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Analysis and Performance Evaluation of the ZEM/ZEV Guidance And Its Sliding Robustification For Autonomous Rendezvous in Relative Motion

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Abstract

Devising closed-loop guidance algorithms for autonomous relative motion is an important problem within the field of orbital dynamics. In this paper, we study the guided relative motion of two spacecraft for which one of them is executing an autonomous rendezvous via the ZEM/ZEV feedback guidance and its robustified Optimal Sliding Guidance (OSG) counterpart. Starting from the classical Clohessy-Wiltshire (CW) model, we systematically analyze the ability of the ZEM/ZEV feedback guidance to generate closed loop trajectories that drive the deputy spacecraft to the chief satellite and evaluate its performance in terms of target accuracy and propellant consumption. It is shown that the guidance gains and the time of flight predicted by the theoretical solution generates a class of feedback trajectories that are accurate but suboptimal with respect to the open-loop fuel-optimal solution. Indeed, a parametric study shows that a different set of gains may generate relative guided trajectories that yields fuel consumption closer to the ideal optimal. The guidance algorithms are also demonstrated to be accurate in guiding the relative motion of the deputy toward a chief spacecraft in highly elliptical orbit where the Linearized Equations of Relative Motions (LERM) are employed to compute the Zero-Effort-Miss (ZEM) and Zero-Effort-Velocity (ZEV) necessary to compute the acceleration command as prescribed by the theory.

Keywords: Relative Motion, Closed-loop Guidance, Optimal control

1. Introduction

Devising closed-loop guidance and control algorithms for autonomous relative motion is an important problem within the field of orbital dynamics. Over the past twenty years, a large variety of control schemas for chief-deputy relative motion in circular and elliptical orbits have been proposed and studied. Such algorithms included impulsive and continuous control using both Cartesian and orbital element formulations. For example, Schaub et al. [1],[2] provided the basis for devising impulsive feedback control algorithms using mean orbital elements. More recently, following a similar line of though, Anderson and Schaub [3] devised an N-impulse control schema for formation flight in geostationary orbit using a non-singular element description. On the continuous control side, both linear and non-linear feedback approaches have been considered. Naasz et al.[4] solved the relative motion control problem via the H2/Hinf approach using the Clohessy-Wiltshire equations. Massari and Zamaro [5] proposed a control algorithm based on the solution of the state-dependent Riccati Equation. Queiroz et al. [6] presented a non-linear Lyapunov based approach to devise an adaptive controller for multiple spacecraft in formation flight. More recently, Sherrill et al. [7] proposed a method for continuous control of spacecraft formation flight in elliptical orbits based on Lyapunov-Floquet theory. The proposed controller featured a set of time-varying feedback gained to guide the deputy spacecraft toward the rendezvous with the chief satellite.

However, generating closed-loop feedback trajectories that are rooted in optimal control theory is not an easy task. Recently, generalized Zero-Effort-Miss/Zero-Effort-Velocity (ZEM/ZEV) feedback guidance [8] and its robustified version known as Optimal Sliding Guidance (OSG) [9] have been developed and applied for both planetary landing and general space guidance. The ZEM/ZEV feedback guidance has been studied extensively and can be found in the literature for intercept, rendezvous, terminal guidance and landing applications. Such analytical closed-loop guidance has been originally conceived by Battin[10] who devised an energy optimal, feedback acceleration command for powered planetary descent. Ebrahimi et al. [11] introduced the ZEV concept, as a partner for the well-known ZEM and integrated it with a sliding surface for missile guidance with fixed-time propulsive maneuvers. Furfaro et al. extended the idea to the problem of lunar landing guidance and set the basis for the theoretical development of a robust closed-loop algorithm for precision landing. The ZEM/ZEM feedback guidance is attractive because of its analytical simplicity as well as potential for quasi-optimal fuel performance for constant gravitational field. When robustified by a time-dependent sliding term, the resulting OSG can be proven to be Globally Finite-Time Stable (GFTS) in spite of perturbation with known upper bound.

In this paper, we study the guided relative motion of two spacecraft for which one of them is executing an autonomous rendezvous via the ZEM/ZEV feedback guidance as well as its robustifed OSG counterpart. When augmented via time-dependent sliding, the application of Lyapunov stability theory for non-autonomous systems provides the sufficient conditions for GFTS. Indeed, the OSG can be demonstrated to be GFTS for any linear and non-linear relative motion model (e.g. rendezvous in circular orbit or in highly eccentric orbit). Starting from the classical Clohessy-Wiltshire (CW) model, we systematically analyse the ability of the ZEM/ZEV feedback guidance to execute closed-loop maneuvers and its ability to correct disturbances for precision guidance. Comparison with numerically-based, open-loop, fuel efficient solution will provide an assessment of the algorithm to execute not only precise but also quasi-optimal feedback rendezvous trajectories. Additional analysis and comparison with the OSG counterpart will provide an assessment of the need for robustification as function of different rendezvous conditions and different thrusting conditions (unlimited versus limited thrust).

