Using the Finite Element Method to Determine the Temperature Distributions in Hot-wire Polystyrene Cutting

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ABSTRACT:
Hot-wire cutting is a common material removal process used to shape and sculpt plastic foam materials, such as expanded polystyrene (EPS). Due to the low cost and sculpt-ability of plastic foams they are popular materials for large sized (> 1 m³) prototypes and bespoke visual artefacts. Recent developments in robotic foam sculpting machines have greatly increased the ability of hot-tools to sculpt complex geometrical surfaces bringing the subject into the realm of subtractive rapid prototyping/manufacturing. Nevertheless foam cut objects are not being exploited to their full potential due to the common perception that hot-wires are a low accuracy cutting tool. If greater accuracy for hot-wires can be obtained, it could provide a low cost method of producing high value functional engineering parts. Polystyrene patterns for lost foam casting are one such possibility.

A nonlinear transient thermal finite element model was developed with the purpose of predicting the kerf width of hot-wire cut foams. Accurate predictions of the kerfwidth during cutting will allow the tool paths to be corrected off-line at the tool definition stage of the CAM process. Finite element analysis software (ANSYS) was used to simulate the hot-wire plastic foam cutting. The material property models were compiled from experimental data and commonly accepted values found in literature.

The simulations showed good agreement with the experimental data and thus the model is thought to be reliable. The simulations provide an effective method of predicting kerf widths, under steady state cutting conditions. Limitations and further developments to the model are described.

KEYWORDS: Hot-wire cutting, EPS, XPS, Thermal properties, Simulation, Kerf width.

1. INTRODUCTION

Hot-wire foam cutting is the material removal process most commonly used in low volume manufacturing to shape or sculpt polystyrene into desired shapes and sizes. Recent research has lead to the development of a number of foam cutting rapid prototyping and manufacturing systems capable of shaping polystyrene objects with high levels of geometrical complexity [1]. Despite these advances in system design, little research effort has been directed at improving the cut surface accuracy. Increasing the potential accuracy of cut surfaces will allow designers to specify tighter tolerances on foam cut parts. In turn foam cutting could provide an inexpensive method of producing engineering parts. Lost foam casting is one such manufacturing process that could greatly benefit from accurately sculpted foam parts. This imbalance in research focus has stimulated interest in this area by the authors, and a number of papers have been produced to-date as a result [2-8].

To maximise the utility of foam cut objects it is necessary to provide methods for predicting and optimising cut surface characteristics for given input parameters, such as feed rate and wire temperature. The achievable accuracy of cut surfaces largely depends on the ability to predict the kerf width before the tool path is generated. It is possible for the kerf width to vary between 1.4 – 6 times the wire diameter depending on the cutting conditions [9]. If the kerf width is known in advance, then an appropriate wire offset can be incorporated into the tool path, ensuring the cut surface is located in the correct position. The kerf width is directly related to the wire temperature while the wire temperature is a function of electrical power input and feed rate. The surface texture or roughness of the cut surface is loosely dependant on the cell size of the foam. For extruded polystyrene (XPS) the cell size is less than 100 μm resulting in a highly homogenous surface texture. Expanded polystyrene (EPS) is made from pre-expanded beads on the order of 4 mm, much larger than the hot-wire diameter; therefore the surface texture of EPS is related to the intracellular structure and the cell boundaries.

For cuts with sufficiently high energy, the thermal field surrounding the wire is large enough to prevent physical contact with the foam, and the cut can be considered to be purely thermal. For low energy cuts the wire may contact the foam resulting in thermomechanical cutting. The thermal field size depends on input parameters such as electrical power, feed rate, cutting tool geometry and material properties [6]. Despite hot-wire cutting being a common process, little research work has been carried out with the aim of numerically predicting hot-wire cutting conditions.

Numerical simulations of hot-wire cutting was first carried out by Ahn et al using the finite element method (FEM) [7, 8]. The analysis was analogous to laser cutting and welding processes, in which a moving thermal energy source heats a work piece by conduction [9]. In thermal analysis of laser cutting, the heat source is applied to a moving coordinate system and the transient temperature distribution within the workpiece is calculated. The same method was modified for hot-wire cutting and simulated in FE software SYSWELD+. The kerf width was defined as the area contained by an isothermal line at the melting temperature of the foam. The authors used a commonly quoted melting temperature of 240°C for crystalline solid polystyrene. It should be noted however, that EPS is amorphous and therefore does not have a defined melting point. In fact the foam begins to soften at the glass transition temperature between 80-120°C and the cells collapse at approximately 160°C [10]. For these reasons the results gathered while using an isothermal line at 240°C could be seen as somewhat arbitrary and are more useful as showing trends than the actual melted widths. Ahn et al assumed a free air wire temperature of 700°C, measured with an infrared temperature measurement device, was constant throughout the cut [7]. This assumption is somewhat valid for very thin foam sheets; however it becomes less valid for thick foam sheets where the wire is known to cool as much as 400°C whilst cutting [2]. Despite these shortcomings the analytical and empirical results showed good agreement with respect to the melted width of the cut. Unfortunately, Ahn et al have not yet explored whether the model can predict kerf widths for different wire diameters and plastic foams.

After examination of the aforementioned literature a number of attributes were identified that required further investigation and incorporation into future FE models. These attributes are as follows: The finite element (FE) model should allow a wide variety of hot-tools to be simulated including wires of different sizes. The FE model should be accurate for both main types of polystyrene foam, i.e. EPS and XPS. The FE model should use experimentally verified thermal material property data and temperature inputs with minimal assumptions. One of the most difficult aspects of modelling hot-wire foam cutting is obtaining accurate and relevant material properties data. The main use of PS foams is for building insulation so the data is often restricted to temperature ranges near room temperature. Material data for the processing of solid PS can be found in literature for temperatures up to the decomposition temperature of PS. However this information is only applicable to hot-wire cutting above the temperature for which the cells collapse. Further materials testing is required to bridge the gap in the data between the low temperature (foamed) and the high temperature (solid) PS states.

