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Article

The Environmental and Economic Importance of Mixed and Boundary Friction [†]

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[†] This paper is an extended version of our paper published in Taylor, R.I.; Sherrington, I. The Environmental and Economic Importance of Mixed and Boundary Lubrication. In Proceedings of the LUBMAT 2023, Preston, UK, 17–19 July 2023.

Abstract: One route to reducing global CO₂ emissions is to improve the energy efficiency of machines. Even small improvements in efficiency can be valuable, especially in cases where an efficiency improvement can be realized over many millions of newly produced machines. For example, conventional passenger car combustion engines are being downsized (and also downspeeded). Increasingly, they are running on lower-viscosity engine lubricants (such as SAE 0W-20 or lower viscosity grades) and often also have stop–start systems fitted (to prevent engine idling when the vehicle is stopped). Some of these changes result in higher levels of mixed and boundary friction, and so accurate estimation of mixed/boundary friction losses is becoming of increased importance, for both estimating friction losses and wear volumes. Traditional approaches to estimating mixed/boundary friction, which employ real area of contact modelling, and assumptions such as the elastic deformation of asperities, are widely used, but recent experimental data suggest that some of these approaches underestimate mixed/boundary friction losses. In this paper, a discussion of the issues involved in reliably estimating mixed/boundary friction losses in machine elements is undertaken, highlighting where the key uncertainties lie. Mixed/boundary lubrication losses in passenger car and heavy-duty internal combustion engines are then estimated and compared with published data, and a detailed description of how friction is related to fuel consumption in these vehicles, on standard fuel economy driving cycles, is given. Knowing the amount of fuel needed to overcome mixed/boundary friction in these vehicles enables reliable estimates to be made of both the financial costs of mixed/boundary lubrication for today’s vehicles and their associated CO₂ emissions, and annual estimates are reported to be approximately USD 290 billion with CO₂ emissions of 480 million tonnes.

Keywords: lubrication; IC engines; machine elements; friction; wear; modelling



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1. Introduction

In a recent, widely cited paper, Holmberg and Erdemir [1] reported that friction and wear are responsible for approximately 23% of global energy consumption, with 20% being associated with friction and 3% being due to manufacture of replacement parts (needed to replace worn or broken components). Reducing the amount of energy associated with friction and wear will help to improve the energy efficiency of machines and contribute to reduced CO₂ emissions. For machine elements that are predominantly lubricated in the fluid–film lubrication regime, one approach used to reduce friction would be to reduce lubricant viscosity. However, for machine elements that are in the mixed/boundary friction regime, an increase in viscosity can be used to reduce friction (by increasing the thickness of the lubricating film). Alternatively, a friction modifier additive can be employed to reduce the friction coefficient in asperity contacts without changing the degree of contact. (More recently, surface micro-patterning has also been used to increase the thickness of lubricating films to reduce contact and friction forces.) Therefore, it is important, in our view, to better

understand the proportion of friction associated with fluid–film lubrication, compared to that for mixed/boundary lubrication. This paper reports such estimates for passenger cars and heavy-duty vehicles and is an expanded version of a conference paper [2] that was originally delivered at the LUBMAT 2023 International Conference which took place in Preston, UK, in July 2023.

In most internal combustion engines, friction losses in engines and transmissions are primarily due to film shear in hydrodynamic lubrication (in journal bearings, piston assembly) or elastohydrodynamic lubrication (in valve train, gears). However, at high loads and/or under stop–start conditions, mixed/boundary lubrication can also occur.

