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# Conventional and downside CAPM: The case of London stock exchange

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## ABSTRACT

Many studies on asset pricing have highlighted the importance of downside risk, in line with the actual losses of investors. In addition, the capital asset pricing model (CAPM), although presented as a universal theory, may provide significantly different rates of return in bull and bear markets. Using the CAPM under different conditions could be regarded as an alternative measurement and valuation approach to downside risk. This paper investigates conventional and downside approaches to risk taking into account different measures of downside beta coefficients. A further contribution of this research is the development of an alternative approach to testing the CAPM relationship. For this purpose, conditional relationships of the CAPM are proposed in which risk premiums are set separately in bull and bear periods. Using equity data and portfolios from the United Kingdom, we obtained positive and statistically significant downside risk premiums. We observed a slight advantage of downside measures over conventional beta measures. Conditional models provide evidence of a positive risk premium in rising markets and a negative risk premium in falling markets. The robustness analysis in subperiods indicates that these findings are largely unchanged for downside beta coefficients, which is not fulfilled by the model in a variance approach.

## 1. Introduction

The first asymmetric approach to risk in modern finance is attributed to Roy's safety-first criterion (Roy, 1952), which is an approach to selecting an investment portfolio in which risk is equated with the failure to achieve the level of aspiration and in which a rational investor tries to select the portfolio components such that the probability of obtaining a result less than assumed is as low as possible. In the same year, Harry Markowitz (1952) published his work on the symmetrical measure of risk—the variance. However, variance-based portfolio construction models may be limited due to their asymmetric distributions of rates of return and their failure to consider investors' risk aversion (Sing & Ong, 2000).

As early as 1959, Markowitz introduced an algorithm for selecting an effective portfolio that measured risk using semivariance. Semivariance is the lower equivalent of variance and gives rise to a whole group of asymmetric measures known as “downside risk” such as lower partial moments (LPM). Bawa (1975) took the semivariance as a second-order LPM and demonstrated that it is an

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appropriate risk measure for utility functions with decreasing risk aversion. Moreover, he showed that when considering changes in two risky assets or a risky asset and a portfolio, asymmetric covariance measures are considered with deviations below the assumed rate of return. Such a measure, called semicovariance, was proposed by Hogan and Warren (1974). Bawa and Lindenberg (1977) incorporated semicovariance into the structure of n-degree LPM measures often called “asymmetric co-LPMs.” The analysis of the mean–risk relationship in the context of semivariance for the assumed rate of return can also be found in Fishburn (1977).

Based on the work of Markowitz (1952), the capital asset pricing model (CAPM) was further developed by Lintner (1965), Mossin (1966), and Sharpe (1964) by expressing the systematic risk accepted by investors as the beta coefficient. A linear regression describes changes in the price of a given company's shares due to changes in the price of its market portfolio. The use of the beta in the CAPM involves adopting the variance as a measure of risk and disregarding the concept of bottom risk (Markowitz, 1959), which represents the theoretical basis for using semivariance as an appropriate measure of risk to describe investor preferences. Thus, the beta from the conventional CAPM was a return to the variance approach, the concept of downside risk measures was ignored. However, the advantage and popularity of classical beta are probably due to the computational simplicity and intuitiveness of Sharpe's model.

The use of the downside risk approach in models for pricing capital assets and optimizing investment portfolios has been adopted in many theoretical and empirical works. Hogan and Warren (1974) presented one of the first studies in this area using semivariance and semicovariance to illustrate the invariability of the conventional CAPM structure. They provided strong theoretical support for the use of semivariance as a more adequate measure of risk than variance. Bawa and Lindenberg (1977) made a significant contribution to asset pricing in terms of downside risk by presenting the conventional CAPM derivation using the LPM approach and avoiding assumptions about the distribution of the rate of return. They assumed that the measure of systematic risk of a given asset about the rates of return of the market is co-LPM and showed that the conventional CAPM is a special case of the CAPM model using the LPM approach. Harlow and Rao (1989) provided a broad theoretical and empirical study of the use of LPM in asset valuation, presenting a generalization of the mean–LPM relationship (MLPM). Harlow and Rao (1989) also demonstrated the advantage of such a model over the conventional CAPM and proved that pricing models with different risk measures and target rates are special cases of MLPM models. While numerous studies have proven the advantage of the downside approach in risk analysis over the decades, downside beta factors are usually not used in practice. The Sharpe beta coefficient is practically unchanged and still used today; its values for individual shares appear in professional databases such as Thompson Reuters EIKON.

In the context of the conventional beta, the CAPM further claims that this coefficient is an appropriate measure of risk or that the risk–return relationship depends on the positive or negative condition of the stock market (Pettengill, Sundaram, & Mathur, 1995). Taking this into account, an additional contribution of this work to empirical studies of CAPM is to test this model by distinguishing between periods of market growth and periods of market decline. Previous studies have rejected the CAPM (Cheung & Wong, 1992; Östermark, 1991), assuming that the relationship between systematic risk and expected returns are independent of market conditions and that variance is the only relevant risk measure that investors should consider in their investment decisions.

This prompted the authors to research the use of downside risk measures in capital asset pricing on the London Stock Exchange (LSE) to establish whether downside risk is priced in this important European market and whether investors and stock analysts should not be limited to the Sharpe beta, conventional CAPM, and other extended pricing models with symmetric risk measures.

The main objective of this study was to test the CAPM relationship with systematic risk measures and realized returns for companies quoted on the LSE using variance and downside frameworks. The problems with the CAPM's assumptions highlighted above indicate the need for different approaches to testing this model. In this paper, the authors propose testing the CAPM in two ways: first, by using the conventional beta from Sharpe's model versus the downside risk associated with achieving returns below the assumed level and second, by using the conditional relationships adopted in the Pettengill et al. (1995) approach in which good and bad periods in the market are analyzed separately.

This paper is organized as follows. Section 2 briefly reviews the literature on testing the CAPM using symmetrical and downside approaches. Section 3 describes the dataset and methodology as well as the hypotheses of conditional and unconditional cross-sectional regressions. Section 4 presents the research findings and discussion. The last section offers some conclusions.

## 2. Literature review

The CAPM is a mathematical model that captures the link between systematic risk and the anticipated return for assets, especially equities. The CAPM approach is frequently used in finance to price risk securities and generate predicted returns for assets based on their risk and the cost of capital. Despite several research publications challenging traditional methodologies such as linear factor models, practitioners and academics alike continue to resort to landmark models like the CAPM due to its appealing simplicity (Ang, Chen, & Xing, 2006; Ayub, Kausar, Noreen, Zakaria, & Jadoon, 2020; Chhapra & Kashif, 2019; Ling, Sun, & Wang, 2020). Asset pricing is, at its core, a contentious matter. Since various investors attribute a lower weight to positive deviations from the mean than to negative deviations, the mean-semivariance relationship has been offered as an alternative approach to the mean-variance approach in portfolio analysis and asset pricing in recent years. An alternative measure to conventional beta is the beta coefficient based on the LPM, and it is called the “downside beta coefficient.” Derived from this is downside CAPM (D-CAPM), which is applied in the finance literature to quantify the cost of capital by identifying the risk–return connection.