2. Guidance Model and Algorithm Development

2.1 Relative Motion Guidance Model

The relative motion of a deputy spacecraft with respect to the chief satellite is commonly described in the Local-Vertical Local-Horizontal (LVLH) coordinate frame. The latter is attached to the chief satellite. In the usual representation of the LVLH coordinate frame, x is directed as the chief satellite radial direction, z is oriented in the direction of the chief's angular momentum (orbital), and y is consequently oriented such that the LVLH frame is right orthogonal and right-handed. In this framework, the $x-y$ coordinates describe the deputy in-plane motion and the z coordinate describes the out-of-plane motion. For highly eccentric orbits, the equations of relative motion can be described using a linearized model, commonly known as Linearized Equations of Relative Motion (LERM):

$$\ddot{x} - 2\dot{f}\dot{y} - \left(\dot{f}^2 + 2\frac{\mu}{r^3}\right)x - \ddot{f}y = a_{Cx} + a_{px} \tag{1}$$

$$\ddot{y} + 2\dot{f}\dot{x} + \ddot{f}x - \left(\dot{f}^2 - 2\frac{\mu}{r^3}\right)y = a_{Cy} + a_{py} \tag{2}$$

$$\ddot{z} + \frac{\mu}{r^3}z = a_{Cz} + a_{pz} \tag{3}$$

Where f is the true anomaly of the chief orbit, μ is the gravitational parameter of the central body, r is the orbital radius of the chief, a_{Cx}, a_{Cy}, a_{Cz} are the components in the LVLH framework of acceleration command (feedback) and a_{px}, a_{py}, a_{pz} are the components of the perturbing acceleration. The latter may include higher-order terms not considered in the linear dynamics and additional modelled perturbing acceleration different than the two-body Newtonian term (e.g. higher-order gravitational harmonics, solar radiation pressure, third-body perturbation, etc.). The equations can be rewritten in a more compact form using a state-space formulation:

$$\dot{\boldsymbol{r}} = \boldsymbol{v} \tag{4}$$

$$\dot{\boldsymbol{v}} = \begin{bmatrix} \left(\dot{f}^2 + 2\frac{\mu}{r^3}\right) & \ddot{f} & 0 \\ -\ddot{f} & \left(\dot{f}^2 - 2\frac{\mu}{r^3}\right) & \mathbf{0} \\ 0 & 0 & -\frac{\mu}{r^3} \end{bmatrix}\boldsymbol{r} + \begin{bmatrix} 0 & 2\dot{f} & 0 \\ -2\dot{f} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\boldsymbol{v} + \boldsymbol{a}_C + \boldsymbol{a}_p \tag{5}$$

Here, we set $\boldsymbol{r} = [x, y, z]^T$, $\boldsymbol{v} = [\dot{x}, \dot{y}, \dot{z}]^T$, $\boldsymbol{a}_C = [a_{Cx}, a_{Cy}, a_{Cz}]^T$ and $\boldsymbol{a}_p = [a_{px}, a_{py}, a_{pz}]^T$. The CW equations are customarily obtained by setting the chief eccentricity equal to zero, resulting the following:

$$\ddot{x} - 2n\dot{y} - 3n^2x = a_{Cx} + a_{px} \tag{6}$$

$$\ddot{y} + 2n\dot{x} = a_{Cy} + a_{py} \tag{7}$$

$$\ddot{z} + 3n^2z = a_{Cz} + a_{pz} \tag{8}$$

Or

$$\dot{\boldsymbol{r}} = \boldsymbol{v} \tag{9}$$

$$\dot{\boldsymbol{v}} = \begin{bmatrix} 3n^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -n^2 \end{bmatrix} \boldsymbol{r} + \begin{bmatrix} 0 & 2n & 0 \\ -2n & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{v} + \boldsymbol{a}_C + \boldsymbol{a}_p \quad (10)$$

Both models will be employed to evaluate the application of the proposed guidance algorithms for relative motion in circular and eccentric orbit. A Theory section should extend, not repeat, the background to the article already dealt with in the Introduction and lay the foundation for further work. In contrast, a Calculation section represents a practical development from a theoretical basis.

2.2 Generalized ZEM/ZEV and Its Robustification: Theoretical Algorithm

The ZEM/ZEV feedback guidance algorithm has its roots in optimal control theory. Indeed, the proposed closed-loop guidance is determined analytically by finding the acceleration command that satisfies the unconstrained energy-optimal control problem in a constant (or time-varying) gravitational field and assuming no perturbation is acting on the spacecraft. The following definitions hold true:

Def: We define Zero-Effort-Miss (**ZEM**) as the distance (vector) the spacecraft misses the target if no acceleration command is executed after time t. Formally:

$$\boldsymbol{ZEM}(t) = \boldsymbol{r}_f - \boldsymbol{r}(t_f), \boldsymbol{a}_C(\tau) = \boldsymbol{0}, \; \tau \in [t, t_f] \quad (11)$$

