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## Predicting the effects of dimensional and material stiffness variations on a compliant bistable microrelay performance

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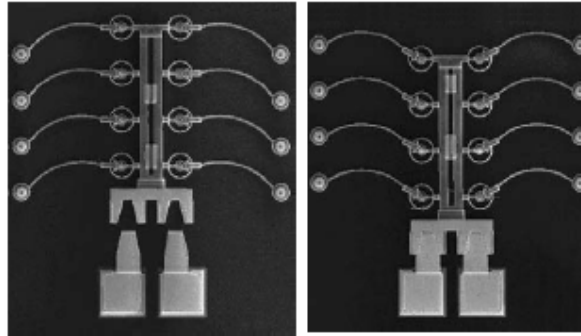
**Abstract.** In this paper we investigate the effects of dimensional and material stiffness variations on a microrelay performance. A linear displacement bistable micromechanism is modelled by pseudo-rigid-body model method and fully characterized. To find the effects of dimensional and material stiffness variation, an analysis of mechanical error is used. The method is a stochastic one and takes into account the random nature of variations. Variations of the contact force and required power of a microrelay example is obtained by the method introduced and the performance of the microrelay is determined. The method introduced is a simple, effective and general that may be used at the design level.

### 1. Introduction

Micromachining processes used to fabricate microrelays have inherent uncertainties. Dimensional and material properties of these microrelays will have variations in comparison with their nominal values when fabrication process finishes. Predicting the effects of length tolerances, cross-sectional shape and stiffness variation on the performance of microrelays is of great importance at the design level. Microrelays have unlimited applications wherever switching functions are needed. They have the potential to reduce the size, weight, power consumption and cost of products in which they are used and have the ability to be batch produced. In the trend of miniaturization, there will be a promising market in automobiles, measurement and automatic test equipment, robotics, cell phones and especially in telecommunications. Micromachined relays may also allow for the interconnection of large relay arrays during fabrication, thus reducing other fabrication steps currently needed for conventional relays.

Several MEMS relays of various types have been developed [1-3]. Among them, bistable microrelays have been seen very useful because of the advantages they offered [4]. One of these microrelays is the linear displacement bistable microrelay [5]. This bistable microrelay uses a linear displacement bistable micromechanism (LDBM) to gain its function.

Little work has been done to model the effects of parameters variation on micromechanisms and especially on microrelays performance. Wittwer *et al* considered the effects of uncertain material and dimensional parameters on surface micromachined gauges [6]. An uncertainty analysis for a bistable micromechanism has been done by Wittwer *et al* [7]. Bahrami and Ghanbari introduced a stochastic model to determine the mechanical error of a compliant bistable micromechanism due to the dimensional and material parameters variation and joint clearances using a direct linearization method [8].



**Figure 1.** Scanning electron microscope photographs of a LDBM microrelay shown in open and close states [5]

In this work initially a linear displacement bistable micromechanism is modeled with pseudo-rigid-body model (PRBM) method and its characteristics are specified in section 2. In section 3 the mechanical error theory that this work is based on is discussed. Section 4 contains the formulations of the micromechanism to determine the effects of material and dimensional parameters variation on its performance with an illustration by a numerical example. In section 5 the performance of the microrelay with considering the effects of variations is discussed. It has been shown that these effects are significant. The method used to predict the performance of the microrelay is very simple, useful and general. The method can be used to determine all microrelay parameters variation due to the material stiffness and dimensional variations.

