Effect of tooth microgeometry profile modification on the efficiency of planetary hub gears

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Effect of Tooth Microgeometry Profile Modification on the Efficiency of Planetary Hub Gears

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Abstract: Planetary hub systems offer desired speed and torque variation with a lighter, compact and coaxial construction than the traditional gear trains. Frictional losses are one of the main concerns. Generated friction between the mating teeth flanks of vehicular planetary hubs under varying load-speed conditions is one of the main sources of power loss. Modification of gear tooth geometry as well as controlling the surface topography are the remedial actions to reduce friction and hence the power loss.

The paper studies the effect of tooth crowning and tip relief upon system efficiency. It includes an analytical elastohydrodynamic analysis of elliptical point contact of crowned spur gear teeth, also including the effect of direct contact of asperities on the opposing surfaces. Tooth contact analysis (TCA) is performed to obtain contact footprint shape as well as contact kinematics and load distribution. A parametric study is carried out with the expounded model to observe the effect of gear crowning and tip relief with different levels of gear surface finish upon planetary hubs' power loss.

Keywords— Transmission efficiency, Gear tooth modification, Planetary wheel hub system, Surface finish

1-Introduction

Planetary gear sets are used in many applications, ranging from gas turbines to automotive power trains. They provide a large set of different transmission ratios. Improved efficiency relative to fixed axes transmission systems is one of the most important advantages of planetary gear sets [1].

Transmission losses are one of the main concerns in any gearing applications. Another concern is noise and vibration of gearing systems, mainly due to low damping characteristics of lubricant

film in gear teeth pair contacts under the usual elastohydrodynamic conditions in medium to high loads. This was first demonstrated by Dareing and Johnson [2], experimenting with a pair of representative wavy surfaced discs. Numerical analysis carried out under the same conditions by Mehdigoli et al [3] agreed with the findings of Dareing and Johnson [2] and showed the lightly damped nature of Elastohydrodynamic contacts. The study in [3] showed that a tribodynamic model is required to simultaneously study vibration and efficiency of gearing systems. A large number of numerical and experimental studies have been devoted to power losses of transmission systems of different configurations under transient dynamic conditions. These include the work of Li and Kahraman [4] for the case of spur gear pairs and that of De la Cruz et al [5] for the case of helical gears of vehicular transmission systems. In the case of the former a model based on mixed-elastohydrodynamic (EHL) regime of lubrication was used. In the case of the latter a transient elastohydrodynamic analysis was carried out.

Mohammadpour et al. [6] proposed an integrated multi-body dynamics and lubricated contact mechanics model to predict the transient behaviour of efficiency and noise, vibration and harshness (NVH) for hypoid gear pairs of vehicular differential system. They showed that NVH refinement and transmission efficiency can lead to contrary requirements. As the regime of lubrication remained predominantly in non-Newtonian elastohydrdynamics, the tribological contacts could be represented by frictional characteristics obtained through combined analytical and experimental studies by Evans and Johnson [7], thus reducing the computational burden of numerical analysis of the meshing teeth pairs. In such an approach, the lubricant film thickness is estimated using extrapolated lubricant film thickness formulae, such as that presented by Chittenden et al [8]. This approach approximates the transient contact dynamics with instantaneous quasi-steady solution of the lubricated contact problem. This approach is computationally quite efficient [9] and as De la Cruz et al [5] show conforms well with the full numerical solution. The same approach is reported by Fatourehchi et al. [10] to estimate gear contact power loss in high performance transmission systems, using different gear teeth modifications and their effects on gear power loss and system durability.

With regard to planetary gears, Talbot et al. [1] investigated the power losses under various operating speeds and torques, inlet oil temperature, numbers of planets and surface roughness of meshing teeth pairs. Their experimental results indicated that mechanical power loss decreases

with a reduction in the oil sump temperature and improved surface roughness. Marques et al. [11] carried out experimental investigation of a wind turbine planetary gear system, measuring the system power loss with different gear oils and operating conditions. They compared the measured results with a numerical model. Inalpolat and Kahraman [13] presented a dynamic model of the planetary gears for automotive transmission, but used a dry contact model taking into account elastic deformation of mating teeth with their initial separation and their rigid body approach. Therefore, their model was suited for dynamic study, not for friction of lubricated contacts. Recently, Mohammadpour et al [13] presented a tribodynamic model of planetary gears for hybrid powertrain systems, studying the effect of power source mode upon transmission efficiency as well as NVH refinement.

Planetary hub systems of trucks and off-highway vehicles are subject to high loads at low operating speeds. These promote the worst tribological conditions, resulting in contact pressures of the order of 1.2 GPa and sub-micrometre lubricant film thickness. Planetary hub systems are also particularly compact, yielding highly concentrated tooth meshing contacts. Therefore, a methodical approach capable of predicting the parameters which affect planetary hub gears efficiency is the key to achieve efficient systems.