Estrada (2002, 2007), using developed and emerging markets, showed that downside measures better explain the variability of returns in cross-sectional relationships than conventional betas. Post and Van Vliet (2006) proved that portfolio ineffectiveness results from disregarding moments other than variance. In addition, they showed that downside risk measures are crucial for explaining high average stock returns. Ajrapetova (2018) analyzes Estrada's study to show the effectiveness of traditional and alternative asset pricing models in explaining cross-sectional asset returns, with an emphasis on the risk–return relationship. He argued that investors in

emerging markets should prioritize overall risk measurements over systematic risk measurements. Furthermore, downside beta has outperformed typical CAPM beta in systematic risk assessments. Ang et al. (2006) argued that a higher value on downside risk requires greater remuneration when holding equities with a high sensitivity to market downturns. They discovered that the cross-section of stock returns represents a 6-percentage-point annual downside risk premium. In falling markets, stocks that co-vary significantly with the market have greater average returns. The payoff for taking on downside risk is not the only recompense for typical market beta; it may also be explained by co-skewness or liquidity risk, or by size, value, or momentum. Models using downside beta have more explanatory power than those using traditional CAPM for New York Stock Exchange stocks (Chen, Chen, & Chen, 2009). Galagedera (2009a) demonstrated that the association between the two systematic risk measures is dependent on the volatility of the market portfolio returns and the deviation of the target rate from the risk-free rate. Galagedera (2009b) examined the cross-sectional relationship using two measures of risk in the downside approach; namely, downside beta and downside co-skewness. Both downside risk measures performed poorly compared to the CAPM beta in developed markets. In emerging markets, there is evidence to suggest that downside co-skewness may be a better measure of risk compared to the CAPM beta and downside beta. Atilgan and Demirtas (2013) found a substantial positive link between monthly predicted market returns and downside risk in developing economies using fixed-effects panel data regressions.

Tsai, Chen, and Yang (2014) investigated whether expected returns depend on the CAPM and downside betas. They used a dynamic conditional correlation model on a sample of developed countries and showed that downside betas explained a substantial part of the volatility in expected asset returns better than CAPM betas. Atilgan, Bali, Demirtas, and Gunaydin (2019) reexamined the relationship between various downside risk metrics and future stock returns of 26 developed markets and found that there was no statistically significant positive relationship between systematic downside risk and cross-sectional equity returns and that the relationship is generally negative. Furthermore, they argued that the relationship between downside risk and future returns is strongly negative at the portfolio level, but it is flat at the stock index level. The significance of downside risk in asset pricing was examined on the Chinese stock market (Ali, 2019) and showed a positive risk premium for downside variability in the medium and long term. Rutkowska-Ziarko, Markowski, and Pyke (2019) investigated whether accounting betas and downside accounting betas both had an impact on the average rate of return in a capital market of 27 Polish construction companies listed on the Warsaw stock market. Their findings demonstrated that for the Polish construction company sector, investors received a positive risk premium associated with accounting betas and downside risk. Chhapra and Kashif (2019) used a dataset of 901 firms on the Pakistan Stock Exchange from 2000 to 2016 to investigate the implications of preference by a risk-averse investor for higher moments and downside risk. They argued that investors preferred firms with negative co-skewness, positive co-kurtosis, and downside risk as they yielded a greater risk premium. Furthermore, they indicated that co-skewness, co-kurtosis, and downside beta were key risk factors, but that only downside beta was genuinely priced above and beyond what covariance risk could explain, and that the CAPM did not significantly capture market risk premium, implying that other risk measures exist for the Pakistan Stock Exchange. Extensive research in other emerging markets such as the Slovenian, Croatian, and Serbian markets demonstrated that downside risk statistically and significantly explains mean returns (Momcilovic, Zivkov, & Vlaovic-Begovic, 2017). Ayub et al. (2020) tested a new 6-factor downside beta CAPM by adding a momentum factor and replaced beta in the 5-factor model using downside beta as a proxy for downside risk. Using data from the Pakistan Stock Exchange (PSX-100), they argued that the momentum factor was rejected in the beta-based 6-factor model only. They believed that the downside beta 6-factor model was a better option for investors compared to the beta-based 6-factor model in the area of asset pricing models. Hoque and Low (2020) implied that investors are penalized for their downside exposure to these risk factors and that such an inference is consistent with the risk preference explanation of prospect theory.

Downside risk measures are widely used in optimization issues in the construction of investment portfolios. To minimize downside risk, Pla-Santamaria and Bravo (2013) constructed a mean-semivariance efficient frontier model, which moved toward the portfolio selection of stocks using Dow Jones stocks with daily prices for 2005–2009. Klebaner, Landsman, Makov, and Yao (2017) showed that for portfolios with prespecified anticipated returns, minimizing downside risk leads to the same solution as minimizing variance. They argued that there is a significant difference between the portfolios obtained using the mean-semivariance efficient frontier model and portfolios of equal expected returns obtained using the conventional Markowitz mean-variance efficient frontier model. Salah, Chaouch, Gannoun, De Peretti, and Trabelsi (2018) argued that the downside risk (DSR) model for portfolio optimization overcomes the drawbacks of the conventional mean-variance model in terms of the asymmetry of returns and the risk perception of investors. According to Foo and Eng (2000), a traditional Markowitz (1952) portfolio optimization has two major flaws. First, when asset returns are skewed, mean-variance portfolio optimization is ineffective. Second, risk aversion among investors is overlooked. A generalized LPM framework offers a more efficient risk metric that focuses exclusively on the departure below a prespecified rate of return. This research demonstrates how to simply create downside risk models using spreadsheet programs and how to include investor risk aversion in a downside risk asset optimization model.

Boasson, Boasson, and Zhou (2011) presented seven exchange-traded index funds that mimicked various categories of securities such as government bonds, municipal bonds, investment-grade bonds, high-yield bonds, real estate bonds, and mortgage-backed securities to compare and test the differences between the optimal portfolios and asset allocations constructed out of the mean-semivariance approach and the traditional mean-variance approach. They argued that the mean-semivariance approach provided certain desirable benefits unavailable when using a traditional mean-variance approach. Specifically, optimization under the conditions of the semivariance model produces different portfolio strategies that at least maintain and at best improve the expected return of the portfolio using the traditional mean-variance model while minimizing its downside risk exposure. Ling et al. (2020) proposed a robust multi-period portfolio selection model based on downside risk with an asymmetrically distributed uncertainty set in which the downside losses of a portfolio were controlled by the LPM. They showed that there was an optimal solution with a robust model that could generate a given probability guarantee for individual and joint stochastic constraints.

The effect of the asymmetrically distributed uncertainty set on the performance of the optimal solution is analyzed using the comparative static method. [Otim, Dow, Grover, and Wong \(2012\)](#) found evidence that information technology (IT) investments and their timing influenced organizational downside risk. Transformational and informational IT investments only led to a reduction in downside risk if they led to strategic IT investments in the industry. For competitive necessities such as IT investments that automated business functions, a reduction in downside risk was realized by investing in parity with industry participants.