In this paper, the thermal characteristics of hot-wire cutting are investigated by experimental and numerical analyses. The aim of the experiments was to provide critical thermal material properties and accurate wire temperatures for use as inputs to the FE model. The numerical simulations were carried out using FE software ANSYS. Based on the calculated temperature distribution around the wire, and the assumption the cells completely collapse above 160°C [10], the kerf width were predicted for varying cutting parameters. Finally the predicted kerf widths are compared against experimental kerf width data to determine the efficacy of the FE model.

2. EXPERIMENTS

The experiments fall into two categories: Materials testing, aimed at determining the thermal properties models for expanded polystyrene (EPS) and extruded polystyrene (XPS) foams. And cutting trials aimed at providing empirical temperature data for inputs to the numerical model and kerf width data for verifying the simulation results.

2.1 Thermal characteristics of polystyrene foam

Central to the study of plastic foam cutting mechanics is the way in which plastic foams degrade with increasing temperature. For any FEA to be accurate it is important to know how the thermal properties change with temperature and also what temperatures cause the various physical transitions present in plastic foam cutting.

Dynamic Mechanical Analysis (DMA) was used to determine the glass transition temperature of the foam. DMA is a technique that measures the modulus and energy dissipation of materials as they are periodically stressed. Various material properties can be measured by subjecting a sample to frequency and temperature effects over time [11]. DMA, using TA instruments: model Q800, was carried out on both EPS and XPS foams to determine the glass transition temperature (T_g). The samples were slowly heated from 20 to 140 °C subjected to frequencies varying from 1 to 50 Hz at a rate of 5 °C/min. Using this method, T_g was found to be between 103 and 118 °C for EPS and XPS respectively. For temperatures over T_g the foams rapidly lose their stiffness, therefore T_g represents a lower threshold for which the wire temperature must be above if mechanical ripping is to be avoided.

Mehta et al investigated the thermal degradation of foamed polystyrene patterns whilst studying the expendable pattern casting (EPC) process and found a number of interesting degradation characteristics applicable to the thermal cutting of EPS foam [10]. The experimental procedure involved submerging EPS samples in water or wax which had been heated to a specific temperature. Water was used for the temperature range between 50 and 100°C while wax was used for temperatures between 100 and 170°C. Subsequently the samples were prepared for imaging with a scanning electron microscope (SEM). Up to temperatures of 100°C the visual appearance of the cell structure does not change. As the temperature increases above 110°C the cells begin to collapse rapidly. At 160°C the cells were shown to completely return to there unexpanded state. The collapse temperatures were found to be the same regardless of EPS density and cell size. Therefore for plastic foam cutting it is sensible to assume that when the wire is sufficiently hot to cause the adjacent foam to heat over 160°C then the foam will reduce away from the wire to its pre-expanded state. As the foam is highly porous this constitutes a significant reduction in volume. It can now be said with some confidence that cuts made with wire temperatures between T_g and 160°C will result in thermomechanical cutting while temperatures over 160°C will result in purely thermal cutting conditions.

At sufficiently high temperatures PS degrades into shorter hydrocarbon molecules in the form of gases and ash. Thermo-gravimetric analysis (TGA) was used to determine this degradation temperature using TA instruments model: STD Q600. The TGA results show PS volatilisation begins at about 275°C with maximum volatilisation rates occurring between 400 to 420°C as shown in figure 1. PS below 400°C will necessarily transfer heat to surrounding foam by conduction. When PS is heated above 400°C it is removed from the system in the form of gas, taking with it significant embedded energy. For this reason it is reasonable to assume a FE model based on conductive heat transfer is less valid for temperatures over 400°C.

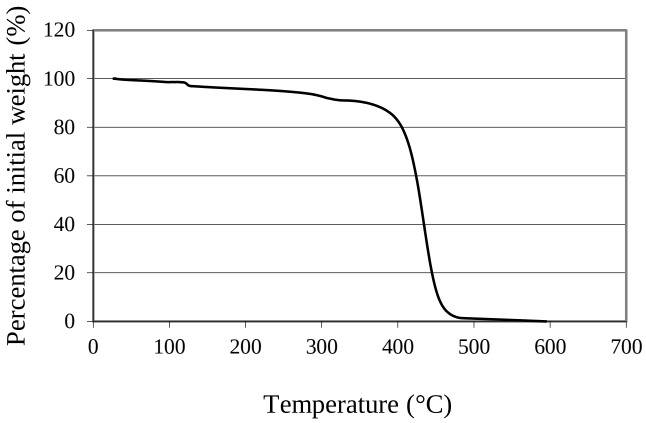

Figure 1. TGA results showing percentage weight loss as a function of temperature.

Also of great importance to the FE model are the temperature dependant conductivity and specific heat. Hot Disk Analysis (model: Hot Disk AB) was used to calculate the conductivity of the foams. The average conductivities for EPS and XPS at 20°C were found to be 0.0327 W/mK and 0.0316 W/mK respectively. These values are consistent with those found in literature.

