Mereosemiotics: Parts and Signs

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Abstract

For descriptions of cognitive processes, including process models for research data provenance and simulation workflow metadata, a formal notation is developed on the basis of the foundational ontological paradigm of mereosemiotics, *i.e.*, the combination of mereotopology with Peircean semiotics. To demonstrate the viability of the approach, this is applied to extend the pre-existing OWL ontology for a *physicalistic interpretation of modelling and simulation – interoperability infrastructure* (PIMS-II) by a modal first-order logic axiomatization.

Keywords

applied ontology, process data technology, mereotopology, semiotics, knowledge representation

1. Introduction

Understanding and characterizing research workflows, *i.e.*, *cognitive processes* that yield a research outcome, is essential to reproducibility [1, 2, 3, 4]. Moreover, it is the reliability of the employed processes and procedures that motivates trust in the research outcome, *cf.* the discussion of epistemic grounding by Williams [5], of warrant transmission by Symons and Alvarado [6], and of computational reliabilism by Durán *et al.* [7, 8]. Accordingly, research data infrastructures can only support reproducibility and reliability if provenance metadata are available and can be exchanged in a standardized form [9, 10, 11, 12, 13, 14, 15]: "The quality of metadata determines the reusability," as Wulf *et al.* [15] assert. Consequently, much of the recent work on semantic artefacts in engineering, natural sciences, and scientific computing has focused on research workflow descriptions; in materials modelling, in particular, a system of connected standardized provenance descriptions at multiple levels has been developed, encompassing MODA ("model data") tables [16] for a semi-formal descriptive annotation of simulation results targeting human-to-human communication, the *ontology for simulation, modelling, and optimization* (OSMO) as a domain-ontology version of MODA [17, 18, 19], the *physicalistic interpretation of modelling and simulation – interoperability infrastructure* (PIMS-II) as a mid-level ontology for cognitive processes [20, 21], and process topology based approaches from ProMo for mapping process models to Petri nets and high-level I/O notations [20].

FOUST 2021: 5th Workshop on Foundational Ontology, held at JOWO 2021: Episode VII The Bolzano Summer of Knowledge, September 11–18, 2021, Bolzano, Italy
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The *elementary multiperspective material ontology* (EMMO), which is work in progress and available as a beta version,[1] is used as a foundational ontology [22, 23, 24]. Both for the present first-order axiomatization of mereosemiotics and for PIMS-II, the associated OWL implementation, many *basic design choices exclusively serve the purpose of remaining close to the EMMO*, so that the present work can support EMMO-based interoperability in line with objectives from the Horizon 2020 and Horizon Europe work programmes. Where there are minor deviations from the EMMO approach, care has been taken to ensure that they do not stand in the way of implementing straightforward crosswalks between the two ontologies that work reliably for typical use cases. No attempt will be made here to provide a philosophical or metaontological justification of nominalism, physicalism, Peirceanism, mereotopology, rejection of the perdurant-endurant distinction, *etc.*; first, because it is the EMMO developers who need to be credited with designing the foundations of this paradigm [22, 23, 24], and second, because some advantages and challenges pertaining to this approach have already been discussed elsewhere [25, 26].

The semantic artefacts mentioned above jointly rely on combining mereotopology with Peircean semiotics, *i.e.*, on the foundational ontological paradigm of mereosemiotics [25]. The present work proposes a system of axioms in modal first-order logic for mereosemiotics. These axioms concern cognitive processes in particular as well as cross-domain concepts and relations provided by the PIMS-II mid-level ontology in general. The article is structured as follows: Section 2 discusses mereotopology and the way in which *nominalism*, *spatiotemporal monism*, *continuity of spacetime*, and *linearity of time* are implemented. Section 3 introduces the present approach to formalizing Peircean semiotics, based on *semiotic monism*, coherently integrating it with mereotopology. Section 4 addresses modal relations and propositions, giving an expression to *necessitism* and *mereotopological essentialism*, and introduces the kinds of collectives that are defined in PIMS-II; it also discusses the pre-existing system of mereosemiotic chain relations from the *VIMMP Primitives* [18] (VIPRS) that is included in PIMS-II to support domain-to-mid-level ontology alignment. Conclusions to be drawn from this work are formulated in Section 5.

2. Mereotopology

2.1. Metaontological Motivation

The following remarks are formally *not part of the present axiomatization* of mereosemiotics, which is given in Sections 2.2 to 4.3. However, they may help to motivate the axioms and provide a suggested interpretation for its concepts and relations on the basis of necessitism, *cf.* the detailed discussion by Williamson [27], four-dimensionalism [28], and spatiotemporal monism [25] as proposed, *e.g.*, by Williams [29]; while Muller [30] does not use the term *spatiotemporal monism*, he similarly proposes to rely on "space-time histories of objects as primitive entities" instead of continuant-occurrent dualism.

It is proposed to construct the domain Δ for the ontology as follows:[2]

[1] The name of that foundational ontology has recently changed, retaining the acronym "EMMO," which had originally been introduced to abbreviate *European materials and modelling ontology*.

[2] Alternatively, where it is beneficial to rely on a model that is enumerable, the domain can also be constructed

1. $\mathbb{R}^4$ is given an interpretation as dimensionless spacetime, with the first three coordinates q_1, q_2, q_3 of some four-dimensional point $\mathbf{q} = (q_1, q_2, q_3, q_4) \in \mathbb{R}^4$ being understood as spatial and the fourth coordinate q_4 being understood as temporal.
2. $S \subseteq \mathbb{R}^4$ is 4D-complete if and only if for each $\mathbf{q} \in S$ there are linearly independent $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4 \in \mathbb{R}^4$ such that $\mathbf{q}' \in S$ for all $\mathbf{q}' - \mathbf{q} = \sum_{1 \leq i \leq 4} v_i \mathbf{r}_i$ where $0 < v_i \leq 1$.
3. $S \subseteq \mathbb{R}^4$ is simple if and only if it consists of finitely many connected components.
4. $S \subset \mathbb{R}^4$ is 4D-delimited if and only if *a)* S is a closed set and *b)* for some $r \in \mathbb{R}$, $(\mathbf{q}' - \mathbf{q})^2 \leq r^2$ for all $\mathbf{q}, \mathbf{q}' \in S$ and *c)* S and $\mathbb{R}^4 \backslash S$ are 4D-complete and simple.
5. $S \subset \mathbb{R}^4$ is temporally closed if and only if $\{q_4 \mid \mathbf{q} \in S\}$ is a closed subset of $\mathbb{R}$.
6. By some relation between physical and dimensionless spacetime, the physical universe is mapped to the connected 4D-delimited set $\omega \subset \mathbb{R}^4$, the *actual and necessary (dimensionless) universe*; the employed relation should be such that its inverse, from ω to physical spacetime, is surjective and preserves continuity if physical spacetime is continuous, or adjacency if physical spacetime is discrete. It does not matter how exactly this is done as long as ω is 4D-delimited and consists of a single connected component, and as long as ω is the dimensionless universe *necessarily*; accordingly, by the present construction, ω is a continuum even if the physical universe is regarded as consisting of discrete elements. As a closed set, ω includes its boundary, the three-dimensional hypersurface $\delta\omega \subset \omega$.
7. The domain of the ontology is $\Delta = \{x \subset \omega \backslash \delta\omega \mid x$ is 4D-delimited and temporally closed$\}$. The elements of Δ are that which exists; they are referred to as *objects.*[3]
8. An object $x \in \Delta$ is an *item* if it is a single connected component and a *mereotopological collective* if it consists of multiple connected components; in the latter case, the (maximal) connected components of x are its *mereotopological members.*
9. For any two objects $x, y \in \Delta$, the criterion for proper parthood is $\dot{\mathsf{P}}xy \Leftrightarrow x \subset y$,
10. the criterion for temporal precedence is given by $\hookrightarrow_{\mathsf{t}}xy \Leftrightarrow \forall \mathbf{q} \in x \; \forall \mathbf{q}' \in y : \; q_4 \leq q'_4$,
11. two objects temporally coextend, $\equiv_{\mathsf{t}}xy$, if and only if $\{q_4 \mid \mathbf{q} \in x\} = \{q'_4 \mid \mathbf{q}' \in y\}$,
12. and the criterion for spatiotemporal connectedness is $\mathsf{C}xy \Leftrightarrow (x \cap y) \neq \emptyset$.