Def: We define Zero-Effort-Velocity (**ZEV**) as the velocity (vector) the spacecraft misses the target velocity if no acceleration command is executed after time . Formally:

$$\boldsymbol{ZEV}(t) = \boldsymbol{v}_f - \boldsymbol{v}(t_f), \boldsymbol{a}_C(\tau) = \boldsymbol{0}, \; \tau \in [t, t_f] \quad (12)$$

With ZEM and ZEM formally defined, one can formally solve the following energy optimal guidance problem, i.e. *find the acceleration command $\boldsymbol{a}_C$ as function of $\boldsymbol{ZEM}$ and $\boldsymbol{ZEV}$ that minimizes the energy-optimal cost (quadratic control effort):*

$$J(\boldsymbol{a}_C) = \int_t^{t_f} \boldsymbol{a}_C(\tau)^T \boldsymbol{a}_C(\tau) d\tau \quad (13)$$

Subject to the dynamical equations of motion as physical constraints

$$\dot{\boldsymbol{r}} = \boldsymbol{v} \quad (14)$$
$$\dot{\boldsymbol{v}} = \boldsymbol{g}(t) + \boldsymbol{a}_C \quad (15)$$

with initial conditions at time t $\boldsymbol{r}(t), \boldsymbol{v}(t)$ and final conditions $\boldsymbol{r}_f, \boldsymbol{v}_f$. Here, the acceleration command is assumed to be unbounded, i.e. no constraints in the acceleration (thrust) magnitude. The problem can be solved by a straightforward application of the Pontryagin's Minimum Principle (PMP) to determine a set of necessary conditions for the existence of an optimal solution. For this specific case, the resulting Two-Point Boundary Value Problem (TPBVP) has an analytical solution. Indeed the acceleration command can be expressed as linear function of $\boldsymbol{ZEM}(t)$ and $\boldsymbol{ZEV}(t)$ and t_{go} as follows:

$$\boldsymbol{a}_C = \frac{k_R}{t_{go}^2} \boldsymbol{ZEM}(t) + \frac{k_V}{t_{go}} \boldsymbol{ZEV}(t) \quad (16)$$

Here, the optimal guidance gains are found to be $k_R = 6$ and $k_V = -2$. Following D'Souza [13] or Guo [14], an alternative formulation of the generalized ZEM/ZEV guidance algorithm can be determined ("DrDv formulation"):

$$\boldsymbol{a}_C = -\frac{6}{t_{go}^2}(\boldsymbol{r}(t) - \boldsymbol{r}_f) - \frac{4}{t_{go}}(\boldsymbol{v}(t) - \boldsymbol{v}_f) - \boldsymbol{g} \quad (17)$$

The two formulations of the guidance algorithm are perfectly equivalent only in the case of constant gravitational field. The time-to-go is determined by applying the transversality condition $H(t_f) = 0$. The later generally results in a quartic equation that yields only one feasible positive solution [13].

Following Wibben and Furfaro [12], the proposed guidance can be robustified by a straightforward inclusion of a sliding control mode into the guidance, yielding the so-called Optimal Sliding Guidance (OSG):

$$\boldsymbol{a}_C = \frac{k_R}{t_{go}^2} \boldsymbol{ZEM}(t) + \frac{k_V}{t_{go}} \boldsymbol{ZEV}(t) - \Phi sgn(\boldsymbol{s})) \quad (18)$$

Here, one defines a time-dependent sliding surface as follows:

$$\boldsymbol{s}(t) = \boldsymbol{ZEV} + \frac{k_R}{k_V t_{go}} \boldsymbol{ZEM} \quad (19)$$

The sliding dynamics is shown to obey the following dynamics

$$\dot{\boldsymbol{s}} + \frac{2}{t_{go}} \boldsymbol{s} = -\Phi sgn(\boldsymbol{s}) \quad (20)$$

Importantly, OSG can be shown to be Globally Finite-Time Stable (GFTS) against perturbations and unmodelled dynamics with known upper bound. For the theoretical proof of the GFTS see Wibben and Furfaro [9]

2.3 Guidance Adaptation to Autonomous Relative Motion

The theoretical development of the generalized ZEM/ZEV feedback law if strictly energy optimal in the case of constant gravity. In such a case, Guo et al.[8] showed that the guidance algorithm comes to be few percent more expensive than the open-loop, fuel efficient solution. Furthermore, the authors demonstrated that for space guidance its fuel efficiency depends on the gravity gradient, i.e. wherever high gravity gradients are experienced, the guidance algorithm deviates quite significantly from optimality (both energy and fuel efficiency). Despite the efficient drawback, the ZEM/ZEV guidance represents a simple analytical algorithm that can be easily mechanized for relative motion. Indeed, the adaptation is fairly straightforward. In the proper ZEM/ZEV formulation, one is simply required to propagate forward the equation of motion from time t until t_f to compute online the required $\boldsymbol{ZEM}$ and $\boldsymbol{ZEV}$ at each guidance cycle. For circular orbits, one can take advantage of existing analytical solution for the CW model. Conversely, elliptical orbits may require numerical propagation of the LERM model.

