5%, 10% and 7.5% tail sizes indicates that a 40% drop in share price over a single day of trading can be expected to occur on ‘average’ (for these firms) about every four to six years.

Huisman et al. (2001) propose a complex procedure that essentially can be viewed as a weighted average of tail indices (exceedence probabilities) calculated from different sizes of the tail region, which eliminates the ad-hoc choices generated by estimating the exceedence probability over a range of tail sizes. However, even where a single estimate of an exceedence probability is produced, most users of this information will want to know the associated confidence intervals for the statistic (based on the upper and the lower values of the tail index). Thus, an advantage of the simple approach of providing estimates for a range of tail sizes is that the range of estimates themselves can be viewed as proxy confidence intervals.

The exceedence probabilities calculated in this paper provide an estimate of the likelihood of sudden falls in share price, as well as providing a measure of the volatility of the different shares considered. The results in Table 4 suggest that the six firms can be grouped (provisionally) into two groups. The first group is made up of Dole, Kraft and Safeway. Based on the results obtained in this paper, the share prices of these firms (in comparison to the others) appear to be less volatile and less likely to suffer large and sudden declines. The second group includes Afton, Del Monte, and Vita. The share prices of these firms appear to be more volatile compared to those of the first group. It is useful to compare and contrast the firms classification in these two groups.

One noticeable difference between the two groups is that all the representatives of the more stable group of stocks are traded on NYSE (see Table 5), while the most volatile stock is Afton, which is traded in Toronto. This raises the possibility that the institutional characteristics of the different stock exchanges are an important influence on volatility. It may be that the registration requirements of the different stock exchanges act to segment the stock market. Alternatively, the operation of 'circuit breaker' and 'trading halt' mechanism may differ across stock exchanges. Further, more detailed research that is beyond the scope of this paper is required to investigate whether these factors influence share price volatility. It is probable that the most important difference between the two groups is in their levels of turnover (see Table 5). The average turnover of group one is measures in $billions, while the average turnover of group two is measured in $millions. This provides a clear suggestion that the level of a firm's turnover influences share price volatility. Other than to acknowledge the fact that food processing firms are found in both groups, there is little to be gained from examining the relationship between the firms' business type and stock volatility.

6. Conclusions

This paper demonstrates the utilisation of the concept of the tail index in testing whether the stable Paretian hypothesis is the true underlying model generating market returns for a selection of food firms and in calculating exceedence probabilities for given critical levels of loss (or share price decline).

In empirical terms, testing the stable Paretian hypothesis is equivalent to testing the IID hypothesis. Our results based on a selection of food firms unequivocally reject this hypothesis. Nevertheless this does not in any sense imply market inefficiency, since the semi-martingale form of the 'efficient markets hypothesis' still holds. The rejection of the stable Paretian hypothesis in this paper agrees with the findings of Lux (1996) following an examination of leading German stocks. Indeed, the emerging picture from recent literature is that, despite initial appearances, the distributions of stock returns are not characterised by the stable laws.

An important advantage of the tail index is that it can be used in calculating extreme quantiles and associated probabilities for share price movements, which means the risks of a large decline (of a given magnitude) in a share's price can be estimated. Such information is extremely important for investors, particularly those investors who may have to liquidate their assets at short notice. The analysis carried out in this paper indicates that a 40% drop in share price over a single day of trading can be expected to occur on average about every four to six years in the case of the firms considered. The analysis indicated that the food firms considered could be divided into two groups based on the exceedence probabilities calculated. One group included firms where the likelihood of a large decline in share price was greater than that of the other group. An examination of the characteristics of the firms in each group revealed that the levels of turnover for the firms in the group with the greater likelihood of a large decline in share price were considerably lower when compared to those of the firms in the other group.

The method of using tail index estimates to obtain exceedence probabilities has been demonstrated in this paper using company stocks, but could be used to assess the risk of holding any market asset. Indeed, we see no reason why the procedure should not be used to assess the risk associated with trading in agricultural commodities. While it is possible to manage the price risk associated with some agricultural commodities through hedging strategies, there are many agricultural commodities for which this is not possible. For these commodities good knowledge of the downside risk of holding these commodities is all the more important. The storability of many agricultural commodities means that farmers have the option of delaying sale when they believe prices may improve. The risk in doing so is that prices may fall, thus we believe that estimating the tail index of agricultural commodity price distributions with a view to calculating exceedence probabilities may provide farmers with valuable information for decision making purposes.

