Elastohydrodynamic analysis of hypoid gear contacts with variable geometry and kinematics

Elastohydrodynamic Analysis of Hypoid Gear Contacts With Variable Geometry and Kinematics

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Nomenclature

h	: Film thickness
h_{c0}	: Central contact film thickness
R_{zx}	: Equivalent radius of contact along the direction of minor axis of elliptical footprint
R_{zy}	: Equivalent radius of contact along the direction of major axis of elliptical footprint
p	: Pressure
U	: Speed of entraining motion
C.R.C	: Constant Radii of Curvature
V.R.C	: Variable Radii of Curvature
x and y	: Cartesian coordinate system.

Greek Symbols

α	: Lubricant pressure-viscosity coefficient
η	: Lubricant dynamic viscosity at pressure *p*
η_0	: Lubricant dynamic viscosity at atmospheric pressure
θ	: Angle of lubricant entrainment into the contact
ρ	: Lubricant density at pressure *p*
ρ_0	: Lubricant density at atmospheric pressure

1. Introduction

Elastohydrodynamic Lubrication (EHL) has received much attention for several decades. Numerous investigations have been carried out in order to gain further insight under different operating conditions and in various applications. Particularly, hypoid and bevel gears as key components in automotive drive trains have attracted much research work. The study of these contacts requires realistic surface geometry, contact kinematics and applied normal load. One of the key complexity of hypoid and bevel gear teeth-pair conjunctions is their variable geometry and kinematics through meshing cycle. Due to the complexity of these conjunctions, Tooth Contact Analysis (TCA) is the only effective tool to obtain the required input (contact geometry and kinematics) which vary through mesh. This paper aims to present an EHL model, combined with TCA. The model is able to capture the effects of varying contact radii of curvature, including the effect of EHL, an approach not hitherto reported.

There has been a large volume of research literature on EHL of hypoid gears in recent years, including the works reported by Mohammadpour et al [1], Paouris et al [2] and Fillot et al [3]. Mohammadpour et al [1] and Paouris et al [2] used TCA, based on dry TCA, whilst Fillot et al [3] approached EHL in a general form. All these approaches only consider constant radii of curvature (CRC). In reality, consideration should be given to the use of variable radii of curvature (VRC).

2. Methodology

TCA [4] has been used effectively by many researchers in order to obtain the required input information for a tribological study [5]. As already noted, these include the instantaneous contact geometry of gear teeth pair, applied normal load and contact kinematics. However, most tribological models use a simplified single value for each of abovementioned parameters, thus neglecting the effect of variable geometry and contact kinematics (speed of entraining motion of lubricant into the contact as well as the relative sliding velocity of contacting surfaces). Although the instantaneous variations may be considered as small, they do deviate from an assumed average value. This can play an important role in predicting a more realistic estimation of lubricant film thickness, friction and power loss. In the current work, a new method is provided to take into account these instantaneous variations in the solution of Reynolds equation. The realistic distribution of each parameter is obtained using a TCA tool.

There are numerous methods available for conducting an EHL analysis. In this paper a robust model proposed by Mohammadpour et al [3] is used. The model is modified in order to incorporate the variation in the radii of curvature data set. Then, a comparison is made between using the CRC data set and that of VRC.

Mohammadpour et al [3] model incorporates the Reynolds equations for the EHL analysis as:

$$\frac{\partial}{\partial x}\left[\frac{\rho h^3}{\eta}\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{\rho h^3}{\eta}\frac{\partial p}{\partial y}\right]=6U\left\{\cos\theta\frac{\partial}{\partial x}[\rho h]+\sin\theta\frac{\partial}{\partial y}[\rho h]\right\} \quad (1)$$

where U is the speed of lubricant entraining motion. This is considered to be constant at any instant of time, thus only the effect of VRC is investigated. It should be noted that the squeeze film effect is omitted from equation 1. Inclusion of squeeze film velocity enhances the load carrying capacity of the contact [6].

The EHL model uses piezo-viscosity [7] and compressibility functions [8] for lubricant viscosity and density respectively. For lubricant dynamic viscosity:

$$\eta=\eta_0\exp\left\{\left[\left(\ln\eta_0+9.67\right)\times\left(1+5.1\times10^{-9}p\right)^Z\right]-\left[\ln\eta_0+9.67\right]\right\} \quad (2)$$

where:

$$Z=\frac{\alpha c_p}{\left(\ln\eta_0+9.67\right)}=\frac{\alpha}{5.1\times10^{-9}\left(\ln\eta_0+9.67\right)},\ \text{as}\ c_p=1.96\ \text{MPa} \quad (3)$$

For density [8]:

$$\rho=\rho_0\left(1+\frac{0.6\times10^{-9}p}{1+1.7\times10^{-9}p}\right) \quad (4)$$

Additionally equation (5) provides the elastic film shape as:

$$h(x,y)=h_{c0}+s(x,y)+\delta(x,y) \quad (5)$$

s is the undeformed conjunctional profile as:

$$s(x,y)=\frac{x^2}{2R_{zx}}+\frac{y^2}{2R_{zy}} \quad (6)$$

Note that with CRC, the equivalent radii of contact R_{zx} and R_{zy} [9] are treated as constants. With the VRC model these radii vary spatially.