The DrDv formulation does not need any onboard numerical or analytical propagation but it relies on the knowledge of the "instantaneous" environmental modelled acceleration $\boldsymbol{g}(\boldsymbol{r},\boldsymbol{v})$. For the CW model, one obtains the following expression:

$$\boldsymbol{g}(\boldsymbol{r},\boldsymbol{v}) = \begin{bmatrix} 3n^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -n^2 \end{bmatrix}\boldsymbol{r} + \begin{bmatrix} 0 & 2n & 0 \\ -2n & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\boldsymbol{v} \tag{21}$$

Conversely, for the LERM model, one obtains the following:

$$\boldsymbol{g}(\boldsymbol{r},\boldsymbol{v}) = \begin{bmatrix} \left(\dot{f}^2 + 2\frac{\mu}{r^3}\right) & \ddot{f} & 0 \\ -\ddot{f} & \left(\dot{f}^2 - 2\frac{\mu}{r^3}\right) & \boldsymbol{0} \\ 0 & 0 & -\frac{\mu}{r^3} \end{bmatrix}\boldsymbol{r} + \begin{bmatrix} 0 & 2\dot{f} & 0 \\ -2\dot{f} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\boldsymbol{v} \tag{22}$$

In addition, mechanizing the OSG to the relative motion guidance requires computing the sliding surface at each guidance cycle and the definition of a proper sliding gain Φ which needs to be tuned as function of the upper bounds of the perturbing acceleration and the desired reaching time (design parameters).

3. Results

In this section, we evaluate the ability of the proposed guidance to drive the deputy toward the chief spacecraft for autonomous rendezvous. Performances are analysed in terms of accuracy and fuel efficiency.

3.1 Initial Simulations: Comparison between ZEM/ZEV and DrDv formulations

The first step toward the analysis of the proposed algorithm within the framework of the guided relative motion dynamics is to analyse the behaviour of the algorithm within the CW model. Indeed, the CW equations, represented by the various formulations described by Eq. (6)-(10), have been implemented in a MATLAB simulation environment to simulate closed-loop trajectories as generated by the generalized ZEM/ZEV guidance algorithm using both available formulations (see Eq. (16) and Eq.(17)). It is assumed that the chief satellite is in an initial circular orbit at an altitude of $7500\ km$. The deputy satellite is located in an initial relative position $\boldsymbol{r}(0) = [7047m,\ 5136m,\ 5013m]^T$ in the LVLH coordinate frame. The initial relative velocity is $\boldsymbol{v}(0) = \left[-2.4\,\frac{m}{s}, -13.7\frac{m}{s}, 4.08\,\frac{m}{s}\right]^T$. The deputy satellite is driven by the generalized ZEV/ZEV feedback algorithm to rendezvous with the chief satellite, i.e. the target point is the origin of the LVLH coordinate system to be achieved with zero terminal velocity. The deputy spacecraft is assumed to have a mass of $2000\,kg$, exhibiting a propulsion system with specific impulse of $I_{sp} = 204\ sec$. No specific constraint on the thrust/acceleration command is considered for this initial set of simulations. Moreover, no perturbing acceleration is considered, i.e. $\boldsymbol{a}_p = \boldsymbol{0}$. Fig.1 shows the histories of the guided trajectory components for both ZEM/ZEV (Eq. (16)) and DrDv (Eq.(17)) formulations. Fig.2 shows the histories of the guided velocity components.

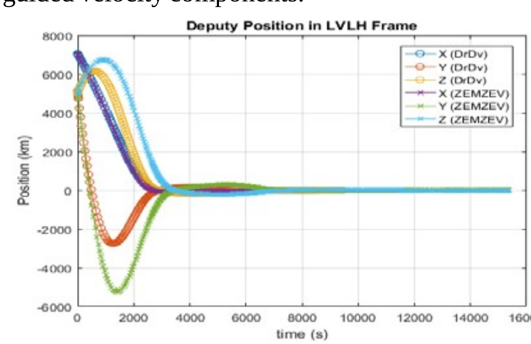

Fig. 1: Histories of the deputy position components in

the LVLH relative frame

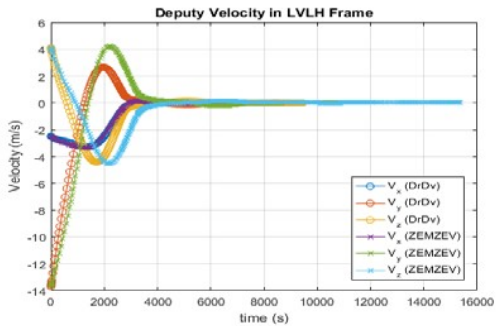

Fig. 2: History of the deputy velocity components in the LVLH relative frame

As clearly demonstrated by the plots, both formulations of the algorithm accurately drive the deputy spacecraft to rendezvous with the chief, i.e. both relative position and velocity are driven to zero. Fig.3 shows the history of the closed-loop acceleration command. Fig. 4 shows the history of the spacecraft mass.

