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Short Communication

Prediction of Friction Coefficients in Mixed Lubrication Regime For Lubricants Containing Anti-Wear and Friction Modifier Additives

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Abstract

Many laboratory tribology test machines are available for evaluating the effect of different lubricants and different operating conditions on friction. For the Mini Traction Machine (MTM) there is much published data that shows how the measured friction coefficient varies with operating conditions and lubricant type. Fully formulated lubricants containing the anti-wear additive ZDDP have often been found to have a significantly higher friction coefficient, which persists to higher speeds, compared to base oils (lubricants with no additives). Recent work has found that the surface roughness of ZDDP tribo-films can evolve to become significantly higher than that of the surfaces they are deposited on. When the measured friction coefficients of lubricants tested in the MTM machine are suitably normalized and plotted against the λ ratio (which is equal to the oil film thickness separating the moving surfaces divided by the combined surface roughness) then the curves for various different lubricants lie on a "master curve" which enables reliable friction estimates to be made for lubricated contacts in the mixed lubrication regime. A simple modification to this approach also allows for the calculation method to be extended to lubricants that contain friction modifier additives.

Keywords

mixed lubrication, lubricants, anti-wear additives, friction modifier, friction

1 Introduction

The testing of lubricant performance for friction and wear in the full-scale machines that they are intended to be used in is an extremely expensive and time-consuming activity, and measurements need to be made on many machines to obtain statistically relevant data.

Therefore, much testing of lubricant formulations to determine how they influence friction and wear, is carried out in laboratory tests, using a variety of tribology test equipment. Such tests are often quicker, cheaper, and only require small amounts of test lubricant. Because of the tightly controlled conditions used in such tests, the repeatability and reproducibility of data can be reduced to acceptable levels.

Scientific reviews of tribology test equipment used in lubricant evaluation and for the measurement of the impact of different formulations on friction and wear have been reported by Lee et al. [1] and Budinski [2]). Both uni-directional tests are available (such as pin-on-disk [3] and Mini-Traction Machine (MTM) tests [4]) as well as reciprocating tests (such as the Plint TE-77 [5], the Optimol SRV-5 [6], the RTEC Instruments Multi Function Tribometer [7] and the High Frequency Reciprocating Rig (HFRR) [8]).

An advantage of a uni-directional tribometer is that the speed of the disk can be increased over a wide range. For example, in the Mini Traction Machine [4], the speed can be increased from around 5 mm/second to over 3000 mm/second. Thus, friction measurements can be made from boundary through mixed lubrication almost to elastohydrodynamic lubrication, depending on the applied load and lubricant temperature used in the tests. Mixed and boundary lubrication is important in many machine elements (such as gears, valve train systems, rolling element bearings etc.) and understanding how friction is influenced by lubricant formulations under such conditions is a topic of much research interest [9-23]. Measurements of the friction behaviour of lubricants using test equipment allows theoretical methods to predict mixed friction to be checked and validated to confirm their reliability.

The lubrication condition in a contact is commonly assessed by the "lambda ratio" (λ) which is the ratio of the oil film thickness in the contact to the combined root mean square (RMS) roughness of the surfaces. Broadly speaking if $\lambda < 1$, the contact would be in the boundary lubrication regime, whereas if $1 < \lambda < 3$, the contact is in the mixed lubrication regime, and if $\lambda > 3$ the surfaces are assumed to be almost completely separated (although since roughness has a statistical distribution, often assumed to be Gaussian, there is still a small chance of asperity contact for $\lambda > 3$).

Recent work [10, 24, 25] has shown that, for a range of lubricants containing no additives (i.e, base oils) and lubricants containing ZDDP anti-wear additives, the normalized friction coefficient, when plotted against λ, forms a "master curve" from which mixed/boundary friction can be estimated relatively simply. It was also found [10, 24, 25] that widely used contact equations used in predicting friction in the mixed/boundary regime (such as the Greenwood-Williamson [26] and Greenwood-Tripp [27] models) significantly underestimated the amount of mixed/boundary friction in a contact, compared to MTM experimental data.

This paper extends the earlier work by these authors [10] to further explore the consequences of the "master curve" and also to include friction modified lubricants. (Note that further background information relating to the behaviour of anti-wear and friction modifier additives is available elsewhere [28, 29]).

2 Experimental methods

Details of the Mini Traction Machine (MTM), or similar machines, and their operation for measuring friction for different lubricants and different lubricant additives, including grease, have been reported by numerous authors [4, 9-23].