Accordingly, for any $x, y \in \Delta$, their spatiotemporal fusion is an object, $x \cup y \in \Delta$. However, the universe does not exist ($\omega \notin \Delta$), an empty object does not exist ($\emptyset \notin \Delta$), and the complements of objects are not objects ($x \in \Delta \Rightarrow \omega \backslash x \notin \Delta$).

Ontological commitment, following Quine [33, paragraph 49], here extends exclusively to individuals, namely, to objects as introduced above. That which exists is that which is accessible to quantification in first-order logic; this quantification is always over Δ which will be omitted for brevity. The inverse relation of Q will be denoted by Q^-, and the product notation QQ' will be employed for chain relations such that $\mathsf{QQ}'xy \Leftrightarrow \exists z\, (\mathsf{Q}xz \wedge \mathsf{Q}'zy)$. Implementing **nominalism**, existence is not attributed to concepts and relations, which are formalized here as unary and binary predicates, not as individuals. However, textual labels corresponding to concepts and relations, in particular, their internationalized resource identifiers (IRIs), can exist in the ontology; this construction is used in the OWL DL demonstrator implementation PIMS-II [21].

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by using $\mathbb{Q}$ everywhere instead of $\mathbb{R}$.

[3] It is expected that research data infrastructures and digital platforms that employ the present ontology will predominantly or exclusively deal with *bona fide objects* as discussed by Vogt [31], which have *bona fide boundaries* [32]; such entities are straightforwardly covered by the domain Δ.

2.2. Spatiotemporal Connectedness and Proper Parthood

In the literature a great variety of potential axiomatizations of mereotopology have been explosed in detail [28, 30, 32, 34, 35]; the aim of the construction from Section 2.1 is to motivate axioms that yield a strong mereotopology that is easy to handle in view of the application in Section 4.1, mainly through a strong version of **spatiotemporal monism** [25, 29, 30]. Five relations[4] constitute the basis of this mereotopology: isSpatiotemporallyConnectedWith (denoted C), isProperPartOf (denoted $\dot{\mathsf{P}}$), isMereotopologicalMemberOf (denoted $\dot{\leq}$), temporallyPrecedes (denoted $\hookrightarrow_{\mathsf{t}}$), and temporallyCoextendsWith (denoted $\equiv_{\mathsf{t}}$). In particular, overlap can then be constructed as $\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}$, the product $\dot{\mathsf{P}}\dot{\mathsf{P}}^{-} \equiv \top_{\mathsf{Q}}$ is the complete relation, and $\dot{\mathsf{P}}$ is idempotent, all of which simplifies the system of mereosemiotic chain relations discussed in Section 4.1; necessitism and mereotopological essentialism support the discussion of modal relations, *cf.* Section 4.2.

Spatiotemporal connectedness C is reflexive and symmetric [34]

$$\forall x\, \mathsf{C}xx \ \wedge\ \forall xy(\mathsf{C}xy \ \rightarrow\ \mathsf{C}yx); \tag{1}$$

nothing is connected with everything, but all objects are connected indirectly

$$\forall x \exists y\, \neg \mathsf{C}xy \ \wedge\ \forall xy\, \mathsf{C}^2 xy; \tag{2}$$

$\mathsf{C}^2 \equiv \top_{\mathsf{Q}}$ is complete. C is constitutive of identity [34], proper parthood, and fusion [35]

$$\forall xy\, (\forall z(\mathsf{C}xz \ \leftrightarrow\ \mathsf{C}yz) \ \rightarrow\ x = y)\,, \tag{3}$$

$$\forall xy\, \Big((\forall z\, (\mathsf{C}xz \ \rightarrow\ \mathsf{C}yz) \wedge x \neq y) \ \leftrightarrow\ \dot{\mathsf{P}}xy\Big)\,, \tag{4}$$

$$\forall \mathbf{x}\, (\forall y\, (\sigma y \mathbf{x} \ \leftrightarrow\ \forall z(\vee_i \mathsf{C}x_i z \ \leftrightarrow\ \mathsf{C}yz)) \wedge \exists y\, \sigma y \mathbf{x})\,; \tag{5}$$

fusion σ here takes multiple arguments, $\mathbf{x} = x_1 \cdots x_n$ being a sequence of $n \geq 1$ variables.

Proper parthood $\dot{\mathsf{P}}$ is asymmetric and transitive, everything is a proper part of something, and $\dot{\mathsf{P}}\dot{\mathsf{P}}^{-} \equiv \top_{\mathsf{Q}}$ is complete (any two objects "underlap," *i.e.*, are joint proper parts of a greater object), all of which are deducible from Axioms (2), (4), and (5). As an expression of **continuity of spacetime**[5] and the strong supplementation principle [36],

$$\forall xy\, \Big(\forall z(\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}xz \ \rightarrow\ \dot{\mathsf{P}}^{-}\dot{\mathsf{P}}yz) \ \rightarrow\ \dot{\mathsf{P}}^2 xy\Big)\,. \tag{6}$$

The maximal items that are proper parts of a mereotopological collective are its members

$$\forall xy\, \Big(\Big(\mathsf{Item}{:}x \ \wedge\ \dot{\mathsf{P}}xy \ \wedge\ \neg\exists x'(\mathsf{Item}{:}x' \ \wedge\ \dot{\mathsf{P}}x'y \ \wedge\ \dot{\mathsf{P}}xx')\Big) \ \leftrightarrow\ \dot{\leq}\, xy\Big)\,, \tag{7}$$

and an object is an item[6] if and only if it cannot be split into disconnected parts

$$\forall x\, (\forall yz(\sigma xyz \ \rightarrow\ \mathsf{C}yz) \ \leftrightarrow\ \mathsf{Item}{:}x)\,. \tag{8}$$

[4] See the Appendix for a list of relations relevant to this work.
[5] By the construction from Section 2.1, this does not require actual physical spacetime to be continuous.
[6] See the Appendix for a list of concepts relevant to this work.

Temporal precedence $\hookrightarrow_{\mathsf{t}}$ is asymmetric and transitive

$$\neg\exists xy(\hookrightarrow_{\mathsf{t}}xy \wedge \hookrightarrow_{\mathsf{t}}yx) \wedge \forall xyz\,((\hookrightarrow_{\mathsf{t}}xy \wedge \hookrightarrow_{\mathsf{t}}yz) \rightarrow \hookrightarrow_{\mathsf{t}}xz)\,, \tag{9}$$

and it carries over from objects to their proper parts

$$\forall xy \left(\hookrightarrow_{\mathsf{t}}xy \rightarrow \left(\forall x'(\dot{\mathsf{P}}x'x \rightarrow \hookrightarrow_{\mathsf{t}}x'y) \wedge \forall y'(\dot{\mathsf{P}}y'y \rightarrow \hookrightarrow_{\mathsf{t}}xy')\right)\right); \tag{10}$$

Axioms (9) and (10) entail that the relations $\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}$ and $\hookrightarrow_{\mathsf{t}}$ are disjoint. Everything precedes something, everything is preceded by something, and by **linearity of time**, any two objects have proper parts that are in a temporal precedence relation with each other

$$\forall x\exists y \hookrightarrow_{\mathsf{t}}xy \wedge \forall x\exists y \hookrightarrow_{\mathsf{t}}yx \wedge \forall xy\exists x'y' \left(\dot{\mathsf{P}}x'x \wedge \dot{\mathsf{P}}y'y \wedge (\hookrightarrow_{\mathsf{t}}x'y' \vee \hookrightarrow_{\mathsf{t}}y'x')\right). \tag{11}$$

Temporal coextension $\equiv_{\mathsf{t}}$ is defined by equivalent applicability of $\hookrightarrow_{\mathsf{t}}$ to proper parts

$$\forall xy \left(\forall zz' \left(\exists x'(\dot{\mathsf{P}}x'x \wedge \hookrightarrow_{\mathsf{t}}zx' \wedge \hookrightarrow_{\mathsf{t}}xz') \leftrightarrow \exists y'(\dot{\mathsf{P}}y'y \wedge \hookrightarrow_{\mathsf{t}}zy' \wedge \hookrightarrow_{\mathsf{t}}yz')\right) \leftrightarrow \equiv_{\mathsf{t}}xy\right) \tag{12}$$

so that it follows from Axioms (9) to (12) that $\equiv_{\mathsf{t}}$ and $\hookrightarrow_{\mathsf{t}}$ are disjoint. The chain $\equiv_{\mathsf{t}}\dot{\mathsf{P}}xy$ (equivalently, $\dot{\mathsf{P}}\equiv_{\mathsf{t}}xy$) can be used to express that x extends over a temporal subinterval compared to y, and $\equiv_{\mathsf{t}}\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}xy$ (equivalently, $\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}\equiv_{\mathsf{t}}xy$) characterizes temporal overlap.