The current study presents a parametric analysis on the effects of different extents of tooth crowning and tip relief on the planetary hub gears' power loss. It also takes into account the influence of roughness of meshing surfaces upon system efficiency.

2- Planetary hub configuration

Figure 1 shows a schematic representation of planetary gear wheel hub system, including the power flow from the differential gearbox through to the wheel hub. It also shows the transmission ratio, torques and speeds at the different stages of the axle system. Power is transmitted through the sun gear, attached to the input shaft. The ring gear is fixed to the housing. The output power is transmitted to the wheels through the carrier. The planetary system comprises three planet gears. It is assumed that there is no misalignment in the planetary system, and the input power from the sun gear is equally divided among the planets.
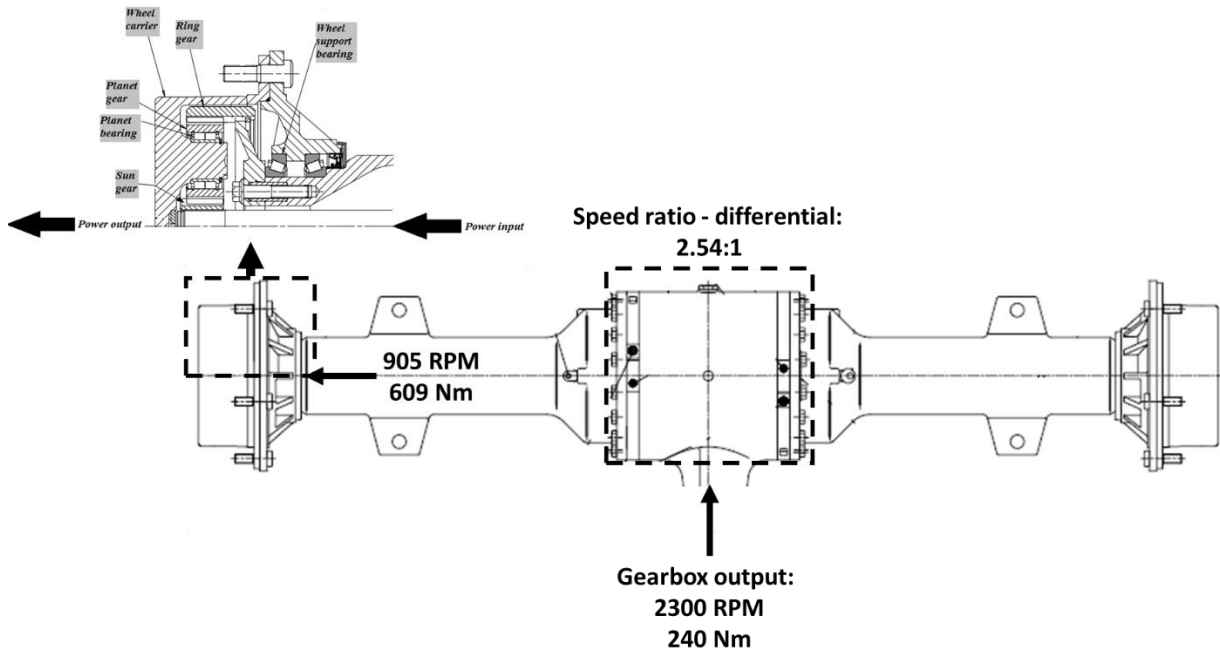
Figure 1: Schematic representation of the planetary wheel hub gears

3- Methodology

The integrated methodology combines Tooth Contact Analysis (TCA) and an analytical elastohydrodynamic contact model. For a complete meshing cycle, the instantaneous radii of curvature, rolling and sliding contact velocities and the normal meshing contact loads are obtained through TCA. These parameters form the input to the elastohydrodynamic model in order to obtain viscous and boundary friction contributions for the planetary hub system power loss for a complete meshing cycle.

3.1 – TCA

The developed TCA model is comprises contact analysis using finite elements, based on the approach of Vijayakar [14], and Xu and Kahraman [15]. The TCA model is used to obtain the instantaneous contact geometry, rolling and sliding velocities and load share per teeth pair [16] for simultaneous meshing of sun-planet and planet-ring contacts in the planetary hub system.

3.2 – The elastohydrodynamic contact model

The planetary wheel hub system of trucks and off-highway applications are subject to high contact pressures up to 1.2 GPa, which can result in asperity interactions within the mating teeth pair contacts. Therefore, mixed regime of lubrication would be expected. Friction in the mixed regime of lubrication comprises two contributions; viscous shear of the thin lubricant film and direct interaction of asperities on the opposing boundary solid surfaces. Therefore, the instantaneous power loss is obtained due to these sources of generated friction as:

$$P_{loss} = (f_v + f_b)U_s \tag{1}$$

3.2.1 – Boundary friction

The thin lubricant films in the elastohydrodynamic contact of gear teeth pairs are usually of the order of surface roughness of contacting surfaces, particularly due to the high generated heat and usually starved inlet boundary conditions in practice [17].

