A brief review on frictional pressure drop reduction studies for laminar and turbulent flow in helically coiled tubes

Andrew Michael Fsadni, Justin P.M. Whitty, Matthew A. Stables

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Title: A brief review on frictional pressure drop reduction studies for laminar and turbulent flow in helically coiled tubes

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A brief review on frictional pressure drop reduction studies for laminar and turbulent flow in helically coiled tubes

Abstract

This review, summarises the pertinent literature on drag reduction (DR) in laminar and turbulent flow in coiled tubes. Due to their compact design, ease of manufacture and superior fluid mixing properties, helically coiled tubes are widely used in numerous industries. However, flow through coiled tubes yields enhanced frictional pressure drops and thus, drag reduction is desirable as it can: decrease the system energy consumption, increase the flow rate and reduce the pipe and pump size. The main findings and correlations for the friction factor are summarised for drag reduction with the: injection of air bubbles and addition of surfactant and polymer additives. The purpose of this study is to provide researchers in academia and industry with a concise and practical summary of the relevant correlations and supporting theory for the calculation of the frictional pressure drop with drag reducing additives in coiled tubes. A significant scope for future research has also been identified in the fields of: air bubble and polymer drag reduction techniques.

Keywords: Helically coiled tube, drag reduction, frictional pressure drop, surfactants, polymer solutions.

1. Introduction

Due to their compact design, ease of manufacture and high efficiency in heat and mass transfer, helically coiled tubes are widely used in a number of industries and processes such as in the food, nuclear, aerospace and power generation industries and in heat recovery, refrigeration, space heating and air-conditioning processes. Due to the formation of a secondary flow, which inherently enhances the mixing of the fluid, helically coiled tube heat exchangers are known to yield improved heat transfer characteristics when compared to straight tube heat exchangers. The secondary flow, which finds its origins in the centrifugal force, is perpendicular to the axial fluid direction and reduces the thickness of the thermal boundary layer. However, for single and multiphase flows, the secondary flow yields a substantial increase in the frictional pressure drop, which often results in diminished system efficiencies (due to enhanced pumping power requirements). For air-water two-phase bubbly flow in helically coiled tubes, Akagawa et al. (1971) reported frictional pressure drops in the range of...
Figure 1: Schematic representation of helical pipe characteristics.

1.1 to 1.5 times greater than those in straight tubes, ceteris paribus, whilst, with the use of nanofluids, such a penalty could nullify the enhanced efficiencies gained with the dispersion of nanoparticles in the base fluid (Aly, 2014). Moreover, due to the secondary flow, the flow characteristics are significantly different to those in straight tubes. Whereas in straight tubes the transition from laminar to turbulent flow occurs at Reynolds numbers in the region of 2500, the transition in curved tubes takes place at higher Reynolds numbers. The critical Reynolds number (Eq. (1)) is used to determine the transition of the flow from laminar to turbulent flow (Ito, 1959).

$$Re_{crit} = 2E4 \delta^{0.32}$$  \hspace{1cm} (1)

where $\delta$ is the curvature ratio defined through Eq. (2).

$$\delta = \frac{d_I}{D_c}$$  \hspace{1cm} (2)

For $\delta^{-1} < 8.6E2$ whilst for $\delta^{-1} > 8.6E2$, $Re_{crit}$ for a curved tube is equal to that for a straight pipe.

Another dimensionless number, unique to coiled tubes, is the Dean number, given in Eq. (3). It is used to characterise the flow in curved tubes and quantifies the magnitude of the secondary flow due to the centrifugal force (Mohammed and Narrein, 2012).

$$De = Re \sqrt{\delta}$$  \hspace{1cm} (3)

The performance of coiled tubes is a complex function of the coil design parameters (Fig. 1) as well as the resultant pressure drop. Therefore, drag reduction (DR) techniques could be particularly beneficial for systems with curved tubes. Intriguingly, whilst numerous investigations have been reported on DR in straight channels and pipelines with the: injection of air bubbles (Nouri et al., 2013; Fujiwara et al., 2004), dispersion of surfactants (Gasljevic and Matthys, 1997) and polymers (Wei and Willmarth, 1992; Al-Sarkhi and Hanratty, 2001), there is a paucity of research in the field of curved tubes. Moreover, researchers have reviewed the frictional DR techniques in straight channels and pipes (Merkle and Deutsch, 1992; Al-Sarkhi, 2010; Murai, 2014) whilst the sole study that reviewed DR in curved tubes was presented by Broniarz-Press et al. (2007). However, the latter focussed on the application of DR surfactant and polymer additives and hence, did not provide a holistic review of the relevant studies. The aim of the current study is to critically review the experimental and numerical studies done on DR in single-phase (water) laminar and turbulent flow through coiled tubes. Such studies are categorized in three sections, representing the pertinent techniques reported. Moreover, this paper complements the earlier review undertaken by the authors of the present study (Fsadni and Whitty, 2016), as it further elucidates the underpinning physics of air-water bubbly flow through curved tubes. It is the authors’ hope that this review will be useful to both academics and industry based engineers through the provision of a concise report on the relevant current knowledge.

2. Injection of air bubbles

Over the past 40 years, the injection of microbubbles in the turbulent boundary layer has been investigated by numerous investigators, with the first study reported by McCormick
and Bhattacharyya (1973) who investigated the DR to a submersible hull. As summarised in Table 1, Shatat et al. (2009a&b) were the first to investigate DR with the injection of air bubbles in laminar and turbulent low through helically coiled tubes. They reported a diminished DR efficiency (Eq. (4)) over that of straight tubes. Such results were more significant with higher curvature ratios whilst, the DR increased with higher air volumetric void fractions (VF) and decreased with higher Re numbers (Fig. 2). Moreover, DR was limited to turbulent flow. Similar results were reported by Saffari et al. (2013) who measured a 25% DR at a VF of 0.09 in turbulent flow bubbly flow. The latter study did not investigate the DR with straight tubes. However, their experimental parameters are comparable to those used by Nouri et al. (2013) who reported a DR of 35% for a VF of 0.09 in straight tubes.

\[ DR = 100 \left( \frac{f_l - f_{tp}}{f_l} \right) \]  

where \( f_l \) is the Fanning friction factor for single-phase flow and \( f_{tp} \) is the friction factor for two-phase flow.