3. Cognitive Processes

3.1. Cognitive Steps as Peircean Triads

The present approach to formalizing steps of a cognitive process as *triads*, inspired by Peirce [37, 38, 39, 40] and more recently Sowa [41, 42] and Goldbeck *et al.* [22], is outlined[7] in recent work [20]. Representation of an object y (referent) by a sign x (representamen), denoted $\mathsf{R}xy$, is understood in terms of a triadic cognition κ

$$\forall\kappa(\mathsf{TriadicCognition}{:}\kappa \leftrightarrow \exists e_1e_2e_3\,\mathsf{3}\kappa e_1e_2e_3), \tag{13}$$

$$\forall\kappa e_1e_2e_3 \left(\mathsf{3}\kappa e_1e_2e_3 \leftrightarrow (\dot{\mathsf{E}}e_1\kappa \wedge \ddot{\mathsf{E}}e_2\kappa \wedge \dddot{\mathsf{E}}e_3\kappa)\right), \tag{14}$$

i.e., a cognitive step that has three elements, above, e_1, e_2, and e_3, where $\mathsf{3}$ is a quaternary predicate relating the three elements to the cognition; $\dot{\mathsf{E}}$ (isFirstElementIn), $\ddot{\mathsf{E}}$ (isSecondElementIn), and $\dddot{\mathsf{E}}$ (isThirdElementIn) relate individual elements to the cognition.

It is not explicitly required here for any of these elements to be unique. On the one hand, Peirce's formulations would suggest this where he speaks of "the sign," "the object," and "the interpretant" [38]. On the other hand, Peirce also develops the understanding that for every semiosis, "there obviously are two objects, the object as it is in itself (the monadic object), and the object as the sign represents it to be (the dyadic object)" and "also three interpretants;

[7] See also Francisco Morgado *et al.* [23] and the report [21] on Borgo's and Kutz' example scenarios [43].

namely, *1)* the interpretant considered as an independent sign of the object, *2)* the interpretant as it is as a fact determined by the sign to be, and *3)* the interpretant as it is intended by, or is represented in, the sign to be" [44, p. 373]. It may be best to understand these multiplicities as single, unique individuals that are here merely viewed in different ways or, alternatively, as collectives that have multiple members, *cf.* Section 4.3. However, for some applications it does help to permit multiple distinct individuals in the same triadic role, *e.g.*, where a step of a research workflow addresses several "objects of research," *cf.* Schembera and Iglezakis [11, 12].

Implementing **semiotic monism** [25] by asserting that anything can act as a representamen *and* also as a referent (in different contexts), no specific concepts are needed to distinguish between indviduals that take these roles, since the same entity can occur on both sides of a representation relation. However, multiple types of cognitive steps need to be distinguished depending on the roles played by triadic elements and the way in which they are connected to each other within a cognitive process. Participation in a process, isParticipantIn (denoted $\ddot{\mathsf{P}}$), requires overlap[8] ($\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}$, see above) and is antireflexive

$$\forall xy(\ddot{\mathsf{P}}xy \rightarrow \dot{\mathsf{P}}^{-}\dot{\mathsf{P}}xy) \wedge \neg\exists x\, \ddot{\mathsf{P}}xx, \tag{15}$$

$$\forall x(\exists y\ddot{\mathsf{P}}yx \leftrightarrow \mathsf{Process}{:}x), \tag{16}$$

and the part of the taxonomy that is relevant for the present purpose is given by[9]

$$\begin{gathered}\forall x(\mathsf{Process}{:}x \rightarrow \mathsf{Item}{:}x) \wedge \forall\kappa(\mathsf{Cognition}{:}\kappa \rightarrow \mathsf{Process}{:}\kappa) \wedge \\ \forall\kappa(\mathsf{CognitiveStep}{:}\kappa \rightarrow \mathsf{Cognition}{:}\kappa) \wedge \forall\kappa(\mathsf{TriadicCognition}{:}\kappa \rightarrow \mathsf{CognitiveStep}{:}\kappa) \wedge \\ \forall\kappa\left((\mathsf{Perception}{:}\kappa \vee \mathsf{Interpretation}{:}\kappa \vee \mathsf{Metonymization}{:}\kappa) \rightarrow \mathsf{TriadicCognition}{:}\kappa\right).\end{gathered} \tag{17}$$

The elements that act as a representamen (denoted $\dot{\mathsf{R}}$) in a cognitive step κ are engaged in representation (R, see above) with the elements that act as a referent (denoted O) in κ

$$\forall\kappa so\left((\dot{\mathsf{R}}s\kappa \wedge \mathsf{O}o\kappa) \rightarrow \mathsf{R}so\right). \tag{18}$$

In a semiosis, these elements are the sign s, the object o, and the interpretant s' [38]

$$\forall\kappa sos'\left(\left(3\kappa sos' \wedge (\mathsf{Perception}{:}\kappa \vee \mathsf{Interpretation}{:}\kappa)\right) \rightarrow (\dot{\mathsf{R}}s\kappa \wedge \mathsf{O}o\kappa \wedge \dot{\mathsf{R}}s'\kappa)\right), \tag{19}$$

where s and s' act as representamina, and o is their joint referent: The sign is the input of the cognitive step, the interpretant is its output, the object is what both are about.

A case distinction is needed for *participation* of triadic elements in triadic cognitions, by relation $\ddot{\mathsf{P}}$ as introduced above, requiring *physical presence* and hence spatiotemporal overlap. Both perceptions and interpretations are semioses; what distinguishes them is whether or not

[8] In particular, if x is included spatially in y for some time, but not for all time, it means that x and y overlap. By virtue of spatiotemporal monism, there is no dualism of "continuants" (or "endurants") and "occurrents" (or "perdurants"); in line with the EMMO, processes are not "perdurants," and participants are not "endurants." It is possible for the same object to be a process *and* to participate in a process.

[9] For definitions, *cf.* the Appendix and the ontology at http://www.molmod.info/semantics/pims-ii.ttl.

the object needs to be present. Representamina need to be present in general, whereas the referent only needs to participate in a perception

$$\forall \kappa s(\dot{\mathsf{R}} s\kappa \rightarrow \ddot{\mathsf{P}} s\kappa) \wedge \forall \pi o \Big((\mathsf{O} o\pi \wedge \mathsf{Perception}{:}\pi) \rightarrow \ddot{\mathsf{P}} o\pi \Big) . \tag{20}$$

In a metonymization [45], a sign s that represents o is attributed to a new referent o'

$$\forall \mu o s o' \Big((\mathsf{3} \mu o s o' \wedge \mathsf{Metonymization}{:}\mu) \rightarrow (\mathsf{O} o\mu \wedge \dot{\mathsf{R}} s\mu \wedge \mathsf{O} o'\mu) \Big) . \tag{21}$$