Greenwood and Tripp [18] presented a method for prediction of boundary friction contribution with an assumed Gaussian distribution of asperity peaks under mixed or boundary regimes of lubrication. This is a function of the Stribeck's oil film parameter: $1 < \lambda = \frac{h_{c0}}{\sigma} < 2.5$ which specifies the fraction of the load carried by the asperities in the contact footprint. Thus [18]:

$$W_a = \frac{16\sqrt{2}}{15}\pi(\xi\beta\sigma)^2\sqrt{\frac{\sigma}{\beta}}E'AF_{5/2}(\lambda) \tag{2}$$

where, the statistical function $F_{5/2}(\lambda)$ for a Gaussian distribution of asperities is obtained as [9, 19]:

$$F_{5/2} = \begin{cases} -0.004\lambda^5 - 0.057\lambda^4 - 0.29\lambda^3 - 0.784\lambda^2 - 0.784\lambda - 0.617 & \text{for } \lambda < 2.5 \\ 0; & \text{for } \lambda \geq 2.5 \end{cases} \tag{3}$$

For steel surfaces, the roughness parameter ($\xi\beta\sigma$) is generally in the range 0.03–0.07. σ/β which is defined as average asperity slope [9], is in the range of 10^{-4} to 10^{-2}. For the current study: $\xi\beta\sigma = 0.055$ and $\sigma/\beta = 10^{-3}$.

Friction generated by asperities interaction should be taken into account in mixed and boundary regimes of lubrication. A thin adsorbed film at the summit of the asperities or entrapped in their inter-spatial valleys is subjected to non-Newtonian shear, thus boundary friction f_b is obtained as:

$$f_b = \tau_L A_a \tag{4}$$

where, τ_L is the lubricant's limiting shear stress:

$$\tau_L = \tau_0 + \varepsilon P_m \tag{5}$$

where, the mean (Pascal) pressure P_m is:

$$P_m = \frac{W_a}{A_a} \tag{6}$$

The asperity contact area is expressed as [18]:

$$A_a = \pi^2(\xi\beta\sigma)^2 A F_2(\lambda) \tag{7}$$

The statistical function $F_2(\lambda)$ is calculated as [9, 19]:

$$F_2(\lambda) = \begin{cases} -0.002\lambda^5 - 0.028\lambda^4 - 0.173\lambda^3 + 0.526\lambda^2 - 0.804\lambda - 0.500 & for\ \lambda < 2.5 \\ 0; & for\ \lambda \geq 2.5 \end{cases} \tag{8}$$

3.2.2 – Viscous friction

Evans and Johnson [7] developed an analytical method to obtain viscous friction in elastohydrodynamic contacts, where the coefficient of friction is calculated as:

$$\mu = 0.87\alpha\tau_0 + 1.74\frac{\tau_0}{\bar{p}} ln\left[\frac{1.2}{\tau_0 h_{c0}}\left(\frac{2K\eta_0}{1+9.6\xi}\right)^{\frac{1}{2}}\right] \tag{9}$$

where, ξ is:

$$\xi = \frac{4}{\pi}\frac{K}{h_{c0}/R}\left(\frac{\bar{p}}{E'RK'\rho'c'U_r}\right)^{1/2}$$

The lubricant film thickness under the instantaneous operating conditions is obtained using the regressed extrapolated lubricant film thickness formula of Chittenden et al. [8]:

$$h_{c0}^* = 4.31 U_e^{0.68} G_e^{0.49} W_e^{-0.073} \left\{ 1 - exp\left[-1.23 \left(\frac{R_y}{R_x}\right)^{2/3} \right] \right\} \tag{10}$$

where, the non-dimensional governing groups are expressed as:

$$U_e = \frac{\pi \eta_0 U}{4 E_r R_x} , \; W_e = \frac{\pi W}{2 E_r R_x^2} , \; G_e = \frac{2}{\pi}(E_r \alpha) , \; h_{c0}^* = \frac{h_0}{R_x}$$

Therefore, the generated viscous friction, using equation (9) becomes:

$$f_v = \mu W \tag{11}$$

4- Results and discussion

The planetary hub gear set of the JCB Max-Trac rear differential is studied here. The input torque to the sun gear from the differential is 609 Nm at the speed of 906 rpm.

The results are presented in two parts. The first part deals with the effect of longitudinal crowning, whilst the second part shows the influence of gear teeth tip relief modification. Two parameters are involved in the tip relief modification: the extent of tip relief and the length relieved region. The results for cyclic meshing power loss are for both the axle's planetary wheel hub sets.