If the minimum oil film thickness separating the surfaces is h_{min} (m), and the root mean square roughness of each surface is σ_1 (m) and σ_2 (m), then the combined root mean square roughness is σ (m), where $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$. The λ ratio can then be defined as $\lambda = h_{min}/\sigma$. (It should be noted that recent interesting work has been published that considers other ways to define a λ ratio [3].) Clearly, at some point, the fluid film will be large enough that the rough surfaces are completely separated, and at this point, and also for thicker oil films, the friction in the contact will be determined by the properties of the fluid lubricant. It is generally assumed within the tribological community and described in tribology textbooks [4] that fluid film lubrication is thought to occur when $\lambda > 3$ (although it should be mentioned that some researchers have seen the impact of rough surfaces for values of λ greater than 3 [5]). Mixed lubrication, where some of the contact load is carried by the liquid lubricant and some of the load is carried by the surface asperities, is generally assumed to take place provided $1 < \lambda < 3$, and boundary lubrication (where all the contact load is carried by the asperities) is assumed to occur if $\lambda < 1$ [4]. In many machine elements, it is possible to calculate the minimum oil film thickness in a contact by solving Reynolds' equation, assuming perfectly flat interface surfaces, and so, if the surface roughness is known, it is conventional to then estimate the λ ratio based on the film thickness estimate and the surface roughness to determine the likely level of surface contact arising in hydrodynamic, mixed, and boundary lubrication.

As an example, consider the piston rings of an internal combustion engine. The piston assembly of an internal combustion engine is often thought to contribute about 50% or so to total engine friction [6], and the piston rings are thought to contribute 50% (or more) to total piston assembly friction [6], so the piston rings themselves (and most engines have three piston rings) could contribute around 25% to total engine friction. Now, if F (N) is the friction force of the piston ring, h_{min} (m) is the minimum oil film thickness under the piston ring, W (N) is the load acting on the piston ring, and η (Pa.s) is the viscosity of the lubricant, then hydrodynamic analysis of piston lubrication [6] leads to

$$h_{min} \propto \sqrt{\frac{\eta U}{W}} \quad (1)$$

$$F \propto \sqrt{\eta U W} \quad (2)$$

Similarly, Taylor [7] has reported that the hydrodynamic power loss of highly loaded short journal bearings is proportional to $\eta^{0.75}$. Therefore, one way to decrease hydrodynamic friction losses in piston rings and journal bearings is to reduce lubricant viscosity. Table 1 shows trends in passenger car lubricant viscosity grades over the last 20–30 years, demonstrating that viscosities recommended by engine manufacturers have indeed tended to decrease over time (and are still getting lower) to reduce engine friction.

Although reducing lubricant viscosity is an effective way to reduce power loss by viscous shear and, therefore, passenger car engine friction (since such engines are predominantly lubricated hydrodynamically), the minimum oil film thickness separating the moving, rough surfaces is also reduced, leading to increased incidence and magnitude of mixed/boundary lubrication and metal–metal or boundary contact under higher loads and/or, stop–start conditions.

Table 1. Typical viscosities of different lubricant viscosity grades, showing the decrease in engine oil viscosity over the last 20–30 years (V_k is kinematic viscosity, and HTHS is the high-temperature high shear viscosity of a lubricant measured at a temperature of 150 °C and a shear rate of 10^6 s^{-1}).

Grade	Typical V_{k40} (cSt)	Typical V_{k100} (cSt)	Typical HTHS Viscosity (mPa·s)	Approx Viscosity (mPa·s) at −15 °C	Approx Year of Use
SAE 20W-50	144.8	17.8	4.1	5900	Before 1990
SAE 15W-40	114.3	14.9	3.5	2900	1990
SAE 10W-30	72.3	10.8	3.2	1900	1995
SAE 5W-30	57.4	9.9	2.9	1100	2000
SAE 0W-20	44.4	8.3	2.6	700	2015
SAE 0W-8	26.4	5.5	1.9	≈250	Future