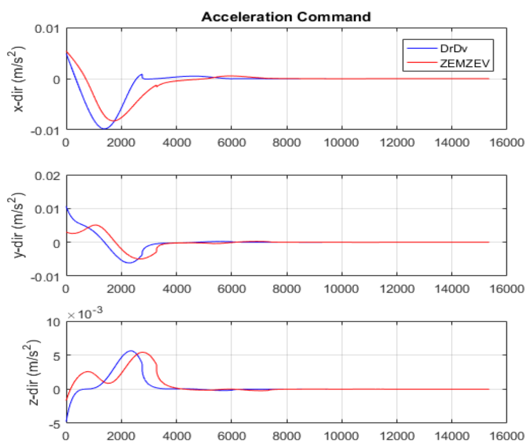

Fig. 3: Histories of the components of the acceleration command for both formulations

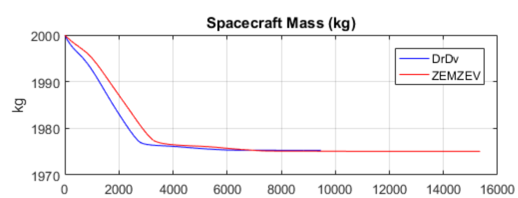

Fig. 4: History of the spacecraft mass

The implementation and mechanization of the two formulations are different. Indeed, whereas the DrDv formulation requires only knowledge of the current relative position, velocity and instantaneous environmental acceleration (Eq. (17)), the ZEM/ZEV formulation requires an on-board computation of the $\boldsymbol{ZEM}$ and $\boldsymbol{ZEV}$ for each guidance cycle. Both misses have been computed by using the known analytical solution of the CW equations to speed-up the simulation time. Although the feedback acceleration command and consequently trajectories and velocities are slightly different, they seem to exhibit similar final result. Indeed, the final spacecraft mass is very close (the computed difference in the final mass is about 0.21 kg). However, the ZEM/ZEV formulation results in a longer flight time, i.e. $15352\ sec$ versus $9465.7\ sec$ reported for the DrDv formulation. As shown in Fig.3, the latter exhibits a higher peak of the acceleration command but generally distributed over a shorter flight time.

3.2 Limited Thrust Case: Comparison with the Optimal Solution

In the second set of simulations, a fuel-efficient, open-loop solution with limited thrust magnitude has been numerically computed to evaluate the performances of the guidance algorithm in terms of propellant mass consumption. In this case, the same initial conditions, initial deputy mass and propulsion systems have been assumed. As in the previous simulation, the goal is to drive the deputy relative position and velocity to zero, i.e. rendezvous with the chief spacecraft. Importantly, the maximum thrust of the magnitude is assumed to be $\|\boldsymbol{T}\|_{max} = 16N$. The fuel-efficient open-loop solution has been numerically computed using the optimal control software package called General Pseudospectral Optimal Control Software (GPOPS [14]) which enables a rapid prototyping of the fuel-efficient guidance problem in a MATLAB platform.

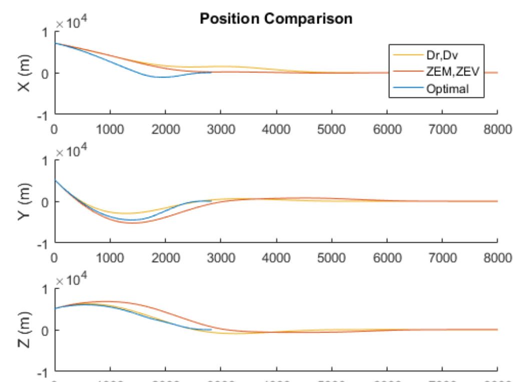

Fig. 5: Histories of the feedback ZEM/ZEV guidance position as compared with the GPOPS open-loop, fuel-optimal solution.

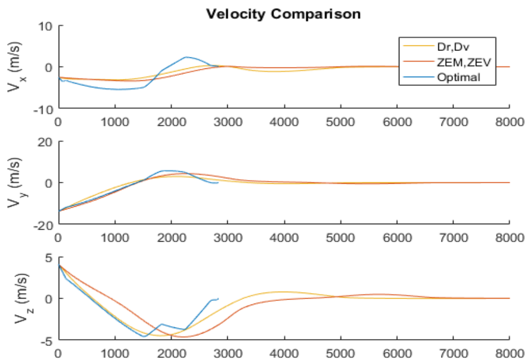

Fig. 6: Histories of the feedback ZEM/ZEV guidance velocity as compared with the GPOPS open-loop, fuel-optimal solution.

Fig. 5 and Fig. 6 shows the histories of the two ZEM/ZEV guided trajectories and velocities as compared to the GPOPS fuel-optimal solution. As clearly noted, the closed-loop trajectories are completely different than the optimal solution. The fuel-optimal solution shows a lower flight time when compared the proposed guidance algorithms. Fig. 7 shows the histories of the spacecraft mass and the computed magnitude of the thrust command. As expected, the optimal solutions exhibits an on-off type of behavior and achieves the desired rendezvous state using three finite burns (blue lines).