For a straight vertical pipe, Fujiwara et al. (2004) reported that, with a high VF in the near-wall region, the turbulence intensity and Reynolds stress are reduced in a wide region of the pipe. The turbulence energy dissipation occurs around the bubbles due to bubble-induced eddies, whilst the diminished fluid density in the near-wall region reduces the shear stress, thus resulting in a lower system frictional pressure drop. Saffari et al. (2013) reported that in curved tubes, higher Re numbers and curvature ratios, result in larger centrifugal forces which force the lower density phase (air bubbles) to migrate towards the inner tube wall region. Resultantly, the shear stress at the inner tube wall region is lower than that at the outer wall region. Hence, the uneven distribution of the air bubbles at higher Re numbers and curvature ratios results in a diminished DR efficiency.

These studies are in a general agreement with relevant theory and numerous DR studies reported for channel and straight tube flow. Moreover, there is significant scope for further research in DR (in coiled tubes) as a function of the bubble diameter. In fact, for
straight tubes and channels, some controversy surrounds the impact of bubble size on the DR, where some investigators reported the DR to be a strong function of the bubble diameter (Liu 1993; Murai et al., 2007) while other investigators reported the DR to be independent of the bubble diameter (Moriguchi and Kato, 2002; Shen et al., 2006). The relation of the bubble induced DR studies with those reviewed for two-phase gas-liquid frictional pressure drop characteristics in coiled tubes (Fsadni and Whitty, 2016) remains indeterminate. In fact, the latter investigations reported a general agreement with the Lockhart and Martinelli correlation for straight tubes, with the two-phase frictional pressure drop multiplier in excess of unity.

3. Surfactant additives

Surface-active agents (surfactants) are low molecular weight, viscous, non-polymer, water-based chemicals that tend to accumulate at a surface and diminish interactive forces between the molecules of the base fluid, thus reducing the surface tension. Inaba et al. (2005) reported that surfactant additives form a network structure of rod-like micelles which absorbs the turbulent energy with its flexibility and deformation, thus leading to a flow laminarisation effect. Hence, surfactants enhance the elastic properties of the fluid with the resultant increase in DR. Unlike polymer based fluids, the mechanical degradation of the micelle network at high shear stresses is completely reversed at a low flow rate. All the studies reviewed reported a DR limited to the transition and turbulent flows, with a reduced DR in curved tubes when compared to straight tubes, ceteris paribus. Such findings were attributed to the formation of the secondary flow which is largely unaffected by the surfactant additive. Gasljevic and Matthys (1999) reported that for a velocity range of 2-5m/s, the secondary flow effects were separated from the turbulence effects through the use of the turbulence reduction – drag (TRD) method given in Eq. (5). This yielded a TDR of 70% (turbulence suppression) for both coiled and straight tubes (Fig. 3). In contrast, Broniarz-Press et al. (2003) reported that the tube curvature effect on the friction factor was diminished due to the damping of the secondary flows streams. A broad analogy can be made with nanofluid flow in coiled tubes where, nanoparticles were also attributed to the mitigation of the secondary flow (Fsadni and Whitty, 2016).

\[
TRD = \frac{F_{Cbf,th} - F_{Cbf,im}}{F_{Cbf,im}}
\]  
(5)

where \( l_m \) refers to the laminar flow of the base fluid (without the DR additives) at the same \( Re \) number and \( F_{th} \) refers to the turbulent flow of the base fluid.
Figure 3: Friction reduction in terms of DR and TRD for a coiled and straight pipe (Gasljevic and Matthys, 1999 Fig. 4).

At laminar flow conditions, Weber et al. (1991) and Gasljevic and Matthys (2009) reported an increase in the frictional pressure drop (compared to water). This was attributed to the enhanced solution viscosity. There is a general agreement amongst the studies reviewed that lower coil curvatures and higher surfactant concentrations yielded higher DR efficiencies. Moreover, Kamel and Shah (2013) reported that at higher concentrations, surfactant solutions are more resistant to mechanical degradation and hence, yield higher DR efficiencies at increased Re numbers. Therefore, Broniarz-Press et al. (2002) reported that DR is a strong function of the surfactant concentration, with DR evident above a critical concentration. Inaba et al. (2005) reported that the dynamic nature of surfactant DR additives render them particularly relevant for heating systems. However, such comments should be considered in light of the fact that these additives are known to yield reduced heat transfer coefficients. Kostic (1994) attributed this phenomenon to the non-homogenous turbulence resulting from the flow-induced anisotropy of the highly structured micelle network. Weber et al. (1991), Inaba et al. (2000&2005), Aly et al. (2006) and Kamel and Shah (2013) presented correlations for the calculation of the friction factor in surfactant solutions. Due to the Non-Newtonian properties of these solutions (C>3,000ppm), correlations were developed as a function of the modified or generalised Re and De numbers.

4. Polymers additives

Toms (1948) reported that the addition of minute concentrations of high-molecular weight, long chain and flexible polymers to a Newtonian solvent can yield significant DR properties. Whilst it is widely accepted that the DR efficiency is a strong function molecular weight and distribution, molecular structure and solubility, the underpinning physics are known to be complex and not well-understood (Gallego and Shah, 2009). Factors such as shear thinning, viscoelasticity and molecular stretching have been suggested to diminish the turbulence in the fluid (Bird et al., 1987), thus resulting in DR.