[Hammoudeh, Araújo Santos, and Al-Hassan \(2013\)](#) investigated the market downside risk of six significant individual assets, including four precious metals, oil, and the Standard & Poor's (S&P) 500 index. Using combinations of these assets, three optimal portfolios and their efficient frontiers within a value-at-risk (VaR) framework were constructed, and the returns and downside risk for these portfolios were analyzed. They argued that VaR-based performance measures ranked the most diversified optimal portfolio as the most efficient and the pure precious metals as the least efficient. Additionally, [Ali, Badshah, and Demirer \(2022\)](#) found that downside risk (VaR) significantly drove the returns of hedge funds.

Early studies often showed pessimistic conclusions regarding the CAPM postulates. The results of CAPM tests may depend on whether we consider the sensitivity of assets to positive and negative market changes in the testing methodology. One of the approaches to testing risk–return relationships in different market conditions is to use asymmetric response models. [Kim and Zumwalt \(1979\)](#) incorporated the two-beta model with the up and down-market responses of securities and confirmed a significant premium for accepting downside risk. Similar extended response models have been used in further studies ([Harlow & Rao, 1989](#); [Pedersen & Hwang, 2007](#)). [Pedersen and Hwang \(2007\)](#) using equity data from the United Kingdom demonstrated the utility of downside risk in explaining equity returns. Their research showed that classical beta explained the rates of return in only a small proportion of equities and concluded that downside risk is a significant risk factor explaining cross-sectional variation in returns. [Huang \(2019\)](#) used beta and downside beta to study trends in stock returns in the United Kingdom related to downside risk with a special focus on stock returns during financial crises. According to the findings, downside risk has a considerable beneficial influence on current stock returns while having a negative influence on future returns. In contradiction to conventional wisdom, when equities are ranked by beta, there is an inverse link between risk and return. Stock returns in the United Kingdom are very time sensitive, especially during financial crises.

An alternative approach to testing conditional CAPM in bull and bear market conditions was provided by [Pettengill et al. \(1995\)](#). They investigated the relationships described by the CAPM model, estimating its parameters separately in periods with positive and negative market excess returns. The study concluded that the unconditional CAPM is rejected for the sample period, but they provided evidence of an inverse relationship of returns to the beta coefficients in periods with negative excess returns. The CAPM testing approach proposed by [Pettengill et al. \(1995\)](#) has been extensively used in research in other capital markets. Using this method, [Fletcher \(1997\)](#) studied the UK stock market and discovered a significant beta-return relationship, although the values of estimated upwards and downwards risk premiums differed. [Galagedera, Henry, and Silvapulle \(2003\)](#) showed a positive relationship on the Australian Stock Exchange between beta and returns in a rising market and the opposite relationship in a falling market. They also confirmed a significant relationship between co-skewness and returns and demonstrated that this relationship depended on the skewness of the market portfolio's distribution. Similar results were obtained by [Nurjannah, Galagedera, and Brooks \(2012\)](#) based on companies listed on the Indonesian Stock Exchange. They showed significant beta-return relationships using conditional relationships dependent on the sign of the excess market portfolio returns over the risk-free rate. [Bilgin and Basti \(2014\)](#) empirically tested both the unconditional and conditional versions of the CAPM in the Istanbul Stock Exchange for the period of 9 years between 2003 and 2011. They argued that this is an unrealistic assumption and that the validity of the model in its standard (unconditional) form is repeatedly rejected by empirical testing. [Markowski \(2020\)](#) used a conventional and downside risk strategy to validate the CAPM in the Polish capital market. He argued that unconditional regressions provided evidence of an existing risk premium associated with co-skewness and downside beta and confirmed the validity of the downside CAPM. Furthermore, a study based on conditional relationships found that risk–return relationships depend on the state of the stock market. A separate procedure for estimating market risk premiums has been practiced and supported in many other developed and emerging capital markets (e.g., [Durand, Lan, & Ng, 2011](#); [Elsas, El-Shaer, & Theissen, 2003](#); [Isakov, 1999](#); [Karacabey & Karatepe, 2004](#); [Tang & Shum, 2003, 2006, 2007](#)).

There is very little research in the literature showing the impact of downside measures in pricing risk on the LSE. Indeed, there are no studies in which conditional relationships are compared with the D-CAPM for different kinds of downside betas. Therefore, this paper is an important contribution to the existing knowledge as it seeks to fill these gaps in the literature.

### 3. Data and methodology

#### 3.1. The dataset

The closing share prices between January 1, 2008 and December 30, 2020 for companies listed on the LSE were collected and analyzed for this study. Eighty-eight companies from the Financial Times Stock Exchange Group (FTSE) 100 index that were quoted continuously during the research period were considered for analysis. The FTSE 100 index was used as an approximation of a market portfolio. All data were collected from the Thomson Reuters Refinitiv Eikon database. The 1-month rate of return was calculated using a rolling procedure; one trading day was used as the step of change. A time series of monthly rates of return were calculated for every company according to the following equation:

$$R_{it} = \frac{N_{i,t+s} - N_{it}}{N_{it}}, \quad (1)$$

where  $R_{it}$  is the rate of return on the  $i$ -th security at time  $t$ ;  $s$  is the length of the investment process expressed in trading days,  $N_{it}$  is the listed value (in this article, the closing price) of the  $i$ -th security at time  $t$ ; and  $N_{i,t+s}$  is the listed value of the  $i$ -th security after  $s$  days of investing starting at time  $t$ .

In addition to individual securities, equally weighted portfolios were also analyzed in this study. Portfolios were formed from securities sorted in ascending order according to the given type of beta. Each portfolio consisted of 10 securities that were created using a rolling procedure. The entire sample of 88 securities was used, and 79 portfolios were created for every considered kind of beta. Four approaches for calculating the beta were considered. One of them was the Sharpe (1964) beta for risk measured by variance. The other three were the downside betas proposed by Harlow and Rao (1989), Bawa and Lindenberg (1977), and Estrada (2002). In the study, each portfolio consisted of 10 securities and was formed by a rolling procedure. In that way, for the entire sample of 88 securities, 79 portfolios could be created for every considered type of beta. The research period includes subperiods of decline in financial markets and subperiods of growth. The financial crisis (i.e., 2008–2010) and the crisis caused by the coronavirus 2019 disease (COVID-19) pandemic were covered by the data collected. Such subperiods should allow us to determine the risk premium in the long term while experiencing changing unpredictability in financial markets, especially in the case of the LSE, which is one of the most important European stock exchanges.