Usually, the load is set to 20 or 30 N, the oil temperature is varied between around 40°C to 120°C, the speed is varied from about 5 mm/second to over 3000 mm/second, and the slide-roll ratio (SRR) is often in the range 50-100%. (Where the slide-roll ratio is defined as $2(U_1 - U_2)/(U_1 + U_2)$, where U_1 and U_2 are the speeds of the moving surfaces, and so the magnitude of the SRR vary from 200% - for pure sliding – to 0% - for pure rolling).

Typical MTM friction measurement results [9] are shown in Fig. 1, where the friction coefficient is measured for a Group III base oil (XHVI 5.2) and a fully formulated SAE 10W-40 engine oil. This data was measured at a load of 20 N, a lubricant temperature of 105°C and a slide-roll ratio of 100%. The engine oil contained ZDDP anti-wear additives, and it is seen that the use of such additives leads to higher friction coefficients, which persist to a higher speed, compared to the friction measured for the base oil (which contains no additives).

Figure 2 shows data from an in-depth study [10] on ZDDP containing lubricants, which explores how the friction coefficient changes as the ZDDP tribo-film forms on the surface. It is interesting to note that the data at 0 minutes (when no ZDDP tribo-film is present on the surfaces) is very similar to the base oil data of Fig. 1. It was also noted that the surface roughness of the surfaces covered with the tribo-films changed with time. The combined surface roughness values (R_q) are given in Table 1.

The data shown in Figs. 1 and 2 can be used to extract the proportion of boundary friction in a mixed friction contact. To do this, note that typically, the maximum friction coefficient, f_o, occurs at the lowest speeds, and that the friction coefficient usually "plateaus out" at the highest speeds at a value of f_{EHD}. This latter friction coefficient is assumed to be that due to elastohydrodynamic lubrication. If f is the measured friction coefficient, then the quantity $(f - f_{EHD})/(f_o - f_{EHD})$ can be calculated. This quantity will have a maximum value of one at the lowest measurement speeds and a value of zero at the highest measurement speeds. The authors of reference [10] have referred to this quantity as being effectively the proportion of boundary friction in a mixed/ friction contact.

In addition, rather than plotting the data against the entrainment speed, as is done in Figs. 1 and 2, the data can be plotted against the λ ratio. This can be done since the speeds, loads, lubricant temperature, component geometry and component mechanical properties are known, and so the oil

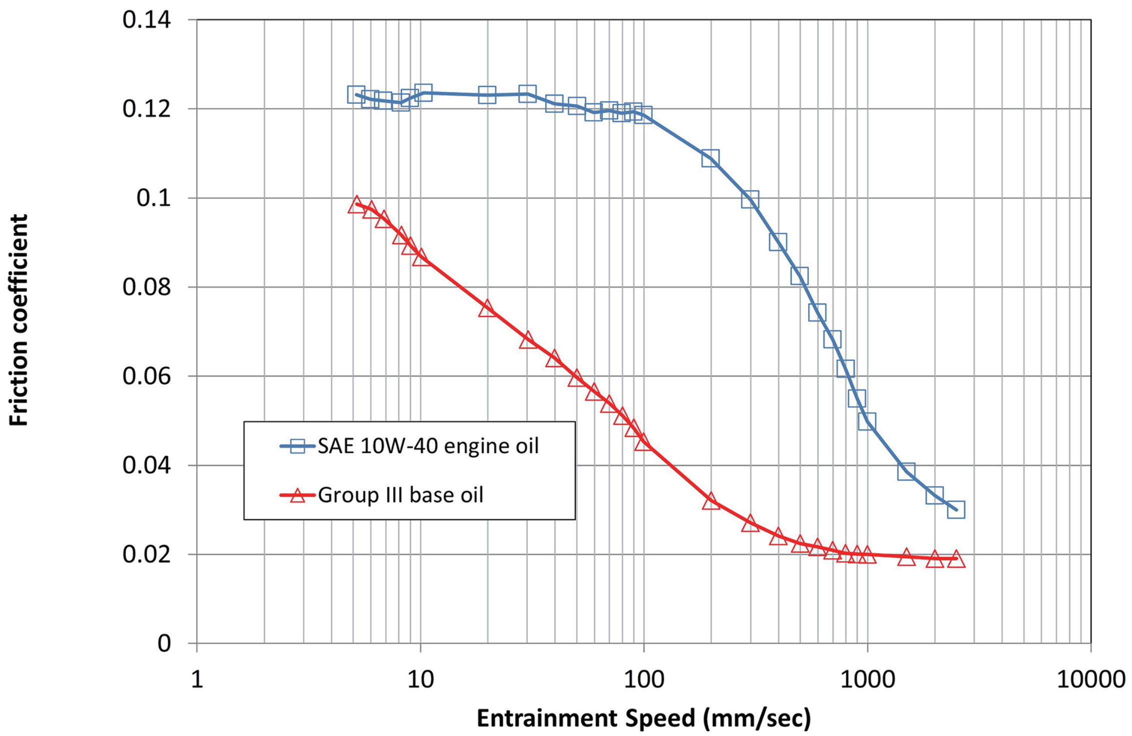

Fig. 1 Friction coefficient versus entrainment speed (mm/s) for two different lubricants, measured in the Mini Traction Machine [9] (load = 20 N, lubricant temperature = 105°C, and slide roll ratio = 100%)