Shah and Zhou (2001) stated that the DR mechanism of polymers occurs at the boundary layer and therefore is typically more effective in smaller tube diameters. Moreover,
in agreement with the findings reported for air-bubble injection, DR efficiency decreases with higher coil curvatures. This is inherent to the effects of the centrifugal force on the fluid flow. DR is also a function of the ability of the polymer to resist thermal and mechanical degradation. Shah et al. (2006) reported that at a volume concentration of 0.07%, the widely used partially hydrolysed polyacrylamide (HPA) copolymer (Nalco ASP-820) yielded the highest DR (65%). At this concentration, it was assumed that the fluid behaviour is quasi-Newtonian. This concentration was subsequently used by Gallego and Shah (2009) and Ahmed Kamel (2011). Gallego and Shah presented a unique generalised friction pressure correlation for DR polymer solutions in coiled tubes. Their correlation assumed that the appropriate characteristic polymer solution viscosity is relative to the zero shear rate viscosity, that is, the shear stress required to deform the polymer molecule from its equilibrium state.

The effect of the polymer concentration is also function of the specific physical conditions of the flow. Resultantly, Shah and Zhou (2001) reported that for large tubes and low flow rates, high concentrations of polymer additives increased the fluid drag and delayed the onset of DR (Fig. 4). For small diameter tubes, the opposite effect was reported and thus, a higher polymer concentration increased the DR.

The effect of elevated temperatures on the DR of polymers in coiled tubes was investigated by Gallego and Shah (2009) and Ahmed Kamel (2011) who reported that, in contrast to the findings for straight tubes, DR remained quasi-constant (Ahmed Kamel) or increased (Gallego and Shah) with temperature. It is widely accepted that with polymer solutions in straight tubes, elevated temperatures yield a drop in the DR. This is due to a combination of factors, such as the deterioration of the solvent-polymer interaction and the diminishing of the macromolecule size (Clifford and Sorbie, 1985; Nesyn et al., 1989). In view of this complexity and the paucity of studies for curved tubes, Gallego and Shah (2009) and Ahmed Kamel (2011) concluded that the origins of their results are indeterminate and thus require further investigation. In contrast to the numerous studies on polymer DR additives to gas-liquid flows in straight tubes (Sylvester and Brill, 1976; Al-Sarkhi and...
Soleimani, 2004), there are no related studies for coiled tubes. This presents further scope for future research in the field of two-phase flow in coiled tubes.

<table>
<thead>
<tr>
<th>Investigators &amp; Methodology</th>
<th>Year</th>
<th>Flow configuration &amp; coil geometry</th>
<th>Mean bubble size</th>
<th>Void fraction or concentration</th>
<th>Drag reduction</th>
</tr>
</thead>
</table>
| Shatat et al. Experimental   | 2009a & b | $d_r=20\text{mm}$  
$D_c=800,400,200\text{mm}$  
$\delta=0.025,0.05,0.1$  
$H=40\text{mm}$  
$1,000<Re<100,000$  
$We<1.0$  
Laminar and turbulent bubbly flow | $d_{b,\text{min}}=0.06\text{mm}$  
$d_{b,\text{max}}=0.174\text{mm}$  
No deformation of bubbles. | $0.21<VF<0.44\%$ | 16% for $\delta=0.025$. For a straight pipe 51% $DR$, ceteris paribus. DR effect starts at the critical $Re$ number. DR increases with $VF$ for all cases. The curvature of the coils had a negative effect on drag reduction. The $Re$ number corresponding to the maximum DR was shifted to a higher value (compared to a straight tube). This shift increased with an increase in the curvature of the coil. |
| Saffari et al. Experimental  | 2013  | $d_r=12,19,19\text{mm}$  
$D_c=200\text{mm}$  
$\delta=0.06,0.095$  
$H=24\text{mm}$  
$P=0.101\text{MPa}$  
$10,000<Re<50,000$  
Turbulent bubbly flow | $d_{b,\text{min}}=0.27\text{mm}$  
Bubble diameter decreased at higher $Re$ numbers. At lower $Re$ numbers, bubbles were less spherical in shape (less rigid). This is due to the influence of flow stress and reduced surface tension (in comparison to the smaller bubbles). | $0.01<VF<0.09$ | DR increased with $VF$ with a maximum of 25% at a $VF$ of 9%. DR diminished with higher $Re$ numbers. At a low $VF$ of 1%, a DR of 9% was measured. DR diminished with an increase in the curvature of the coil. |
| Saffari and Moosavi Numerical (Eulerian-Eulerian multiphase model) | 2014  | $d_r=16,25,40\text{mm}$  
$D_c=100,200\text{mm}$  
$\delta=0.08,0.125,0.20$  
$H=20,60$  
$15,000<Re<80,000$  
Turbulent bubbly flow | $d_{b,\text{min}}=0.1\text{mm}$  
No deformation of bubbles. | $0.01<VF<0.09$ | Due to a reduction in the mixture density, higher $VF$ yields lower pressure drops, shear stress and friction coefficient. |
<table>
<thead>
<tr>
<th>Source</th>
<th>Year</th>
<th>Parameters</th>
<th>Fluid Properties</th>
<th>Conditions</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weber et al.</td>
<td>1991</td>
<td>$d=10.5,16.5\text{mm}$ $D_c&lt;454\text{mm}$ $0.105&lt;d_c&lt;0.036$, $N=12,18,34,39$ $1,500&lt;Re&lt;100,000$ $6,750&lt;Re_{crit}&lt;9,480$ $30^\circ\text{C}&lt;T&lt;90^\circ\text{C}$ Laminar and turbulent</td>
<td>$C=62.5;250;1,000 \text{ppm}$ Habon in water.</td>
<td>For laminar flow, surfactant additives increased the fluid drag. For turbulent flow the increase in DR with $C$ was marginal. DR in curved tubes diminished at a lower $Re$ value than that in straight tube, ceteris paribus.</td>
<td></td>
</tr>
<tr>
<td>Gastljevic and Matthys</td>
<td>1999</td>
<td>$d=2\text{mm}$ $D_s=200\text{mm}$ $\delta=0.01$ $1.8&lt;V&lt;7\text{m/s}$ $T=25^\circ\text{C}$ Laminar and turbulent</td>
<td>$C=2,000 \text{ppm}$ SPE95285 (Same viscosity as water)</td>
<td>DR in coiled tube is 30%, in a straight tube 60%, ceteris paribus. Calculated 70% reduction in turbulence effects for both straight and coiled tubes. At $V&gt;5\text{m/s}$ DR effect diminishes due to micelle degradation.</td>
<td></td>
</tr>
<tr>
<td>Inaba et al.</td>
<td>2000</td>
<td>$d=17.7\text{mm}$ $D_s=177,300,9,442,5,885\text{m}$ $\delta=0.02,0.04,0.059,0.1$ $400&lt;Re&lt;200,000$ $10^\circ\text{C}&lt;T&lt;25^\circ\text{C}$ $\theta=45^\circ,90^\circ,180^\circ,270^\circ$ Laminar and turbulent</td>
<td>$C&lt;1,773$ ppm Dodecytrimethylammonium Chloride ($C_{12}H_{25}N(CH_3)_3=263.89$) and Sodium Salicylate ($C_7H_5NaO_3=160.10$) in water</td>
<td>No DR at laminar flow conditions, whilst DR at turbulent flow conditions was less in relation to that in a straight pipe. At a $C$ of 561ppm no DR was measured. Due to the suppression of turbulence vortexes, the heat transfer coefficient was less than that for water.</td>
<td></td>
</tr>
<tr>
<td>Broniarz-Press et al.</td>
<td>2002</td>
<td>$0.0219&lt;\delta&lt;0.0792$ $1,200&lt;Re_{krit}&lt;30,000$ $70&lt;De^*=3,000$ $T=303,323,333K$ Laminar and turbulent</td>
<td>$WC=0.1,0.25$% Cationic Hexadecyltrimethylammonium chloride (HTAC) and</td>
<td>DR is only evident above a critical $C$. This contrasts to polymers where DR is significant with minute $C$ of polymer additives.</td>
<td></td>
</tr>
</tbody>
</table>

