3D millimeter-Wave Indoor Localization

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Abstract—The 3D nature of modern smart applications has imposed significant 3D positioning accuracy requirements, especially in indoor environments. However, a major limitation of most existing indoor localization systems is their focus on estimating positions mainly in the horizontal plane, overlooking the crucial vertical dimension. This neglect presents considerable challenges in accurately determining the 3D position of devices such as drones and individuals across multiple floors of a building let alone the cm-level accuracy that might be required in many of these applications. To tackle this issue, millimeter-wave (mmWave) positioning systems have emerged as a promising technology offering high accuracy and robustness even in complex indoor environments. This paper aims to leverage the potential of mmWave technology to achieve precise ranging and angling measurements presenting a comprehensive methodology for evaluating the performance of mmWave sensors in terms of measurement precision while demonstrating the 3D positioning accuracy that can be achieved. The main challenges and the respective solutions associated with the use of mmWave sensors for indoor positioning are highlighted, providing valuable insights into their potential and suitability for practical applications.

Index Terms—3D, indoor, localization, millimeter-Wave

I. Introduction

The explosive growth of the Internet of Things and the emergence of many location-based services (LBS) and mobile smart applications make localization an even more important key-enabling technology in the Information and Communications Technology (ICT) world while many of these LBS impose very high 3D localization accuracy requirements. Several approaches have been proposed during the last few decades to address the challenges of indoor localization however most of them only estimate positions on a horizontal $(x - y)$ plane and neglect the vertical (z) dimension. This lack of vertical infromation could lead into problems, such as the inability to determine whether a device is held up high or in a pocket etc. Accurate 3D positioning is also critical in scenarios such as drone-assisted crop seeding, search and rescue operations, and wireless communication [1], where sub-meter or cm-level accuracy is likely essential.

To address this demand, millimeter-wave (mmWave) positioning systems have emerged as a promising technology, offering high accuracy and robustness in complex environments. mmWave is currently used in some Wi-Fi systems

(e.g. IEEE802.11ad) while it is planned to be used in 5G communications due to it flexibility to use wider bandwidths and hence its strong potential in achieving much higher data rates and capacity. mmWave systems typically operate in frequencies between 26 to $300GHz$. Their very large availability of bandwidth which leads to fine timing (and hence ranging) resolution and together with the use of massive phase array antennas that allow the estimation of the phase (and hence angle estimation) could be used for achieving cm-level 3D positioning accuracy or better [2]. In this work, we capitalize on the potential of mmWave technology to accurately provide ranging and angling information, and sustain the momentum of ongoing research efforts in this topic by demonstrating its suitability to achieve cm-level accuracy, while presenting the most important challenges it imposes.

The remainder of this paper is organized as follows: in Section II the recent related works and developments in 3D localization using mmWave technology are presented while Section III describes the methodology and setup used for the experimentation including the details of challenges and difficulties faced during the implementation and proposes solutions to overcome them. Section IV presents the results of the range and angle precision analysis conducted using two off-the-shelf mmWave sensors as well as the accuracy achieved using two 3D positioning approaches. Finally, in sections V and VI we provide a critical discussion and conclusion.

II. Related Work

The modern nature of smart applications that require the precise estimation of the location in 3 dimensions, stimulated a growing research interest and activity during the last few years to develop/investigate 3D positioning methods using the most promising up-to-date technologies. The authors of this paper provide in [3] a complete survey of 3D indoor localization techniques and approaches where 3D where many of of these modern technologies are discussed and evaluated. The most relevant works to this paper a mentioned here. For instance, in [4] the authors have theoretically derived the Cramér-Rao Bound (CRB) on position and rotation angle estimation uncertainty from mmWave signals from a single transmitter, in the presence of scatterers. They have demonstrated that in open Line of Sight (LoS) conditions, it is possible to estimate the target's position and orientation angle, by exploiting the information coming from the multipath, though at a