When mixed/boundary lubrication occurs, a simple approach for calculating the friction coefficient can be to assume that a portion of the total load, W (N), is carried by the rough asperities, W_A (N) and that the remainder of the load, W_F (N), is carried by the fluid pressure. If the respective friction coefficients are written as f_A and f_F and if the total friction force is F_{TOTAL} (N) and the overall friction coefficient is f , then it is possible to write

$$W = W_A + W_F \quad (3)$$

$$F_{TOTAL} = f_A W_A + f_F W_F \quad (4)$$

$$f = \frac{F_{TOTAL}}{W} = f_A \frac{W_A}{W} + f_F \frac{W_F}{W} \quad (5)$$

If X is defined to be W_A/W , then X is a measure of the proportion of mixed/boundary lubrication in a contact. When hydrodynamic conditions hold $X = 0$, and in this case, the rough surfaces are completely separated by a fluid film, so $W_A = 0$. In addition, when there is no fluid film separating the surfaces, $W_F = 0$ and so $X = 1$, as expected. The overall friction coefficient, f , can thus be written as

$$f = f_A X + f_F (1 - X) \quad (6)$$

Such an expression has been previously reported by Olver and Spikes [8].

It should be noted that whilst the simple expression above has been widely used to predict mixed/boundary friction in components such as valve trains [9] and piston rings [10], modifications to the equation would be needed for contacts where the adhesion of asperities occurred.

In calculating mixed/boundary friction, it is of great interest to know how X varies with the λ ratio. The next section explains how different models can be used to estimate X versus λ .

2. Models for Mixed/Boundary Lubrication Friction

Numerous well-known models for mixed/boundary friction employ surface contact models, which assume that contacting asperities deform elastically [11–14]. Although, at first sight, this seems to be a drastic oversimplification, in fact, there is much experimental evidence for this being approximately true for most contacts, once the “running-in” of the component has completed. It is well known that new components “run-in”, and usually, during this process, the highest asperities are truncated by material loss and plastic deformation, and the component, once “run-in”, will usually have a lower root mean square surface roughness and higher surface hardness, compared to the new component [15]. It can also be the case that the overall shape (form) of the component can change during

“running-in” (for example, a curved piston ring can become flatter [16]), and such changes in shape can result in a higher oil film thicknesses at critical positions (for example, at the top dead centre, piston lubrication is primarily due to the “squeeze” effect [17], and at this position, a flatter piston ring will result in a thicker oil film). Any plastic deformation is thought to have occurred during the “running-in” process, which is only a small fraction of a typical machine element’s lifetime, and so, in fact, the assumption of elastic deformation is much more reasonable than appears at first sight. Of course, some components may not “run-in” as described, and in such cases, the components will usually fail quickly. In recent work based on indentation experiments [18], it was found that plastic deformation persisted well after the surface roughness profile had reached a steady state, and so the assumption that asperities mainly deform elastically is still an active area of research.

It was found by a number of researchers that by assuming elastic deformation of rough surfaces and by assuming a statistical distribution of asperity properties and heights, the real contact area was proportional to the applied load [11–14], as found experimentally. As an example of such a model, the Greenwood–Williamson model describes a rough model surface (consisting of hemispherical asperities with specific peak height distributions) in static contact against a flat surface. If the separation of the flat surface from the centre line average of the rough surface is d (m) (whose rms surface roughness is σ (m)), and if the asperity height probability distribution is assumed to be exponential, then the load, $W(d)$ (N), supported by the asperities is reported [12] to be

$$W(d) \propto \exp\left(-\frac{d}{\sigma}\right) \quad (7)$$

On the other hand, when the rough surface was assumed to have a Gaussian distribution of asperity heights, it was found that $W(d)$ was given by [12] as

$$W(d) \propto F_{3/2}\left(\frac{d}{\sigma}\right) \quad (8)$$

where

$$F_n(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty (s-u)^n \cdot \exp\left(-s^2/2\right) ds \quad (9)$$