2.2 Description of the cutting trials

2.2.1 Experimental Apparatus

The cutting trial apparatus comprised of an industrial robot, hot-wire cutting device, thermocouple, electrical power supply and foam sample holding fixture. These system components are introduced and discussed below:

The workpiece manipulation device was a Kuka KR6/2, 6-axis articulated robot which was fitted with the foam sample holding fixture. The robot effected the accurate linear movement of the foam samples over the statically mounted hot-wire, at predefined velocities.

The purpose built hot-wire cutting device supported a representative 140 mm length of Nikrothal N80 wire. The wire was tensioned using a pneumatically actuated (cylinder) tensioning device. Thermal expansion and contraction of the wire was accommodated by setting the cylinder pressure to a constant value which thus provided constant wire tension throughout the tests. The cutting device was statically mounted on a heavy steel frame.

An Omega thermocouple (Ø 0.1" sheathed K-type, part number KQIN-18U-12) was fixed onto the hot-wire where it was used to monitor the local wire temperature. As the electrical potential, applied across the hot-wire, adversely affected the thermocouple readings, the driving power supply (detailed below) was momentarily switched off while temperature readings were being taken. For a given unmodulated current setting the modulated current was controlled to ensure that an equivalent power was delivered.

Electrical heating of the wire was achieved by way of a programmable regulated power supply (ITECH, Model IT6831). The unit adopted for the tests could deliver a maximum current of 10A at 18V.
Figure 2 below shows a close up view of the purpose built cutting device, foam workpiece, thermocouple and wire tensioning device (far right).

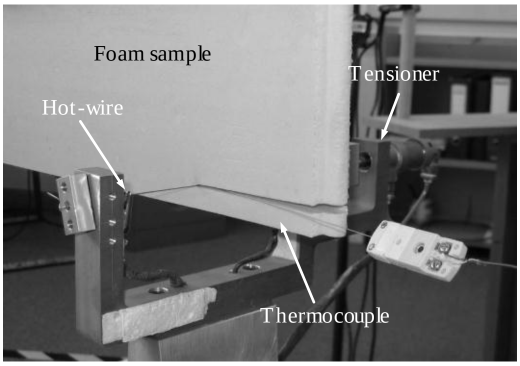

Figure 2. Test apparatus with thermocouple measuring active hot-wire cutting temperature.

2.2.2 Experimental Procedure

Two different types of plastic foam were employed in the cutting trials. They were 26 kg/m³ Expanded Polystyrene (H-Grade Polyfoam by Bondor) and 30 kg/m3 Extruded Polystyrene (Styrofoam by DOW).

The cutting trials consisted of over 400 individual cuts from which a significant number of measurements were made. The general cutting procedure for all the cutting tests was as follows:

A sheet of material was held vertically in the clamping device which was manipulated by the robot (step A). Power was supplied to the hot-wire by fixing the average current and allowing the voltage to float. The temperature of the wire was then allowed to stabilise (step B). A robot programme was run to initiate the linear cutting of the foam sample at a predetermined feed-rate (step C). As a result of step c above, a 10 mm sliver of material was pared from the parent sheet by the hot-wire. The robot programme was then necessarily modified to advance the cutter path by 10 mm (step D). Steps b-d were then repeated to create a sliver of foam with near identical cutting conditions on each side. This sliver was then used to calculate the kerf widths by subtracting the sample thickness from the known 10 mm robot increment. Steps b-d were then repeated measuring the temperature in the centre of the sample throughout the cut (E). This was done independently of the kerf sample cuts to ensure the thermocouple did not introduce errors into the measured kerf surfaces. Overall, three cuts were made for each set of cutting conditions.

Steps b to e were then repeated changing the test conditions (current/wire temperature, feed rate, wire diameter and foam type) as necessary.

Previous work by the authors found that hot-wire cutting is a transient phenomenon and as a result the cutting conditions change along the length of the cut [2]. For this reason all kerf width and temperature measurements were made approximately 280 mm from the start of the samples to ensure steady-state conditions had been reached.

2.2.3 Results and discussions of the experiments

The cutting trials provided kerf width and wire temperature data for steady-state cutting conditions. With regard to increasing the feed rate both the kerf width and the wire temperature were found to decreased linearly. Increasing the power input had the opposite effect of increasing the kerf width and wire temperature. In order to consider the influence of the feed rate and power input together, the effective heat input is calculated as follows:

$$Q_{eff} = \frac{P}{V} \qquad \text{(Eq. 1)}$$

Where:
Q_{eff} = the effective heat input (J/m)
P = the electrical power input per meter (W/m)
V = the wire velocity relative to the foam (m/s)

Q_{eff} represents the amount of energy used by the wire to create a unit area of cut. It is measured in J/m^2 and was first formulated by Ahn et al [8].

Figure 3 shows trend lines for kerf width and wire temperature plotted against Q_{eff} for EPS foam cut with 0.64 diameter Nichrome wire. The equations relating the measured characteristics are as follows:

$$\textit{Kerf width} = 0.0964 \times Q_{eff} + 0.343 \qquad \text{(Eq. 2)}$$
$$\textit{Wire temperature} = 17.7 \times Q_{eff} + 68.8 \qquad \text{(Eq. 3)}$$

The R^2 values for each data set are 0.9267 and 0.9345 respectively, signifying an clear linear correlation. Similar linear relationships were found to exist for XPS and for 0.36 mm and 0.91 mm wire diameters.