Figure 2 is a schematic representation of teeth longitudinal crowning and tip relief modification. In order to study the effect of surface roughness on the gear pair power loss, the gear tooth surface roughness for different longitudinal crowning and tip relief modification are varied from 0.4 μm to 3.6 μm.
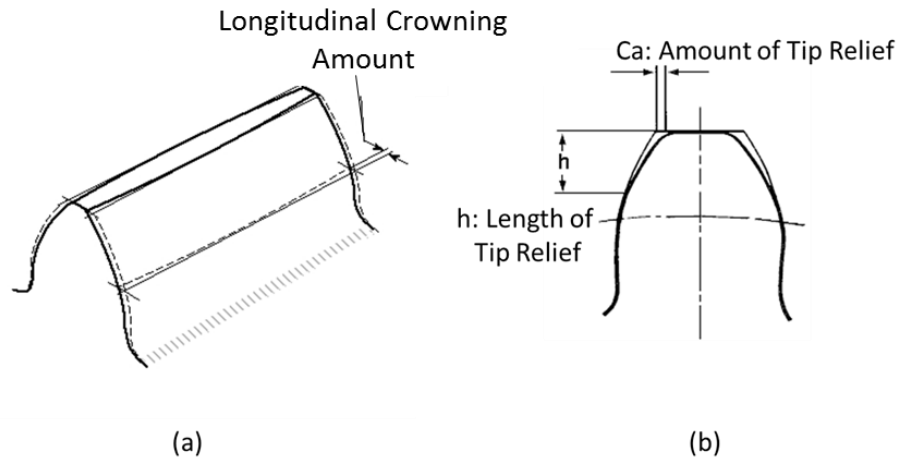
Figure 2: Schematic representation of gear teeth modification; (a) crowning (b) tip relief

4.1- Effect of tooth profile longitudinal crowning

Idealised spur gears with finite line contact geometry are very sensitive to misalignment and manufacturing errors which cause edge loading of their contacts, leading to edge stress generated pressure spikes, similar to the straight-edged roller bearings [20, 21]. One repercussion of this is localised wear or fatigue spalling (pitting) due to inelastic sub-surface stresses [22], another is increased gear noise. Like rolling element bearings, where their sharp edges are crowned or relieved by a dub-off radius to reduce the edge stress discontinuity [23], the simplest way to avoid the edge loading is through longitudinal crowning of gear teeth surfaces. Several investigators have studied improvements in the meshing contact stress distribution for misaligned spur gears through crowning [24, 25].

In order to study the effect of longitudinal crowning on power loss, the magnitude of longitudinal crowning is varied between 50% -150% of the current in-field design. The amounts of applied longitudinal crowning used in the current study are listed in Table 1.

Table 1: Amount of applied longitudinal crowning for different scenarios

Scenario	Crowning amount [%]		
	Sun	Planet	Ring
1	150	150	150
2	125	125	125
3 (current design)	100	100	100

4	75	75	75
5	50	50	50

In order to study the simultaneous effect of longitudinal crowning and surface roughness on power loss, a map of these values is generated. Figure 3 shows the total power loss with different longitudinal crowning and surface roughness. Figure 4 represents percentage change in the total power loss with respect to the current base design. Referring to Figures 3 and 4, the power loss of the planetary gear sets can be reduced by 5%. However, considering the high efficiency of these gear sets, the absolute value of this reduction only amounts to 45W per meshing cycle. This gain in efficiency should be set against the entailing manufacturing costs, indicating little incentive for implementation.
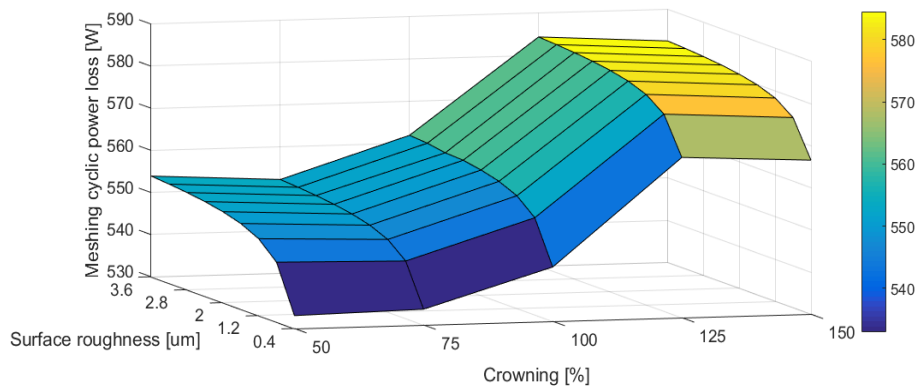