## 2. Linear displacement bistable micromechanism

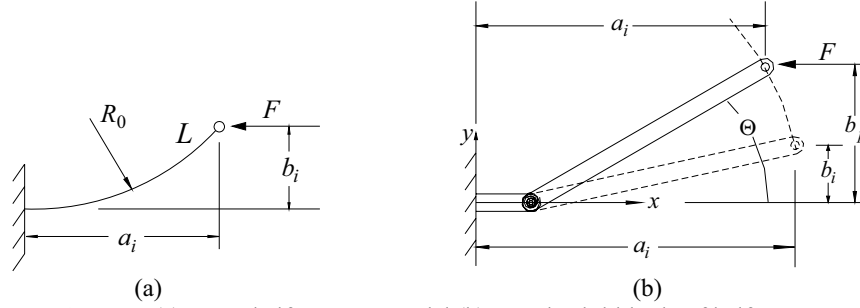
There are several advantages in using a bistable mechanism as a relay. First, the contact force is higher in the closed position. This can be accomplished by placing the contact pad of the relay at the position where the force is high, i.e. close to its unstable state. This is because if the mechanism is stopped at this position, it is actually trying to reach the second stable state and will push against the contact pad providing higher force and reliable contact. Second, because of this quality, power is only required to move from one stable state to another and no power will need to be supplied when in the ON state. The higher contact force can also reduce bounce on closing. Third, bistable mechanism produces large gaps between contacts which is desirable for high voltage switching and high breakdown voltages.

A linear displacement bistable micromechanism is composed of several functionally binary pinned-pinned (FBPP) segments attached to a central slider with the segments symmetric about the center line of the slider, as illustrated in figure 1. As the slider moves down, the FBPP segments are compressed, resulting in a motion to snap back to its first stable state. When the force exerted by the segments to the slider, it snaps to its second stable state. This behavior is the result of use of compliant segments. Compliant structures could replace mechanisms that rely solely on pin joints and springs to achieve their motion. This would allow microrelay to be fabricated more easily and operate more reliably.

However, predicting the force-deflection relationship of compliant mechanisms is not straightforward. Exact solution results in complex elliptic integrals. Accurately approximating, a pseudo-rigid-body model can be used to characterize the force-deflection relationship of the FBPP segment [9]. Because of symmetry, the arc can be divided at the midpoint for analysis (figure 2a). The pseudo-rigid-body models of the half FBPP segment is shown in figure 2b.

## 3. Mechanical error analysis

Since tolerances and clearances have a random nature, an appropriate stochastic method is required to determine the mechanical error. In order to evaluate the error of an output variable of a mechanism, the variance of that output variable must be calculated. Since, in general, the output variable is an implicit and nonlinear function of random variables; it is expanded in Taylor series. Because tolerances on variables are much smaller than their mean values, second and higher order terms can be



**Figure 2.** (a) FBPP half segment model (b) Pseudo-rigid-body of half segment neglected. Hence, the variance of the  $i$ th output variable  $Y_i$  is estimated using the following equation

$$\text{var}(Y_i) = \sum_{j=1}^m \left[ \left( \frac{\partial Y_i}{\partial z_j} \right)_{\text{mean}(z_j)} \right]^2 \text{var}(z_j) \quad i = 1, 2, \dots, n \quad (1)$$

where  $z_j$  is the  $j$ th random variable,  $n$  and  $m$  are the number of output and random variables, respectively. The partial derivative  $\partial Y_i / \partial z_j$  denotes the sensitivity of the  $i$ th output variable with respect to the  $j$ th random variable that must be evaluated at the mean value of random variables.

Sensitivities can be derived from the below equation [8]:

$$[S] = -[H_Y]^{-1} [H_z] \quad (2)$$

$$S_{ij} = \frac{\partial Y_i}{\partial z_j}$$

where  $[H_z]$  and  $[H_Y]$  are the matrices of first-order partial derivatives of mechanism equations with respect to the random and output variables, respectively and can be found from:

$$[H_z] = \frac{\partial h_i}{\partial z_j} \quad (3)$$

$$[H_Y] = \frac{\partial h_i}{\partial Y_j}$$

$z$  is the random variable vector and  $Y$  is the vector of dependent output variables and  $h_i$  is the  $i$ th linkage equation.

#### 4. Mechanism model

In order to construct  $h_i$ s required for equation (3) to evaluate errors due to the dimensional and material stiffness variations, the model shown in figure 2 is considered. Equations are written in a suitable format for use in equation (3).  $a_i$  and  $b_i$ , the initial horizontal and vertical coordinates of the half segment end respectively, may be obtained from:

$$h_1 = a_i - R_0 \sin K_0 = 0 \quad (4)$$

$$h_2 = b_i - R_0 (1 - \cos K_0) = 0 \quad (5)$$

where  $R_0$  is the segment's radius of curvature and  $K_0$  is the initial curvature of the segment and may be calculated from:

$$h_3 = K_0 - \frac{L}{R_0} = 0 \quad (6)$$

the  $L$  is the arc-length of the half segment.