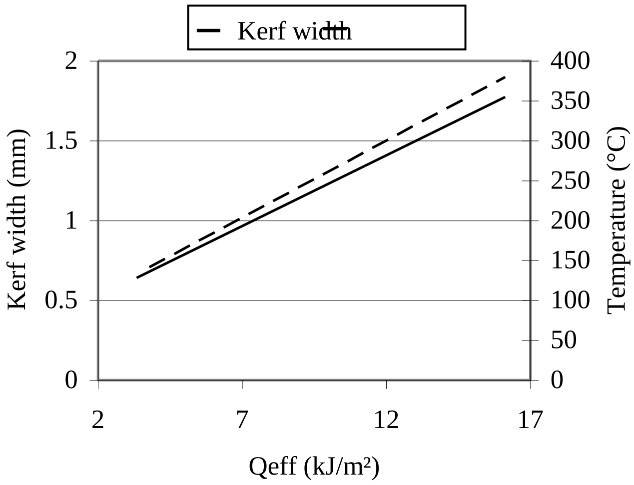

Figure 3. Trend lines for kerf width and wire temperature vs. effective heat input (EPS foam cut with 0.64 mm diameter Nichrome wire).

As the kerf width and the wire temperature are linearly related to Q_{eff} it follows mathematically that the kerfwidth is also linearly related to the wire temperature. This relationship is fundamental to the FE model developed in the next section.

3. NUMERICAL ANALYSIS

Hot-wire foam cutting was simulated using ANSYS finite element software. The objective of the simulations is to provide a method for predicting the kerf width based on known wire temperatures. By comparing the predicted kerf widths with experimental data an assessment on the accuracy of the material data and the validity of the assumptions can be made.

3.1 Modelling of the hot-wire

The assumed geometry of the hot-wire cutting process is shown in figure 4. The following assumptions were made in developing the finite element model:

1. The temperature distribution across the centreline is symmetrical.
2. The foam material is isotropic and homogeneous.
3. The work piece is initially at 25°C (298K)
4. Thermal expansion is negligible; therefore the model geometry is constant.
5. The conductivity of the foam is low and therefore heat transfer on the surfaces far away from the wire is negligible. The surfaces can be treated as adiabatic.
6. The hot-tool is surrounded by foam at all times; therefore conduction is the only mode of heat transfer.
7. Thermal degradation of the PS plastic at temperatures over 400°C is assumed negligible.
8. The work piece geometry is 10 mm high and 30 mm wide. As the conductivity of foam is very low only the material closest to the wire increases in temperature. Thus the area around the wire that needs to be simulated is quite small.
9. The kerfwidth is represented by the maximum width of the isotherm at the temperature at which the cells collapse. A value of 160°C was adopted from literature [10].

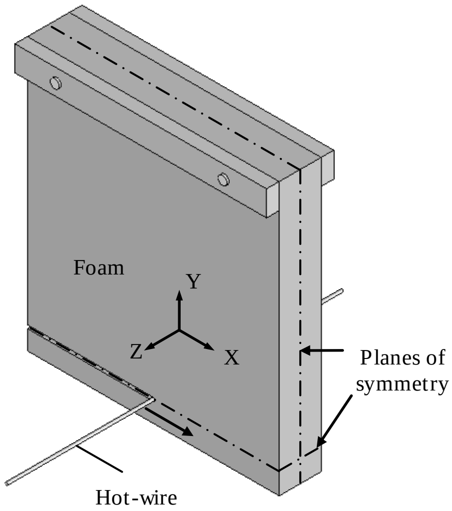

Figure 4. Graphical representation of the hot-wire cutting process.

3.2 Finite Element formulation

Consider a homogeneous two-dimensional medium and a temperature distribution $T(x, y)$ which is expressed in Cartesian coordinates. Using the law of conservation of thermal energy the temperature distribution in an infinitesimally small control area defined in the medium is governed by the following general differential equation:

$$\rho c_P \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \dot{q} \qquad \text{Eq. (4)}$$

Equation 4 is the two-dimensional form of the *heat diffusion equation*. This equation provides the basic tool for heat conduction analysis. From its solution, the temperature distribution $T(x, y)$ is obtained as a function of time. To solve the above differential equation, it is necessary to define the boundary conditions on selected surfaces and nodes.

For a specified nodal temperature, T_n,

$$T_{(x,y,t)} = T_n \ \text{ for } t > 0 \qquad \text{Eq. (4a)}$$

For initial nodal temperatures, T_i,

$$T_{(x0,y0)} = T_i \ \text{ for } t = 0 \qquad \text{Eq. (4b)}$$

For specified surface heat flux, q'',

$$\nabla T = 0 \qquad \text{adiabatic heat flux} \qquad \text{Eq. (4c)}$$

where $\nabla T = \left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y}\right)$.

The problem is nonlinear due to temperature-dependant material properties. The weak form of equation 4 can be derived and rearranged with integration by parts. For the general nonlinear transient case (for example, the hot-wire foam cutting case), the formulations are written as follows:

$$[C_P(T)]\{\dot{T}\}+[K_c(T)]\{T\}=\{R_{T_s}(T,t)\}+\{R_Q(T,t)\} \qquad \text{(Eq. 5)}$$

Equation 5 shows the element matrices and heat load vectors are both temperature and time dependant, and a solution by an iterative, time marching scheme is required. The full Newton-Raphson iteration method was adopted.

3.3 Thermal properties of EPS foam

The material models for the FEA were developed with a combination of experimental testing and values from literature. The thermal material properties for EPS and XPS used in the FEA are shown in figure 5. At temperatures over 160°C the foam cells collapse returning the foam to a high density (non-foamed) state. At this stage there is a large change in the conductivity of the foam due to the removal of air gaps within the foam. A conductivity of 0.16 W/mK was used for all temperatures over 160°C [12]. This value is valid for both plastic types as they are chemically, thermally and mechanically very similar at this stage. The specific heat data for temperatures up to 400°C was taken from literature [13] and is assumed to be the similar for all PS plastics. The specific heat increases linearly from room temperature to the glass transition temperature of 110°C. The glass transition activation energy is responsible for the jump in specific heat between 110 and 160°C, after which the trend continues linearly up to 400°C. In all cases after 400°C the thermal material properties are considered to have constant values.