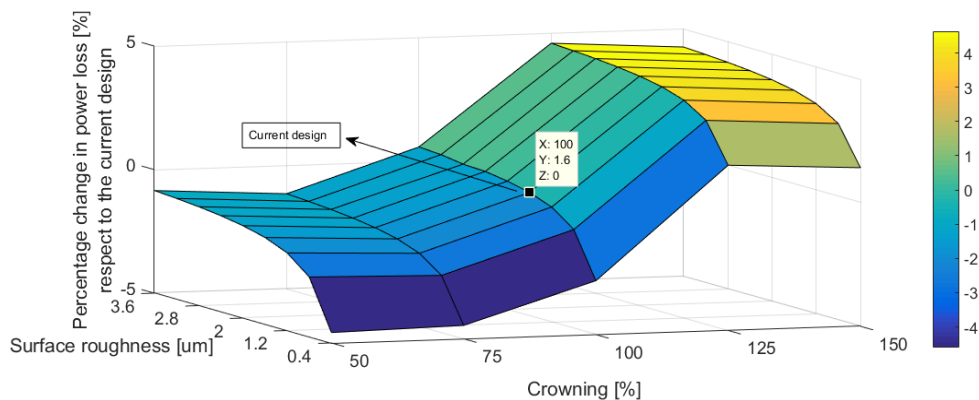Figure 3: Meshing cyclic power loss with different surface topography and crowning


Figure 4: Percentage change in power loss with different crowning and surface roughness

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Finally, Figure 5 shows the cyclic precession of contact footprint for different crowning cases for both the planet-ring and the sun-planet contacts. It can be noted that by increased crowning, the contact is concentrated in the centre of the teeth flanks. This