On the Reological Properties of Human Blood

Whitty, Justin and Wain, Richard

Available at http://clok.uclan.ac.uk/25378/


It is advisable to refer to the publisher’s version if you intend to cite from the work.

10.31031/RMES.2018.06.000644

For more information about UCLan’s research in this area go to http://www.uclan.ac.uk/researchgroups/ and search for <name of research Group>.

For information about Research generally at UCLan please go to http://www.uclan.ac.uk/research/

All outputs in CLoK are protected by Intellectual Property Rights law, including Copyright law. Copyright, IPR and Moral Rights for the works on this site are retained by the individual authors and/or other copyright owners. Terms and conditions for use of this material are defined in the http://clok.uclan.ac.uk/policies/
On the Rheological Properties of Human Blood

Wain R and Whitty J*

School of Engineering, UK

*Corresponding author: Whitty J, John Tyndall Institute, School of Engineering, Preston, PR1 2HE, UK

Introduction

Blood is a suspension of various solid cells in a liquid plasma, formed from a solution of proteins and minerals. The rheological properties of blood are influenced by individual characteristics of these components as well as factors such as haematocrit, amongst others [1,2]. Whilst plasma itself is a Newtonian fluid, addition of the blood cellular components alters its rheological behaviour towards a shear-thinning non-Newtonian fluid [1,2]. No single rheological model can capture all aspects of this complex fluid, and as such particular aspects can be modelled individually [2,3]. To this end, differing rheological models are applied when predicting flow characteristics depending on area of the circulatory system of interest. For example, blood is often approximated to a shear-thinning non-Newtonian fluid in arterial flows, though has also been commonly modelled using the Cross [4–7] and Carreau-Yasuda [5–11] models; their constitutive relationships being defined respectively by:

\[ \Phi(\dot{\gamma}) = \mu_\infty + \frac{\mu_\infty - \mu_1}{1+(\lambda\dot{\gamma})^n} \] (1)

and

\[ \Phi(\dot{\gamma}) = \mu_\infty + \frac{\mu_\infty - \mu_1}{1+(\dot{\gamma})^m} \] (2)

where \( \Phi(\dot{\gamma}) \) is the so-called effective fluid viscosity, \( \mu_\infty \) is the low shear viscosity, \( \mu_1 \) (~2–4 cP) is the high shear viscosity, \( \lambda \) is the shear time constant and \( n \) is a shaping constant termed the Yasuda index. Whilst \( n \in (0,1) \) is the power index with values less than unity indicating shear thinning behaviour, \( \lambda \) values approaching unity approximate Newtonian behaviour. In this work we have used multivariate optimization routines resident in the SciLab data driven modelling open source software to approximate each of these. The values obtained being compared with the more traditional non-Newtonian constitutive relations proposed by Ostwald-de Waele [12] & Sisko [13] defined respectively as:

\[ \Phi(\dot{\gamma}) = \frac{K}{(\dot{\gamma})^n} \] (3)

and

\[ \Phi(\dot{\gamma}) = \mu_\infty + \frac{\mu_\infty - \mu_1}{(\dot{\gamma})^m} \] (4)

where \( K \) is the viscosity consistency and \( \mu_1 \) is a low (non-zero) shear viscosity. To ensure dimensional consistency, each of these are presented here in slightly modified forms to those initially described. As such, they should be more correctly referred to as Modified Ostwald-de Waele [12] and Modified Sisko [13] fluids respectively.

Method

Parameters were approximated for each of the before mentioned rheological models using compiled experimental data from the literature [14–17]. This required a multivariate optimization of the square of the error function, namely:

\[ err^2 \{ \mu(\dot{\gamma}) \} = \sum_{m=1}^{N} [\Phi(\dot{\gamma}) - \mu_m]^2 \]

where \( \mu_m \) is the mth measurement of the blood viscosity, \( \Phi(\dot{\gamma}) \) is constitutive relationship, i.e. equation (1), (2), (3) or (4), and \( N \) is the total number of data points. For each of the rheological models we simultaneously fit the free parameter set to experimental data using a variation of the Nelder-Mead optimization algorithm (i.e. \texttt{fminbnd} routine) in SciLab open-source data driven modelling

\[ \text{https://scilab.in/fossee-scilab-toolbox/optimization-toolbox} \]
To preserve structural identification of the Modified Sisko and Carreau-Yasuda fluid models the low shear viscosity terms ($\mu_l$ and $\mu_0$, respectively) were fixed to 150 cP \cite{8,11,19}. The literature maintaining that these terms have no physiological relevance so the calculations that follow are unaffected by this assumption. Cross fluid bounds were used as $[1,300]$, $[0.01,1]$ and $[0.01,100]$ for $\lambda$, $n$ and $\mu_\infty$ respectively. For the Modified Sisko and Carreau-Yasuda models identical bounds were used for these parameters as for the Cross fluid. In the case of the latter, an additional bound $[0.01,5]$ was used to represent the Yasuda index ($\alpha$). In the case of the Ostwald-de Waele fluid, $K$ in equation (3) is effectively set to $(150 - 2.3) \approx 148$ cP in order to render a structurally identifiable model with two free parameters. Refined accuracy being obtained via application of an analogous constrained multivariate optimization algorithm. That is, the \texttt{fmincon} routine with initializations set to those previously evaluated from the \texttt{fminbnd} SciLab command. Each of the calculations allowed for 30000 interactions in a time of 16.7 minutes. A convergence tolerance of $10^{-10}$ being set for the initial \texttt{fminbnd} calculation with default values being left for the \texttt{fmincon} evaluation which followed.