### 3.2. Systematic risk measures

According to the conventional CAPM, the main source of risk is the economic situation of the market, which is reflected in the market portfolio, including all available assets. In practice, stock indices approximate such a portfolio. The primary, systematic measure of risk is the beta factor, which expresses the sensitivity of changes in a given asset to changes in market conditions. The CAPM postulates a linear function between the expected returns of assets and the systematic risk expressed by beta coefficients. This relationship can be given as follows:

$$E(R_i) = R_f + \beta_i(E(R_M) - R_f) \quad (i = 1, \dots, N), \tag{2}$$

where  $E(R_i)$ ,  $E(R_M)$  is the expected return on the  $i$ -th asset and the market portfolio, respectively;  $R_f$  is the risk-free rate; and  $\beta_i$  is the beta coefficient for the  $i$ -th asset, which reflects the sensitivity of a given asset to changes in the stock index as a proxy of the market portfolio.

By contrast, the D-CAPM, unlike the conventional approach, identifies risk only as a deviation below the assumed rate of return. Downside risk measures are based on semi-measures like the semivariance of returns and semicovariance or, more generally, the LPM of degree  $n$  (with  $n \geq 1$ ), and they are defined as follows:

$$LPM_i^n = \frac{1}{T-1} \sum_{t=1}^T (LPM_{it})^n, \tag{3}$$

where:

$$LPM_{it} = \begin{cases} 0 & \text{for } R_{it} \geq \tau \\ R_{it} - \tau & \text{for } R_{it} < \tau \end{cases} \tag{4}$$

and  $R_{it}$  is the rate of return on the  $i$ -th asset in period  $t$ ;  $T$  is the length of the time series; and  $\tau$  is the target rate. An important issue for assessing downside risk measures is the target rate, which indicates the required rate by the investor. This theory distinguishes between many forms of downside beta coefficients, differentiating them from the equation and the target rate. When the required rate  $\tau$  equals the risk-free rate and  $n = 2$ , the downside beta coefficient can be written as follows (Bawa & Lindenberg, 1977; Hogan & Warren, 1974):

$$\beta_i^{BL} = \frac{E\{(R_{it} - R_f) \bullet \min[(R_{Mt} - R_f); 0]\}}{E\{\min[(R_{Mt} - R_f); 0]^2\}} \tag{5}$$

It is a common solution to take the risk-free rate (i.e., the minimum required rate of return) as the point at which there are no losses and no profits (Zebrowska-Suchodolska & Karpio, 2020). The special case of the above coefficient in Eq. (5) is the point at which the target rate is zero (Nurjannah et al., 2012), as given by the following equation:

$$\beta_i^{BL}(R_f = 0) = \frac{E[R_{it} \min(R_{Mt}; 0)]}{E[\min(R_{Mt}; 0)]^2}. \tag{6}$$

Harlow and Rao (1989) developed the problem of the LPM as the  $n$ -th order generalized LPM. They indicated that investors' risk assessment of securities should consider various acceptable target rates. One such level for determining the downside measure is the average rate of the market portfolio. The downside beta of the Harlow and Rao formula uses the covariance between asset  $i$  and the market portfolio as follows:

$$\beta_i^{HR} = \frac{E[(R_{it} - E(R_i)) \min(R_{Mt} - E(R_M); 0)]}{E[\min(R_{Mt} - E(R_M); 0)]^2}. \tag{7}$$

The third proposition of the downside beta is the coefficient determined by Estrada (2002) and is written as follows:



$$\beta_i^E = \frac{E[\min(R_{it} - E(R_t); 0)\min(R_{Mt} - E(R_M); 0)]}{E[\min(R_{Mt} - E(R_M); 0)]^2} \tag{8}$$

This coefficient, unlike the one mentioned previously, takes into account the skewness of returns. In this research, the coefficients described in Eqs. (6)–(8) were employed to calculate the downside betas to analyze the risk–return relationships.

### 3.3. Unconditional relationships of conventional and downside CAPM

The study of the relationship between the measures of systematic risk and the returns of individual securities and portfolios was carried out in accordance with a two-stage regression analysis using the classical Fama and MacBeth (1973) procedure. In the first stage, a whole sample period was considered and the conventional beta coefficient and proposed downside betas were estimated in accordance with the relationships in Eqs. (6)–(8).

In the second stage, regression analysis was applied to the cross-sectional series in which the dependent variables were the realized assets and portfolios returns and the independent variables were the systematic risk measures estimated in the first stage. The unconditional cross-sectional relationships were estimated for each month of the sample period (Nurjannah et al., 2012) as follows:

$$R_{it} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i + \eta_{it} \quad (i = 1, \dots, N); (t = 1, \dots, T), \tag{9}$$

$$R_{it} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i^{HR} + \eta_{it} \quad (i = 1, \dots, N); (t = 1, \dots, T), \tag{10}$$

$$R_{it} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i^{BL} + \eta_{it} \quad (i = 1, \dots, N); (t = 1, \dots, T), \tag{11}$$

$$R_{it} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i^E + \eta_{it} \quad (i = 1, \dots, N); (t = 1, \dots, T), \tag{12}$$

where  $\lambda_{0t}$ ,  $\lambda_{1t}$  represent the parameters of equation  $t$ ;  $\eta_{it}$  is the random error term of the  $t$ -th equation;  $N$  is the number of assets or portfolios; and  $T$  equals the length of the entire period.

Eqs. (9)–(12) provide the possibility of using statistical methods to test hypotheses regarding asset pricing theory. The average values of parameter  $\lambda_1$  for the accepted beta coefficient should correspond to the market risk premium, and they are expected to be positive. The relevant sets of hypotheses regarding the parameters of the unconditional relationships of the CAPM are presented in Table 1.

These hypotheses were tested using the  $t$ -test for average value with a one- or two-sided critical area according to the following equation:

$$t = \frac{\bar{\lambda}_i}{\hat{\sigma}_{\lambda_i} / \sqrt{T}} \quad (i = 0, 1), \tag{13}$$

where  $\bar{\lambda}_i$  denotes the average value of estimated  $\lambda_i$ , and  $\hat{\sigma}_{\lambda_i}$  is the standard deviation of estimated  $\lambda_i$  over  $T$  periods.

### 3.4. Conditional conventional CAPM relationships based on market conditions

The conditional relationships of the CAPM consist of a separate estimation and verification of this model in which there are positive and negative market returns. It is conditional on the sign of the market return, and the CAPM equation in the testable version is in the following form:

$$R_{it} = \delta\lambda_{0t}^U + (1 - \delta)\lambda_{0t}^D + \delta\lambda_{1t}^U\hat{\beta}_i + (1 - \delta)\lambda_{1t}^D\hat{\beta}_i + \eta_{it}, \tag{14}$$

where  $\delta$  is a dichotomous variable used to determine the positive and negative market return, then  $\delta = 1$  if  $(R_{Mt}) > 0$  and  $\delta = 0$  if  $(R_{Mt}) < 0$ ;  $\lambda_{0t}^U$ ,  $\lambda_{0t}^D$ ,  $\lambda_{1t}^U$ ,  $\lambda_{1t}^D$  is the parameters of equation  $t$ ; and  $\eta_{it}$  is the random error term of the  $t$ -th equation.