is safer to avoid any edge stress discontinuity. Therefore, in terms of surface fatigue and useful life some gain can be expected due to reduced sub-surface stressing [22]. However, the reduced cross-section can lead to increased root stresses with higher contact pressures as the result of reduced contact footprint dimensions, requiring further in-depth analysis.
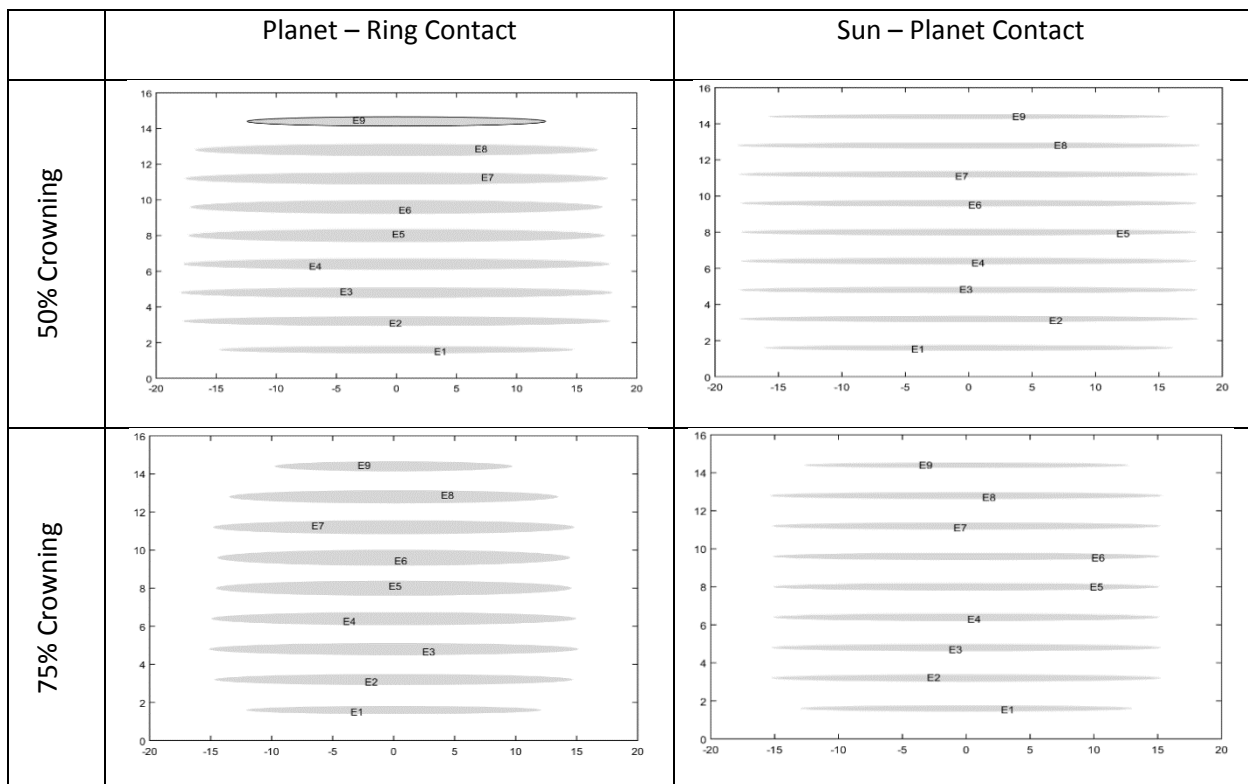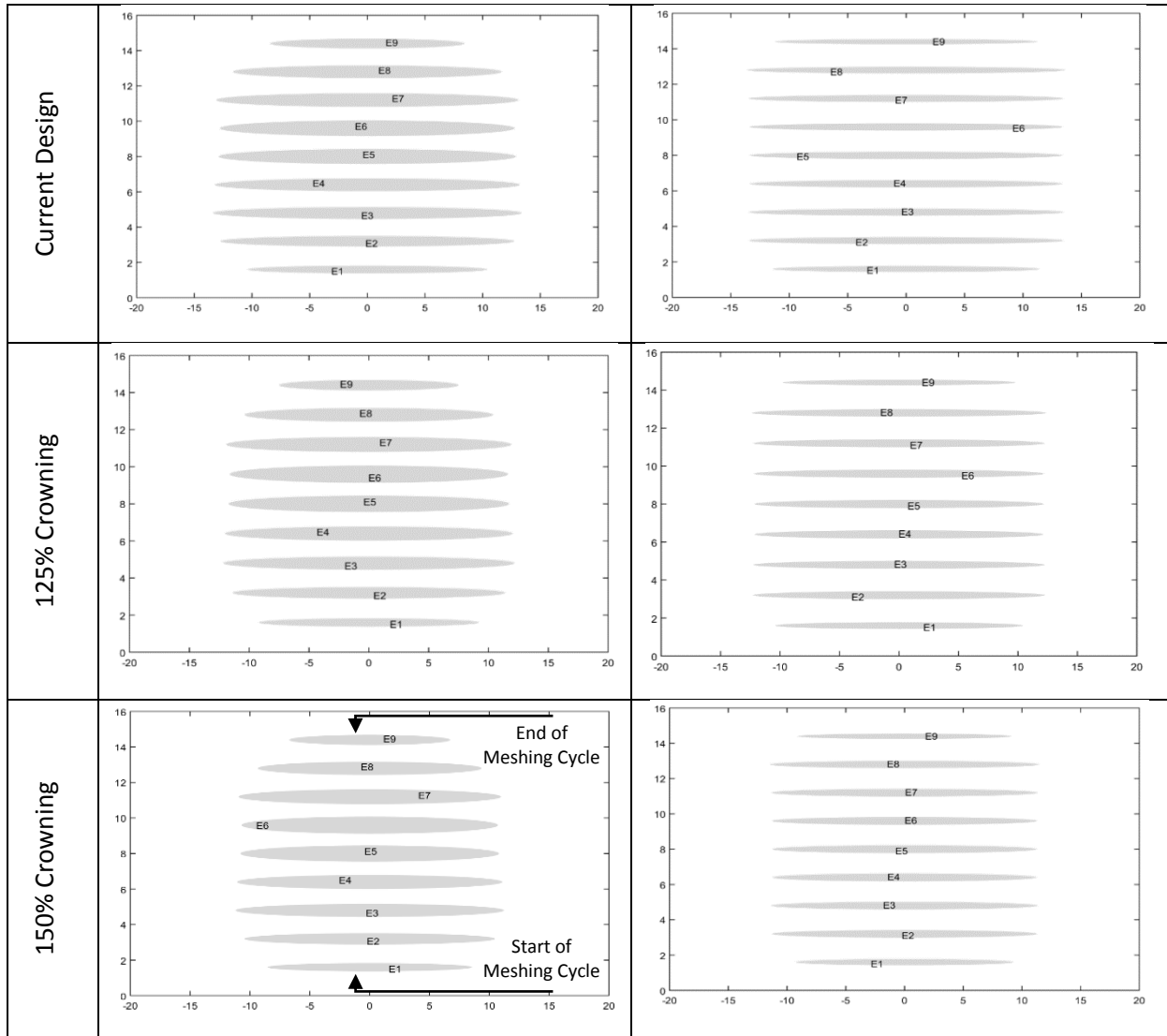
Figure 5: Contact footprints for different longitudinal crowning case studies