All food firms need to operate within the limitations of cash flow constraints. Because of this a sudden extreme loss ensuing from the fall in the price of some market asset they are currently holding, may compromise their financial abilities to meet cash outflows. In such a situation the calculation of the exceedence probabilities associated with this critical level of loss may dramatically improve risk management practices with regard to decisions about whether or not to hold certain market assets.

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Table 1. Point Estimates of the tail index, α.

Size of the tail (%)	Total Sample	Effective sample	15	12.5	10	7.5	5
afton+	1288	357	2.4576	3.0515	3.7484	4.0676	4.0894
afton-	1288	323	2.2479	2.7311	2.5977	2.4630	1.8633
del monte+	889	378	2.3402	2.9194	2.8570	2.5818	3.1436
del monte-	889	401	2.8694	3.3070	3.2839	4.6705	6.9572
dole+	4436	1911	2.8212	2.9004	3.0291	3.2024	3.3518
dole-	4436	1958	2.6234	2.7709	2.8461	3.0074	3.1102
kraft+	292	145	5.2253	4.6748	4.3556	3.4057	3.0995
kraft-	292	126	3.2410	3.3533	3.7801	3.7355	4.4710
safeway+	3101	1375	3.1848	3.1873	3.3269	3.4766	3.8183
safeway-	3101	1385	2.9121	3.0530	2.9969	3.1101	3.3114
vita+	828	181	2.6648	2.8378	3.2566	3.0535	2.9606
vita-	828	190	3.7077	3.4534	3.2687	4.6273	3.6073

Table 2. The 95% confidence intervals for the tail index point estimates

Tail size (%)	15	12.5	10	7.5	5
afton+	(2.36842,2.54681)	(2.93046,3.17261)	(3.58262,3.91418)	(3.85665,4.27854)	(3.82968,4.34916)
afton-	(2.16185,2.33404)	(2.61703,2.84517)	(2.47518,2.72029)	(2.32964,2.59646)	(1.73678,1.98982)
del monte+	(2.25858,2.42192)	(2.80757,3.0312)	(2.73431,2.97969)	(2.45313,2.7104)	(2.94987,3.33727)
del monte-	(2.77201,2.96685)	3.18443,3.42953)	(3.14678,3.4211)	(4.44622,4.8947)	(6.5406,7.37383)
dole+	(2.77854,2.86391)	(2.85229,2.94843)	(2.97299,3.08525)	(3.13386,3.27089)	(3.26396,3.43962)
dole-	(2.5842,2.66261)	(2.72558,2.81628)	(2.79406,2.89816)	(2.94368,3.07103)	(3.02956,3.19086)
kraft+	(4.90331,5.54725)	(4.35737,4.99221)	(4.02198,4.68928)	(3.09983,3.71166)	(2.74733,3.45166)
kraft-	(3.02842,3.45363)	(3.10736,3.59922)	(3.47640,4.08379)	(3.37683,4.09425)	(3.96304,4.97903)
safeway+	(3.12791,3.24178)	(3.12482,3.24977)	(3.2542,3.3997)	(3.38874,3.56452)	(3.69983,3.93686)
safeway-	(2.86026,2.96385)	(2.99349,3.11247)	(2.93157,3.06214)	(3.03186,3.18833)	(3.20939,3.41343)
vita+	(2.52366,2.80600)	(2.67241,3.00321)	(3.04298,3.47023)	(2.81958,3.28738)	(2.67635,3.24494)
vita-	(3.51545,3.90001)	(3.25725,3.64963)	(3.06107,3.47629)	(4.28797,4.96671)	(3.28332,3.93136)

Table 3. Chi square tests for identical left and right tails of the statistical distributions for market returns and their significance levels

Tail size (%)	15	12.5	10	7.5	5
afton					
q[1]	0.18486	0.24036	2.1999	3.2677	6.1810
Prob(q)[2]	0.08829	0.11324	0.66711	0.80483	0.95452
del monte					
q	1.1536	0.3553	0.35174	5.6543	7.6071
Prob(q)	0.43832	0.16277	0.16127	0.94082	0.97771
Dole					
q	0.75741	0.24844	0.37107	0.28212	0.26688
Prob(q)	0.31525	0.11681	0.16934	0.13156	0.12492
Kraft					
q	2.0185	0.74561	0.10597	0.032395	0.36239
Prob(q)	0.6355	0.3112	0.051605	0.016067	0.16573
safeway					
q	0.81802	0.15684	0.74474	0.63484	0.69364
Prob(q)	0.33569	0.075426	0.3109	0.27197	0.29307
vita					
q	1.3554	0.38244	0.000106	1.0776	0.14777
Prob(q)	0.49221	0.17405	5.31E-05	0.41657	0.071223

Note 1. q is the chi-square test statistic.