Parameters  $\lambda_1^U$  and  $\lambda_1^D$  in Eq. (14) express the market beta risk premium, and they should be positive in periods with positive market returns and negative in periods of negative market returns. The set of hypotheses for all parameters of conditional CAPM relationships is presented in Table 2.

The rejection of the null hypothesis in both cases indicates the occurrence of systematic relationships between the beta coefficients

**Table 1**  
Hypotheses for the parameters of unconditional CAPM relationships.

Risk measure	The null hypothesis	The alternative hypothesis
$\beta_i$	$H_0 : E(\lambda_1) = 0$	$H_1 : E(\lambda_1) > 0$
$\beta_i^{HR}, \beta_i^{BL}, \beta_i^E$	$H_0 : E(\lambda_1) = 0$	$H_1 : E(\lambda_1) > 0$
Constant term	$H_0 : E(\lambda_0) = 0$	$H_1 : E(\lambda_0) \neq 0$

Source: Authors' own study.

and the realized returns of securities or portfolios.

#### 4. Research results and discussion

##### 4.1. The analysis of the whole period using unconditional CAPM

The results comparing the relationship between the Sharpe beta and realized returns across securities and portfolios are considered using regression analysis. The estimates of the parameters of cross-sectional single unconditional regressions according to Eq. (9) are shown in Table 3. For the Sharpe beta, the R-squared value is higher for portfolios than for individual securities. The results reveal that for both individual stocks and portfolio, the regression coefficients  $\lambda_0$  and  $\lambda_1$  are statistically significant ( $p$ -value < 0.05). Investors in the LSE are rewarded with a positive market risk premium of approximately 0.003% per month.

Next, the risk premium is estimated in the unconditional models using the downside approach. The results comparing the relationship between downside betas and realized returns according to Eqs. (10)–(12) across securities and portfolios are reported in Table 4. Estimates of parameters  $\lambda_0$  are statistically significant for all models ( $p$ -value < 0.05). The estimated results of this parameter are positive and range between 0.0023% and 0.0083% per month. It is consistent with the assumption in Eq. (6) that the risk-free rate equals zero. Estimations of parameter  $\lambda_1$  are positive and statistically significant for downside betas calculated by the Harlow–Rao and Estrada propositions ( $p$ -value < 0.05). The estimated results of this parameter for  $\beta_i^{HR}$  and  $\beta_i^E$  are 0.0046%–0.0065% per month. These values are much higher than the estimates of premiums obtained in the conventional CAPM. In the case of the downside beta calculated by the Bawa–Lindenberg formula, the parameter  $\lambda_1$  is not statistically significant. This parameter is negative for individual securities. For every downside beta, the R-squared value is higher for portfolios than for individual securities just as with the conventional beta.

Similar results have been obtained for the Warsaw Stock Exchange during the period between January 2010 and December 2017 (Rutkowska-Ziarko & Markowski, 2020). For the Polish capital market, a positive risk premium was identified for the conventional and downside Harlow–Rao betas. A negative risk premium occurred for the Bawa–Lindenberg beta. For all downside betas, the R-squared values were higher for portfolios than for individual securities.

##### 4.2. The analysis of the whole period using conditional CAPM

In this section, the conditional relationships of the CAPM defined by Eq. (14) are estimated. The tested sample period was characterized by 1790 observations with positive ( $R_{Mt} > 0$ ) and 1462 observations with negative ( $R_{Mt} < 0$ ) monthly market returns. The relationship tests the hypothesis of a positive relationship return beta in periods with a positive market return and the hypothesis of a negative relationship return beta in periods with a negative market return. The results of these estimations are reported in Table 5.

The results of the conditional relationship show that the average estimations of the parameter  $\lambda_{1t}^U$  for the beta coefficient are positive, ranging from 0.0313% to 0.0327% per month, respectively, for individual securities and portfolios. The estimations are statistically significant at the level of significance  $\alpha = 0.01$ . The average estimations of the parameter  $\lambda_1^D$  are as they are expected. The mean values of  $\lambda_1^D$  are negative and range from –0.0314% to –0.0333% per month and they are statistically significant at the level of 1%.

The results indicate that the average value of systematic risk premium is significantly higher than zero in periods of the positive market return and significantly lower than zero in periods of the negative market return. Similar results were confirmed in the Australian capital market (Galagedera et al., 2003), the Indonesian Stock Exchange (Nurjannah et al., 2012), and the Russian Stock Exchange (Teplova & Shutova, 2011).

The explanatory power of conditional relationships, as for unconditional relationships, is much higher for portfolios than individual securities. The results of conditional CAPM relationships allow a conclusion that investments with high beta coefficients in periods with a positive market return (with a negative market return), achieve higher rates of return (lower rates of return) than investments with relatively lower beta coefficients. Empirical correspondence of the results with the CAPM in the context of expected values is presented in Figs. 1 and 2.

This study finds a positive relationship between the risk measure coefficients and average returns in periods of positive market returns and a negative relationship in periods of negative market returns. The results provide strong evidence that the higher the value of the risk measure, the higher the absolute average returns. It should be noted that the rates of return and beta coefficients for portfolios, being the average value of these measures for individual securities, show the trends of the analyzed relationships much more clearly. This is due to the much lower volatility of portfolio returns than individual stocks. The portfolio investments used in this study are significantly diversified, thus the impact of the specific risk of individual shares is substantially reduced. Therefore, the total risk of the portfolio is decreased.

**Table 2**  
Hypotheses for the parameters of conditional CAPM relationships.

Risk measure	The null hypothesis	The alternative hypothesis
$\beta_i$	$H_0 : E(\lambda_1^U) = 0$ $H_0 : E(\lambda_1^D) = 0$	$H_1 : E(\lambda_1^U) > 0$ $H_1 : E(\lambda_1^D) < 0$
Constant term	$H_0 : E(\lambda_0) = 0$	$H_1 : E(\lambda_0) \neq 0$

Source: Authors' own study.

**Table 3**  
Estimates of the unconditional CAPM relation in the conventional framework.

Coefficient	Individual securities			Portfolios				
	Mean	t-Stat	p-value	avg. R <sup>2</sup>	Mean	t-Stat	p-value	avg. R <sup>2</sup>
Model: $R_{it} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i + \eta_{it}$								
$\lambda_{0t}$	0.0066	7.177	0.0000	0.084	0.0068	7.927	0.0000	0.328
$\lambda_{1t}$	0.0031	2.236	0.0127		0.0033	2.330	0.0099	

Source: Authors' analysis.