Surfactant solutions & Foam fluids

\[ f_{\text{Darcy}} = 6.75\left(\frac{D_c}{d_c}\right)^{-0.56} \theta^{0.146} De^{-0.5} \]

$0.02<\delta<0.05; 45^\circ<\theta<270^\circ; C>1,000 \text{ppm}$ ($SD=9.17\%$)
<table>
<thead>
<tr>
<th>Study</th>
<th>Year</th>
<th>Conditions</th>
<th>Fluid Properties</th>
<th>Results</th>
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</thead>
<tbody>
<tr>
<td>Broniarz-Press et al.</td>
<td>2003</td>
<td>$0.0219 &lt; \delta &lt; 0.0792$, $1200 &lt; Re_{gen} &lt; 30000$, $70 &lt; De &lt; 3000$, $T = 303,313,333 K$</td>
<td>Laminar and turbulent</td>
<td>DR only evident when the molar mass is above a critical value. Cylindrical micelles stabilise the mechanisms of curved flow. DR increases with higher turbulence.</td>
</tr>
<tr>
<td>Inaba et al.</td>
<td>2005</td>
<td>$d = 14.4 mm$, $D_s = 540 mm$, $\delta = 0.0267$, $H = 32 mm$, $N = 10$, $100 &lt; G_z/G_s &lt; 10000$, $5^\circ C &lt; T &lt; 20^\circ C$, Laminar and turbulent</td>
<td>Non-Newtonian viscoelastic fluid.</td>
<td>DR observed in turbulent and pseudolaminar flows. Surfactant additives diminished the tube curvature effect on the friction factor. This was attributed to the damping of the secondary flow streams.</td>
</tr>
</tbody>
</table>

With polymer additives, DR is only evident when the molar mass is above a critical value. Cylindrical micelles stabilise the mechanisms of curved flow. DR increases with higher turbulence.

**Additional notes:**
- **Broniarz-Press et al.**
  - Experimental study with specific conditions and observations.
  - Fluid properties: Laminar and turbulent with specified ranges for molar mass, temperature, and other parameters.
  - Results: DR observed under specific conditions.

- **Inaba et al.**
  - Experimental study with specific conditions and observations.
  - Fluid properties: Non-Newtonian viscoelastic with conditions for temperature, concentration, and other parameters.
  - Results: DR observed under specific conditions, with implications for friction factor and tube curvature effects.
where:

\[
\frac{f_{c, Darcy}}{f_{st, Darcy}} = D e^{0.42 \cdot \frac{T_{nd}}{T_{critical} + 0.11 \cdot T_{nd}}}
\]

\[
T_{nd} = \frac{T_{actual}}{T_{critical} + 0.11 \cdot T_{nd}}
\]

\[
C_{nd} = \frac{c_{actual}}{c_{critical} + 0.11 \cdot T_{nd}}
\]

### Aly et al. 2006

**Experimental**

- \(d=14.4\text{mm}\)
- \(D_c=320,540,800,\text{mm}\)
- \(0.018<\delta<0.045\)
- \(H=32\text{mm}\)
- \(N=10\)
- \(1.000<Re^{'<100,000}\)
- \(5^\circ\text{C}<T<20^\circ\text{C}\)
- Laminar and turbulent

Newtonian fluids for \(C<3,000\text{ppm}\).