Table 1: Comparison between ZEM/ZEV algorithm and GPOPS optimal solution

	Final Mass (kg)	*Time of Flight (sec)*
ZEM/ZEV	1967	18455
DrDv	1956	10944
Fuel-Optimal	1992.6	2837.1

As shown in Table 1, the guidance algorithm perform in a suboptimal fashion. Such result was expected as the analytical closed-loop guidance is strictly energy optimal only in the case of constant gravity field. While the solution is quasi-fuel-optimal for guided landing in large planetary bodies, the performance in terms of propellant consumption in relative motion are not expected to be excellent. Indeed, in spite of targeting accuracy, the high spatial variation exhibited by the acceleration environment (R.H.S. of Eq.(6)-(10)) heavily influences the fuel performance of the proposed guidance. However, for the limited thrust case, the ZEM/ZEV formulation seems to outperform the DrDv formulation, yielding an 11 kg saving in mass propellant consumption. Indeed, the thrust magnitude histories (Fig. 7) shows that, toward the initial part of the guided flight, the DrDv formulation tend to saturate faster than the ZEM/ZEV counterpart and keeps the maximum thrust for longer flight time. Conversely, the ZEM formulation guidance tends to exhibit higher thrust values toward the mid to end of the guided flight, although at much lower thrust levels.

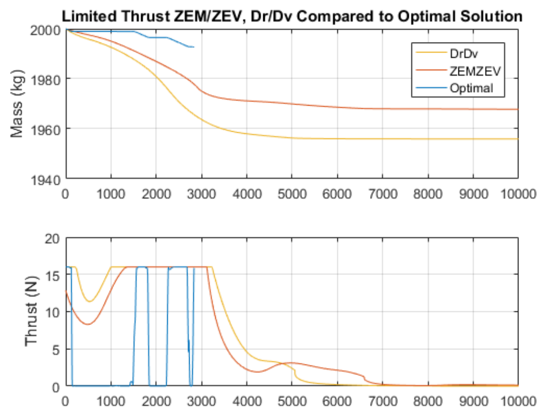

Fig. 7: Histories of deputy spacecraft mass and thrust magnitude for the ZEM/ZEV guidance in both formulations and the fuel-efficient guided trajectory computed via GPOPS

3.3 Guidance Parametric study

The proposed guidance is shown to be sub-optimal in terms of fuel efficiency. The theory shows that the guidance gains need to be $k_R = 6$ for $k_V = -2$ for the ZEM/ZEV formulation and $k_R = -6$ for $k_V = -4$ for the DrDv formulation. Whereas the gains are optimal for the constant gravity field case, it may not be the case for relative motion. The on-line computation of the t_{go} generally determines the optimal time of flight, which again may not be generally optimal for relative motion.

A parametric study has been conducted to explore the fuel-consumption performance as function of k_R, k_V and t_F. An initial set of simulations have been conducted by executing a guided rendezvous using the DrDv formulation with the same previously considered initial conditions. In this case, the value of k_R is kept fixed to the theoretical optimal value (-6) and k_V, t_F are varied parametrically. Fig. 8 shows that the spacecraft propellant consumption exhibits a minimum for $k_V = -3.25$ and $t_F = 2100$ sec. A second set of simulations have been considered by keeping the theoretical value of $k_V = -4$ fixed and varying k_R, t_F. Fig. 9 shows that the deputy spacecraft propellant mass consumption exhibits a minimum for $k_R = -14$ and $t_F = 3400$ sec. Importantly, keeping fixed the k_V gain yields a lower propellant consumption.

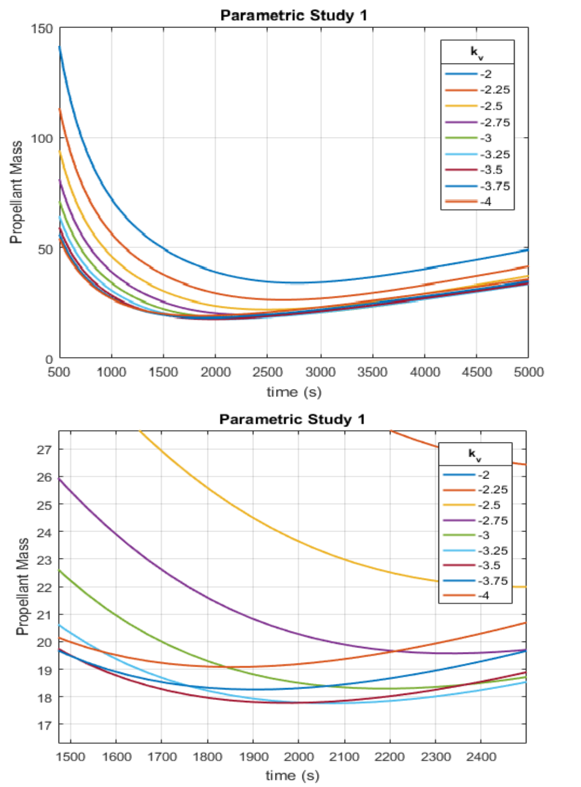

Fig. 8: Propellant Mass as function of the time-of flight with k_V as parameter (k_R is kept fixed). The bottom panel is a zoomed toward the minimum.