significant performance penalty. Also, the authors of [5] have demonstrated the benefits of array antennas in identifying the orientation of a device. Finally, due to this high sensitivity of the mmWave technology, positioning accuracy seems to be strongly correlated with the distance away from the target to be positioned. For instance, the authors of [6] have conducted AoA and signal measurements in a $35m$ by $65.5m$ open space and have achieved a position accuracy ranging from $16cm$ to $3.25m$. Positioning research using this mmWave technology is still in the early stages but early theoretical findings and some practical experiments demonstrate its strong potential towards achieving the very high accuracy required by modern smart applications. In another piece of work, the authors in [7] propose a multipath-assisted localization (MAL) model based on the mmWave radar to achieve the localization of indoor electronic devices. The model fully considers the help of the multipath effect when describing the characteristics of the reflected signal and precisely locates the target position by using the MAL area formed by the reflected signal. At the same time, for the situation where the radar in the traditional Single-Input Single-Output (SISO) mode cannot obtain the 3D spatial position information of the target, the advantage of the MAL model is that the 3D information of the target can be obtained after a mining process of the multipath information. Experiments show that the proposed MAL model enables the mmWave multipath positioning model to achieve a 3D positioning error within $15cm$. A virtualized indoor office scenario with only one mmWave base station (BS) is considered in [8]. User equipment (UE) motion feature, mmWave line-of-sight (LoS), and first-order re-flection paths' AoA-ToA are fused for indoor positioning. Firstly, an improved least mean square (LMS) algorithm that combines motion messages is proposed to refine the multipath AoA estimation. Furthermore, a modified multipath unscented Kalman filter (UKF) is proposed to track UE's position in the scenario. The information exchanges of the two stages not only consist of estimates (position, AoA) but also the variance of position. Based on the simulation results, the proposed methods provide 2 times LoS-AoA estimation gains and centimeter 3D positioning accuracy respectively of around $60cm$. Besides, this strategy is capable of positioning tasks with insufficient anchor nodes.

III. METHODOLOGY

A. System Overview

The methodological framework to investigate the research question posed in the introduction is presented in this section, describing the experimental system setup and equipment used while emphasizing on the particular challenges that the available mmWave products impose towards achieving the desired 3D accuracy. The integrated system was used to perform a precision analysis and compare the two predominantly-used mmWave ranging sensors currently in the market and thereafter use the ranging/angular information to conduct positioning using both a 3D multilateration and an improved 3D triangulation approach.

1) Equipment: The two mmWwave radar sensors that were used for the precision analysis were the Texas Instruments (TI) IWR1642BOOST and Infineon Distance2Go. The TI sensor is equipped with 4 receiving (Rx) and 2 transmitting (Tx) antennas operating at frequencies between $76\text{-}81GHz$ with a 120-degree field of view and ranging capabilities of up to 72 meters. In contrast, the Infineon Distance2Go mmWave sensor is equipped with 1 Rx and 1 Tx antenna and operates between $24\text{-}26GHz$ with a field of view of 20 degrees and a maximum detection range of around 20 meters. While the TI sensor performs range and angle measurements, the Infineon one can only measure range. The experimental setup involved utilizing a DJI Air 2S drone as the target for ranging and angular measurements. It is a compact drone with dimensions of $183.0 \times 77.0 \times 253.0mm$.

2) Experimental Setup: Both the precision analysis and the 3D positioning accuracy experimentation were carried out in an $8.85 \times 6.85m$ engineering laboratory the top-view of which is shown in Fig. 1. The precision analysis was conducted to compare the ranging and angular capabilities of the two mmWave sensors. The sensor under test was placed in location **G** and range measurements were collected every $0.5m$ while the drone was flying in a straight line in front of the sensor (0.5 to $8m$). To assess the ability of the sensors to conduct range measurements at different angles, the orientation of the sensor was systematically varied from 0 to 60 degrees (15-degree step). This comprehensive analysis aimed to gather precise data on the sensors' precision, resolution, and reliability at different distances and angles. Also, the precision of the TI sensor in measuring the angle of departure was evaluated using the same setup.