- 250<\(C<5,000\) ppm
- Mixture of non-ionic surfactant olelyldihydroxy ethylamineoxide (ODEAO, \(C_2H_{41}NO_3=371\)) 90%, & cetyldimethylaminodiacetic acid betaine (CDMB, \(C_2H_{41}NO_2=327\)) 10% as a zwitterion surfactant in water.

- \(\delta_{nd} = 0.35\text{mm}\)
- \(\delta_{st} = 0.45\text{mm}\)

- \(K_{nd} = \frac{T_{actual}}{T_{critical}(275K)}\)
- \(K_{st} = \frac{c_{actual}}{c_{critical} + 0.11 \cdot T_{nd}}\)

- \(T_{nd} = \frac{T_{actual}}{T_{critical} + 0.11 \cdot T_{nd}}\)

- \(C_{nd} = \frac{c_{actual}}{c_{critical} + 0.11 \cdot T_{nd}}\)

- \(f_{Darcy} = \frac{1378^{0.62} \cdot (1 + 0.94 \cdot \frac{C}{c_c} - 0.34 \cdot T_c^{-1.37})}{(1.56 + \log D e^{'})^{5.73}}\)

(SD=10%)

1<\(T_c<1.065\); 4<\(C_c<14\); 0.018<\(\delta<0.045\)

### Gasljevic and Matthys 2009

**Experimental**

- \(d=12\text{mm}\)
- \(\delta=0.043,0.067,0.116\)
- \(0.96V<7\text{m/s}\)
- \(T=25^\circ\text{C}\)
- Laminar and turbulent

- Non-Newtonian viscoelastic fluid.

- \(C=2,000\text{ppm}\)
- Cationic surfactant Ethoquad T-13 & 2,000ppm NaSl as a counterion.

- DR for turbulent flow in the range of 30-40% was measured.

- This is less than that in a straight pipe where 75% DR was measured, ceteris paribus.

- DR decreased with higher curvature ratios.

- For the coil with the highest curvature, at \(V=0.9\text{m/s}\), the pressure drop increased in relation to that of water. This was attributed to the higher viscosity of the surfactant solution in relation to water at a shear rate of 500s\(^{-1}\).

### Kamel and Shah 2013

**Experimental**

- \(11.0<d<63.5\text{mm}\)
- \(360<D_c<2,850\text{mm}\)
- \(0.018<\delta<0.031\)
- \(20,000<Re<200,000\)
- Turbulent

- Non-Newtonian viscoelastic fluid.

- VC=1.5,2.3,4 %
- Tallowalkyla midopropyl

- DR is significant in coiled tubes and increases with \(C\), with a significant
dimethylamine oxide viscoelastic surfactant (VES) containing 50-65% WC active surfactant, 25-40% propylene glycol and water as solvents. Increase above a VC of 2%. Higher C also exhibit higher resistance to mechanical degradation. Surfactant based fluids are more resistant to shear degradation than polymer based fluids. Larger tube diameters and smaller curvature ratios yield larger DR.

\[
f_{\text{Fanning}} = (-32,200.42\delta^3 + 1,830.62\delta^2 + 0.32)Re_{\text{gen}}^{0.7210956^3 - 316.976 - 0.55} \]

where:

\[
Re_{\text{gen}} = \frac{d^2 V^{2-n} \rho}{8 n^{-1} \kappa}
\]

<table>
<thead>
<tr>
<th>Wang et al.</th>
<th>Numerical</th>
<th>2015</th>
<th>(d=7.3\text{mm}) (D_c=203\text{mm}) (\delta=0.036) (V=3\text{m/s})</th>
<th>Compressible Non-Newtonian foam fluid.</th>
<th>65&lt;(\Gamma&lt;98) The secondary flow effect (vortex roll) of the foam fluid is smaller than that of water.</th>
</tr>
</thead>
</table>

**Polymer solutions**

| Barnes and Walters | Experimental | 1969 | \(d=8.96\text{mm}\) \(D_c<3000\text{mm}\) \(0<Q<80\text{cm}^3/\text{s}\) \(T=20^\circ\text{C}\) Spiral coil Laminar and turbulent | Non-Newtonian viscoelastic fluid. | VC=0.025,0.03,0.10% Polyacrylamide (P250); Polyethylene oxide (Polyox SR305) and Guar Gum. | Easier to pump viscoelastic liquids in curved tubes. Suppression of turbulence with polymer additives which renders the flow almost laminar. Curvature enhances DR in the transition region, whilst it reduces DR at high Re numbers. |
| --- | --- | --- | --- | --- | --- |

| Kelkar and Mashelkar | Experimental | 1972 | \(d=12.5\text{mm}\) \(D_c=665\text{mm}\) \(\delta=0.019\) \(H=38\text{mm}\) \(N=6\) 10<Re<100,000 Laminar and turbulent | Non-Newtonian viscoelastic fluid. | 50<\(C<500\text{ppm}\) Polyacrylamide (AP30&ET59 7) 0.76<\(n<1.00\) | DR limited to turbulent flow. DR increases with polymer C up to a critical Re when DR diminishes. |
| --- | --- | --- | --- | --- | --- |

\[
\beta = 0.2 + \frac{0.8}{1 + \left(\frac{N'_{de}}{0.6}\right)^{0.6}}
\]

where:

\[
N'_{de} = \frac{\left(\frac{\delta^2}{\kappa}\right)^{0.75}}{\left(\frac{\gamma}{\rho}\right)}
\]