In a later model, due to Greenwood and Tripp [13], where it was assumed both contacting surfaces were rough, it was found that

$$W(d) \propto F_{5/2}\left(\frac{d}{\sigma}\right) \quad (10)$$

The above equations can be recast in terms of the proportion of mixed/boundary lubrication, X , by dividing $W(d)$ by $W(0)$, and so the above equations become (where we have also written $d/\sigma = \lambda$)

$$X = \exp(-\lambda) \quad (11)$$

$$X = \frac{F_{3/2}(\lambda)}{F_{3/2}(0)} \quad (12)$$

$$X = \frac{F_{5/2}(\lambda)}{F_{5/2}(0)} \quad (13)$$

A later model, due to Bush et al. [14], which assumed paraboloid asperity geometry, gave a different equation for X as shown below:

$$X = \operatorname{erfc}\left(\frac{\lambda}{\sqrt{2}}\right) \quad (14)$$

Rough surface contact models that assume elastic deformation of asperities strictly only apply when the real area of contact is small (less than a few %). However, rough

surface contact models are also available at the opposite limit, where real contact areas are high [19]. These models were originally motivated by the study of rubber surfaces. It is of interest to note that elastic deformation models tend to predict that the pressure in the contact varies with separation of the surfaces, d , according to

$$P_{elastic} \propto \exp\left(-\frac{d^2}{\sigma^2}\right) \quad (15)$$

On the other hand, rough surface contact models that assume plastic deformation [19] tend to find that the pressure varies according to

$$P_{plastic} \propto \exp\left(-\frac{d}{\sigma}\right) \quad (16)$$

According to Persson [19], the variation of contact pressure for the plastic model is generally in better agreement with experimental data.

In practice, however, the Greenwood and Tripp model is still widely used today for predicting the load carried by asperities in rough surface contacts. A comparison of a selection of the above models is shown in Figure 1. When $\lambda = 1$, the value of X is predicted to be approximately 0.368 for the exponential Greenwood and Williamson model [12], about 0.317 for the Bush et al. model [14], and only about 0.131 for the Greenwood and Tripp model [13]. It is clearly of interest, when predicting mixed/boundary friction, to clarify which of these different values of X is more reliable.

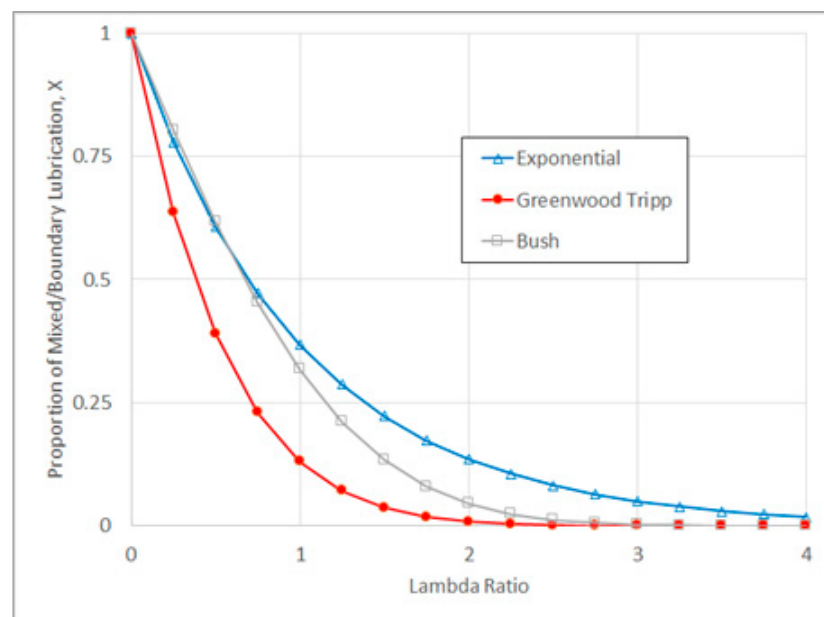


Figure 1. Graph showing the predicted variation of X (the proportion of mixed/boundary lubrication in a contact) versus the λ ratio for three commonly used rough surface contact models.