For positioning accuracy experimentation, the positioning system comprises mainly of a number of TI mmWave sensors each of which is connected to a Raspberry Pi 4 that serves as a gateway collecting the data from each TI sensor and sending it to the central PC for processing. Each sensor has its own Raspberry Pi 4 where the data string is sent through a TCP connection and parsed. A number of TI sensors were deployed in the corners of the lab while position estimation was done using two approaches: (1) 3D Multilateration (4 sensors at locations **A, B, C, F**) and (2) an improved 3D-triangulation (5 sensors at locations **A, D, B, C, E**) approach. Eight ground-truth points (**1-8**) were randomly selected across the lab space. Each point was meticulously marked, and their corresponding coordinates were recorded. The drone was positioned precisely on these marked points and subsequently lifted to hover over them at various heights. These heights were also carefully noted down for subsequent analysis. While the drone hovered over each point, the range and angle measurements from each sensor were sent to a central PC that produces the metadata needed to perform 3D positioning calculations using the two approaches mentioned above. This setup allowed for a direct comparison of the accuracy and performance of the two methods for real-time 3D positioning, providing valuable insights into capabilities and suitability for both the methods and the technology for practical applications.

Fig. 1. mmWave 3D Positioning Experimental Setup

B. Challenges

During the development and setup of the positioning system described above, several challenges have emerged regarding the usage of mmWave sensors which could potentially cause significant difficulties when these are used for positioning. This section describes all these challenges and subsequently explains the solutions we implemented to overcome them.

1) Accuracy and Sensing: Although mmWave sensors have been introduced to be used mainly for ranging measurements for the automotive industry, they have emerged as a promising radar-like technology for indoor positioning applications due to their high accuracy in estimating distance (and angles) to objects mainly because of the availability of a very wide bandwidth on mmWave frequencies and the availability of phase antenna arrays on the sensor board. However, the accuracy of mmWave sensors is highly dependent on the sensing conditions, such as the scattering caused due to reflective surfaces, the angle of incidence, and the distance between the sensor and the target object. In addition, the complexity of the indoor environment including multipath effects, can affect the accuracy of mmWave sensing. Therefore, careful consideration of the sensing conditions and the deployment of mmWave sensors is essential to achieve high accuracy in indoor positioning applications. Our experimentation has indicated that the presence of metallic objects in the close vicinity of the target or within the field of view of the sensor causes problems.

2) Stationary Positioning: In addition to the sensing constraints, the fact that these sensors rely strongly on the Doppler-effect principle, challenges emerge when stationary targets need to be detected. To be sensed by a mmWave radar sensor, an object must be constantly in motion for the sensor to be able to detect the Doppler shift and distinguish it from stationary objects and background noise. To overcome this challenge, researchers are currently exploring several approaches. One promising solution could be the fusion of mmWave data with information collected from inertial sensors. For our experiments, this limitation was overcome by making

the drone make small movements around the target location, collecting multiple measurements from a single location which were then averaged.

3) Multi-object Detection/Clustering: An inherent limitation of the off-the-shelf mmWave sensors compared to systems that use receivers on the target is the fact that they operate based on the radar principle reducing the capability of identifying correctly specific objects. The mmWave sensor emits electromagnetic waves at high frequencies that bounce off surrounding objects and return as echoes. By analyzing the time delay and amplitude of these echoes, the sensor can determine the location and characteristics of the objects in the environment relative to each sensor. These echoes, however, can become mixed together in complicated environments with multiple objects, making it difficult to differentiate and identify specific objects. This becomes especially more challenging when using multiple sensors to identify a position of a specific object in the presence of other moving or stationary objects. The solution to this multi-object identification is clustering. Literature reports various clustering approaches that can be used for this purpose [9]–[11].

The clustering technique used in this work to identify a specific target is known as the z-score method [12], which is widely employed for identifying and managing outliers in datasets. This method begins by calculating the mean and standard deviation of the dataset and then computes the z-score for each data point, measuring its deviation from the mean in terms of standard deviations. By establishing a threshold, typically based on a certain number of standard deviations away from the mean, outliers can be identified and subsequently removed from the initial detected objects list to obtain a new filtered list of clustered points.