<p>| Mashelkar and Devarajan | Experimental | 1976 | (d=12.48,12.49,12.50\text{mm}) (92.3&lt;D_c&lt;1,282\text{mm}) (0.01&lt;\delta&lt;0.135) (H=38.1\text{mm}) 3&lt;N&lt;40 | Non-Newtonian viscoelastic fluid. | 0.01&lt;(C&lt;0.5%) Carboxymethyl cellulose (CMC), Polyacrylamid | The PEO and PAA polymer yielded the best DR, even at the lowest C. This was attributed to the fluid |</p>
<table>
<thead>
<tr>
<th>Author/Sources</th>
<th>Year</th>
<th>d&lt;sub&gt;t&lt;/sub&gt;</th>
<th>d&lt;sub&gt;t&lt;/sub&gt; &lt; 14mm</th>
<th>0.033 &lt; d &lt; 0.082</th>
<th>152 &lt; L/d &lt; 410</th>
<th>N = 3-4</th>
<th>60 &lt; De &lt; 2,000</th>
<th>10 &lt; G&lt;sub&gt;s&lt;/sub&gt; &lt; 400</th>
<th>Laminar and Turbulent</th>
<th>10 &lt; Re&lt;sub&gt;avg&lt;/sub&gt; &lt; 100,000</th>
<th>70 &lt; De &lt; 400</th>
<th>40 &lt; Wi &lt; 950</th>
<th>Laminar and Turbulent</th>
<th>Water</th>
<th>e (PAA-AP-30)</th>
<th>0.354 &lt; n &lt; 0.99</th>
<th>Polyethylene oxide (PEO-WSR-301)</th>
<th>0.871 &lt; n &lt; 0.99</th>
<th>Elasticity.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oliver and Asghar</td>
<td>1976</td>
<td>6.72 &lt; d &lt; 14.0mm</td>
<td>0.033 &lt; d &lt; 0.082</td>
<td>152 &lt; L/d &lt; 410</td>
<td>N = 3-4</td>
<td>60 &lt; De &lt; 2,000</td>
<td>10 &lt; G&lt;sub&gt;s&lt;/sub&gt; &lt; 400</td>
<td>Laminar</td>
<td>Non-Newtonian viscoelastic fluid. Solvent: Water.</td>
<td>250 &lt; C &lt; 2,500 ppm</td>
<td>Polyacrylamide Separan AP273 in water and a 56/44 (WC) glycerol/water solution with 500 ppm Separan AP273.</td>
<td>Some DR due to the partial suppression of the secondary flow.</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Rao</td>
<td>1993</td>
<td>d&lt;sub&gt;t&lt;/sub&gt; = 9.35mm</td>
<td>98 &lt; D&lt;sub&gt;c&lt;/sub&gt; &lt; 247mm</td>
<td>0.038 &lt; d &lt; 0.095</td>
<td>H = 19.5mm</td>
<td>8 &lt; N &lt; 20</td>
<td>10,000 &lt; Re &lt; 60,000</td>
<td>Turbulent</td>
<td>Non-Newtonian viscoelastic fluid. Solvent: Water.</td>
<td>C = 50,100,200 ppm</td>
<td>Polyacrylamide (Praestol 2273TR)</td>
<td>Higher DR with higher polymer C and smaller coil curvatures.</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Azouz et al.</td>
<td>1998</td>
<td>d&lt;sub&gt;t&lt;/sub&gt; = 30mm</td>
<td>pH = 9, 10, 11</td>
<td>100 &lt; Re&lt;sub&gt;avg&lt;/sub&gt; &lt; 100,000</td>
<td>Laminar and Turbulent</td>
<td>Non-Newtonian viscoelastic fluid. Solvent: Water.</td>
<td>C = 35, 40 lb/kgal</td>
<td>Linear Guar gum &amp; Hydroxypropyl Guar (HPG), Crosslinked Guar gum &amp; Hydroxypropyl Guar (HPG) with 12% sol. of boric acid as crosslinking agent.</td>
<td>For borate-crosslinked HPG, the pressure gradient is a strong function of pH and the tube length. For borate crosslinked guar, the pressure gradient is pH dependent but is not effected by the tube length.</td>
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</tr>
</tbody>
</table>

\[ f_{p, fanning} = f_s (1 - 0.03923 W^{0.2488}) \]

\[ f_{s, fanning} = (9.069 - 9.438n + 4.374n^2)D^{0.5}D_t^{(-7.68 + 0.122n)} \]

0.35 < n < 1
| Shah and Zhou | 2001 | $d_t=25.4,38.1,60.3\text{mm}$ | $D_c=121.92,182.88,281.94\text{mm}$ | $\delta=0.0113,0.0165,0.0169$ | $P_{max}=34.47\text{MPa}$ | $4,000<Re_{tov}<200,000$ | Laminar and Turbulent | Non-Newtonian viscoelastic fluid. Solvent: Water | Guar $C=2.397$ kg/m³ | $0.642<n<0.72$ | $C=3.595$ kg/m³ | $0.527<n<0.55$ | $C=4.793$ kg/m³ | $0.433<n<0.48$ | $3$ partially hydrolysed polyacrylamide (PHPA), $C=2.397$ kg/m³ | $0.355<n<0.38$ | $4$ Xathan gum $C=1.198$ | $0.472<n<0.48$ | $9$ | $C=2.397$ | $0.381<n<0.43$ | $9$ | $C=4.793$ | $0.277<n<0.34$ | $3$ hydroxyethylcellulose (HEC) $C=2.397$ | $0.6<n<0.668$ | $C=3.595$ | $0.494<n<0.54$ | $5$ | $C=4.793$ | $0.42<n<0.443$ | DR of polymer solutions decreases with the curvature ratio. Xathan and PHPA yielded the best DR properties. HEC resulted in no DR. Higher DR with smallest tube diameters. For the largest tube diameter, higher polymer $C$ decreased the onset of the DR whilst the opposite effect was reported for the smallest tube diameter. |  |
| Shah et al. | 2006 | $d_t=11\text{mm}$ | $D_c=35.60,57.24,109.97\text{mm}$ | $\delta=0.01,0.019,0.031$ | $N=3,6$ | $22,000<Re_t<155,000$ | Turbulent | For $0.01<C<0.07\%$ fluid is assumed to be Newtonian. Non-Newtonian viscoelastic fluid for $C>0.07\%$. Solvent: Water | Nalco ASP-820 (PHPA) $0.01<VC<0.1\%$ | $0.814<n<1.00$ | Optimum VC of ASP-820 is $0.07\%$. At $0.07\%$, ASP-820 yields a DR of $75\%$ in straight tube and $65\%$ in coiled tube, ceteris paribus. Increase in flow rate increases the DR while the opposite effect was reported for an increase in curvature. An increase in the polymer $C$ or curvature ratio delays the onset if DR. |
\[ f_{p,\text{Fanning}} = A' \delta^B \left( \frac{1.0}{Re^C} \right) \]