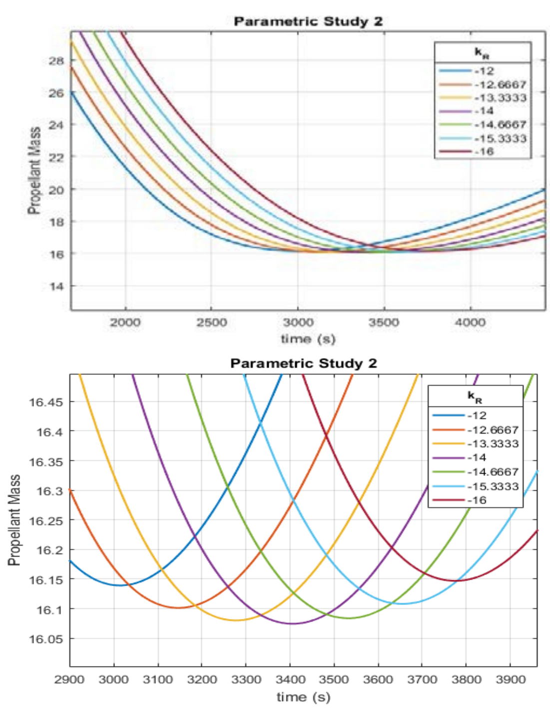

Fig. 9: Propellant Mass as function of the time-of flight with k_R as parameter (k_V is kept fixed). The bottom panel is a zoomed toward the minimum.

Finally, the mass of consumed propellant and the final spacecraft mass have been computed varying at the same time both guidance gains and time of flight. Fig. 10 shows a contour plot of the propellant mass as function of the guidance gains for $t_F = 5000\ sec$. The simulations have been conducted on a finer grid in an attempt to find the minimum propellant by brute computational force. However, it is found that the mass of propellant exhibits many local minima. On the chose grid, we were able to find the best minimum at $M_{prop} = 13.42\ kg$ for $k_R = -45.15, k_V = -6.35, t_F = 6200\ sec$. This is equivalent to a final spacecraft mass of $1986.58\ kg$ which is closer to the ideal fuel-efficient open loop solution that yields a final mass of $1992.6\ kg$ with a total difference of about $6\ kg$ of propellant. The latter shows that the theoretical optimal gains are not quasi-optimal for the relative motion problem.

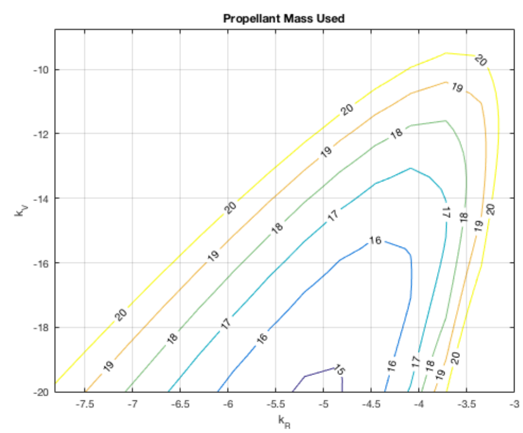

Fig. 10: Contour plot of the propellant mass as function of k_R and k_V ($t_F = 5000\ sec$ is kept fixed).

3.4 OSG Guidance Simulation

The generalized ZEM/ZEV feedback guidance robustified with the sliding mode (i.e. OSG see Eq. (18)) is theoretically shown to be GFTS against perturbing accelerations with known upper bound. A simulation has been considered to show the effect of the sliding mode on the guidance algorithm. Using the same initial conditions for relative motion in circular orbit and propulsion system, OSG-drive relative motion have been simulated for a limited thrust magnitude of $16N$. Here it is assumed a sliding gain $\Phi = 0.001\ m/sec^2$ although no specific perturbing acceleration is considered Fig. 11 shows the deputy spacecraft mass as function of time for both the ZEM/ZEV formulation and OSG. For this specific case and in absence of perturbing accelerations the ZEM/ZEV guidance outperforms the OSG in terms of mass propellant.

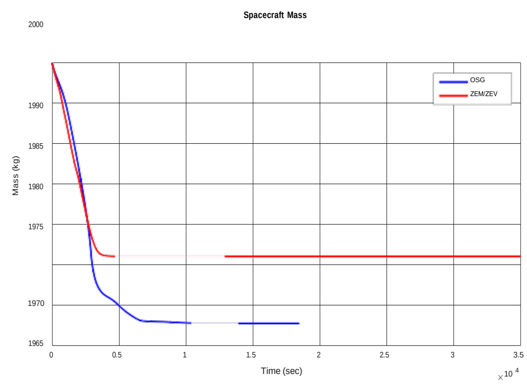

Fig. 11: Histories of the deputy spacecraft mass for the ZEM/ZEV formulation and OSG

As shown in Fig. 12, OSG seems to saturate faster than the ZEM/ZEV counterpart a, which is mostly responsible for wasting more propellant mass.