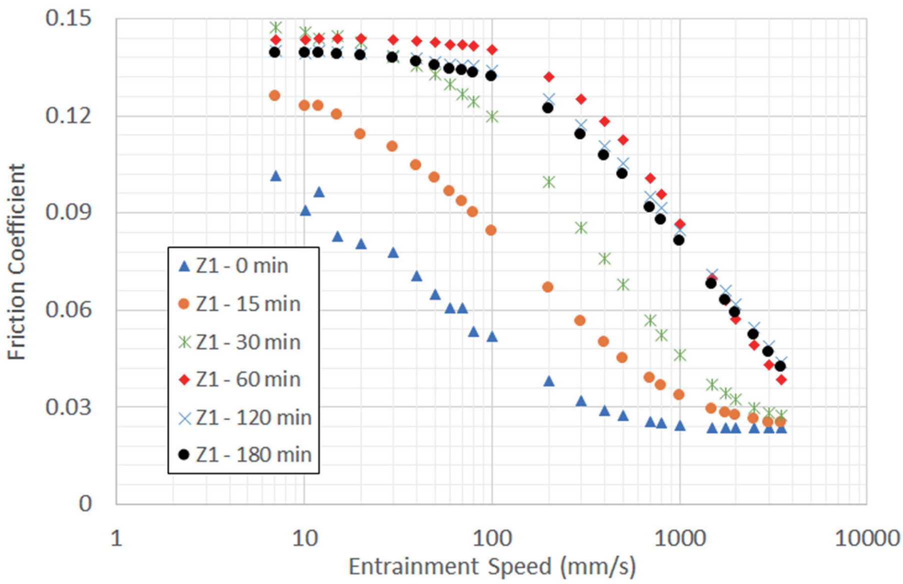

Fig. 2 MTM friction coefficient measurements of ZDDP containing lubricants (load = 31 N, lube temp = 100°C and SRR = 50%) [10]

Table 1 Combined root mean square surface roughness (R_q) of tribo-film covered surfaces [10]

Time (minutes)	Combined surface roughness R_q (nm)
0	5.66
15	17.07
30	39.12
60	63.33
120	56.93
180	47.31

film thickness can be calculated in the usual way using point contact elastohydrodynamic theory [30]. The pressure viscosity coefficients of the lubricants (the α values) may be estimated using data from Gold et al. [31]. The roughness values contained in Table 1 can be used with the data of Fig. 2 to estimate the Lambda ratio. For the data of Fig. 1 it was assumed that the combined surface roughness for the base oil test was 5.67 nm whereas that for the SAE 10W-40 engine oil was 56.7 nm. Since the oil film thickness and combined surface roughness values are known, the λ ratio can easily be calculated, and the results of the analysis are shown in Fig. 3.