where \( A', B \) & \( C \) are constants given in Shah and Ahmed Kamel, (2005) and is valid for \( VC = 0.07\% \).

\[ ME = \pm 6\% \]

### Zhou et al. Experimental

<table>
<thead>
<tr>
<th>Year</th>
<th>( d_i = )</th>
<th>( D_i = )</th>
<th>( \delta = )</th>
<th>( N = )</th>
<th>( VC = )</th>
<th>( C = )</th>
<th>DR in coiled tubing is diminished (by 10-30%) in relation to that in a straight tube, ceteris paribus. DR in coiled tubing is increased with higher ( Re ). This contrasts to the case of straight tubes, where DR diminishes at higher ( Re ). DR increased with ( C ) of Xanthan. Curvature delayed the onset of DR as a result of the delay in the onset of turbulence.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>11.05mm</td>
<td>12.14,29.67,47.70,91.64mm</td>
<td>0.010,0.019,0.031,0.076</td>
<td>3,6,7</td>
<td>5,000&lt;(Re_{cr})&lt;100,000</td>
<td>Laminar and turbulent</td>
<td>Non-Newtonian viscoelastic fluid. Solvent: Water</td>
</tr>
</tbody>
</table>

### Gallego and Shah Experimental

<table>
<thead>
<tr>
<th>Year</th>
<th>( d_i = )</th>
<th>( D_i = )</th>
<th>( \delta = )</th>
<th>( T = )</th>
<th>( VC = )</th>
<th>( C = )</th>
<th>DR decreases with curvature. DR in coiled tubes is lower than that in straight tubes, ceteris paribus. At 0.07% ASP-820, DR is 77% in a straight tube and 64% in the coiled tube (79%&amp;59% for ASP-700). The increase in ( T ) resulted in a decrease of DR in straight tubes. The opposite effect was measured in coiled tubes (DR=45%,52%&amp;55% at 21.1,37.7,54.4°C respectively for ASP-820) DR decreases with tube roughness in both straight and coiled tubes (64% to 60% for coiled tube).</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>11.20.57mm</td>
<td>35.60,57.24,109.97,18.28cm</td>
<td>0.01, 0.0113, 0.019, 0.031</td>
<td>21.1,37.7,54.4°C</td>
<td>0.05,0.07, 0.10,0.15%</td>
<td>0.75&lt;n&lt;1.00</td>
<td>For 0.01&lt;C&lt;0.07% fluid is assumed to be Newtonian. Non-Newtonian viscoelastic fluid for ( C &gt; 0.07% ). Solvent: Water</td>
</tr>
</tbody>
</table>

\[ N_{De} = \frac{1.6675 \times 10^{-3} \left( f_{s,\text{Fanning}} Re_s \right)^{1.4084} \left( \frac{B\lambda u}{d_e} \right)^{1.42305} \left( \frac{\rho \mu_s}{\rho_p \mu_o} \right)^{0.1129}}{1 + 1.0974 \times 10^{-3} \left( f_{s,\text{Fanning}} Re_s \right)^{0.7511}} \]
\[
N_{De} = \left( \frac{f_{Fanning}}{f_{p,Fanning}} \right)^2 - 1
\]

\[\text{(ME} = \pm 10\%\)]

| Shah and Zhou | Experimental | 2009 | \(d=12\text{mm}\) | \(D_r=146,356,572, 1100 \text{ mm}\) | \(\delta=0.01,0.019,0.031,0.076\) | \(N=3,3,7\) | \(3,700<Re_{gen}<11,500\) | Laminar and turbulent | Non-Newtonian viscoelastic fluid. Solvent: Water | \(1.198<C<3.59\) | \(5 \text{ kg/m}^3\) | Guan gum, 0.482<\(n<0.81\) | 9 | Hydroxypropyl Guar (HPG), 0.485<\(n<0.80\) | 5 | Xanthan gum 0.310<\(n<0.71\) | 7 | Significant DR with all three polymer fluids. Curvature reduces the DR and delays the onset of DR. |
| Ogugbue and Shah | Numerical | 2011 | \(\delta=0.3,0.5,0.6,0.8\) | \(\varepsilon=0.25,0.5,0.75,0.96\) | \(100<Re_{gen}<10,000\) | Laminar and turbulent | Non-Newtonian viscoelastic fluid. Solvent: Water | \(C=20,30,40,60\) | \(\text{lb/Mgal}\) | Guan | 0.335<\(n<0.66\) | 6 | DR increases with increased eccentricity (50% reduction for fully eccentric annular section) Higher C increased the frictional pressure drop for laminar flow. For turbulent flow, all C resulted in a significant DR. |
| Ahmed Kamel | Experimental | 2011 | \(d=11\text{mm}\) | \(D_r=579\text{mm}\) | \(\delta=0.019\) | \(T=22.35,38^\circ C\) | \(20,000<Re<200,000\) | \(P_{max}=6.9\text{MPa}\) | Turbulent | Properties assumed to be quasi-Newtonian. Solvent: Water | \(\text{Nalco ASP-820 (PHPA)}\) | \(\text{VC}=0.07\%\) | \(n = 1.0\) | DR in the range of 30-80%. At elevated T, the DR effect is diminished in straight tubes while it remains quasi-constant in coiled tubes. |