4.2- Effects of tip relief

At the beginning and the end of a meshing cycle with no tip relief, an impact and sharp rise in the contact pressure occurs as a pair of new teeth comes into contact. In order to attenuate this effect the involute profile in the tip region is relieved. The optimum length of tip relief region enables smooth load variation from one pair of teeth to the next. The extent of tip relief in length and amount should be determined (see Figure 2(b)).

4.2.1- Effects of tip relief amount

In order to study the effect of tip relief amount on the gear pair power loss, the amount of tip relief is changed from 25% to 150% of the current base design values. Amounts of tip reliefs for different cases are shown in Table 2.

Table 2: Amount of tip relief for different scenarios

Scenario	Tip relief amount [%]		
	Sun	Planet	Ring
1	150	150	150
2	125	125	125
3(Current-Design)	100	100	100
4	75	75	75
5	50	50	50
6	25	25	25

The map of these results is illustrated in Figure 6. This figure shows the total power loss with different tip relief amount and surface roughness. Figure 7 represents percentage change in total power loss with respect to the current design. According to these results, the overall power loss can be reduced by 12% in comparison with the current design. This value, in absolute term, is nearly 70W. The results reveal that the effect of tip relief on the surface power loss is much more pronounced.
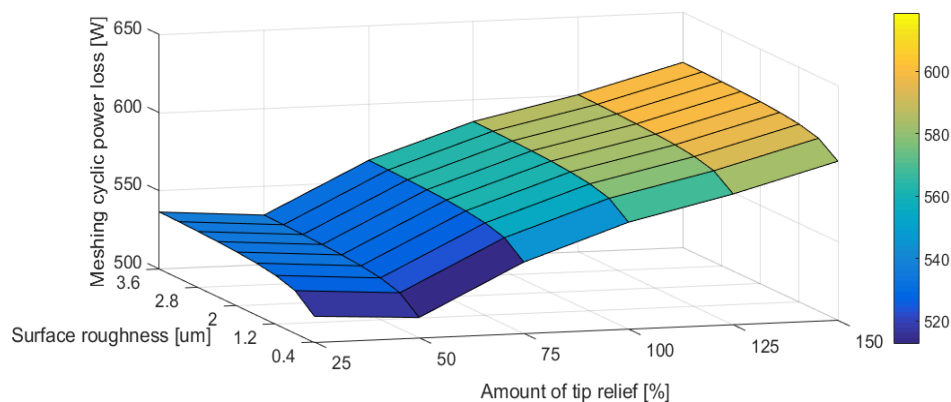

Figure 6: Meshing cyclic power loss with different surface topography and tip relief amount
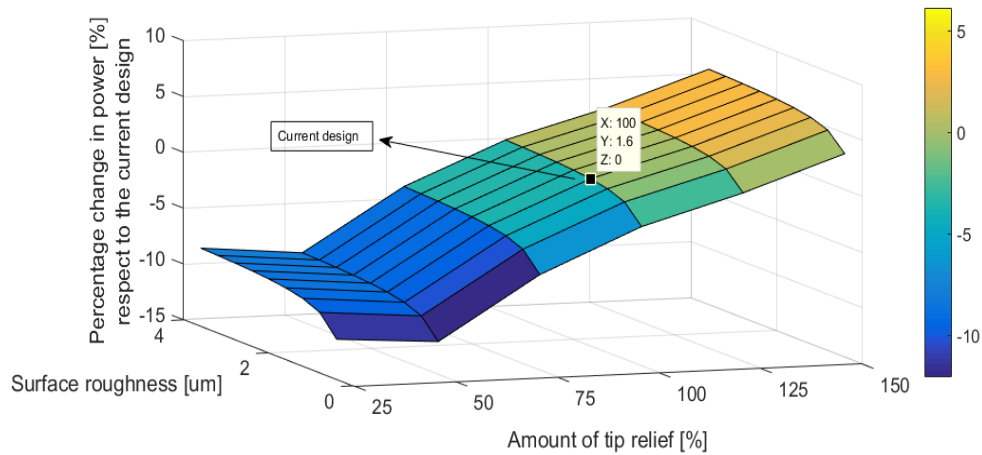
Figure 7: Percentage change in power loss with different surface roughness and tip relief amount

4.2.2- Effect of tip relief length

In order to study the effect of change in the length of tip relief (specified as h in Figure 2(b)) on the gearing power loss, the relief length is reduced from the current design (base value) by 25% of its value. Table 3 shows changes in the length of tip relief for different cases.

Table 3: Length of tip relief for different scenarios

Scenario	Length of tip relief [%]		
	Sun	Planet	Ring
1(Current-Design)	100	100	100
2	75	75	75
3	50	50	50
4	25	25	25