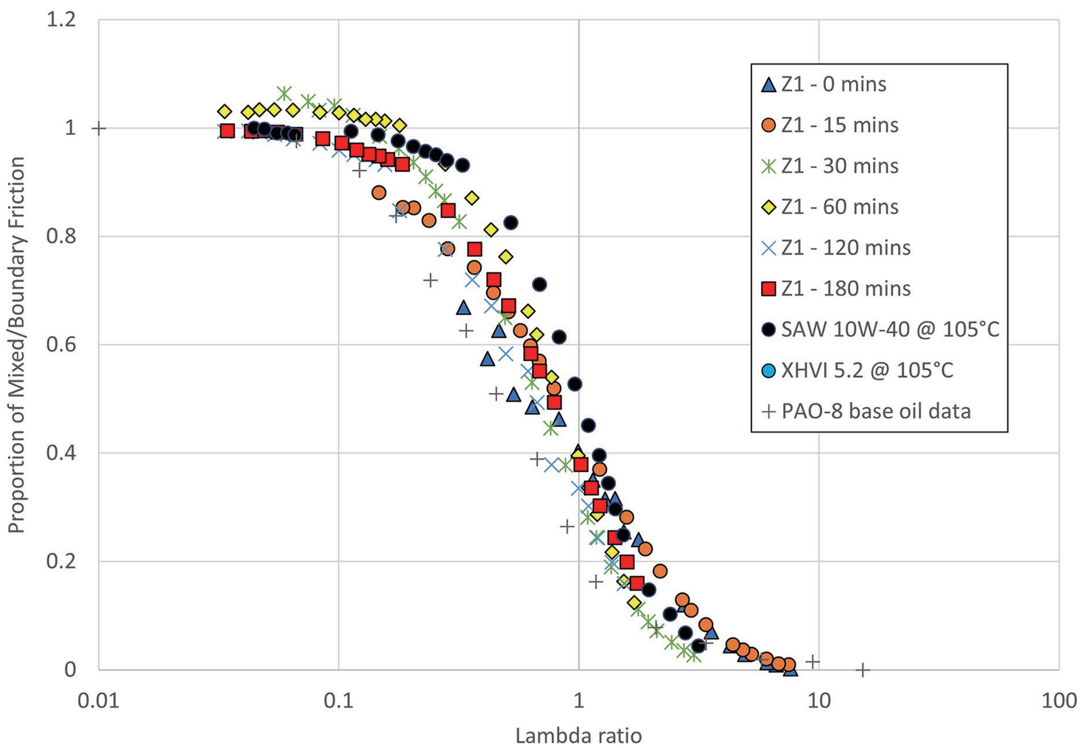

Fig. 3 Re-plot of data from Figs. 1 and 2 to show the proportion of mixed/boundary friction versus λ ratio

Recently, Taylor and Sherrington [25] have shown that the data in Fig. 3 can be fitted rather well with a "reverse S-curve" function of the following form:

$$X = \frac{1}{(1+\lambda^k)^a} \tag{1}$$

Equation (1) gives a good fit to the experimental data of Fig. 3 if k ≈ 3/2 and a ≈ 4/3. Excel's Solver function was used to find the values of k and a which gave the "best" fit to the data (by minimizing the least squares error between Eq. (1) and the experimental data). The values of k and a, found in this way, are: k = 1.581 ; a = 1.301. Note that these values are slightly different to those reported in [25], mainly because a different set of experimental data was used for the fit.

Figure 4 replots the experimental data with the curve fit from above.

As mentioned above, the values of k and a reported in reference [25] were k = 1.453 and a = 1.32, whilst the best fit values for the data of Fig. 4 were found to be k = 1.581 and a = 1.301. For convenience, a reasonable fit can be obtained using k ≈ 3/2 and a ≈ 4/3. Although the reader may be perturbed by these differences in the possible values of k and a, it is apparent from Fig. 5 that curves with these different values of k and a are all very similar in form and given the spread in experimental data of this nature which is conventionally experienced, all the curves in Fig. 5 give acceptable fits.

Some further comments are worth making on the types of functions that could be used to fit the experimental data of Fig. 3. As pointed out by Taylor and Sherrington [25], the type of curves needed are "reverse S-curves". Mathematically, reverse S-curves are continuous functions, with a negative gradient, apart from when $\lambda = 0$ or when $\lambda \to \infty$, when the gradient is zero, and whose value is 1 when $\lambda = 0$ and zero when $\lambda \to \infty$.

Simple differentiation shows that the curve described by Eq. (1) satisfies the above criteria provided k > 1. It is interesting to note that a function of similar form to Eq. (1) was previously reported for describing the level of boundary friction in the mixed regime, by Olver and Spikes [32]. However, the curve they proposed, that $X = (1 + \lambda)^{-2}$, although similar to Eq. (1), is not, in fact, a reverse S-curve since its gradient is non-zero when $\lambda = 0$.

In a similar way, Greenwood and Williamson [26] proposed that $X = \exp(-\lambda)$. Again, although this function may provide a reasonable fit to experimental data, it is not a physically correct model, since it is not a reverse S-curve as its gradient is non-zero when $\lambda = 0$.

Functions that are reverse S-curves and can potentially provide acceptable fits to experimental data include the complementary error function (erfc(x)) which has been used in previous mixed/boundary friction studies [33], and the functions $F_{3/2}(x)$ and $F_{5/2}(x)$ which were proposed by Greenwood and Williamson [26] and Greenwood and Tripp [27].

A comparison of the different type of functions used to predict the proportion of boundary lubrication in mixed friction lubrication, as a function of λ was recently reported by Taylor [24].

It is also worth commenting that the use of S-curves (which are also known as logistic curves) have recently been used to more accurately model wear using an extension of the Archard wear equation [34].

3 Results for friction modified lubricants

The experimental data described in the previous section showed that base oils (lubricants containing no additives) and ZDDP containing lubricants can fit onto a "common" curve when the proportion of mixed/boundary lubrication is plotted against the λ value.