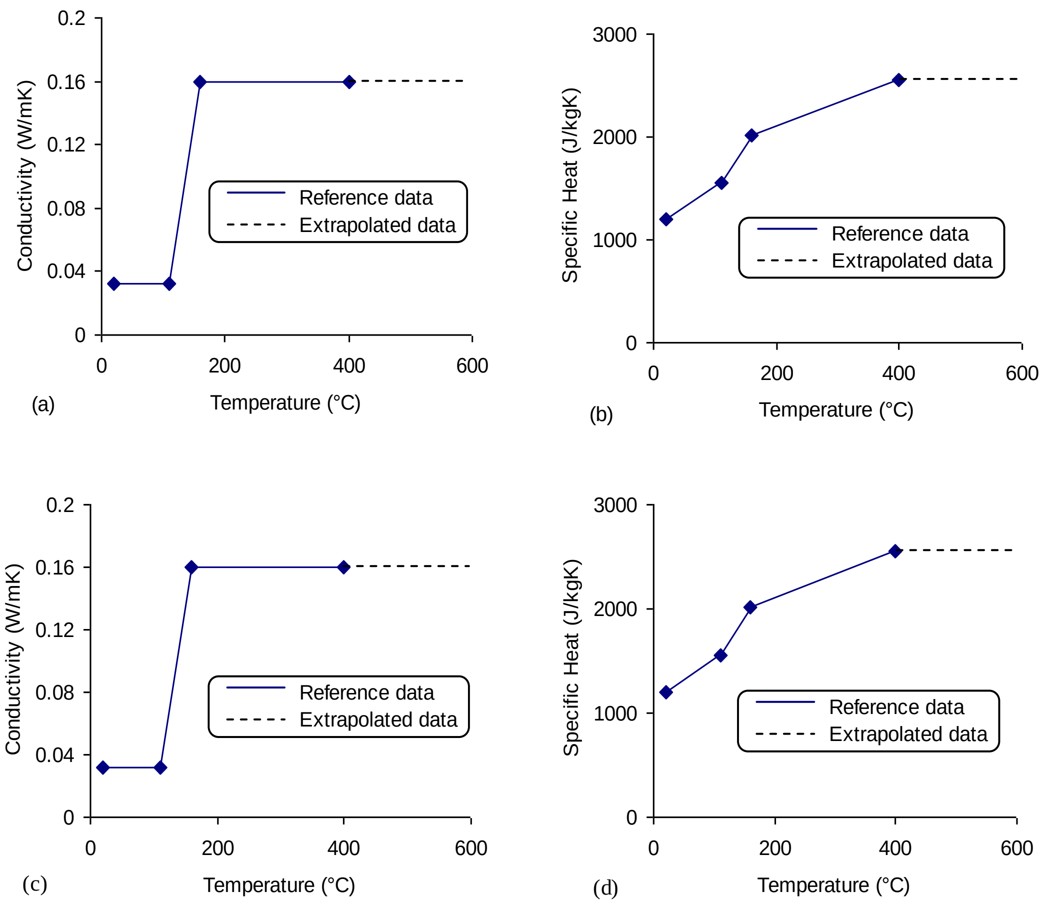

Figure 5. Thermal material properties of PS. a) EPS conductivity b) EPS specific heat c) XPS conductivity d) XPS specific heat.

Density values for EPS and XPS where determined experimentally and found to be 26 kg/m³ and 30 kg/m³ respectively.

3.4 Mesh generation for finite element analysis

Due to symmetry in the cutting plane only half of the foam block (i.e. approximately 5 mm x 30 mm) is used for generating the mesh (figure 6). The mesh pattern is made of uniform rectangles as this simplifies the application of the boundary conditions and analysis of the results. The element size is directly related to the size of the cutting tool (i.e. six elements are used to simulate the top half of each hot-wire regardless of wire diameter). The width and height of the foam block is chosen to be exact multiples of the element size to ensure that the elements are not automatically resized by the software; this is important as the kerf width is determined by counting the number of elements inside the 160°C isotherm. The element chosen for this FEA is a two dimensional quadrilateral element with four nodes (PLANE55).