Peirce requires a "real, physical connection of a sign with its object, either immediately or by its connection with another sign" [37]; a sign "must be affected in some way by the object which it signified or at least something about it must vary as a consequence of a real causation with some variation of its object" [39]. In particular, permitting the application of signs to contingent or hypothetical phenomena, Peirce states that a real causal connection is present if a "cause which precedes the event also precedes some cognition of the mind" [40, p. 142]; therein, the event (the occurrence of which may be contingent) is the referent, and the common cause is something that contributes causally to the occurrence of the event and to the cognition by which the representamen is generated. Accordingly, the "real causal connection" between the referent and the representamen needs to be preserved, *i.e.*, it must be ensured that the old referent has a causal connection (hasCausalConnectionWith, denoted $\dot{\mathsf{C}}^{\star}$) with the new referent

$$\forall \mu o o' \Big((\mathsf{O} o\mu \wedge \mathsf{O} o'\mu \wedge \mathsf{Metonymization}{:}\mu) \rightarrow \dot{\mathsf{C}}^{\star} o o' \Big) . \tag{22}$$

The relation $\dot{\mathsf{C}}^{\star}$ is constructed as the reflexive and transitive closure of the symmetric and antireflexive relation hasDirectCausalConnectionWith (denoted $\dot{\mathsf{C}}$)

$$\forall xy \Big(\dot{\mathsf{C}}^{\star} xy \leftrightarrow \Big(\exists z (\dot{\mathsf{C}}^{\star} xz \wedge \dot{\mathsf{C}} zy) \vee \dot{\mathsf{C}} xy \vee x = y \Big) \Big) . \tag{23}$$

Constitutivity $\ddot{\mathsf{C}}$, *cf.* Axioms (41) and (42), participation $\ddot{\mathsf{P}}$, and direct grounding $\hookrightarrow$

$$\forall xy \Big((\ddot{\mathsf{C}} xy \vee \ddot{\mathsf{P}} xy \vee \hookrightarrow xy \vee \dot{\mathsf{C}} yx) \rightarrow \dot{\mathsf{C}} xy \Big) \wedge \neg \exists x \, \dot{\mathsf{C}} xx, \tag{24}$$

are sufficient criteria for the presence of a direct causal connection. Future work might explore a more coherent formalization of causal connectedness in modal terms [46, 47].

3.2. Epistemic Grounding

Peirce introduces chain formation out of individual triadic cognitions as follows: "the interpretant, or third, cannot stand in a mere dyadic relation to the object, but must stand in such a relation to it as the [first] representamen itself does. [...] The third must [...] be capable of determining a third of its own. [...] All this must equally be true of the third's thirds and so on endlessly" [38]. For a conceptual analysis of typical research workflows in computational engineering, *cf.* Lenhard and Hasse [48, p. 73f.] as well as Lenhard and Küster [3, Section 2]; it is explained in previous work [20] how MODA [16] and OSMO [17] can be mapped to the

EMMO [23, 24] through the PIMS-II mid-level ontology to translate such workflows into chains of Peircean triadic cognitive steps.

When a triadic cognition κ directlyGrounds another triadic cognition λ, denoted $\hookrightarrow\kappa\lambda$, this means that step λ reuses a representation relation $\mathsf{R}so$ from the preceding step κ

$$\forall\kappa\lambda\left((\hookrightarrow\kappa\lambda \,\wedge\, \mathsf{TriadicCognition}{:}\kappa) \,\rightarrow\, \exists so(\dot{\mathsf{R}}s\kappa \,\wedge\, \dot{\mathsf{R}}s\lambda \,\wedge\, \mathsf{O}o\kappa \,\wedge\, \mathsf{O}o\lambda)\right), \quad (25)$$

cf. Fig. 1; for instance, κ could be a modelling step from which a molecular model s for system o is obtained as an interpretant, and λ could be a molecular simulation of system o where the model s is used to represent o. There needs to be a ground[10] g_λ that represents both κ and λ, where g_λ isGroundFor λ, denoted $\ddot{\mathsf{R}}g_\lambda\lambda$ and subsumed under R

$$\forall g_\kappa\kappa(\ddot{\mathsf{R}}g_\kappa\kappa \,\rightarrow\, \mathsf{R}g_\kappa\kappa). \quad (26)$$

In the present example, the ground g for λ could be the assertion that "step κ epistemically grounds step λ by parameterizing the molecular model that is subsequently employed as a simulation input." The reasoning according to which one cognitive step grounds another can itself be given the form of a sequence of cognitive steps; *e.g.*, consider an explanation of how λ is epistemically grounded, providing a justification for a cognitive process where κ directly grounds λ. As in the example above, the grounds g_κ and g_λ then both represent κ. The preceding ground g_κ comes into existence before the new ground g_λ, such that g_κ is the sign and g_λ is the interpretant in a *grounding interpretation* $\vec{\iota}_\lambda$

$$\forall\kappa\lambda\left(\hookrightarrow\kappa\lambda \,\rightarrow\, \exists\vec{\iota}_\lambda g_\kappa g_\lambda(\ddot{\mathsf{R}}g_\kappa\kappa \,\wedge\, \ddot{\mathsf{R}}g_\lambda\lambda \,\wedge\, \mathsf{3}\vec{\iota}_\lambda g_\kappa\kappa g_\lambda \,\wedge\, \mathsf{Interpretation}{:}\vec{\iota}_\lambda)\right), \quad (27)$$

cf. Fig. 1. The representation relation $\mathsf{R}g_\lambda\kappa$ is reused in a *grounding metonymization* $\vec{\mu}_\lambda$ by which g_λ is assigned the grounded step λ as its new referent by $\mathsf{3}\vec{\mu}_\lambda\kappa g_\lambda\lambda$. In general,

$$\forall\kappa\lambda\left(\hookrightarrow\kappa\lambda \,\rightarrow\, \exists\vec{\mu}_\lambda g_\lambda(\ddot{\mathsf{R}}g_\lambda\lambda \,\wedge\, \mathsf{3}\vec{\mu}_\lambda\kappa g_\lambda\lambda \,\wedge\, \mathsf{Metonymization}{:}\vec{\mu}_\lambda)\right), \quad (28)$$

cf. Fig. 1. Ultimately, a triadic cognition is grounded epistemically if there is an accepted presupposition that logicallyPrecedes it (denoted $\hookrightarrow^+$) directly or indirectly

$$\forall\kappa\Big(\big((\mathsf{Presupposition}{:}\kappa \,\vee\, \exists\bot(\hookrightarrow^+\bot\kappa \,\wedge\, \mathsf{Presupposition}{:}\bot)\big) \,\leftrightarrow\, \mathsf{GroundedCognition}{:}\kappa\big) \,\wedge$$
$$(\mathsf{GroundedCognition}{:}\kappa \rightarrow \mathsf{Cognition}{:}\kappa) \,\wedge\, (\mathsf{Presupposition}{:}\kappa \rightarrow \mathsf{CognitiveStep}{:}\kappa)\Big), \quad (29)$$

where $\hookrightarrow^+$ is the transitive closure of $\hookrightarrow$

$$\forall\vartheta\lambda\left(\hookrightarrow^+\vartheta\lambda \,\leftrightarrow\, \left(\exists\kappa(\hookrightarrow^+\vartheta\kappa \,\wedge\, \hookrightarrow\kappa\lambda) \vee \hookrightarrow\vartheta\lambda\right)\right). \quad (30)$$

[10] To Peirce [38], a *ground* is "an idea" that explains how a sign relates to an object. The present use of the term applies this specifically to an explanation that provides *epistemic grounding* to a cognition, as understood by Williams: "Epistemic grounding is a matter of *reliability*. A belief is epistemically grounded [... if and only if] it is formed via a process that in fact makes it likely to be true" [5]. Similarly, Jubb argues in favour of "distinguishing between what some position commits one to – logical grounding – and what may be useful in assessing whether it is a sensible position at all – epistemic grounding" [49].