\[
f_{Fanning} = 0.00378 \frac{d_{eit}}{d_{lot}} + \frac{3.7374}{Re_{gen}} + \frac{4042}{2Re_{gen}} - 0.00124
\]

\[\text{(ME} = \pm 5\%\)]

\[
\frac{DR_T}{DR_a} = 1.0
\]

\[\text{(ME} = \pm 2.1\%\)]

Table 1: Review of the experimental and numerical work

5. Conclusions
The studies reviewed have demonstrated that, due to the secondary flow, which increases with curvature, DR in coiled tubes is diminished when compared to straight tubes. However, a significant DR can be still be achieved with the introduction of: bubbles (9-25%), surfactant (30-59%) and polymer (circa 30-80%) additives. DR is a strong function of the surfactant concentration and the air volume fraction whilst with polymer additives DR efficiency is dependent on the molecular weight, structure and solubility. DR is generally present in flows with $Re$ numbers in excess of the critical number. However, at elevated $Re$ numbers DR diminishes. This is due to the higher centrifugal forces (air bubbles and polymers) and mechanical degradation with high shear stress (surfactants). A number of authors have presented correlations for the calculation of the friction factor which are typically a function of: curvature ratio, $Re$ and $De$ numbers and the additive concentration.

Due to their low molecular weights, viscous properties and resilience to mechanical degradation, surfactant based fluids are generally considered to be superior to polymer based fluids. Hence, surfactants are suitable for a variety of applications such as district cooling and heating systems. A significant scope for future research has been elucidated for DR in coiled tubes with the injection of air bubbles (impact of bubble size and relation with the Lockhart and Martinelli correlation) and the application of a combination of methods, such as the use of polymer and surfactant additives with bubbly flow.

**Acknowledgments**

The authors of the current investigation would like to thank the University of Central Lancashire UK, for facilitating the completion of this study as well as the various authors who have been contacted during the course of this study.

**Notation List**

- $C$: concentration (ppm)
- $Cc$: non-dimensional surfactant concentration (-)
- $Cst$: empirical constant (-)
- $d$: tube internal diameter (m)
- $dr$: drag ratio (-)
- $D$: helix diameter (m)
- $De$: Dean number ($Re\delta^{1/2}$) (-)
- $De'$: modified Dean number ($Re'\delta^{1/2}$) (-)
- $De''$: modified Dean number ($Re_{gen}\delta^{1/2}$) (-)
- $DR$: drag reduction (%) 
- $f$: friction factor (-)
- $FC$: friction coefficient (-)
- $Gz$: Graetz number ($RePr\delta$) (-)
- $Gz'$: modified Graetz number ($Re'Pr'\delta$) (-)
- $H$: pitch (m)
- $K$: rheometric and technical consistency index (Pa s$^n$)
- $L$: length (m)
- $ME$: mean error (%) 
- $n$: power law model flow behaviour index (-)
- $N$: number of turns (-)
- $N_{De}$: Deborah number (-)
- $N_{De}'$: modified Deborah number (-)
\[ P \quad \text{pressure (Pa)} \]
\[ Pr \quad \text{Prandtl number (-)} \]
\[ Pr' \quad \text{modified Prandtl number (-)} \]
\[ Q \quad \text{volume flow rate (m}^3/\text{s}) \]
\[ Re \quad \text{Reynolds number (-)} \]
\[ Re' \quad \text{modified Reynolds number as proposed by Metzner and Reed (1955)} \]
\[ \beta \quad \text{reduced friction factor (-)} \]
\[ \delta \quad \text{curvature ratio (-)} \]
\[ \varepsilon \quad \text{coil eccentricity (-)} \]
\[ \theta \quad \text{angle from inlet of curved pipe (°)} \]
\[ \lambda \quad \text{relaxation time (s)} \]
\[ \mu \quad \text{viscosity (cP)} \]
\[ \mu_o \quad \text{zero shear rate viscosity (cP)} \]
\[ \rho \quad \text{density (kg/m}^3\text{)} \]
\[ \sigma \quad \text{stress (N/m}^2\text{)} \]
\[ \Gamma \quad \text{quality (%)} \]
\[ a \quad \text{ambient temp} \]
\[ b \quad \text{bubble} \]
\[ bf \quad \text{base fluid} \]
\[ c \quad \text{coil} \]
\[ crit \quad \text{critical} \]
\[ DRF \quad \text{drag reducing fluid} \]
\[ eff \quad \text{effective} \]
\[ el \quad \text{elastic} \]
\[ eit \quad \text{external diameter of inner tubing} \]
\[ gen \quad \text{generalised} \]
\[ iot \quad \text{internal diameter of outer tubing} \]
References


Highlights

- Review on pressure drop reduction studies in helically coiled tubes
- Air bubbles, surfactant and polymer additives are effective in diminishing drag
- Drag reduction is diminished in relation to straight tubes
- Drag reduction is predominantly evident in turbulent flow
- Drag reduction diminishes with higher coil curvatures and excessive $Re$ numbers