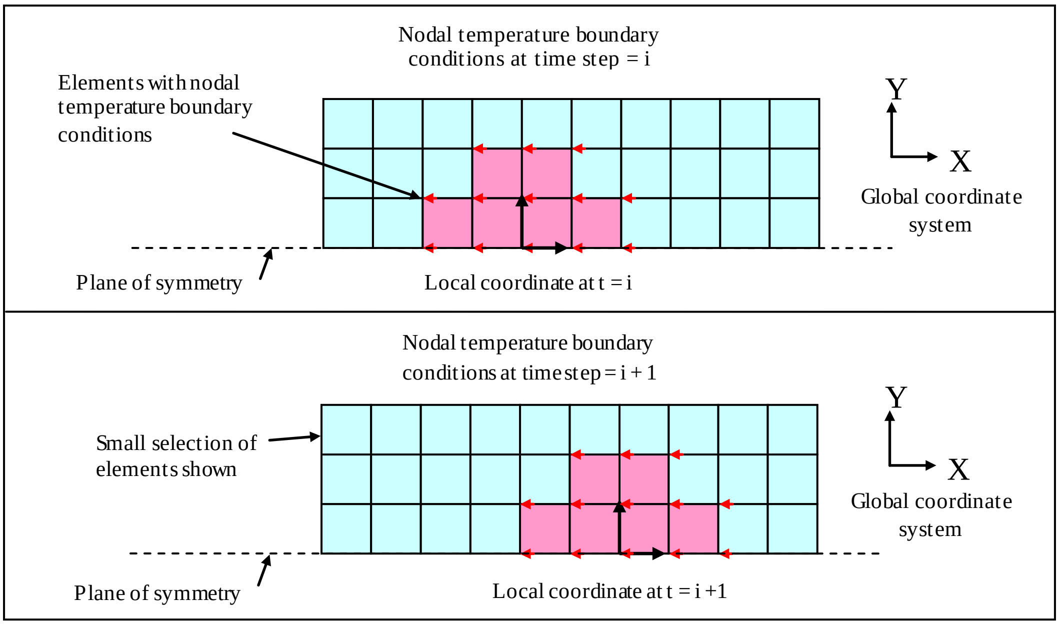

Figure 6. Nodal boundary conditions applied at nodes relative to the local coordinates for two successive time steps.

The hot-wire is simulated by imposing constant temperature boundary conditions on a select group of nodes for a predefined amount of time. After each successive time step the old temperature boundary conditions are deleted and a new set are imposed in a new location as shown in figure 6. The locations of the temperature boundary conditions are determined by moving a set of local coordinates along the foam block a distance of two elements each time step. The group of nodes are selected in such a way as to approximate the shape of the hot-wire being simulated. For example to simulate a hot-wire with a diameter of 0.64 mm a cylindrical local coordinate system is used to select all the nodes within a 360° arc about the origin and within the radius of the wire. The local coordinates are moved a distance of two elements each time step to allow sufficient approximation of the actual continuous process. The time step (*TS*) is calculated by the following equation:

$$TS = \frac{(2 \times elementsize)}{feedrate} \qquad \text{(Eq. 6)}$$

3.5 Results of analysis

This section reports the results of over 25 hot-tool plastic foam cutting simulations using the finite element model developed. The final results of the FEA are evaluated against experimental data and displayed graphically.

Each hot-wire simulation has four main input parameters; wire diameter, material, wire temperature and feed rate. Figure 5 shows the temperature distribution in EPS foam for a 0.64 mm diameter hot-wire with a feed rate of 0.030 m/s and a wire temperature of 252°C. The kerfwidth is calculated by counting the number of vertical elements in the innermost (>160°C) contour and multiplying them by the height of the elements (e.g. for the figure shown; kerf width = 7.6 elements x 0.16 mm/element = 1.2 mm).

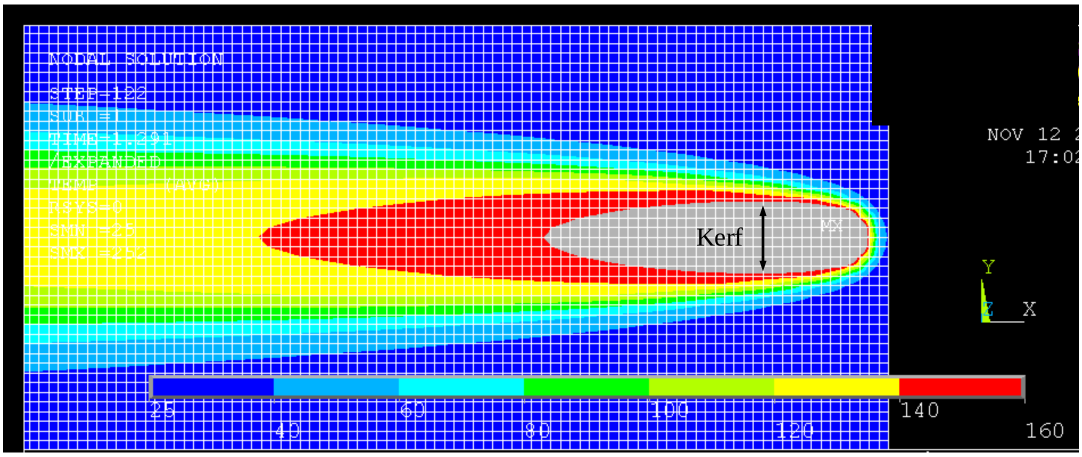

Figure 7. FEA temperature distribution for a hot-wire cut (EPS, 0.64 mm wire, 0.030 m/s).

The results from each simulation were then plotted against the effective heat input and compared against the experimentally determined kerf widths. Figure 6 and 7 show the results for XPS and EPS with a 0.64 mm hot-wire. The FEA kerf widths do not form a perfectly straight line because the temperature inputs came directly from experimentally determined wire temperatures and not from an equation defining a line of best fit. The FEA results match well with the experimental data for both XPS and EPS suggesting the assumptions made in previous sections are valid. For low values of Qeff the FEA kerfwidths are lower than the experimental trend line. It is believed that as the kerf width approaches the diameter of the wire there is more mechanical interaction between the wire and the foam which creates larger than calculated kerfs. For high Qeff values the FEA kerfwidths are greater than the experimental trend line. This is thought to be the result of neglecting the effect of the insulating air gap which grows in size around the wire as the effective heat input increases.