The results in Figure 8 show the total power loss with different length of tip relief and surface roughness. Figure 9 shows the percentage change in the total power loss with respect to the current design. It shows that the power loss can be decreased by up to 10%, representing a 50W reduction per meshing cycle.
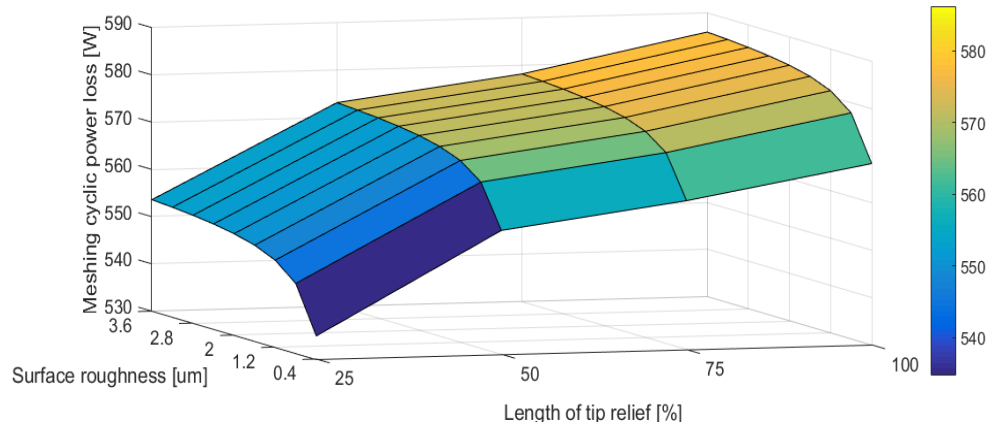
Figure 8: Cyclic meshing power loss with different surface topography and tip relief length
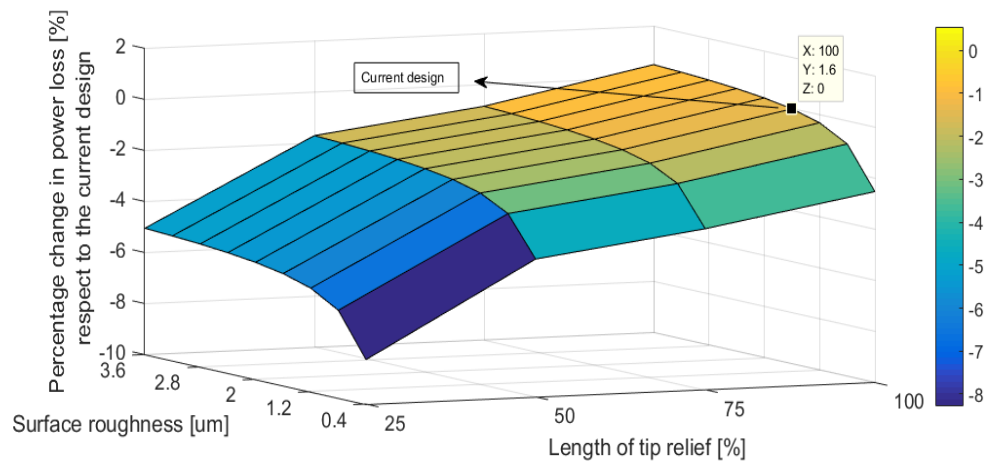

Figure 9: Percentage change in power loss with different surface roughness and tip relief length

5- Conclusions

The study investigates the effect of tooth longitudinal crowning and tip relief (length and amount) modifications on the power loss of planetary wheel hub gears of off-highway vehicles.

The following conclusions are made:

I.   Better surface finish with both longitudinal crowning and tip relief modifications reduces the gear contact power loss. This is because of a reduction in boundary friction.

II.   Reduction of longitudinal crowning by 25% with respect to current design minimises the power loss. This is because of reduced contact pressures and an increased lubricant film thickness, thus reducing the extent of asperity interactions.

III.   In terms of tip relief modification, decreasing both the amount of tip relief (to 50% of the current base design value) and length (by 25%) leads to reduced power loss. This is mostly due to an increase in the duration of single contact time along the meshing cycle with the highest applied load. However, this effect can have adverse effect on the root stresses and potential adverse component reliability. Furthermore, applying tip relief decreases radii of curvature and rolling velocity within double contact region. These are the reasons for a decreased lubricant film thickness and a correspondingly increased friction.

IV.   Finally, the "optimum" scenario corresponds to the case of 75% longitudinal crowning, 50% tip relief amount and 25% tip relief length with 0.4 μm surface roughness average with respect to the current base design, yielding a total power loss of 507 W for both the planetary wheel hub gearing of the axle. The total power loss for the current design is 583 W.

7- Acknowledgements

8- References

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9- Definitions/Abbreviations

A	Apparent contact area
A_a	Asperity contact area
$c^{'}$	Specific heat capacity of the solid surfaces
E_r	Reduced elastic modulus of the contact
$E^{'}$	Reduced elastic modulus of the contact: $(2E_r)/\pi$
EHL	Elastohydrodynamic Lubrication
f_v	Viscous friction
f_b	Boundary friction
h_{c0}^{*}	Dimensionless central lubricant film thickness
h_{c0}	Central lubricant film thickness
K	Lubricant's thermal conductivity
$K^{'}$	Thermal conductivity of the solids
$\bar{p}$	Average (Laplace) contact pressure
P_m	Mean pressure
R	Effective radii of curvature
R_x	Radii of curvature along the direction of sliding
R_y	Radii of curvature along the direction of side leakage
TCA	Tooth Contact Analysis
U_r	Rolling velocity
U_s	Sliding velocity
U	Speed of entraining motion
W_a	Asperity load share

α	Pressure viscosity coefficient
β	Average asperity tip radius
ε	Slope of the lubricant limiting shear stress-pressure dependence
η_0	Lubricant dynamic viscosity at atmospheric pressure
λ	Stribeck's oil film parameter
μ	Coefficient of friction
ξ	Asperity density per unit area
ρ'	Density of solids
σ	Composite RMS surface roughness
τ_0	Eyring shear stress
τ_L	Limiting shear stress