The ability of the IWR1642 sensor to measure the relative range and azimuth of a detected object facilitates this clustering process as it allows the estimation of the relative (x, y) coordinate of the target. As this target is detected from multiple sensors its relative coordinates need to be converted to absolute ones by utilizing the rotation/translation equations shown below (eq. 1-2) in which θ is the absolute orientation of the sensor and x_{trans}, y_{trans} are the 2D coordinates of each sensor relative to the chosen $0,0$ point (see Fig. 1). Once this is done, the measurements from each sensor correspond to the same axes system, and their (x, y) coordinates can be matched to identify the range/angle measurements from the multiple sensors to the same object.

$$x_{abs} = x\cos\theta + y\sin\theta + x_{trans} \tag{1}$$

$$y_{abs} = -xsin\theta + ycos\theta + y_{trans} \tag{2}$$

4) Timing Synchronization: Timing synchronization is critical in mmWave positioning systems that use multiple sensors to accurately determine the location of objects. When multiple sensors are used, they must be synchronized so that they can collectively capture and analyze the echoes returned from the environment. If the sensors are not synchronized, the echoes may arrive at different times, leading to incorrect

and inconsistent measurements, which can result in inaccurate positioning data. The timing synchronization ensures that the sensors are accurately aligned in time, allowing them to capture the echoes simultaneously and consistently. Therefore, timing synchronization is critical to the performance and accuracy of mmWave positioning systems.

To achieve timing synchronization, a timestamp was placed at the beginning of each data string. The timestamp corresponds to the exact recording time, allowing for accurate alignment with the real-time clock. By matching these timestamps with the current time, the data strings within a specific timeframe were then organized into a list. Once the data string list is established, it is then filtered using the clustering technique mentioned previously and utilized to identify a specific object within the environment.

IV. RESULTS

A. Precision Analysis

To evaluate the accuracy and sensing quality of the IWR1642BOOST and the Infineon sensors a range/angle precision analysis experimentation was carried out using the setup described in section III-A2. A drone was flown along a straight line, while a mmWave sensor was placed at different orientations at location **G** as shown in Fig. 1. Given that the Infineon sensor has a relatively narrow field of view (around 20 degrees), the analysis of distance accuracy in comparison to the TI sensor was conducted up to 15 degrees. The results of this comparison are shown in Fig. 2 and a notable observation is the difference in distance errors between the two sensors. Both at 0 and 15 degrees, the TI sensor outperforms the Infineon sensor. Specifically, the TI sensor demonstrates an average distance error of around $0.17m$, whereas the Infineon sensor exhibits a higher error of $0.32m$. While the error remains relatively consistent as the distance increases for both sensors, the analysis indicates a decrease in accuracy with larger angles. At 15 degrees, there is a slight increase in error, approximately $0.05m$, compared to the error at 0 degrees.
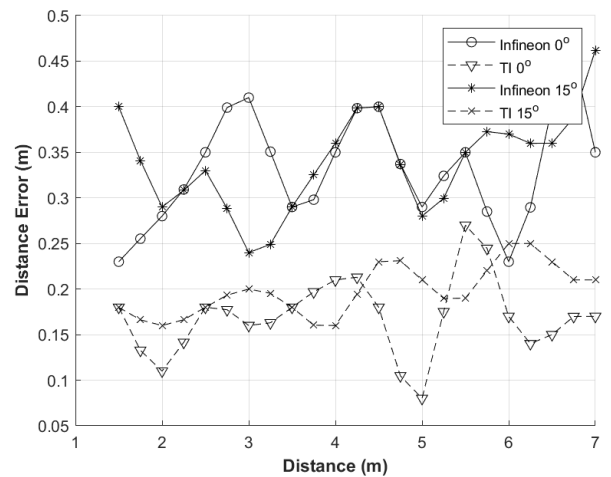
Fig. 2. Infineon Vs TI Distance Accuracy

Following the comparison between the two sensors, the distance and azimuth angle accuracy of the TI sensor were further tested beyond 15 degrees, as depicted in Figures 3 and 4. Figure 3 specifically illustrates the distance error of the TI mmWave sensor across angles ranging from 0 to 60 degrees.