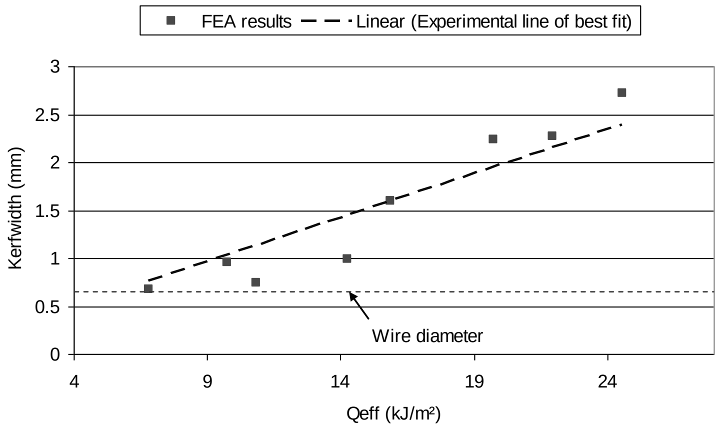

Figure 8. Comparison between FEA and experimental kerfwidths (XPS, 0.64 mm wire).

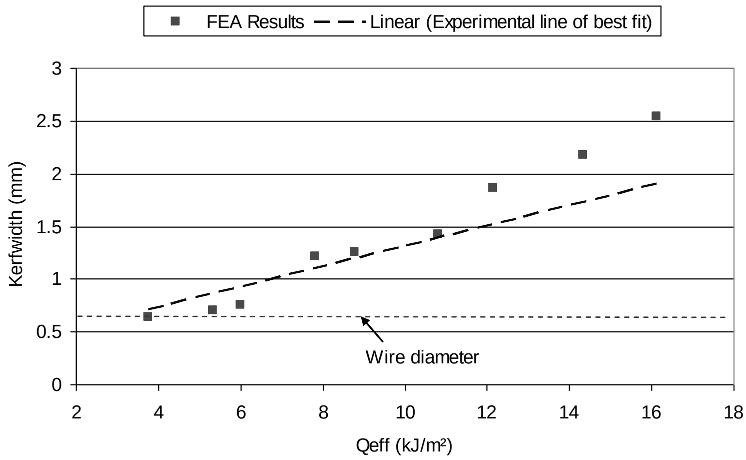

Figure 9. Comparison between FEA and experimental kerfwidths (EPS, 0.64 mm wire).

The results for the 0.36 mm and 0.91 mm wires in XPS are shown below, figures 8 and 9 respectively. The results for the 0.36 mm wire are very close to the experimental data. For low Qeff values the FEA results are above the experimental line of best fit however the actual experimental results are horizontal below 5.5 kJ/m². The range of Qeff values for both the 0.36 and 0.91 mm wires do not go as high as for the 0.64 mm wire graph, therefore the deviation in results at higher kerfwidths is not seen.

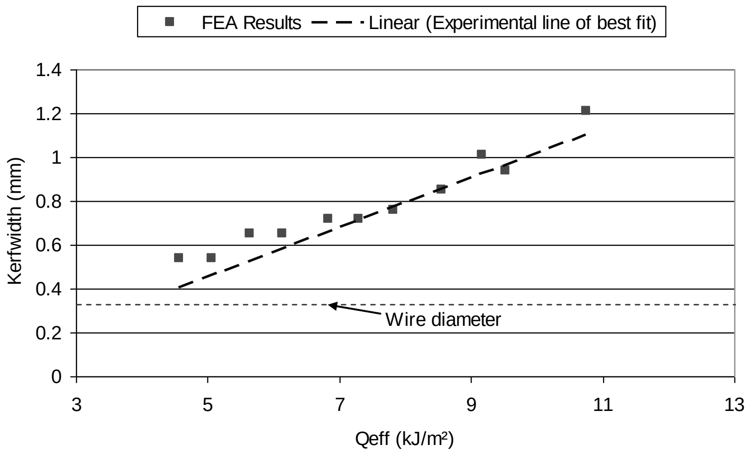

Figure 10. Comparison between FEA and experimental kerfwidths (XPS, 0.036 mm wire)

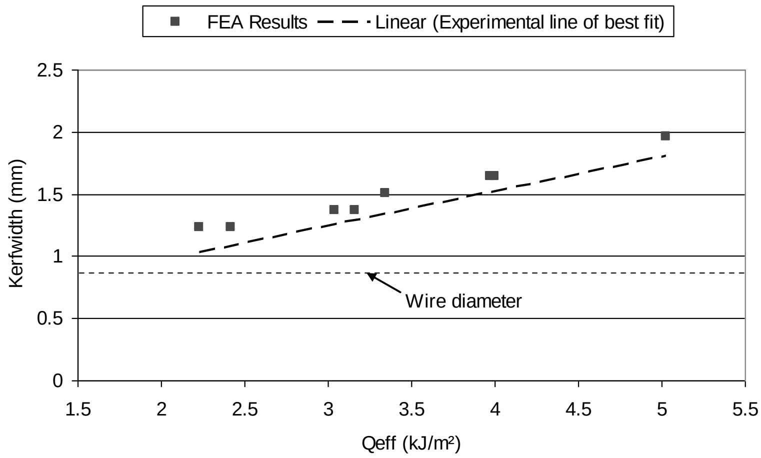

Figure 11. Comparison between FEA and experimental Kerfwidths (XPS, 0.91 mm wire).

The FEA results for the 0.91 mm wire in XPS are all parallel with the experimental line of best fit with a small positive offset. This could be because the range of effective heat inputs is quite low and/or because sum of the assumptions inherent in the model are not valid for large wire diameters.

In summary, the FE model was successful in predicting the kerf width for a wide variety of wire diameters and two different material types. The largest errors occurred at the maximum and minimum effective heat input values due to assumptions that became less valid in those conditions. It is believed that the accuracy of the model could be improved with more accurate temperature inputs.