### Results

In most of the cases investigated, the optimization routine reported that either an optimum solution was found or that a solution was obtained to acceptable accuracy. The notable exception being in the case of the Carreau-Yasuda fluids which initially reported that the maximum number of iterations had been reached. In this case the bounds we altered using successive bisection until a solution to acceptable accuracy was obtained. These values were then used as initialization within the aforementioned SciLab \texttt{fmincon} routine thereby obtaining an optimized solution.

Table 1 compares these calculated parameters with those in the literature \cite{1,3,4,20-23}. Those for Modified Sisko fluid are similar to the Ostwald-de Waele fluid, on the other hand equivalent parameters calculated for the Cross fluid are similar to those of Carreau-Yasuda model. The power index and high shear viscosity values are in good agreement with those quoted in the literature \cite{4,24-26}. Particularly good agreement is demonstrated with respect to values obtained from the Cross and Carreau-Yasuda constitutive relationships and those reported in the literature. This said, analogous values obtained using identical optimization routines for the Modified Ostwald-de Waele and Modified Sisko fluids indicate significantly different values. However, similar values are obtained for the power index and shear time constant when comparing the Modified Sisko and Modified Ostwald-de Waele fluids.

**Table 1:** Rheological parameters to 2 s.f. obtained from fitting combined existing experimental data shown in Figure 1 to various non-Newtonian models. Experimental data is taken from a variety of sources.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\hat{n}$</th>
<th>$\hat{\lambda}$</th>
<th>$\hat{\mu}_\infty$</th>
<th>$\hat{\alpha}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ostwald-de Waele</td>
<td>0.53</td>
<td>34</td>
<td>2.3</td>
<td>-</td>
</tr>
<tr>
<td>Modified Sisko</td>
<td>0.51</td>
<td>33</td>
<td>2.3</td>
<td>-</td>
</tr>
<tr>
<td>Cross</td>
<td>0.24</td>
<td>8.6</td>
<td>3.6</td>
<td>-</td>
</tr>
<tr>
<td>Carreau-Yasuda</td>
<td>0.23</td>
<td>17</td>
<td>3.6</td>
<td>1.7</td>
</tr>
<tr>
<td>Cross \cite{24,25}</td>
<td>0.21</td>
<td>8.2</td>
<td>3.5</td>
<td>-</td>
</tr>
<tr>
<td>Carreau-Yasuda \cite{4,24-26}</td>
<td>0.21</td>
<td>8.2</td>
<td>3.5</td>
<td>0.6</td>
</tr>
</tbody>
</table>

*set value

Discrepancies in the power index calculations are of particular interest from a practical viewpoint. To date the power index has been thought of as a diversion from an idealized Newtonian fluid. The natural assumption being that this quantity is a property of the fluid, hence independent of constitutive relationship, these results certainly suggest that this is not the case. Implying that, at least in the current mathematical forms, the power index is not a property of the fluid.

![Figure 1](image-url) A log-log plot demonstrating four non-Newtonian constitutive models fitted to collated experimental rheological data for blood adapted from the literature.
Figure 1 demonstrates that the empirical data are well predicted by each of the models to varying degrees; the primary source for these data being Ballyk et al. [20]. Crosses represent experimental blood rheological data collated from several sources i.e. Huang [14], Merrill [15], Cokelet [16] & Skalak [17]. Lines represent rheological models i.e. Ostwald-de Waele [12], Cross [27], Carreau [28,29], and Modified Sisko [13], fitted to the compiled experimental data.

As expected each of the models converge at higher (imposed) apparent viscosity values. The models predicting the apparent viscosity of the blood at relatively lower shear rates \( \gamma \leq 50s^{-1} \) very well. At higher shear rates the model predictions diverge. At the highest measured shear rate 950s\(^{-1}\) the Cross and Carreau-Yasuda models predict higher viscosity values of respectively 3.77cP and 3.69cP respectively, with the Sisko model predicting a 12% lower viscosity than that the measurement of 3.6cP. Moreover, Figure 1 demonstrates a deficiency with the Ostwald-de Waele model whereby at high shear rates the apparent viscosity is too low in comparison with the empirical data. Indeed, since this model is unbounded at both low and high shear limits, unlike other shear-thinning models discussed, its use for evaluating blood is limited to a narrow range of shear rates [30], in this case probably between 1s\(^{-1}\) and 45s\(^{-1}\) (Table 1).

**Conclusion**

A combination of optimization routines have been used to evaluate the constitutive non-Newtonian relationships from compilation of data evident in the literature, via minimization of appropriate least squares error function. It was found that the values within each constitutive relationship vary depending on the data set used and non-Newtonian fluid model applied. Specifically, this study has shown that:

A. Using these calculated optimized parameters most of the non-Newtonian models describe the empirical data rather well;

B. The Ostwald-de Waele model is not suitable for predicating the effective blood viscosity at high shear rates;

C. Due to the similarity of the models the Sisko and Ostwald-de Waele similar constitutive non-Newtonian parameters are apparent models; analogous similarities being evident regarding Cross and Carreau-Yasuda fluids;

D. The optimization routine predicts the Carreau-Yasuda and Cross models agree well about analogous parameters as well as with values in the literature;

E. The non-Newtonian constitutive parameters (e.g. \( n, \lambda \), and \( \mu_0 \) and \( \alpha \)) may not be properties of blood.

We should also note that there are more data sets available in the literature. Furthermore, much more advanced technology is available to measure blood viscosity. So, whilst the work herein does raise some interesting observations which have been properly verified, further work is required to validate these findings.

**References**