The localised deflection $\delta(x,y)$ is obtained through solution of elasticity potential equation [6].

3. TCA Results

In order to calculate the required input parameters for the tribological model, a finite element-based TCA model is used. The model is able to predict the variable geometry and contact kinematics for all teeth pairs in simultaneous contact. Figure 1 shows the distribution of radii of curvature in the contact footprint for all engaged teeth pairs, for one time step of simulation.

Figure 1 clearly reveals the significant change in the both the radii of curvature across the flank. Concerning the position at the centre of the flank, figure 1 shows a difference of 36.3% between the CRC and VRC approaches.

The results from the EHL model used in this paper use the radii curvature data from the centre of the contacting flanks.

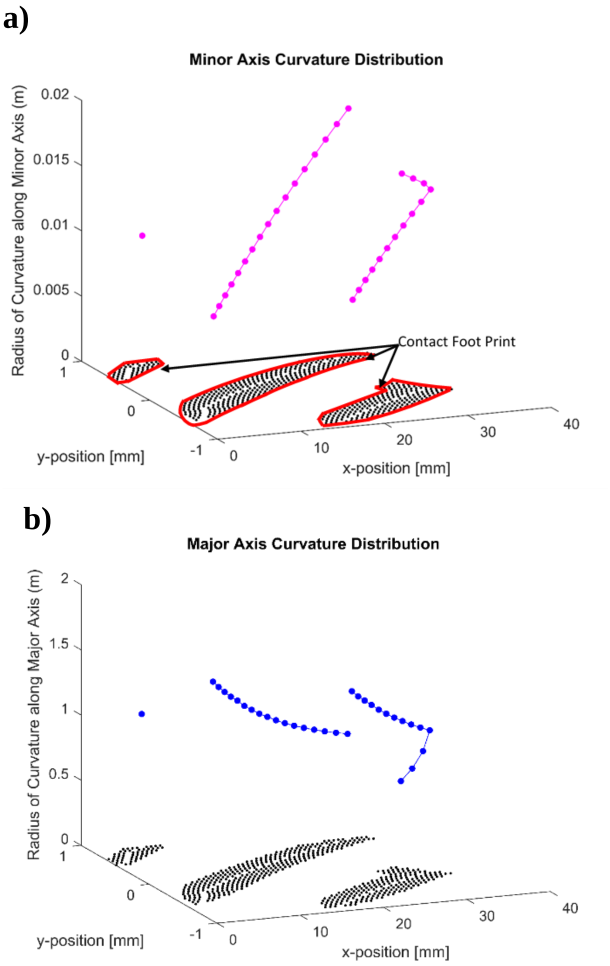

Figure 1: Instantaneous Radius of Curvature distribution along a) Minor Axis b) Major Axis.

4. EHL Results

Figure 2 shows the film shape across the contact for constant and varying radii of curvatures.

With the VRC, the film shape becomes more asymmetrical as opposed to predicted results using CRC. Figure 2b clearly shows a more distorted elliptical contact footprint. This is reasonable due to the nature of teeth engagement in hypoid gears.

The film shape for CRC is also skewed. This is substantially induced by the angled flow entrainment into the contact footprint, but not due to any variable flank radii or contact kinematics [1]. The film shape also shows that the minimum film thickness is observed close to the left side of the contact for VRC, opposite to the minimum film thickness position in CRC. This means that the effect of variable geometry has overcome the effect of the angled flow and has moved the minimum film thickness to the opposite side of the contact. This also implies that opposite side of the flank will exhibit higher pressures than the rest of the flank. Conversely using CRC indicates higher pressures near the centre of the flank. This location of the maximum pressure indicates the critical position in terms of potential wear and durability issues.