Another approach to estimating how X varies with λ is to fit a suitable curve to accurate mixed/boundary friction data. An early attempt using this approach was reported by Olver and Spikes [8], and the following equation was proposed:

$$X = \frac{1}{(1 + \lambda)^2} \quad (17)$$

Recent good-quality experimental data on mixed/boundary friction have become available that show how X varies with λ [20]. Full details of the experiment performed can be found in [20]. By combining data from [20] with similar Mini Traction Machine data from

other sources [7,21], it was found that the data, on a wide range of lubricants (which did not contain friction modifiers), could be fitted reasonably well with the following equation:

$$X = \frac{1}{(1 + \lambda^k)^a} \quad (18)$$

where the values of k and a which give the best fit to the data are $k \approx 3/2$ and $a \approx 4/3$. It should be noted that in the hydrodynamic limit (high λ), both the above equation and that by Olver and Spikes [8] predict that $X \propto \lambda^{-2}$, and both types of equation were based on experimental data rather than from a theoretical model.

The above equation predicts that $X \approx 0.397$ when $\lambda = 1$, which suggests that the widely used Greenwood and Tripp model [13] substantially underestimates the amount of mixed and boundary friction in the range $1 < \lambda < 3$. The finding that the Greenwood and Tripp [13] model underestimates mixed/boundary friction was also recently reported by Leighton et al. [22]. This underestimate of mixed/boundary friction arises at least in part because the Greenwood and Tripp model [13] does not incorporate the stress due to the lateral forces that arise in sliding due to friction. It has been demonstrated that lateral forces which oppose friction serve to increase the real contact area in a process often referred to as “junction growth”. This lateral force tends to increase both the contact spot size and the number of contact spots increasing the contact area to be sheared [23]. Failing to include this effect in a friction model, therefore, results in an underestimate of the predicted friction force.

In terms of modelling mixed/boundary friction, there are three main sources of uncertainty:

- Different mixed/boundary friction models predict different values for the way in which X varies with λ .
- Details of the surface roughness and contact shape after “running-in” are not always available, and so often, the surface roughness and shape of the “new” contact are used in mixed/boundary friction calculations. This will likely lead to an overestimate of the amount of mixed/boundary friction in a lubricated contact. It has been estimated that a “new” piston ring pack has approximately 10–15% higher friction than that compared to when it has fully “run-in” [16]. It has also been pointed out [20] that the presence of thick anti-wear films deposited on a rough surface can alter the surface roughness characteristics, and these details are not usually known.
- The friction coefficient f_A is needed. This will vary depending on the materials used and the operating conditions and often needs to be determined experimentally (or estimated).

It could be argued that application of the Greenwood and Tripp model [13] to “new” components could actually give a reasonable result for mixed/boundary friction, since the overestimate associated with the use of “new” component surface roughnesses could potentially partly cancel out the underestimate associated with the Greenwood and Tripp model [13], although from a scientific point of view, it would be preferable to use a better mixed/boundary friction model and apply it to “run-in” components.

Significantly more details of the analysis method and its application for additivated and non-additivated lubricants, as well as a discussion of the scope of application of the techniques, can be found in reference [24].

3. Mixed/Boundary Friction Estimates for Passenger Car Internal Combustion Engines

In internal combustion engines, there are a range of components which operate in different lubrication regimes. For example, journal bearings are designed to operate primarily in the hydrodynamic lubrication regime, so in normal operation, the moving rough surfaces in the bearing would be expected to be fully separated by a fluid film. However, even in journal bearings, mixed/boundary lubrication could potentially occur at high loads (especially if the lubricant temperature is high), under stop–start conditions

(where speeds are low), and/or if there is fuel dilution [25] of the lubricant (which can result in lower lubricant viscosities). On the other hand, the valve train operates almost entirely in the mixed/boundary lubrication regime, at least when the engine is fully warmed up, since the oil film thickness separating the moving parts is very low (usually less than 0.2 μm) and the λ ratio is usually much less than 3. Under these circumstances, the friction loss of valve trains is often estimated by assuming the valve train is in the mixed lubrication regime, and a suitable friction coefficient is assumed [9]. Between these extremes lies the piston assembly, which is lubricated hydrodynamically for most of its operation, but it is also known that mixed/boundary friction occurs near the top and bottom dead centre positions (positions where the piston speed is zero). Most of the mixed/boundary friction in the piston assembly will occur near to the top dead centre position when combustion occurs, since at this point in the engine cycle, the piston speed is zero, the ring-to-cylinder contact loads are high (due to high combustion chamber pressures), and the oil temperatures are also high (leading to low viscosity). This will also lead to higher levels of piston ring and cylinder wear.