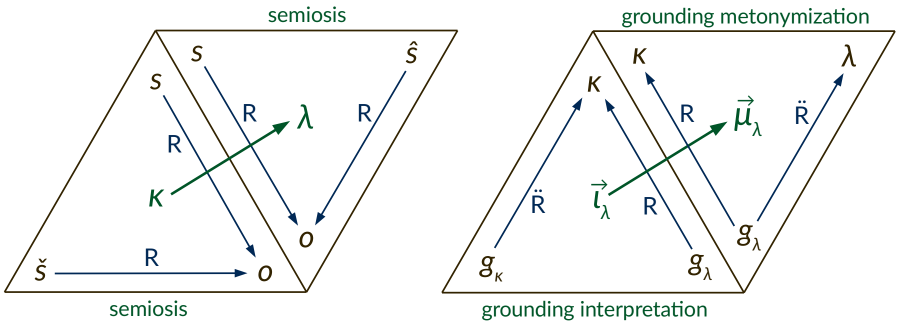

Figure 1: A cognitive chain consisting of two semioses $\kappa \hookrightarrow \lambda$ and the associated grounding chain consisting of a grounding interpretation $\vec{\iota}_\lambda$ followed by a grounding metonymization $\vec{\mu}_\lambda$, conforming with Axioms (25) to (28); concerning diagram notations based on Peircean semiotics, *cf.* Sowa [41, Fig. 3], Chandler [50, p. 34], Klein *et al.* [20, Fig. 5], and the supplementary material [21, Figs. 2–4].

4. Relational and Conceptual Framework

4.1. Mereosemiotic Chain Relations

Relations from domain ontologies, which are comparably specific, often correspond to chains of top-level relations, which are more generic; as Zhou *et al.* [51] put it, they can be "used to 'flatten' the structure of the other ontology by short-cutting a property chain." Since the OWL description logic $\mathcal{SROIQ}$ only permits one-way chain inclusions [52], in what is usually the wrong direction to implement the alignment, it can be useful if constructs for chains out of generic relations are explicitly included in a foundational or mid-level ontology to facilitate an alignment. Here, these abstract relations are $\dot{\mathsf{P}}$ (isProperPartOf), R (isRepresentamenFor), and their inverse relations. Accordingly, the PIMS-II mid-level ontology includes the *mereosemiotic chain relations* that were introduced in previous work when the VIMMP domain ontologies were aligned with the EMMO; these relations were found to be helpful or even necessary in many cases [18, Chapter 5]. The present system of chain relations is thus constituted by the free monoid $\mathbb{M}^\star$ over $\mathbb{M} = \{\dot{\mathsf{P}}, \dot{\mathsf{P}}^-, \mathsf{R}, \mathsf{R}^-\}$, with identity as the neutral element.[11] For any $\mathsf{Q}, \mathsf{Q}' \in \mathbb{M}^\star$

$$\forall xy \left(\exists z (\mathsf{Q}xz \land \mathsf{Q}'zy) \leftrightarrow \mathsf{Q}\mathsf{Q}'xy \right), \tag{31}$$

and inverse chains are obtained by $(\mathsf{Q}\mathsf{Q}')^- \equiv (\mathsf{Q}')^-\mathsf{Q}^-$, *i.e.*, for any $\mathsf{Q}_1, \ldots, \mathsf{Q}_n \in \mathbb{M}$

$$\forall xy (\mathsf{Q}_1 \cdots \mathsf{Q}_n xy \leftrightarrow \mathsf{Q}_n{}^- \cdots \mathsf{Q}_1{}^- yx). \tag{32}$$

As a consequence of the strong mereotopology from Section 2.2, an implementation of the mereosemiotic chain relations can disregard many combinations of elements that are equivalent

[11] For the OWL implementation, the following naming convention is employed: The binary product relations are called overlapsWith ($\dot{\mathsf{P}}^-\dot{\mathsf{P}}$), sharesReferentWith ($\mathsf{R}\mathsf{R}^-$), and sharesRepresentamenWith ($\mathsf{R}^-\mathsf{R}$). The names for higher-order product relations begin with the prefix ms (for "mereosemiotics"), followed by the elements IP for "is proper part of," HP for "has proper part," IR for "is representamen for," and HR for "has representamen." In this way, *e.g.*, the name msIPHRHPIP is used for the product $\dot{\mathsf{P}}\mathsf{R}^-\dot{\mathsf{P}}^-\dot{\mathsf{P}}$.

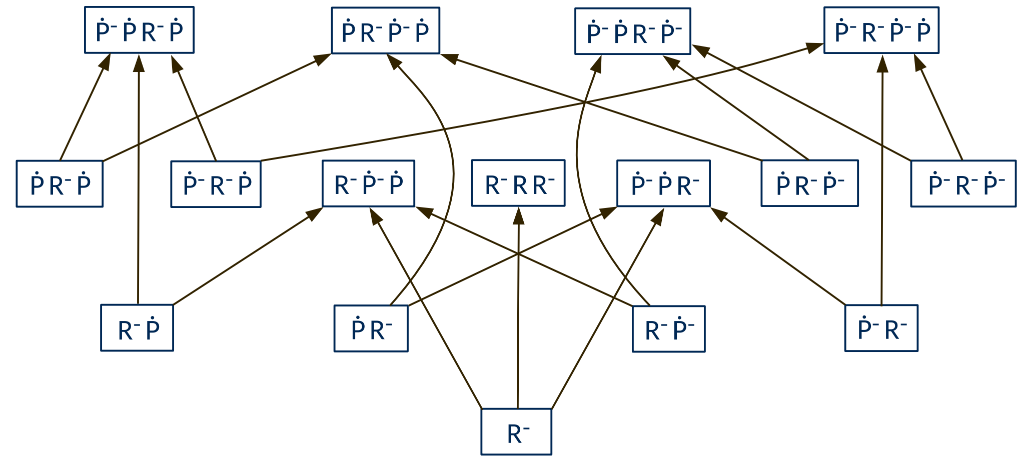

Figure 2: Fragment of the system of mereosemiotic relations; arrows denote relational subsumption.

or redundant. For $\mathsf{Q}, \mathsf{Q}' \in \mathbb{M}^\star$, it follows from Axioms (4), (6), and (31) that

$$\forall xy(\mathsf{Q}\dot{\mathsf{P}}^2\mathsf{Q}'xy \leftrightarrow \mathsf{Q}\dot{\mathsf{P}}\mathsf{Q}'xy),$$

so that chains containing the factor $\dot{\mathsf{P}}^2$ need not be considered; the same applies to $\dot{\mathsf{P}}^{-2}$. Similarly, any chains containing the "underlap" factor $\dot{\mathsf{P}}\dot{\mathsf{P}}^-$ can be eliminated by

$$\forall xy\left(\mathsf{Q}\dot{\mathsf{P}}\dot{\mathsf{P}}^-\mathsf{Q}'xy \leftrightarrow (\exists y'\,\mathsf{Q}xy' \wedge \exists x'\,\mathsf{Q}'x'y)\right),$$

where $\mathsf{Q}, \mathsf{Q}' \in \mathbb{M}^\star$, since $\dot{\mathsf{P}}\dot{\mathsf{P}}^- \equiv \top_\mathsf{Q}$, *cf.* Section 2.2; therefore, any elementary proposition on $\mathsf{Q}\dot{\mathsf{P}}\dot{\mathsf{P}}^-\mathsf{Q}'$ can be replaced with a conjunction over two separate existential propositions on Q and Q' (which remain expressible in $\mathcal{SROIQ}$ description logic) so that it is not necessary to include an explicit declaration of $\mathsf{Q}\dot{\mathsf{P}}\dot{\mathsf{P}}^-\mathsf{Q}'$ in the ontology. By construction

$$\forall xy(\mathsf{Q}_1\mathsf{Q}_2\mathsf{Q}_3xy \rightarrow \mathsf{Q}_1\mathsf{Q}_2\mathsf{Q}_2{}^-\mathsf{Q}_2\mathsf{Q}_3xy),$$

for all $\mathsf{Q}_1, \mathsf{Q}_2, \mathsf{Q}_3 \in \mathbb{M}^\star$, and from mereotopology

$$\forall xy(\mathsf{Q}\mathsf{Q}'xy \rightarrow \mathsf{Q}\dot{\mathsf{P}}^-\dot{\mathsf{P}}\mathsf{Q}'xy),$$
$$\forall xy(\mathsf{Q}\dot{\mathsf{P}}\mathsf{Q}'xy \rightarrow \mathsf{Q}\dot{\mathsf{P}}^-\dot{\mathsf{P}}\mathsf{Q}'xy) \wedge \forall xy(\mathsf{Q}\dot{\mathsf{P}}^-\mathsf{Q}'xy \rightarrow \mathsf{Q}\dot{\mathsf{P}}^-\dot{\mathsf{P}}\mathsf{Q}'xy),$$

for any $\mathsf{Q}, \mathsf{Q}' \in \mathbb{M}^\star$, since $\dot{\mathsf{P}}^-\dot{\mathsf{P}}$ is reflexive and $\dot{\mathsf{P}}$ and $\dot{\mathsf{P}}^-$ are subrelations of $\dot{\mathsf{P}}^-\dot{\mathsf{P}}$, *cf.* Section 2.2, yielding a dense hierarchy of relational subsumptions over $\mathbb{M}^\star$, *cf.* Fig. 2.