**Table 4**  
Estimates of unconditional CAPM relationships in the downside framework.

Coefficient	Individual securities			Portfolios				
	Mean	t-Stat	p-value	avg. R <sup>2</sup>	Mean	t-Stat	p-value	avg. R <sup>2</sup>
Model: $R_{it} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i^{HR} + \eta_{it}$								
$\lambda_{0t}$	0.0050	5.240	0.0000	0.085	0.0049	5.435	0.0000	0.331
$\lambda_{1t}$	0.0046	3.213	0.0007		0.0047	3.503	0.0002	
Model: $R_{it} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i^{BL} + \eta_{it}$								
$\lambda_{0t}$	0.0111	12.930	0.0000	0.081	0.0083	10.221	0.0000	0.329
$\lambda_{1t}$	-0.0013	-0.980	0.8365		0.0016	1.195	0.1161	
Model: $R_{it} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i^E + \eta_{it}$								
$\lambda_{0t}$	0.0023	2.092	0.0365	0.087	0.0032	3.383	0.0007	0.333
$\lambda_{1t}$	0.0065	4.489	0.0007		0.0064	4.633	0.0000	

Source: Authors' analysis.

**Table 5**  
Estimates of conditional CAPM relationships.

Coefficient	Individual securities			Portfolios					
	Mean	t-Stat	p-value	avg. R <sup>2</sup>	Mean	t-Stat	p-value	avg. R <sup>2</sup>	
Model: $R_{it} = \delta\lambda_{0t}^U + (1-\delta)\lambda_{0t}^D + \delta\lambda_{1t}^U\hat{\beta}_i + (1-\delta)\lambda_{1t}^D\hat{\beta}_i + \eta_{it}$									
Up market	$\lambda_{0t}^U$	0.0094	6.897	0.0000	0.076	0.0086	6.981	0.0000	0.320
$\delta = 1$	$\lambda_{1t}^U$	0.0313	17.381	0.0000		0.0327	20.019	0.0000	
Down market	$\lambda_{0t}^D$	0.0032	2.714	0.0067	0.088	0.0045	3.936	0.0000	0.338
$\delta = 0$	$\lambda_{1t}^D$	-0.0314	-18.544	0.0000		-0.0333	-20.246	0.0000	

Source: Authors' analysis.

#### 4.3. Subperiod analysis using unconditional CAPM

In this section, the robustness for the period changes to analyze the risk–return relationship. This research intends to verify the stability of the results across subperiods. It is interesting that the robustness of the relationships studied is the same whether using conventional or downside betas when the sample period is changed. The total sample period was divided into two equal subperiods of 6.5 years each. The subperiods are January 2008–June 2013 and July 2013–December 2020. Table 6 presents the estimation of the unconditional CAPM relationship for Subperiods I and II.

The results of the unconditional CAPM relationships estimated in the subperiods do not give clear conclusions and differ significantly from the results for the entire sample. While the results for the first subsample indicate a significant positive risk premium, in the second subsample, this premium is negative and insignificant, which is not in line with the CAPM. In particular, the results reveal that the estimates of parameters  $\lambda_{0t}$  and  $\lambda_{1t}$  are both statistically significant and positive for individual securities and portfolios, respectively. Although while comparing the relationship between Sharpe beta and mean returns between individual securities and portfolios (Subperiod I), we found evidence that the coefficient values of both  $\lambda_{0t}$  and  $\lambda_{1t}$  are the same as 0.0086 and 0.0030 for individuals securities and for portfolios. The estimated parameters  $\lambda_{0t}$  and  $\lambda_{1t}$  are statistically significant at both a 1% and 10% level of significance. In Subperiod II, the values of  $\lambda_{0t}$  are positive and statistically significant at 1%, and the values of the coefficient are 0.0080 and 0.0087 for individual securities and portfolios, respectively, while the values of  $\lambda_{1t}$  are negative (i.e., -0.0001 and -0.0008) for individual securities and portfolios, respectively (Subperiod II). When comparing both subperiods, we found that the R-squared value was higher for portfolios than individual securities in both Subperiods I and II, while Subperiod I had a higher R-squared value than Subperiod II. Our results are similar to those in the literature that support the view that a static CAPM is unable to explain the cross-sectional variations in average returns, such as Basu and Stremme (2011). The results indicate that there is a significant differentiation in the average values of the risk premium measured by the beta coefficients. The choice of the sample period, in the case of the conventional CAPM, can have a significant impact on both the sign and absolute value of the market risk premium. The negative value of



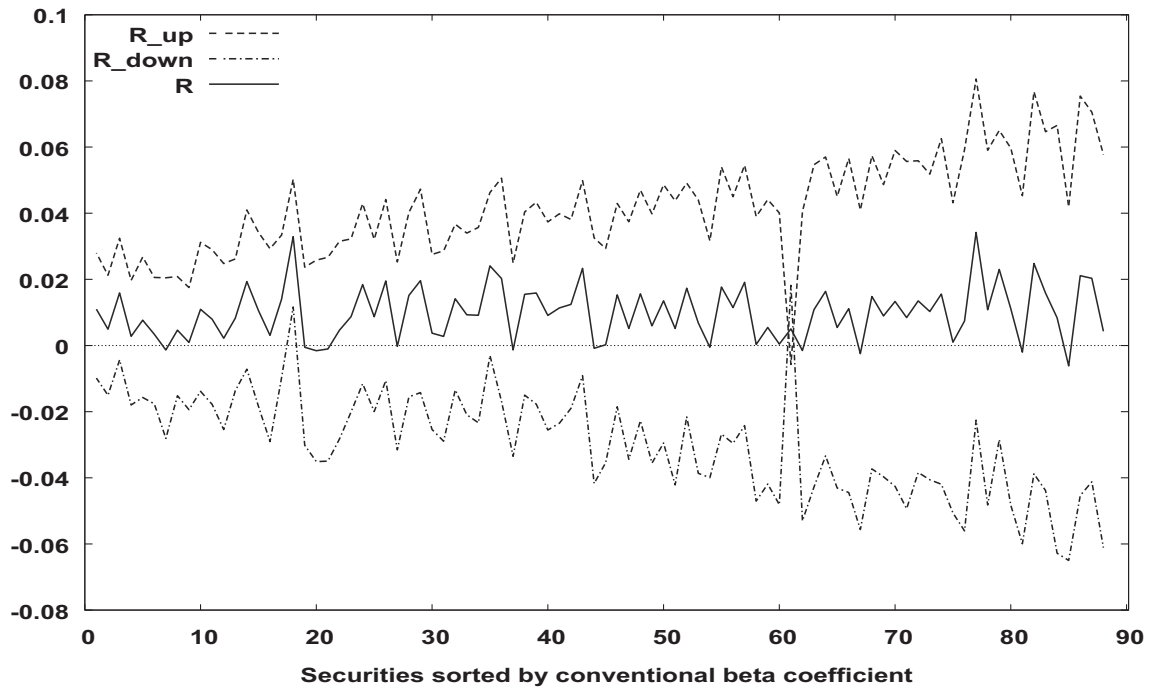


Fig. 1. Average returns of securities sorted by the beta coefficient under various market conditions.  
(Source: Authors' analysis.)