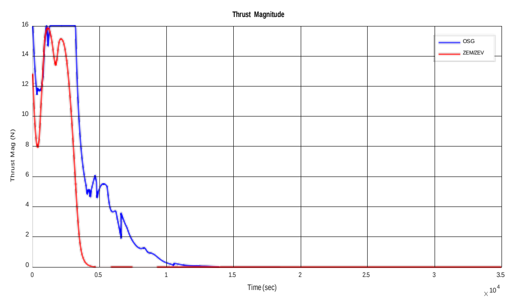

Fig. 11: Histories of the thrust magnitude for the ZEM/ZEV formulation and OSG

However, this initial analysis is not conclusive and performances should be re-evaluated in the context of performance with perturbed acceleration where the sliding may show advantages in terms of accuracy under limited thrust or thrust failures [9].

3.5 ZEM/ZEV guidance for highly elliptical orbit

For highly elliptical orbits, the CW model is not valid anymore. LERM can be employed to simulate a more realistic dynamics of relative motion whenever the distance between the chief and deputy satellites are small. The feedback ZEM/ZEV can be still implemented in both formulations. In this case the LERM equations can be employed for an on-line numerical calculation of the $\boldsymbol{ZEM}$ and $\boldsymbol{ZEV}$ vectors, as well as to evaluate the instantaneous value of the environmental acceleration $\boldsymbol{g}(\boldsymbol{r}, \boldsymbol{v})$. To simulate the ability of the ZEM/ZEV guidance to drive the deputy spacecraft to the rendezvous target using the LERM model, we assumed the chief satellite to be in a highly elliptical orbit with eccentricity $e = 0.15$, perigee at $r_p = 7500\ km$ and inclination $i = 45\ deg$. Initial conditions and spacecraft mass are assumed to be the same as in the previous simulation. The thrust magnitude is assumed to have a maximum of $16N$. Fig. 12 shows the histories of the relative closed-loop trajectory and velocity components of the deputy spacecraft. Fig. 13 show the history of the spacecraft mass and thrust magnitude of the ZEM/ZEV formulation.

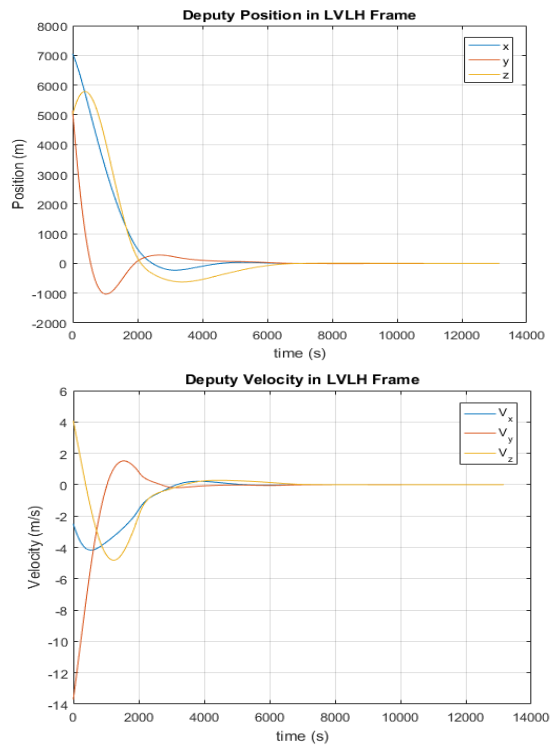

Fig. 12: Histories of the trajectories components and velocity components for the LERM-based ZEM/ZEV guidance algorithm.

Importantly, the LERM-based ZEM/ZEV guidance algorithm accurately drives the deputy spacecraft to the desired target. It is observed that the closed-loop thrust command saturates during the first portion of the guided flight. No comparison with an open-loop optimal solution has been done at this stage.

6. Conclusions

An analysis of the performances of the feedback ZEM/ZEV guidance algorithm and its robustified OSG counterpart have been conducted in the context of chief-deputy relative motion. It is shown that the algorithms under investigation can generate closed-loop trajectories that accurately drive the deputy spacecraft to rendezvous with the chief satellite. However, using the guidance gains and the time-to-go as prescribed by the theory generate closed-loop trajectories that are far from the fuel-optimal value. A parametric study has been conducted to show that the guidance gains and the time of flight can be changed to achieve a closer-to-fuel-optimal performance. However, it is clear that to get performances closer to the fuel-optimal point, the gains and time of flight may be changed adaptively, i.e. as function of the current relative position and velocity. The OSG has been theoretically demonstrated to be GFTS. An initial simulation show that including a sliding mode may help with propellant consumption, although the benefits and advantages of such augmentation need to be the subject of further investigation. The ZEM/ZEV algorithm is shown to accurately work in the case of highly elliptical orbit where the LERM model has been employed to iteratively compute the $\boldsymbol{ZEM}$ and $\boldsymbol{ZEV}$ vectors. Further analysis of the proposed guidance algorithms behavior in perturbed environments will be conducted in the near future.

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