However, friction modified oils do not appear to fit onto

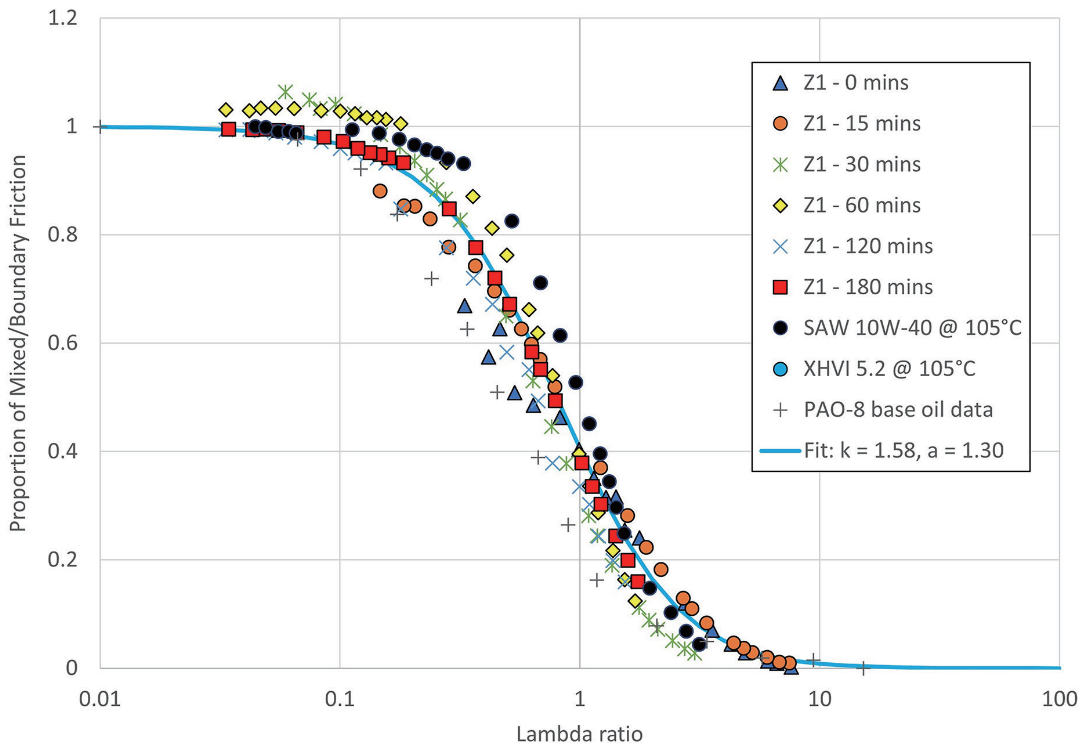

Fig. 4 Re-plot of Fig. 3 with fit to experimental data (from Eq. (1)) shown

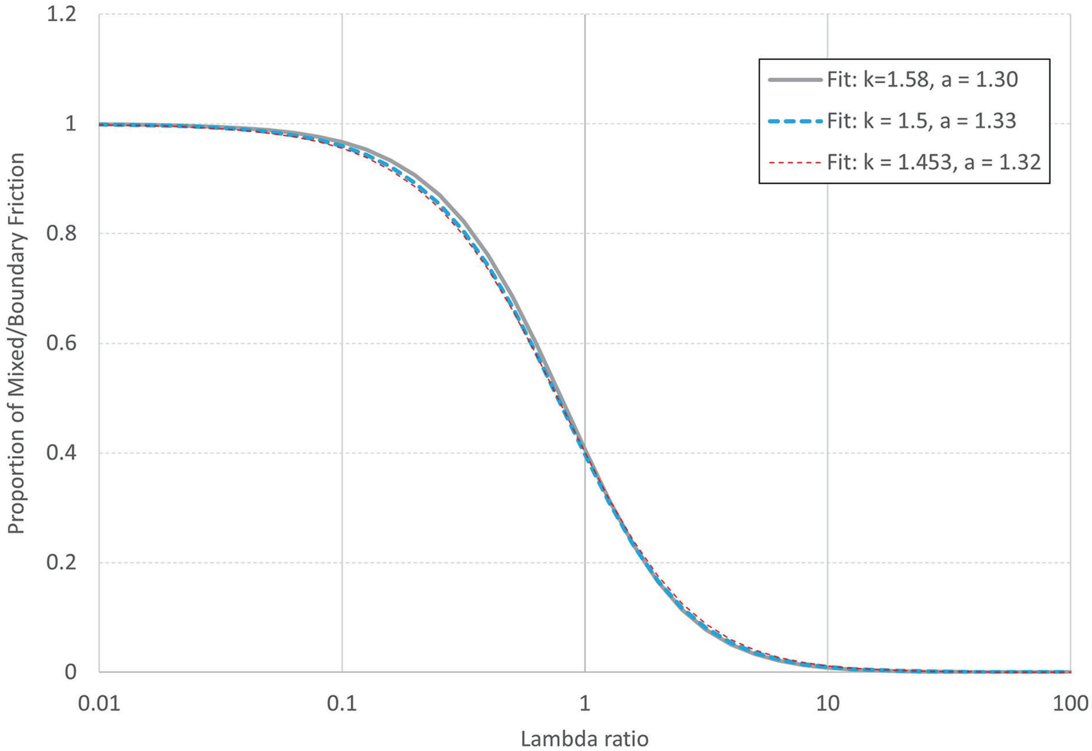

Fig. 5 Graph showing values of Eq. (1) for different values of k and a

this "common" curve. Friction modifiers are not thought to form thick tribo-films and it is generally believed that molecularly thin layers of the friction modifier form on top of surface asperities, and it is the sliding of these low shear films over each other that results in low friction (compared to asperities sliding over each when not covered in such films). Since the friction modifier films are thought to be relatively thin, when such films form on rough metal surfaces, they do not change the λ value significantly, and so any changes to the shape of the friction curve cannot be explained using the same physical argument put forward to explain friction curves developed in the presence of ZDDP anti-wear additives. However, the above understanding of how friction modifiers work suggests that it is the value of f_o that is affected when friction modifiers are used. For lubricants which contain various surface-active additives, such as anti-wear additives, friction modifiers, detergents etc. which are all competing for the surface, the presence of a friction modifier could affect the growth of a tribo-film formed on the surface, and so could affect the surface roughness of the resulting tribo-film.

Published data is available on the friction behaviour of lubricants that contain friction modifier additives [9, 12, 15-18, 20]. Typical data [9], generated after a suitable "run-in" period, and measured in a Mini Traction Machine, is shown in Fig. 6 for an SAE 0W-20 fully formulated engine lubricant (the lubricant contained ZDDP), with two different types of friction modifier, and the comparison with an SAE 10W-40 engine lubricant which does not contain friction modifier is also shown (for these experiments the slide roll ratio used was 100%).

A possible approach for describing friction modified lubricants is to modify Eq. (1) so it becomes:

$$X' = \frac{1}{(c + \lambda^k)^a} \tag{2}$$

Where c = 1 if the lubricant is un-additivated, or contains anti-wear additives only, whilst c > 1 for lubricants that contain friction modifiers. It should be mentioned that the amount of mixed/boundary lubrication in the contact would still be given by Eq. (1), but that when calculating mixed/boundary friction, the relevant equation for predicting the friction force, F, would be:

$$F = f_o . X' . W \tag{3}$$

If f_{TOT} is the measured friction coefficient, this can be predicted using:

$$f_{TOT} = f_o X' + f_{EHD}(1 - X) \tag{4}$$

Since X′ = X if the lubricant does not contain a friction modifier, Eq. (4) is equivalent to the usual expression for calculating friction coefficient [10, 32].

Equation (3) should be compared to the expression F = f_o.X.W which would be used for un-additivated lubricants or for lubricants that contain anti-wear additives only. In essence, for lubricants that contain friction modifiers, the boundary friction coefficient has changed to become f_o.X′/X. As an example, consider a fully formulated lubricant that contains both an anti-wear additive and a molybdenum-based friction modifier. For the fully formulated lubricant SAE 10W-40 engine oil without a friction modifier, shown in Fig. 6, f_o is about 0.127. When the friction modifier is included, the boundary friction coefficient is reduced to a value of 0.04 (for molybdenum based FMs) or 0.07 (for organic based FMs). This would suggest that c can be found (for the molybdenum containing oil) by the equation below (which uses Eqs. (1) and (2) when λ = 0.

$$0.04 = \frac{0.127}{c^a} \tag{5}$$

Earlier discussions have suggested that a ≈ 1.33, which leads to a value of c of approximately 2.38 for the molybdenum containing oil. Similar arguments lead to a value of c of about 1.56 for the organic friction modifier lubricant shown in Fig. 6.

Figure 7 shows a comparison of predicted friction (using Eqs. (2) and (4)) and measured friction coefficient (from data of Fig. 6) versus λ. The values of c above were used to calculate X'. It should be noted that surface roughness values were not given in reference [9] for the friction modified oils, and surface roughness values for the friction modified oils were adjusted to ensure a good fit to the curves predicted by Eq. (2). For the SAE 0W-20 oil with Mo FM, a combined surface roughness of 15 nm gave the best fit, whilst for the SAE 0W-20 with organic FM, the combined surface roughness used was 9 nm (this compares to 5.7 nm for the combined surface roughness of the metal surfaces in the MTM contact, and around 57 nm for ZDDP tribo-film covered surfaces).