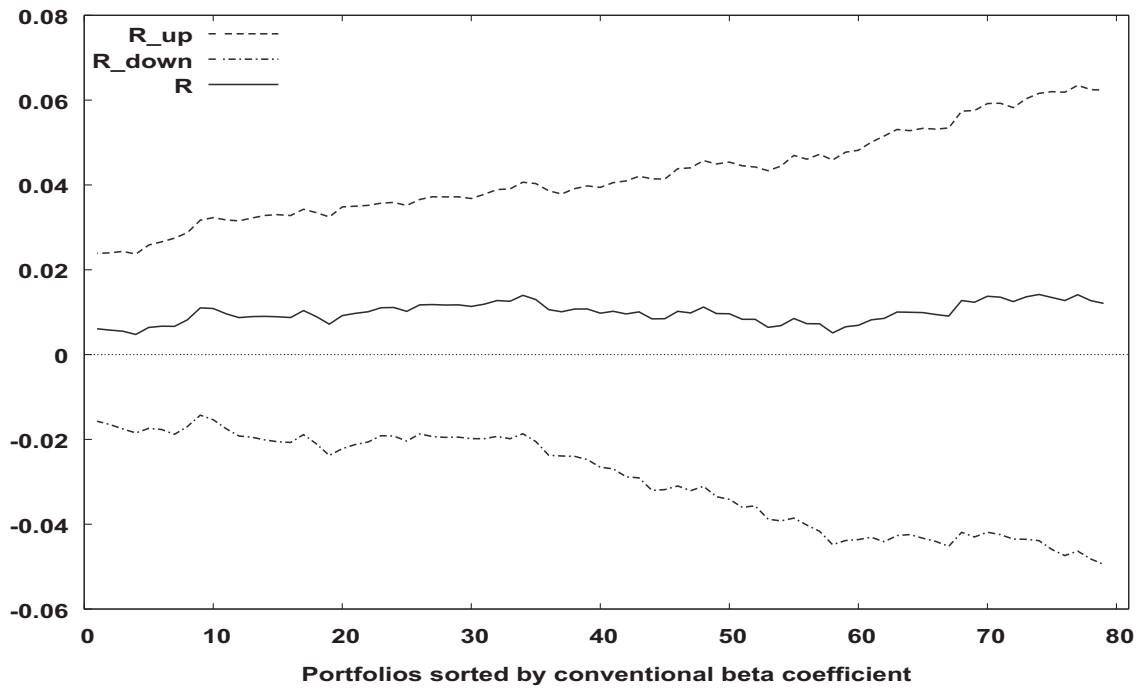


Fig. 2. Average returns of portfolios sorted by the beta coefficient under various market conditions.  
(Source: Authors' analysis.)

**Table 6**

Estimates of unconditional CAPM relation in the conventional framework Subperiods I and II.

Coefficient	Individual securities			avg. $R^2$	Portfolios			
	Mean	t-Stat	p-value		Mean	t-Stat	p-value	avg. $R^2$
Subperiod I								
Model: $R_{it} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i + \eta_{it}$								
$\lambda_{0t}$	0.0086	7.529	0.0000	0.115	0.0086	8.151	0.0000	0.373
$\lambda_{1t}$	0.0030	1.516	0.0649		0.0030	1.636	0.0510	
Subperiod II								
Model: $R_{it} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i + \eta_{it}$								
$\lambda_{0t}$	0.0080	6.306	0.0000	0.067	0.0087	5.770	0.0000	0.292
$\lambda_{1t}$	-0.0001	-0.069	0.5275		-0.0008	-0.430	0.6664	

Source: Authors' analysis.

this premium achieved in the second subperiod may result from much lower average rates of return in the market index compared to the first subperiod.

Table 7 shows the results of the unconditional CAPM relationship in the downside risk framework. It shows the cross-sectional comparative analysis of different D-CAPMs for Subperiods I and II using the Harlow–Rao, Bawa–Lindenberg, and Estrada downside beta coefficients. The results of the D-CAPM are more stable than those of the conventional CAPM.

The Harlow–Rao and Estrada beta coefficients generate positive and significant market premiums for both portfolios and individual securities in the first and second subperiods. The estimated parameters that denote risk premium are positive, ranging from 0.0037 to 0.0052 for individual securities and 0.0026–0.0028 for portfolios, respectively, for Subperiod I, and 0.0029–0.0043 and 0.0044–0.0065 for Subperiod II, respectively. However, the coefficients  $\lambda_{1t}$  under the Bawa–Lindenberg formula are negative and statistically insignificant both for individual securities and portfolios, except for the portfolio estimate in Subperiod I in which the coefficient is positive. The estimate of coefficient  $\lambda_{0t}$  under the Estrada proposition is insignificant for portfolios. In all the models, the Harlow–Rao, Bawa–Lindenberg, and Estrada propositions, the R-squared values of the portfolio models are greater than the individual securities in both Subperiods I and II.

#### 4.4. Subperiod analysis using conditional CAPM

Table 8 shows the conditional CAPM relationship for Subperiods I and II estimated through Eq. (14). The results indicate that for an up market where  $\delta = 1$ , parameters of  $\lambda_{0t}^U$  and  $\lambda_{1t}^U$  both are positive and statistically significant at a 1% level of significance for both Subperiods I and II. In Subperiod I, the estimated parameters  $\lambda_{0t}^U$  and  $\lambda_{1t}^U$  for up market are 0.0107 and 0.0336 for individual securities and 0.0103 and 0.0343 for portfolios. In Subperiod II, coefficient for up market of  $\lambda_{0t}^U$  and  $\lambda_{1t}^U$  are 0.0121 and 0.0256 for individual securities and 0.0114 and 0.0263 for portfolios. The results indicate an interesting outcome that the coefficients for portfolios are higher than for individual securities both for Subperiods I and II.

However, in declining markets ( $\delta = 0$ ), while both coefficient  $\lambda_{0t}^D$  and  $\lambda_{1t}^D$  both are statistically significant for individual securities and portfolio (Subperiods I and II),  $\lambda_{0t}^D$  was positive and  $\lambda_{1t}^D$  had negative market returns. The estimated coefficient values of  $\lambda_{0t}^D$  are 0.0056 and 0.0063 (Subperiod I) and 0.0035 and 0.0057 (Subperiod II) for individual securities and portfolios, respectively, while the coefficient values of  $\lambda_{1t}^D$  are -0.0392 and -0.0400 (Subperiod I) and -0.0282 and -0.0304 (Subperiod II) for individual securities and portfolios, respectively. The relationship between the conventional beta coefficients and realized rates of return are entirely different in rising and falling markets, and they are conditioned by the sign of the market return. The results for Subperiods I and II indicate that during a positive market return period, the beta coefficient generates a positive market risk premium, and that during a negative market return period, the beta coefficient generates a negative risk premium. Moreover, the risk premium value, both in absolute terms and in terms of explanatory power, in the first subperiod is greater than in the second subperiod.

## 5. Conclusion

The main purpose of this study was to test the standard, downside, and conditional CAPM relationships between systematic risk measures and mean returns for companies quoted on the LSE. We used two different approaches: the first approach used the stock prices (1 month rate of return) of individual companies, and the second approach used portfolios constructed from stocks (10 elements equally weighted).