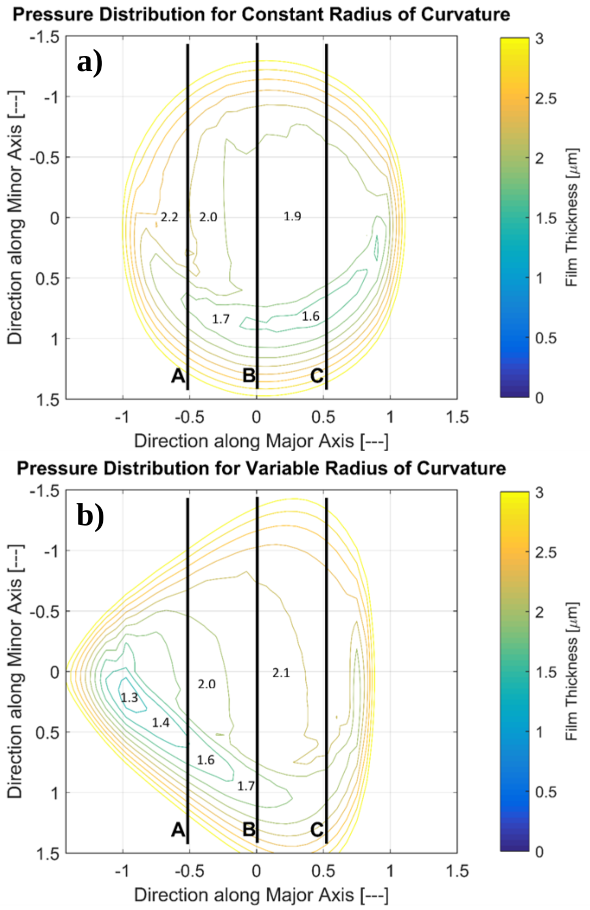

Figure 2: Film thickness shape for a) Constant b) Variable radii of curvature.

For a better understanding of the effect of VRC versus CRC, the film shape along the minor axis of the elliptical contact footprint is shown in figure 3. Film shapes are obtained at the centre of the contact and two positions on the right and left of central region, marked by A, B and C in figure 2. Table 1 shows the percentage difference in the minimum film thickness between constant and variable radii of curvature approaches at each of these positions.

Region	Constant (µm)	Varying (µm)	% Difference
A	1.80	1.55	14.0
B	1.66	1.73	4.5
C	1.61	1.93	20.0

Table 1: Minimum film thickness comparison.

The minimum film thickness at the contact centre shows little change. However, both the inlet and exit of regions of the contact show significant differences.

Not only there is between 4.5% to 20% error using the CRC, the location of the minimum film thickness along the minor axis has also changed.

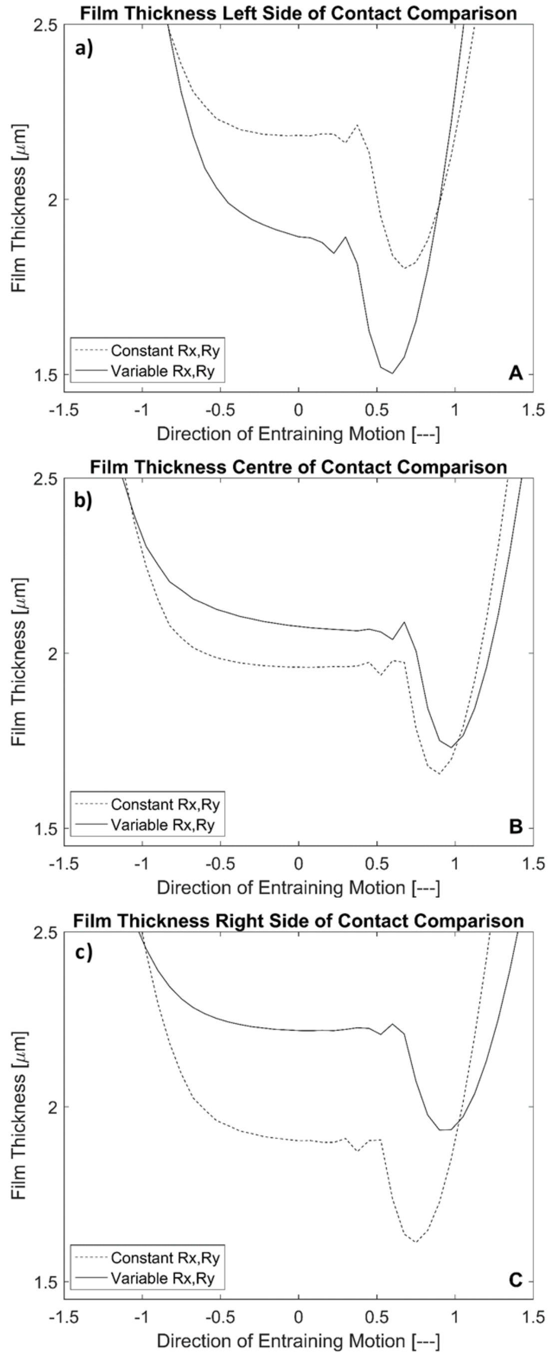

Figure 3: Film thickness across direction of entraining motion equal to a) A= -0.525 b) B= 0 and c) C= 0.525 data cut through for both constant and varying radii of curvature.

Overall across the entire contact, it was found that 1.283µm and 1.612µm are the minimum film thickness values for constant and variable radii curvatures respectively. This contributes a difference of 20.4%, indicating significant difference between CRC and VRC analyses.

5. Conclusion

The EHL methodology outlined in this paper is successfully implemented in conjunctions with variable instantaneous radii of curvature. There is strong evidence that using varying radii can significantly affect the predicted conditions. Using the conventional CRC method, 20% error in under-estimating the minimum film thickness in the conjunction is observed. Furthermore, the effect of angled flow lubricant entrainment upon the location of minimum film thickness appears to be offset by the introduction of VRC. Clearly, this would affect the predicted friction, power loss and efficiency.

6. Acknowledgements

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7. References

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