Upon closer inspection, it becomes evident that while the error remains consistent for each analyzed angle, there is a noticeable and constant increase in error. At 0 degrees, the average distance error stands at $0.17m$, gradually rising to approximately $0.32m$ at 60 degrees. It is worth noting that considering the wide field of view spanning 60 degrees, an error of $0.17m$ may not appear excessively large. However, a limitation is encountered as the sensor ceases to detect objects beyond a range of 6 meters.
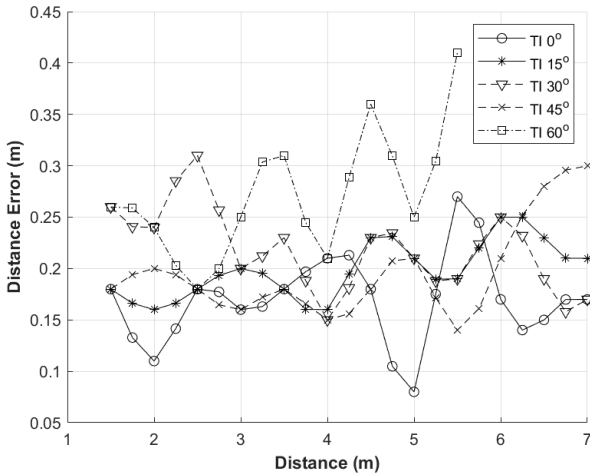
Fig. 3. IWR1642BOOST Distance Accuracy

Following the range-precision analysis, an experiment was conducted to evaluate the azimuth angle precision of the TI sensor. Similar experimental methodology was used, with the object moving away from the sensor while adjusting the sensor angle from 0 to 60 degrees. The results can be seen in Figure 4. During the experiment, the azimuth angle error exhibited variations ranging from 0.5 to 3.5 degrees. Notably, it was observed that the error improved with increasing distance. This improvement can be attributed to the fact that as the object moves farther away, its target size diminishes, making it relatively easier to identify accurately.

B. 3D Positioning

Utilizing the experimental setup described in section III-A2, a set of ranging and angular measurements was collected from TI mmWave sensors while the drone was flown at 8 well-known 3D locations as shown in Figure 1. Using these measurements 3D positioning estimation was conducted both using a multilateration and a 3D-Triangulation approach. Ground-truth location precision is crucial for the validity of this work as it serves as the reference for evaluating the accuracy of the approach. While flying around lab, the drone was instructed to hover at the particular points of interest
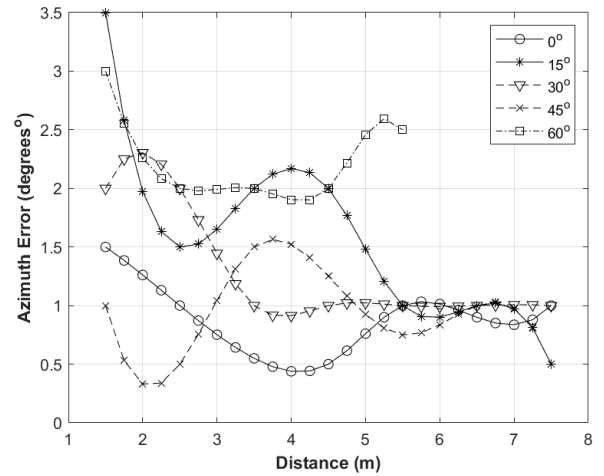

Fig. 4. IWR1642BOOST Azimuth Accuracy

and while hovering the precise location of the drone was determined using a laser distance measuring tool. Basically, the laser tool was secured using a tripod at the location of the drone while the later was hovering and distance measurements were taken to the horizontal and vertical walls of the lab as well as to the floor determining the precise x, y, z location of the drone.

1) 3D Multilateration Approach: Multilateration serves as a fundamental technique for achieving 3D positioning across a wide range of scientific and technological domains. It harnesses distance measurements from multiple reference points to determine the exact location of an object within three-dimensional space by using at least 4 sensors. Through the exploitation of geometric relationships between the object and these reference points, multilateration algorithms facilitate the calculation of intersecting spheres or hyperboloids, ultimately yielding the object's coordinates. In this work, 3D position estimation is done using an algebraic solution of the multilateration problem using ranging measurements collected from 4 TI sensors deployed in the 4 corners of the room in locations **A, B, C** and **F** as shown in Figure 1.