In general, stocks and portfolios with higher conventional or downside betas generated greater mean returns. For all risk factors, the R-squared values were higher for portfolios compared to individual securities. However, for the Bawa–Lindenberg beta, a negative risk premium was identified. The highest explanatory power of the rates on return on the UK capital market in an unconditional relationship was found for the Estrada beta. For all betas, the explanatory power was higher for portfolios than for individual securities, and it oscillated between 0.329 and 0.338 for portfolios. This could mean that all betas considered in the study had the same usefulness in understanding return–risk relation on the LSE. But in the case of the Bawa–Lindenberg beta, the risk premium for both individual securities and portfolios was not statistically significant. Additionally, this premium was negative for individual securities. These

**Table 7**  
Estimates of unconditional CAPM relationships in the downside framework in Subperiods I and II.

Coefficient	Individual securities			Portfolios				
	Mean	t-Stat	p-value	avg. R <sup>2</sup>	Mean	t-Stat	p-value	avg. R <sup>2</sup>
Subperiod I								
Model: $R_{it} = \lambda_{0t} + \lambda_{1t}\widehat{\beta}_i^{HR} + \eta_{it}$								
$\lambda_{0t}$	0.0078	6.405	0.0000	0.115	0.0088	7.479	0.0000	0.385
$\lambda_{1t}$	0.0037	1.783	0.0374		0.0026	1.311	0.0950	
Model: $R_{it} = \lambda_{0t} + \lambda_{1t}\widehat{\beta}_i^{BL} + \eta_{it}$								
$\lambda_{0t}$	0.0131	12.883	0.0000	0.108	0.0114	11.044	0.0000	0.388
$\lambda_{1t}$	-0.0015	-0.757	0.7754		0.0001	0.027	0.4892	
Model: $R_{it} = \lambda_{0t} + \lambda_{1t}\widehat{\beta}_i^E + \eta_{it}$								
$\lambda_{0t}$	0.0057	3.910	0.0000	0.119	0.0089	7.547	0.0000	0.386
$\lambda_{1t}$	0.0052	2.468	0.0068		0.0028	1.422	0.0776	
Coefficient	Individual securities			Portfolios				
	Mean	t-Stat	p-value	avg. R <sup>2</sup>	Mean	t-Stat	p-value	avg. R <sup>2</sup>
Subperiod II								
Model: $R_{it} = \lambda_{0t} + \lambda_{1t}\widehat{\beta}_i^{HR} + \eta_{it}$								
$\lambda_{0t}$	0.0049	4.136	0.0000	0.066	0.0032	2.278	0.0228	0.307
$\lambda_{1t}$	0.0029	1.784	0.0373		0.0044	2.419	0.0078	
Model: $R_{it} = \lambda_{0t} + \lambda_{1t}\widehat{\beta}_i^{BL} + \eta_{it}$								
$\lambda_{0t}$	0.0136	12.004	0.0000	0.065	0.0089	6.949	0.0000	0.314
$\lambda_{1t}$	-0.0061	-3.819	0.9999		-0.0014	-0.776	0.7811	
Model: $R_{it} = \lambda_{0t} + \lambda_{1t}\widehat{\beta}_i^E + \eta_{it}$								
$\lambda_{0t}$	0.0030	2.211	0.0272	0.066	0.0010	0.698	0.4853	0.314
$\lambda_{1t}$	0.0043	2.600	0.0047		0.0065	3.610	0.0001	

Source: Authors' analysis.

**Table 8**  
Estimates of conditional CAPM relationships in Subperiods I and II.

Coefficient	Individual securities			Portfolios					
	Mean	t-Stat	p-value	avg. R <sup>2</sup>	Mean	t-Stat	p-value	avg. R <sup>2</sup>	
Subperiod I									
Model: $R_{it} = \delta\lambda_{0t}^U + (1 - \delta)\lambda_{0t}^D + \delta\lambda_{1t}^U\widehat{\beta}_i + (1 - \delta)\lambda_{1t}^D\widehat{\beta}_i + \eta_{it}$									
Up market	$\lambda_{0t}^U$	0.0107	6.290	0.0000	0.110	0.0103	6.692	0.0000	0.361
$\delta = 1$	$\lambda_{1t}^U$	0.0336	13.338	0.0000		0.0343	14.953	0.0000	
Down market	$\lambda_{0t}^D$	0.0056	4.193	0.0000	0.121	0.0063	4.685	0.0000	0.388
$\delta = 0$	$\lambda_{1t}^D$	-0.0392	-17.405	0.0000		-0.0400	-18.586	0.0000	
Coefficient	Individual securities			Portfolios					
	Mean	t-Stat	p-value	avg. R <sup>2</sup>	Mean	t-Stat	p-value	avg. R <sup>2</sup>	
Subperiod II									
Model: $R_{it} = \delta\lambda_{0t}^U + (1 - \delta)\lambda_{0t}^D + \delta\lambda_{1t}^U\widehat{\beta}_i + (1 - \delta)\lambda_{1t}^D\widehat{\beta}_i + \eta_{it}$									
Up market	$\lambda_{0t}^U$	0.0121	6.459	0.0000	0.076	0.0114	5.311	0.0000	0.281
$\delta = 1$	$\lambda_{1t}^U$	0.0256	11.536	0.0000		0.0263	10.876	0.0000	
Down market	$\lambda_{0t}^D$	0.0035	2.090	0.0369	0.088	0.0057	2.752	0.0061	0.303
$\delta = 0$	$\lambda_{1t}^D$	-0.0282	-11.909	0.0000		-0.0304	-11.318	0.0000	

Source: Authors' analysis.

results enabled us to conclude that downside beta coefficients are no less useful in asset pricing than conventional beta coefficients. Investors in downside risk are rewarded with higher premiums than those who invest in conventional beta risk.

The final part of the research was an estimation of the conditional relationships between the realized rates of return and beta coefficients for periods with positive and negative market returns. The results obtained allowed us to formulate the following conclusions. The relationship between the conventional beta coefficients and the realized rates of return are entirely different in rising and declining markets, and they are conditioned by the sign of the market return. Examining the conditional relationships, the value of the risk premium is positive and significant in periods of a positive situation of the capital market. The premium for systematic risk is significantly less than zero during periods of negative market returns. It should be emphasized that the analysis of the risk–return relationships in negative market return periods can be treated as an alternative approach to asset risk in the downside framework.

The estimates obtained for subperiods showed an advantage of the D-CAPM over the conventional CAPM. Market premiums for downside risk were significantly positive in both subperiods, while market premiums for risk expressed by the beta were positive only in the first subperiod. The results obtained for the subperiods confirmed the legitimacy of considering the CAPM separately in rising and falling markets.

## Author statement

**Anna Rutkowska-Ziarko:** Conceptualization, Methodology, Data curation, Formal analysis, Writing- Original draft preparation. **Lesław Markowski:** Conceptualization, Methodology, Formal analysis, Writing- Original draft preparation. **Christopher Pyke:** Supervision, Writing- Reviewing and Editing. **Saqib Amin:** Writing- Original draft preparation, Visualization.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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