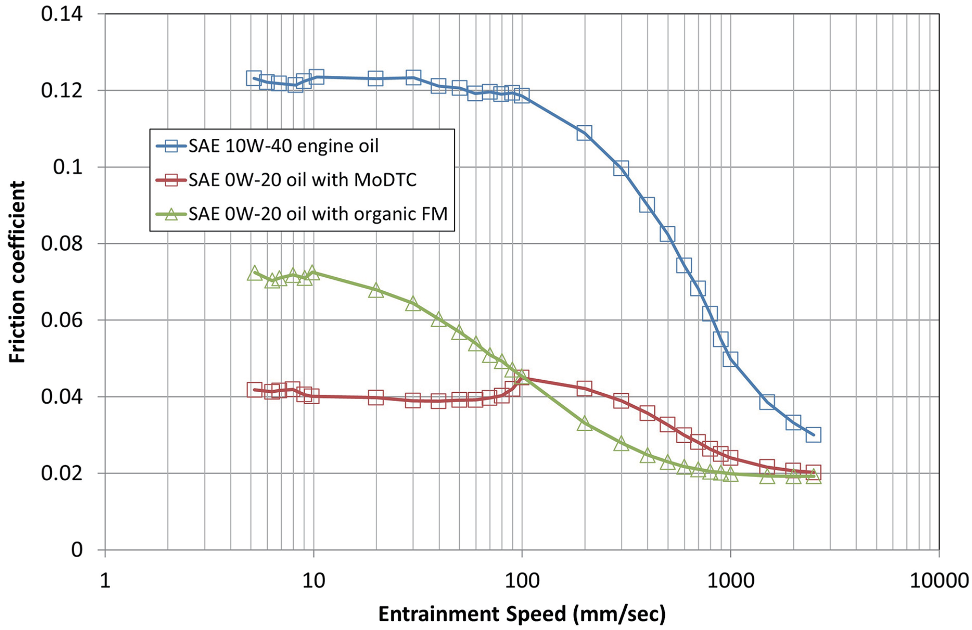

Fig. 6 Friction coefficient versus entrainment speed (mm/s) as measured in the Mini Traction Machine (same conditions as for Fig. 1) for an SAE 10W-40 engine oil (containing ZDDP anti-wear additives) and two SAE 0W-20 friction modifier containing lubricants (for this data, the slide roll ratio used was 100%)

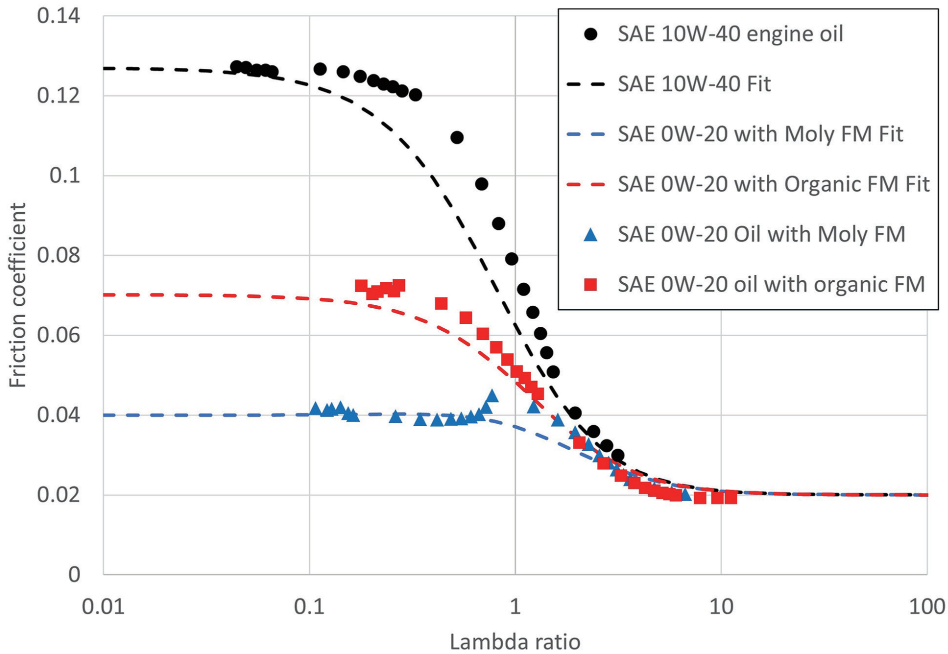

Fig. 7 Comparison of predicted and measured friction coefficient (using measured data of Fig. 6 and Eq. (4). The following values were used for the fitted curves: f_o = 0.127, f_{EHD} = 0.02, k = 1.5, a = 1.33, c = 2.38 for the MoDTC containing oil, and c = 1.56 for the oil containing an organic FM.