4. DISCUSSION

The 2D thermal FE model was considered a success, it consistently provided reliable predictions of kerf width based on wire temperatures. However, for this model to be a truly powerful predictive tool some improvements are required.

The first major limitation with the model is that the wire temperature and Q_{eff} relationship needs to be known in advance, hence some prior empirical work in necessary. This problem can be overcome by using a combination of elemental heat generation (which represents joule heating) and applying an initial wire temperature. If the initial wire temperature is low then, for each time step, the wire will heat up due to internal heat generation. At the start of the next time step the tool will have moved a small distance and will be the final temperature of the previous time step. The tool will then heat up again due to joule heating. After a sufficient number of time steps the wire will reach a constant temperature; because the amount of heat generated within the tool will be balanced by the heat transferred to the passing foam. In this way the initial temperatures may be estimated and the amount of heat generated can be calculated with $P = I^2 R$.

The second major limitation with the model is how its predictions diverge from the experimental values at high and low values of the effective heat input. One possible method of minimising this problem is to incorporate the air gap that surrounds the tool into the model. This could be done by changing the material properties of all elements which are found to exceed the temperature at which the cells collapse (160°C for PS foams) from foam to air. The major difficulty with this solution would be accurately modelling the high density layer represented by the collapsed foam. This is necessary to maintain conservation of mass and energy.

5. CONCLUSIONS

In this paper a finite element model was developed based on 2D nonlinear transient thermal finite element equations. The important outcomes are as follows:

- Material testing was carried out in order to develop a successful thermal properties model for EPS and XPS foam cut with hot-wires.
- Experimental cutting trials were used to obtain steady-state wire temperature and kerf width data for a wide range of cutting conditions. The wire temperature data was then used as a simulations input, while the kerf width data was used to verify the simulation results.
- A nonlinear transient thermal finite element model was developed with the purpose of predicting the kerf width of a wide range of feed rate and power combinations for XPS and EPS materials and tools of different sizes.
- The main assumptions in the FE model are; conduction is the only mode of heat transfer, there is no air gap surrounding the cutting tool and the kerf width is represented by the maximum height of the area enclosed by the 160°C isotherm.
- The hot-wire simulations all showed good agreement with the experimental data. The largest errors were often associated with the maximum and minimum effective heat input values for which the assumptions were less accurate.
- The limitations of the FE model in its current form were discussed as well as possible improvements.

REFERENCES

[1] H. Brooks and D. Aitchison, "A Review of State-of-the-Art Large Sized Foam Cutting Rapid Prototyping and Manufacturing Technologies," *The Rapid Prototyping Journal,* Accepted 20 April 2009.

[2] D. Aitchison, H. Brooks "Rapid Manufacturing Facilitation Through Optimal Machining Prediction of Polystyrene Foam" *Virtual and Physical Prototyping*, 1745-2767, Volume 6, Issue 1, 2011, Pages 41-46

[3] H. L. Brooks and D. R. Aitchison, "Force Feedback Temperature Control for Hot-tool Plastic Foam Cutting." Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, Volume 224, Number 5, 2010, pg 709-719.

[4] D. Aitchison, J. Bain, H. Brooks, A. Posthuma, and M. Taylor, "Transient Temperature Effects in Hot-wire Plastic Foam Cutting," in *4th International Conference and Exhibition on Design and Production of MACHINES and DIES/MOLDS,* Cesme, Turkey, 2007, pp. 219-224.

[5] D. Aitchison, H. Brooks, R. Kandula, B. Kraus, and M. Taylor, "Feed-rate, Temperature and Feed-force Relationships for Foam Plastics Cut by a Taut Hot-wire," in *International Conference on Manufacturing Science and Technology* Melaka, Malaysia: Proceedings of ICOMAST2006, 2006.

[6] D. Aitchison and R. Sulaiman, "Determining the surface form of polystyrene through the coordinate measurement machine," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science,* vol. V217, pp. 839-844, 2003.

[7] H. Brooks and D. Aitchison, "Thermal Plastic Foam Cutting Mechanics for Rapid Prototyping and Manufacturing Purposes," in *9th National Conference on Rapid Design, Prototyping and Manufacturing.*, Lancaster University, 2008, pp. pg. 33-40.

[8] H. Brooks, "Plastic Foam Cutting Mechanics for Rapid Prototyping and Manufacturing Technologies," in *Department of Mechanical Engineering*. Thesis PhD Christchurch: University of Canterbury, 2009.

[9] D. G. Ahn, S. H. Lee, and D. Y. Yang, "Investigation into thermal characteristics of linear hotwire cutting system for variable lamination manufacturing (VLM) process by using expandable polystyrene foam," *International Journal of Machine Tools and Manufacture*, vol. 42, pp. 427-439, 2002.

[10] D. G. Ahn, S. H. Lee, and D. Y. Yang, "A study on the influence of the sloped cutting angle on kerfwidth and part quality in the hotwire cutting of EPS foam for the VLM-rapid prototyping process," *International Journal of Machine Tools and Manufacture,* vol. 43, pp. 1447-1464, 2003.

[11] H. C. Jung, "A Study on Laser Forming Processes with Finite Element Analysis," in *Department of Mechanical Engineering*. vol. Ph.D Christchurch: University of Canterbury, 2006.

[12] S. Mehta, S. Biederman, and S. Shivkumar, "Thermal degradation of foamed polystyrene," *Journal of Materials Science,* pp. 2944-2949, 1995.

[13] B. Wunderlich, A. Boller, I. Okazaki, and S. Kreitmeier, "Modulated differential scanning calorimetry in the glass transition region," *Journal of Thermal Analysis and Calorimetry*, vol. 47, pp. 1013-1026, 1996.