Table I tabulates the error of this multilateration positioning estimation. It appears that an average error in the ranging measurement of $0.19m$ translates into a $0.80m$ 3D positioning error. Another observation is that the algebraic solution fails mostly in the z-axis averaging an error $0.76m$ while the error in the x and y is only $0.13m$ and $0.17m$ respectively.

2) 3D Triangulation Approach: Considering the inaccuracy of the multilateration approach in the z-axis, and capitalizing on the ability of the IWR1642 sensor to measure the azimuth angle, the experimental setup was adjusted, deploying 2 sets of two sensors on top of each other as shown in Figure 5. Sensor **D** is placed on top of **A**, sensor **E** on top of **C**, while sensor **F** was left on its own on the far-most right corner. 3D position estimation is achieved by using a combination of typical triangulation formulation using the azimuth angles

TABLE I
3D MULTILATERATION POSITIONING

Point	Distance Error(m)	XYZ Error(m)			3D Error(m)
		x	y	z	
2	0.20	0.14	-0.20	-0.73	0.77
4	0.15	-0.13	0.19	-0.79	0.82
5	0.16	0.10	0.24	1.17	1.20
9	0.24	-0.08	0.34	0.95	1.01
10	0.21	0.24	0.06	-1.06	1.09
11	0.12	-0.08	0.11	-0.82	0.83
12	0.11	0.04	0.00	0.15	0.15
13	0.29	0.20	0.20	0.42	0.50
Average	*0.19*	*0.13*	*0.17*	*0.76*	*0.80*

measured from the 3 corners while the z-axis coordinate is estimated based on the height formulation (3) below which estimates the height h in the Complexity-Reduced Trilateration Approach (COLA) approach presented in [13].

$$h = z_2 - \frac{d_2^2 - d_1^2 + (z_2 - z_1)^2}{2(z_2 - z_1)} \quad (3)$$

Results tabulated in Table II indicate a significant improvement in the z-axis $(0.11m)$ while there is also a good improvement in the x and y axes (error being $0.09m$ and $0.08m$) bringing the 3D positioning accuracy down to $0.17m$

TABLE II
3D TRIANGULATION POSITIONING

Point	Azimuth Error(m)	XYZ Error(m)			3D Error(m)
		x	y	z	
2	1.02	0.13	0.14	-0.03	0.20
4	0.72	-0.03	0.09	0.16	0.19
5	0.91	-0.08	-0.09	0.14	0.19
9	1.88	-0.04	-0.13	0.16	0.22
10	1.51	0.25	0.11	-0.13	0.31
11	0.91	0.09	-0.08	-0.30	0.32
12	0.39	0.06	0.03	0.04	0.08
13	2.20	0.05	-0.02	-0.03	0.06
Average	*1.20*	*0.09*	*0.08*	*0.11*	*0.17*

V. DISCUSSION

The results of the precision analysis and positioning estimations highlight the potential of mmWave technology for achieving range and angle measurement precision and thereafter high 3D positioning accuracy. The precision analysis revealed that the TI sensor outperformed the Infineon sensor in terms of range and angle measurement precision at a wider field of view. Due to the fact that the Infineon sensor is only able to identify objects up to a 20-degree angle, it becomes evident that this sensor is not appropriate for a positioning system where at least 4 sensors are required to cover the visibility of an entire room. On the other hand, TI sensor has shown very promising results, showcasing ranging precision of $0.17m$ at 0 degrees and a capability of identifying an object at 60 degrees with an accuracy of $0.3m$ up to $6m$. This makes the IWR1642 sensor a good choice for a 3D positioning system.
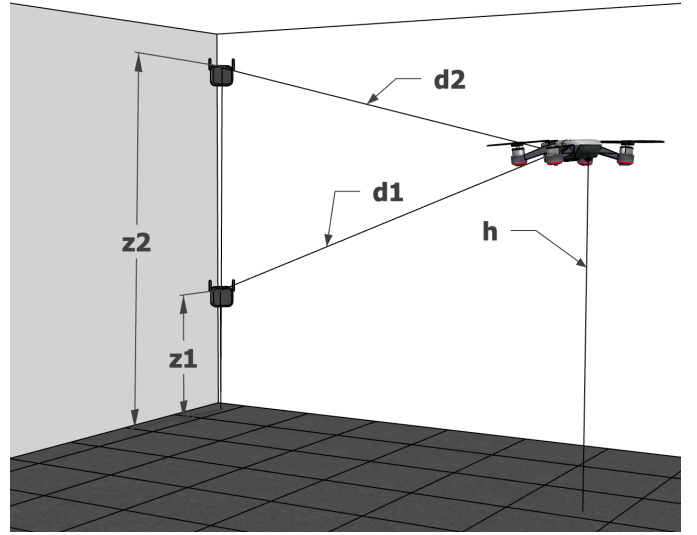