4.2. Absolute and Qualified Necessity

The present ontology permits the application of modal operators *a)* to relations, by which *necessary and possible relations* $\Box\mathsf{Q}$ and $\Diamond\mathsf{Q}$ can be constructed from a relation Q, and *b)* to rules, yielding *laws* as rules of the type $\Box(\varphi \to \psi)$, where φ and ψ are propositions. Beside qualified modal operators $\Box_F$ and $\Diamond_F$, which require a descriptor of the applicable modal context F, it includes *absolute necessity and possibility* $\Box$ and $\Diamond$, which are free of context. The absolute operators satisfy the S_5 axioms of modal logic

$$\varphi \quad \Rightarrow \quad \Box\Diamond\varphi, \tag{33}$$

$$\Box\varphi \quad \Rightarrow \quad (\varphi \wedge \Box\Box\varphi), \tag{34}$$

$$\Box(\varphi \to \psi) \quad \Rightarrow \quad (\Box\varphi \to \Box\psi), \tag{35}$$

where φ and ψ are propositions. Motivated by the construction from Section 2.1, where the domain is *necessarily* Δ (understood here as absolute necessity), **necessitism** is applied; *i.e.*, the existence of an object is a matter of absolute necessity [27]

$$\forall x\, \Box\exists y\, (x = y), \tag{36}$$

and the Barcan formula [27, 53] holds for any proposition φ with the free variables $\mathbf{x}$

$$\Diamond\exists\mathbf{x}\, \varphi \;\Rightarrow\; \exists\mathbf{x}\, \Diamond\varphi. \tag{37}$$

All existing objects exist necessarily because they are here defined *as spatiotemporal entities* and the spatiotemporal extension of the domain Δ is regarded as an absolute necessity; contingency is thereby shifted to the relations between objects. This does not mean that counterfactual propositions need to be absolutely impossible, or that it becomes impossible to speak of contingent phenomena – quite the opposite: This construction is introduced here precisely to solve the paradox that contingent situations (including multiple mutually contradictory scenarios, as in an optimization) can occur as referents in actually occurring cognitive processes [25, Section 3.2]. In such cases, it is safe to say that, *e.g.*, a model s of an undesirable event o, with the representation relation $\mathsf{R}so$, does retain an actually existing referent even if we hope or assume that the event as such will not occur; the spatiotemporal existence of o is nonetheless a necessity.

By the same line of reasoning, **mereotopological essentialism** is supported[12]

$$\forall xy\, ((\Diamond\mathsf{C}xy \to \Box\mathsf{C}xy) \wedge (\Diamond{\hookrightarrow_{\mathsf{t}}}xy \to \Box{\hookrightarrow_{\mathsf{t}}}xy))\,. \tag{38}$$

Analogous rules can then be deduced for *all* mereotopological relations, including $\dot{\mathsf{P}}$, $\dot{\leq}$, and $\equiv_{\mathsf{t}}$, and for the instantiation of the mereotopologically defined concept Item, so that

$$\forall x(\Diamond\mathsf{Item}{:}x \;\to\; \Box\,\mathsf{Item}{:}x).$$

The OWL implementation of PIMS-II [21] realizes modal relations in terms of assertions about the IRIs of the relations, *i.e.*, using individuals that instantiate the concept IRI; in a similar way, a

[12]Varzi refers to this as "a radical form of mereotopological essentialism" [54, p. 1017]. Chisholm [55, p. 145ff.] and Plantinga [56] speak of "mereological essentialism," which unproblematically extends from parthood to connectivity and hence from mereology to mereotopology; however, Chisholm and Plantinga employ constructions based on *possible worlds* and *temporal slicing*, neither of which is done here, which makes it easier to capture mereotopological essentialism by a simple expression such as Axiom (38).

relation can be asserted to be the negation $\neg\mathsf{Q}$ of another relation Q. To define laws, Proposition objects are employed, which have Triple objects (*i.e.*, RDF triples that are reified as PIMS-II articulations) as semiotic members, *cf.* Section 4.3. This permits applying PIMS-II to encode propositions that go beyond the expressive power of OWL DL. Ongoing work on RDF-star [57] suggests that in the future, there will be a new recommendation for the reification of triples, which might then be employed instead.

The relation between absolute and qualified modes of necessity is not straightforward to characterize in general, and no such attempt is made in the present work. Multiple modal frameworks are a requirement for addressing problems such as those posed by McCarthy [58], or hypercontingency ("it may be possible" to construct a warp drive, but it may also be impossible) as discussed by Kaminski [59, p. 351f.]. Qualified necessity can go beyond absolute necessity; *e.g.*, it was not absolutely necessary for Trump to lose reelection, but to a knowledge base that holds this information, it is epistemically necessary. On the other hand, qualified possibility can go beyond absolute possibility: If Jones j and Lewis ℓ, who have never met before, will meet in the future and shake hands, their overall spacetime trajectories are connected, $\mathsf{C}j\ell$, which is then absolutely necessary, $\Box\mathsf{C}j\ell$, due to mereotopological essentialism; however, since we do not know whether that will happen and are therefore *unaware of the shape* of j and ℓ, appropriate use can be made of the qualified modal proposition $\neg\Box_{\Gamma}\mathsf{C}j\ell$ in an adequate context.

4.3. System of Collective Objects

Mereotopology includes the concept of a mereotopological collective, *i.e.*, a spacetime object that is not connected as a whole and is decomposed into maximal connected components, its members, which are items and which are related to the associated collective by the relation $\dot{\leq}$ (isMereotopologicalMemberOf), *cf.* Axiom (7). However, the practical use of this EMMO construction for annotating research data is far too limited to be sufficient [25, Section 3.1]. Jones j and Lewis ℓ from the example above (Section 4.2) might never meet during their lifetime: Then they are spatiotemporally disconnected from each other, $\neg\mathsf{C}j\ell$, and their fusion o (with $\sigma oj\ell$) has two members, $\dot{\leq}\, jo$ and $\dot{\leq}\, \ell o$. Or they might meet and shake hands, in which case their fusion becomes one item. We do not know which is the case, and if we did, it would still require us to handle different pairs of people differently depending on irrelevant circumstantial phenomena.

A viable ontology that deals with research data requires additional kinds of collectives. Here, in line with the general structure of the present approach, SemioticCollective individuals (to which their members are related by $\ddot{\leq}$, isSemioticMemberOf) are introduced to complement the MereotopologicalCollective individuals (objects of the relation $\dot{\leq}$) such that generally, Collective individuals (objects of $\leq$) have at least two members

$$\forall x\, (\exists y \leq yx \;\rightarrow\; \exists yz(\leq yx \,\wedge\, \leq zx \,\wedge\, y \neq z))\,, \tag{39}$$

$$\forall xy\, \big((\dot{\leq}\, xy \,\vee\, \ddot{\leq}\, xy) \,\rightarrow\, \leq xy\big) \;\wedge\; \forall xy(\leq xy \,\rightarrow\, \dot{\mathsf{P}}xy), \tag{40}$$

whereby membership is subsumed under proper parthood. It is common to classify collectives into different kinds depending on how their components or members interact or are assembled into a whole; *e.g.*, Masolo *et al.* [60] propose three types: Pluralities like "Alice and Bob," proper collectives (*e.g.*, forests or organizations), and composites "that have another sort of internal structure" [60]. Canavotto and Giordani [61] distinguish between heaplike and non-heaplike collectives, *e.g.*, contrasting "a bunch of puzzle pieces" (heaplike) against "one puzzle made of those pieces" [61]. Semiotic collectives are characterized by joint action as a representational element, *i.e.*, as a referent or representamen; PIMS-II distinguishes the following four kinds of semiotic collectives:

1. In a Plurality x, the members y (with $\lhd_{\mathsf{p}} yx$) appear together as one representational element, all contributing to this effect in the same way; the members of a plurality can be anything other than another plurality or a structure.
2. In a Structure x, the members y (with $\prec yx$) constitute one representational element, but all in different roles; anything except structures can be a member of a structure.[13]
3. In an Articulation x, to which its members are related by $\lhd_{\mathsf{r}} yx$ ("y realizes x"), different realizations of a single representational element are grouped together; *e.g.*, copies of the same data item on different computers, multiple ways of denoting or encoding the same molecular model, or spoken utterances and written versions of the Lord's Prayer in different languages can be grouped together into articulations. The realizations (members) of an articulation cannot include any semiotic collectives.
4. A Proposition φ has articulations x as its members, denoted $\lhd_{\mathsf{a}} x\varphi$ ("x articulates φ"); this construction is to be employed whenever there are several ways in which the same propositional content was expressed and it is this shared semantic and/or pragmatic content that is relevant, rather than the exact way in which it was stated.

Following the paradigm of semiotic monism, no distinction is made between collectives that appear as a referent or a representamen. Membership is further generalized to *constitutivity* (denoted $\ddot{\mathsf{C}}$), *cf.* Axiom (23), which requires spatiotemporal overlap, and to *underlying* (denoted $\ddot{\mathsf{C}}^+$), the transitive closure of $\ddot{\mathsf{C}}$, which is antireflexive

$$\forall xy(\leq xy \rightarrow \ddot{\mathsf{C}}xy) \land \forall xy(\ddot{\mathsf{C}}xy \rightarrow \dot{\mathsf{P}}^- \dot{\mathsf{P}}xy), \tag{41}$$

$$\forall xy \left(\ddot{\mathsf{C}}^+ xy \leftrightarrow (\exists z(\ddot{\mathsf{C}}^+ xz \land \ddot{\mathsf{C}}zy) \lor \ddot{\mathsf{C}}xy)\right) \land \neg\exists x\, \ddot{\mathsf{C}}^+ xx; \tag{42}$$

e.g., the articulations "200" and "kPa" are constitutive of the articulation "200 kPa." This relation *cannot be subsumed under proper parthood*, since realizations of the same articulation "200" are proper parts of realizations of other articulations, such as "200 K," that are disjoint with "200 kPa." It is left open whether constitutivity is reducible to a construction involving modal relations; work by Vogt [62] suggests that this is challenging, since there are many ways in which one object may be constitutive of another.

5. Conclusion

Arndt *et al.* observe that it is "very common for standardization bodies on all levels (regional, super-regional, international) often to provide an inconsistent, ambiguous set of concepts,

[13]Pluralities are permitted as members of structures, to be employed as follows: If a and b contribute to the appearance of a structure as a referent or a representamen in the same way, while c contributes in a different way, the structure has two members. First, the plurality of a and b; second, c.

terms, and definitions. This is especially true for subjects that are relevant in a wide range of domains" [63]. Ontology-based data technology aims at improving consistency, and facilitating a uniform understanding of basic cross-domain concepts is a task that is usually attributed to foundational ontologies. To achieve this goal, it is necessary to provide a formalization of the underlying ontological paradigm, going beyond description logic which is insufficient to express many typical axioms. Mereotopology in combination with Peircean semiotics, or *mereosemiotics*, is an ontological paradigm that is already in use by one foundational ontology (EMMO [23, 24]), mid-level ontologies including PIMS-II [20, 21], and a great number of domain ontologies, *e.g.*, the eight domain ontologies from the Virtual Materials Marketplace (VIMMP) project [18] the ontologies mentioned by Francisco Morgado *et al.* [23], and many more that are being developed in projects funded from the Horizon 2020 research and innovation programme.

Despite the uptake of this ontological paradigm in data management practice, there is little literature so far on the interaction between spatiotemporal parthood/connectedness on the one hand and cognitive processes consisting of Peircean triads on the other hand; none, to the knowledge of the author, includes an axiomatization comparable to that of the paradigms underlying other foundational ontologies. Addressing this challenge, the axiomatization of the core parts of PIMS-II in modal first-order logic, given in the present work,[14] provides mereosemiotics with an unambiguous formalization. While the present axiomatization does not entail four-dimensionalism, it is consistent with it; thereby, it specifies a coherent approach to integrating semiotics with four-dimensionalism and nominalism (strongly rejected by Peirce, but enforced by the EMMO). By this approach, research data provenance can be denoted in terms of cognitive processes, *e.g.*, experimental procedures or simulation workflows, documenting the reliability and supporting the FAIRness of data that are made available on research data infrastructures.

Supplementary information is made openly accessible through Zenodo [21, 64].

Acknowledgments

This work was funded by the German Research Foundation (DFG) through the National Research Data Infrastructure for Catalysis-Related Sciences (NFDI4Cat), DFG project no. 441926934, within the National Research Data Infrastructure (NFDI) programme of the Joint Science Conference (GWK). It was facilitated by activities of the Innovation Centre for Process Data Technology (Inprodat e.V.), Kaiserslautern, Germany. Discussions in the Metadata4Ing group within NFDI4Ing, DFG project no. 442146713, are acknowledged.

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Appendix: List of Concepts

The following PIMS-II concepts are directly relevant to this work:

- Articulation: A semiotic collective x such that $\exists y \lhd_{\mathsf{r}} yx$. the realizations y of a single articulation x can include literal (written or digital) and non-literal (*e.g.*, spoken) versions; taxonomy: Articulation $\sqsubseteq$ SemioticCollective.
- Cognition: Process in which signs represent objects; taxonomy: Cognition $\sqsubseteq$ Process.
- CognitiveStep: Elementary cognition (*e.g.*, a triad); taxonomy: CognitiveStep $\sqsubseteq$ Cognition.
- Collective: Anything that has members (an x with $\exists y \leq yx$); taxonomy: Collective $\sqsubseteq$ Object.
- GroundedCognition: Cognition that is logically preceded by a Presupposition, *cf.* Axiom (29); taxonomy: GroundedCognition $\sqsubseteq$ Cognition.
- Interpretation: Semiosis that does not require the object to participate and be present physically; taxonomy: Interpretation $\sqsubseteq$ Semiosis.
- Item: Connected component of spacetime, *cf.* Axiom (8); taxonomy: Item $\sqsubseteq$ Object.
- MereotopologicalCollective: Anything that is not an Item, *i.e.*, any x such that $\exists y \dot{\leq} yx$; taxonomy: MereotopologicalCollective $\sqsubseteq$ Collective.
- Metonymization: Semantic change that satisfies Axiom (22) such that there is a "real causal connection" [40, p. 142], $\dot{\mathsf{C}}^{\star}oo'$, between the old referent o and the new referent o'; taxonomy: Metonymization $\sqsubseteq$ SemanticChange.
- Object: All that exists is an Object.
- Perception: Semiosis that requires the object to participate and be present physically, *cf.* Axiom (20); taxonomy: Perception $\sqsubseteq$ Semiosis.
- Plurality: A semiotic collective x such that $\exists y \lhd_{\mathsf{p}} yx$. Members y of a plurality x engage in representation jointly, all *in the same way or role*; taxonomy: Plurality $\sqsubseteq$ SemioticCollective.
- Presupposition: Anchor point for epistemic grounding – some previous cognitive step that is not subject to further scrutiny; taxonomy: Presupposition $\sqsubseteq$ CognitiveStep.
- Process: Connected region (Item) in which one or multiple objects *participate* (have a role), *i.e.*, any x such that $\exists y \ddot{\mathsf{P}} yx$ is a Process, *cf.* Axiom (16); taxonomy: Process $\sqsubseteq$ Item.
- Proposition: A semiotic collective φ such that $\exists x \lhd_{\mathsf{a}} x\varphi$. The members of a proposition φ are articulations that express some joint semantic and/or pragmatic content, namely φ; taxonomy: Proposition $\sqsubseteq$ SemioticCollective.
- SemanticChange: Cognitive step with the structure *old referent – sign – new referent* as in Axiom (21), *cf.* Paradis [45]; taxonomy: SemanticChange $\sqsubseteq$ TriadicCognition.
- Semiosis: Cognitive step with the structure *sign – object – interpretant* following Peirce [38], *cf.* Axiom (19); taxonomy: Semiosis $\sqsubseteq$ TriadicCognition.
- SemioticCollective: An x with $\exists y \ddot{\leq} yx$, *i.e.*, a collective that acts jointly as a representational element (*i.e.*, representamen or referent); taxonomy: SemioticCollective $\sqsubseteq$ Collective.
- Structure: A semiotic collective x such that $\exists y \prec yx$. The members y of a structure x engage in representation together, but all contributing *in different ways or roles*; taxonomy: Structure $\sqsubseteq$ SemioticCollective.
- TriadicCognition: Cognitive step that is constituted by the interaction between three elements, *i.e.*, any κ such that $\exists e_1 e_2 e_3\ 3\kappa e_1 e_2 e_3$; taxonomy: TriadicCognition $\sqsubseteq$ CognitiveStep.