The pin joint distance from ground is determined by the non-dimensional "fundamental radius factor",  $\gamma$ , and is given by  $L(1-\gamma)$ .  $\gamma$  can be obtained from one of below equations, related to the value of  $K_0$  [9]:

$$\begin{aligned} h_4 &= \gamma - 0.8063 + 0.0265K_0 = 0 & 0.500 \leq K_0 \leq 0.595 \\ h_4 &= \gamma - 0.8005 + 0.0173K_0 = 0 & 0.595 \leq K_0 \leq 1.500 \end{aligned} \quad (7)$$

Another important factor to analyze the model is the "characteristic radius factor,"  $\rho$  [10]. The length of the rotating link in the PRBM is determined by  $\rho L$ , which may be calculated from:

$$h_5 = \rho - \sqrt{\left(\frac{a_i}{L} + \gamma - 1\right)^2 + \left(\frac{b_i}{L}\right)^2} = 0 \quad (8)$$

It is necessary to calculate the torsional spring constant to complete the pseudo-rigid-body model of the FBPP segment. The torsional spring constant can be obtained from:

$$h_6 = K - \rho K_\theta \frac{EI}{L} = 0 \quad (9)$$

$E$  and  $I$  are the modulus of elasticity of polysilicon and the area moment of inertia of the beam.  $K_\theta$  for an initially curved segments is given as [9]:

$$h_7 = K_\theta - 2.568 + 0.028K_0 - 0.137K_0^2 = 0 \quad (10)$$

Beams fabricated using MUMPs process have the trapezoidal cross section rather than rectangular [7]. From the trapezoidal moment of inertia about the vertical axis

$$h_8 = I - \frac{h}{48} (w_b^3 + w_b^2 w_t + w_b w_t^2 + w_t^3) = 0 \quad (11)$$

where  $w_t$  is the width of the top surface,  $w_b$  is the width of the bottom surface, and  $h$  is the layer thickness.

From the experimental data [11], the width of the bottom surface can be estimated by assuming an angle for the side wall of the beam:

$$h_9 = w_b - w_t - 2h \tan \alpha = 0 \quad (12)$$

where  $\alpha$  is the angle of the side wall.

The total potential energy of the mechanism is given by:

$$h_{12} = V - 2nK(\Theta - \Theta_i)^2 = 0 \quad (13)$$

where  $n$  is the number of FBPP segments per each side of the slider.  $\Theta$  and  $\Theta_i$  are the pseudo-rigid-body model link angle and its initial value respectively and may be found from:

$$h_{10} = \Theta - \arccos\left(\frac{\sqrt{x_i^2 + (y_i - \delta)^2} - 2(1-\gamma)L}{2\rho L}\right) = 0 \quad (14)$$

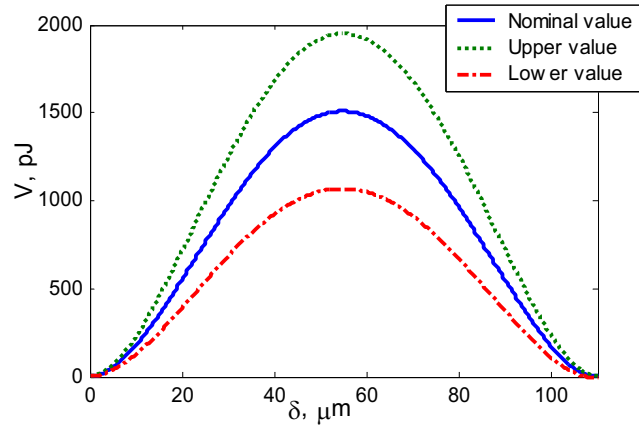
$$h_{11} = \Theta_i - \arccos\left(\frac{\sqrt{x_i^2 + y_i^2} - 2(1-\gamma)L}{2\rho L}\right) = 0 \quad (15)$$

where  $\delta$  is the slider displacement,  $y_i$  and  $x_i$  are the pin-to-pin distance in the  $\delta$  direction and perpendicular to it.