Fig. 5. 3D Triangulation - Sensor Arrangement

The 3D positioning estimation was done using both a 3D multilateration and a triangulation approach. The multilateration approach demonstrated a relatively high 3D positioning error of $0.8m$ in the z-axis estimation. This emphasized the challenges associated with accurately estimating the z-axis using multilateration alone. To address these limitations in z-axis estimation, a 2D triangulation approach utilizing azimuth angles from 3 sensors was used combined with a lateration approach to estimate the height utilizing sensors placed on top of each other. Although only one additional sensor is required to be placed at a higher altitude above one of the existing sensors to be able to estimate the height, we have deployed 2 sets at the two corners of the room to ensure sufficient measurements in case one of these fails to return measurements due to either blockages or long distances. This modification in the sensor setup resulted in a reduction of the z-axis error down to $0.11m$, leading to an overall decrease in the 3D positioning error down to $0.17m$. The errors in the x and y axes also improved, indicating the effectiveness of the triangulation approach in precise 3D positioning estimation.

Dilution of Precision (DOP) plays a crucial role in 3D indoor positioning, as it directly affects the accuracy and reliability of position estimates. While DOP values are commonly considered in the horizontal plane, they are equally important in the vertical plane [14]. Considering this, the reason why our results often exhibit better accuracy in the horizontal plane compared to the vertical plane can be attributed to the distribution of sensors. In the horizontal plane, the sensors are spread out more widely, allowing for better sensor geometry. This improved distribution of sensors results in lower HDOP values, indicating reduced potential for horizontal positioning errors. The IWR1642BOOST mmWave sensor, with its narrow 15-degree elevation field-of-view, poses a limitation on the distribution of sensors in the vertical plane. The narrower vertical perspective leads to a less favorable sensor geometry

and higher VDOP values. As a consequence, the accuracy of height estimation in 3D positioning may be more susceptible to errors and uncertainties.

Comparing the results mentioned in Section II with the findings in [6] and [7], it is evident that our approach yielded a similar level of accuracy. In [6], the authors achieved positioning accuracy ranging from $16cm$ to $3.25m$ using the AoA technique in an open space while the authors of [7] demonstrated an accuracy of $15cm$. Despite the fact that we were operating in a more cluttered environment, we achieved an accuracy of $0.17m$, which is comparable to the aforementioned works. It is noteworthy that our experiment was unique as, to the best of the authors' knowledge, no other similar work utilizing a UAV has been reported in the literature.

VI. CONCLUSION

In this paper, we have demonstrated the potential of mmWave radar sensory technology to be used for accurate cm-level 3D indoor localization. Despite this high accuracy, the technology imposes several challenges, difficulties, and limitations when it comes to setting up and using a multi-sensor positioning system. These challenges include sensing limitations of mmWave sensors, the difficulty of detecting stationary targets, the complexity of multi-object detection, and the need for timing synchronization. These challenges were addressed through careful system design and the implementation of appropriate solutions.

It is also worth noting that while we were conducting this research new sensors were made available by Texas Instruments that can measure elevation in addition to azimuth angles. This implies that only one sensor would be needed to accurately determine the 3D location of a target overcoming a few of the difficulties mentioned above. Also, if this technology is combined with Kalman filtering then the limitations of the multilateration approach could also be overcome. Our future research efforts will focus towards that direction as well as fusing information from mmWave sensors with context received from AI cameras and LIDAR sensors and IMUs.

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