Appendix: List of Relations

The following PIMS-II relations are directly relevant to this work:

name (IRI suffix and label)	symbol	hierarchy
articulates	$\lhd_{\mathsf{a}}$	$\sqsubseteq\ \ddot{\leq}$
directlyGrounds	$\hookrightarrow$	$\sqsubseteq\ \dot{\mathsf{C}} \sqcap \hookrightarrow^{+}$
hasCausalConnectionWith	$\dot{\mathsf{C}}^{\star}$	$\sqsubseteq\ \top_{\mathsf{Q}}$
hasDirectCausalConnectionWith	$\dot{\mathsf{C}}$	$\sqsubseteq\ \dot{\mathsf{C}}^{\star}$
hasProperPart	$\dot{\mathsf{P}}^{-}$	$\sqsubseteq\ \dot{\mathsf{P}}^{-}\dot{\mathsf{P}}$
hasRepresentamen	R^{-}	$\sqsubseteq\ \dot{\mathsf{P}}^{-}\dot{\mathsf{P}}\mathsf{R}^{-} \sqcap \mathsf{R}^{-}\dot{\mathsf{P}}^{-}\dot{\mathsf{P}} \sqcap \mathsf{R}^{-}\mathsf{R}\mathsf{R}^{-}$, *cf.* Fig. 2
isConstitutiveOf	$\ddot{\mathsf{C}}$	$\sqsubseteq\ \dot{\mathsf{C}} \sqcap \dot{\mathsf{P}}^{-}\dot{\mathsf{P}} \sqcap \ddot{\mathsf{C}}^{+}$
isFirstElementIn	$\dot{\mathsf{E}}$	$\sqsubseteq\ \mathsf{E}$
isFusionOf	σ	(relates an Object to an rdf:List)
isGroundFor	$\ddot{\mathsf{R}}$	$\sqsubseteq\ \mathsf{R}$
isImproperPartOf	P	$\sqsubseteq\ \dot{\mathsf{P}}^{-}\dot{\mathsf{P}}$
isMemberOf	$\leq$	$\sqsubseteq\ \ddot{\mathsf{C}} \sqcap \dot{\mathsf{P}}$
isMemberOfPlurality	$\lhd_{\mathsf{p}}$	$\sqsubseteq\ \ddot{\leq}$
isMemberOfStructure	$\prec$	$\sqsubseteq\ \ddot{\leq}$
isMereosemioticallyRelatedTo	$\top_{\mathsf{Q}}$	(complete relation, equivalent to C^{2} and $\dot{\mathsf{P}}\dot{\mathsf{P}}^{-}$)
isMereotopologicalMemberOf	$\dot{\leq}$	$\sqsubseteq\ \leq$
isParticipantIn	$\ddot{\mathsf{P}}$	$\sqsubseteq\ \dot{\mathsf{P}}^{-}\dot{\mathsf{P}} \sqcap \dot{\mathsf{C}}$
isProperPartOf	$\dot{\mathsf{P}}$	$\sqsubseteq\ \mathsf{P} \sqcap \equiv_{\mathsf{t}}\dot{\mathsf{P}}$
isReferentIn	O	$\sqsubseteq\ \mathsf{E}$
isRepresentamenFor	R	$\sqsubseteq\ \mathsf{R}\dot{\mathsf{P}}^{-}\dot{\mathsf{P}} \sqcap \dot{\mathsf{P}}^{-}\dot{\mathsf{P}}\mathsf{R} \sqcap \mathsf{R}\mathsf{R}^{-}\mathsf{R}$
isRepresentamenIn	$\dot{\mathsf{R}}$	$\sqsubseteq\ \mathsf{E} \sqcap \ddot{\mathsf{P}}$
isRepresentationalElementIn	E	$\sqsubseteq\ \top_{\mathsf{Q}}$
isSecondElementIn	$\ddot{\mathsf{E}}$	$\sqsubseteq\ \mathsf{E}$
isSemioticMemberOf	$\ddot{\leq}$	$\sqsubseteq\ \leq$
isSpatiotemporallyConnectedWith	C	$\sqsubseteq\ \equiv_{\mathsf{t}}\mathsf{C}$
isSpatiotemporallyDisconnectedFrom	$\neg\mathsf{C}$	$\sqsubseteq\ \neg\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}$
isTemporallyConnectedWith	$\equiv_{\mathsf{t}}\mathsf{C}$	$\sqsubseteq\ \top_{\mathsf{Q}}$
isTemporallyIncludedIn	$\equiv_{\mathsf{t}}\dot{\mathsf{P}}$	$\sqsubseteq\ \equiv_{\mathsf{t}}\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}$
isThirdElementIn	$\dddot{\mathsf{E}}$	$\sqsubseteq\ \mathsf{E}$
isTriadOf	3	(relates a TriadicCognition to an rdf:List)
logicallyPrecedes	$\hookrightarrow^{+}$	$\sqsubseteq\ \hookrightarrow_{\mathsf{t}} \sqcap \dot{\mathsf{C}}^{\star}$
realizes	$\lhd_{\mathsf{r}}$	$\sqsubseteq\ \ddot{\leq}$
sharesReferentWith	$\mathsf{R}\mathsf{R}^{-}$	$\sqsubseteq\ \mathsf{R}\mathsf{R}^{-}\dot{\mathsf{P}}^{-}\dot{\mathsf{P}} \sqcap \dot{\mathsf{P}}^{-}\dot{\mathsf{P}}\mathsf{R}\mathsf{R}^{-}$
sharesRepresentamenWith	$\mathsf{R}^{-}\mathsf{R}$	$\sqsubseteq\ \mathsf{R}^{-}\mathsf{R}\dot{\mathsf{P}}^{-}\dot{\mathsf{P}} \sqcap \dot{\mathsf{P}}^{-}\dot{\mathsf{P}}\mathsf{R}^{-}\mathsf{R}$
temporallyCoextendsWith	$\equiv_{\mathsf{t}}$	$\sqsubseteq\ \equiv_{\mathsf{t}}\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}$
temporallyPrecedes	$\hookrightarrow_{\mathsf{t}}$	$\sqsubseteq\ \neg\equiv_{\mathsf{t}}\dot{\mathsf{P}}^{-}\dot{\mathsf{P}} \sqsubseteq \neg\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}$
temporallyOverlapsWith	$\equiv_{\mathsf{t}}\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}$	$\sqsubseteq\ \equiv_{\mathsf{t}}\mathsf{C}$
overlapsWith	$\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}$	$\sqsubseteq\ \mathsf{C} \sqcap \equiv_{\mathsf{t}}\dot{\mathsf{P}}^{-}\dot{\mathsf{P}}$
underlies	$\ddot{\mathsf{C}}^{+}$	$\sqsubseteq\ \dot{\mathsf{C}}^{\star}$

(For descriptions *cf.* http://www.molmod.info/semantics/pims-ii.ttl.)