4 Discussion

The previous sections have described how experimental friction data for different lubricants, containing the anti-wear additive ZDDP, when suitably normalized, and plotted against the λ ratio, lie on a "master curve", which enables the relatively straightforward, and reliable estimation of friction in the mixed/boundary lubrication regime.

The proportion of mixed/boundary friction is given by Eq. (1), with $k \approx 3/2$ and $a \approx 4/3$.

Although the work reported here is based on data from a Mini Traction Machine [4], it was shown in reference [25] that this approach also worked well with experimental data from other tribology friction machines.

For lubricants that also contain friction modifiers, Eq. (1) was modified to allow for the friction coefficient to be reduced due to the incorporation of the friction modifier in the lubricant. The proportion of mixed/boundary friction in the contact is still given by Eq. (1), but the value of the boundary friction coefficient is altered due to the presence of a friction modifier. A relatively simple equation (Eq. (4)) is used to predict the overall friction coefficient.

It is interesting to speculate on the value of the parameter c, which is responsible for the reduction in friction coefficient when friction modifiers are included in the lubricant. The mode of operation for friction modifiers is thought to be that they form surface films that are easy to shear. For MoS_2 surface layers, it has been reported that the interfacial shear strength MoS_2 coatings is of the order of 5 MPa [35] under high vacuum conditions, whereas other researchers have quoted values of around 25 MPa for sliding experiments performed in a dry air environment [36]. Shear strength values have also been reported for organic films. Timsit and Pelow [37] reported shear strength values in the range of 7-25 MPa for stearic acid layers sliding on aluminium and commented that these values were in good agreement with the work of Briscoe and Evans [38]. Other useful references on the shear strength of organic films include the work of Briscoe et al. [39], Bailey and Courtney-Pratt [40] and work by Israelachvili and Tabor [41]. It is proposed that c could be regarded as a ratio of shear strengths raised to an appropriate power, i.e. c = (shear strength of ZDDP film/shear strength of FM)$^{\alpha}$. This would then suggest that to get the largest reduction in friction, the friction modifier film would need to have as low a shear strength as possible. Many of the references quoted above are quite old, and it is suggested that improved measurements of the shear strength of lubricant tribo-films (both ZDDP anti-wear and various organic and molybdenum-based friction modifiers) should be carried out to put the above discussion on firmer quantitative ground.

It is also important to note that the λ ratio should be calculated from the roughness of the tribo-films formed on the surface and may well be different from the roughness of the metal surfaces that the tribo-films are formed on.

5 Conclusions

Previous work [25], describing how a "common" curve for mixed friction can be found for base oils and oils that contain ZDDP anti-wear additives, has been extended to also include oils containing friction modifier additives.

The simple expressions presented can be used by engineers and tribologists to make improved estimates of friction losses in the mixed and boundary friction regime. There are two broad general cases, as follows:

- For lubricants that do not contain friction modifiers, the proportion of mixed/boundary friction, X, is found to be given by: $X = (1 + \lambda^k)^{-a}$ where $k \approx 3/2$ and $a \approx 4/3$.
- For lubricants that contain friction modifiers the boundary friction coefficient has been found to be modified to become $f_o.X'/X$, where $X' = (c + \lambda^k)^{-a}$, where $k \approx 3/2$ and $a \approx 4/3$. The friction force, F, is simply calculated using $F = f_o.X'.W$, where f_o is the friction coefficient for the lubricant without a friction modifier, and W is the load. For a lubricant containing a molybdenum-based friction modifier, the value of c which gave a good fit to experimental data was c = 2.38, whereas for a lubricant containing an organic FM, c = 1.56 (in these cases f_o was approximately 0.12). If the lubricant does not contain a friction modifier, c = 1, and the modified equation reverts to the equation previously reported [25].

The other important point to note, as stressed in references [10] and [25], is that it is the surface roughness of the tribo-films that should be used to calculate the λ ratio, and NOT the surface roughness of the underlying metal surfaces. For the relatively smooth surfaces used in the Mini Traction Machine, ZDDP tribo-films, following running-in, are generally significantly rougher than the underlying metal surfaces.

It is suggested that further work is needed to better understand how the factor c is related to the shear strength of the tribo-films formed on the surfaces and also how the details of the lubricant formulation affect the surface roughness of the tribo-films that are formed.

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