The force predictions of the mechanism may be obtained by differentiating the potential energy equation (13). The contact force of the mechanism may be obtained from:

$$h_{13} = F - 4nK(\Theta - \Theta_i) \frac{d\Theta}{d\delta} = 0 \quad (16)$$

For a numerical problem, random and output variables are chosen as  $z = [L, R_0, x_i, y_i, E, w_t, h, \alpha]$



**Figure 3.** Potential energy curve of the mechanism with its upper and lower values

and  $Y = [a_i, b_i, \rho, \gamma, K_0, K_\theta, K, w_b, I, \Theta, \Theta_i, V, F]$ . The mechanism parameters and tolerances have been given in table 1 and are used along with the equations (1) through (16) to evaluate the sensitivities and the errors. Here is assumed that the actual values of all of the random variables are normally distributed with a mean value of the nominal value of the random variable and a standard deviation equal to one third of the symmetric tolerance value. The 3-sigma bands of potential energy and contact force of the mechanism with their nominal values are shown in figures 3 and 4 with respect to the slider displacement,  $\delta$ .

## 5. Relay performance

The performance of a microrelay is specified with some characteristics such as power consumption, contact force, contact resistance, switching time and AC/DC isolation. In this section the performance of the microrelay is discussed from the view of the power consumption and the contact force.

**5.1.1. Power consumption.** The energy needed for the microrelay with its upper and lower bands are shown in figure 3. The value of the potential energy changes in the range of  $1509 \pm 440$  pJ. It means that the power consumption of the microrelay has a wide variable range. This shows the importance of considering the effects of dimensional and material properties variation at the design level and when the relay operates.

**5.1.2. Contact force.** The maximum contact force of the microrelay is changed with the mean value of  $42.21 \mu\text{N}$  and the symmetrical tolerance of  $12.28 \mu\text{N}$ . As seen, the tolerance is large and about 30% of mean value. Not predicting the effects of variations on performance of the microrelay at the design

**Table 1.** Values and tolerances of mechanism parameters

Variable	Value $\pm$ tolerance	Unit
$L$	$144 \pm 3$	$\mu\text{m}$
$R_0$	$122.94 \pm 3$	$\mu\text{m}$
$x_i$	$210 \pm 3$	$\mu\text{m}$
$y_i$	$55 \pm 0.5$	$\mu\text{m}$
$E$	$169 \pm 10$	GPa
$w_t$	$3.45 \pm 0.15$	$\mu\text{m}$
$h$	$3.5 \pm 0.05$	$\mu\text{m}$
$\alpha$	$3.5 \pm 2$	deg
$n$	4	-

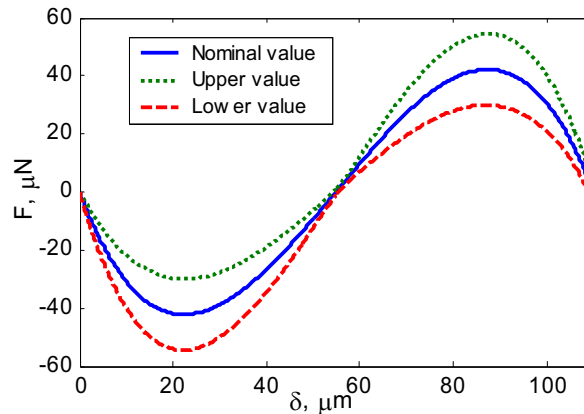


Figure 4. Contact force of the mechanism with its upper and lower values

level will result in a microrelay without desired and specified characteristics.

## 6. Conclusion

It has been shown that analyzing the effects of dimensional and material properties variation is a critical matter at the design level. These effects have been seen significant. As shown, the performance of the microrelay changed greatly with the presence of variations. The method introduced to calculate the effects of variations is a stochastic method that can be used at the design level to determine all outputs variation and predict the relay performance.

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