Note 2. Prob(q) denotes the significance levels of the null that there is no difference between the tail indexes. If Prob(q)>0.95 then the null hypothesis is rejected.

Table 4 Annual Exceedence probabilities for different tail sizes

Tail size (%)		15	12.5	10	7.5	5
	Loss (%)					
Afton	0.15	2.20810	2.31920	1.95200	1.47840	0.91372
	0.20	1.99570	1.38130	1.30170	1.12150	0.85113
	0.25	1.82710	0.99523	0.98647	0.91069	0.79866
	0.30	1.68990	0.78455	0.80027	0.77136	0.75399
	0.35	1.57600	0.65179	0.67726	0.67238	0.71549
	0.40	1.47990	0.56040	0.58989	0.59838	0.87387
Del Monte	0.15	1.23710	0.83061	0.68852	0.21920	0.06399
	0.20	0.94315	0.61325	0.50508	0.14813	0.04052
	0.25	0.76808	0.49021	0.40228	0.11299	0.02997
	0.30	0.65184	0.41099	0.33648	0.09202	0.02398
	0.35	0.56898	0.35567	0.29072	0.07807	0.02011
	0.40	0.50689	0.31484	0.25704	0.06812	0.01740
Dole	0.15	0.75826	0.56924	0.41305	0.26742	0.18518
	0.20	0.58363	0.43392	0.31185	0.19914	0.13649
	0.25	0.47803	0.35334	0.25251	0.15996	0.10900
	0.30	0.40724	0.29985	0.21348	0.13453	0.09133
	0.35	0.35644	0.26172	0.18584	0.11668	0.07900
	0.40	0.31820	0.23316	0.16523	0.10346	0.06991
Kraft	0.15	0.28414	0.20020	0.14086	0.08751	0.06028
	0.20	0.21424	0.15013	0.10493	0.06478	0.04423
	0.25	0.17334	0.12108	0.08430	0.05186	0.03523
	0.30	0.14648	0.10210	0.07090	0.04352	0.02947
	0.35	0.12747	0.08872	0.06150	0.03769	0.02547
	0.40	0.11330	0.07877	0.05453	0.03338	0.02252
Safeway	0.15	0.79095	0.63977	0.58259	0.51231	0.32654
	0.20	0.52023	0.41538	0.37852	0.33221	0.20399
	0.25	0.39183	0.31095	0.28346	0.24856	0.15007
	0.30	0.31682	0.25054	0.22843	0.2002	0.11972
	0.35	0.26761	0.21113	0.19252	0.16867	0.10024
	0.40	0.23281	0.18338	0.16723	0.14648	0.086668
Vita	0.15	1.10890	1.02270	0.90805	1.62570	0.79308
	0.20	0.68560	0.63830	0.55574	0.17313	0.39656
	0.25	0.50185	0.46915	0.40501	0.040868	0.26803
	0.30	0.39908	0.37394	0.32129	0.093229	0.20438
	0.35	0.33339	0.31285	0.26799	0.06459	0.16637
	0.40	0.28775	0.27029	0.23105	0.049853	0.14108

Table 5. Firm Characteristics.

Company	Group	Turnover[1]	Business Type	Stock Market[2]
Afton	2	Can$31m	Franchises	TSX
Del Monte	2	$1,300m	Food Processing	NYSE
Dole	1	$4,500m	Food Processing	NYSE
Kraft	1	$33,800m	Food Processing	NYSE
Safeway	1	$34,000m	Food Retailing	NYSE
Vita	2	$22m	Food Processing	AMEX

Note 1. The turnover figures for each firm are given in US dollars, except in the case of Afton where the figure is in Canadian dollars.

Note 2. TSX indicates the Toronto stock exchange, while NYSE indicates the New York stock exchange and AMEX represents the Chicago stock market.