There has been much published research on the measurement of engine friction [26–33], and although there are differences between different engine designs, there is a broad consensus that, for fully warmed-up engines operating at medium speeds (1500–2500 rpm), the piston assembly makes up around 40–50% of total engine friction, the valve train makes up around 10–20%, and the journal bearings account for about 20–30%. However, at low speeds (many engines idle at around 750 rpm), the valve train contribution to total engine friction has been measured to be as high as 40% for some engines [30,34].

Recently published experimental data on piston assembly friction [34] for a fully warmed-up engine, which separates out the mixed/boundary friction and fluid–film friction from measured data, found that the amount of mixed/boundary friction drops off rapidly as engine speeds increase. This is as expected since oil film thickness, and the λ ratio, both increase with engine speed. An example of such data is plotted in Figure 2, with the engine in the study being a 1990 2.0 L gasoline engine. Clearly, the overall amount of mixed/boundary lubrication will depend greatly on the driving cycle. “City type” driving, with many stop–starts and low speeds, will tend to have more mixed/boundary lubrication compared to motorway driving (where speeds are steady and relatively high). However, the impact of mixed/boundary friction on vehicle fuel consumption is less than may be expected since most mixed/boundary lubrication occurs at “low” speeds. (For individual contacting components, this means that power loss, the product of friction force and sliding speed, is low due to the low speeds.) Consequently, under these conditions, fuel consumption is relatively low.

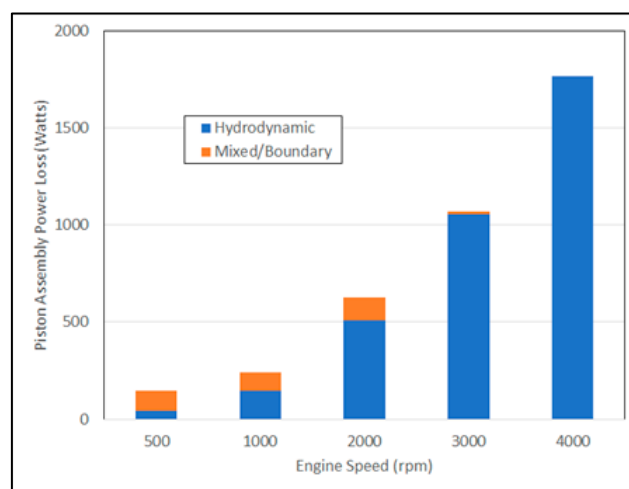


Figure 2. Experimental data (adapted from [34]) for the measured piston assembly power loss (Watts) versus engine speed for a low viscosity SAE 0W-8 lubricant in a 1990s 2.0 L gasoline engine.

Figure 3 shows the total FMEP (friction mean effective pressure, in bars) versus engine speed for the data from [34] and also shows the relative split between fluid–film and mixed/boundary friction (these data are for an SAE 0W-8 lubricant, which does not contain a friction modifier, and for which the sump oil temperature was 93 °C). The data shown in Figure 3 indicate that mixed/boundary friction losses exceed those due to fluid–film friction at speeds lower than about 1250 rpm. The Friction Mean Effective Pressure (FMEP) is a commonly used measure of engine friction and is, essentially, the frictional power loss of the engine (in Watts) divided by the engine speed (in revs per second) and engine displacement (litres). The use of FMEP is a useful way of comparing the friction losses of different-sized engines under different operating conditions.

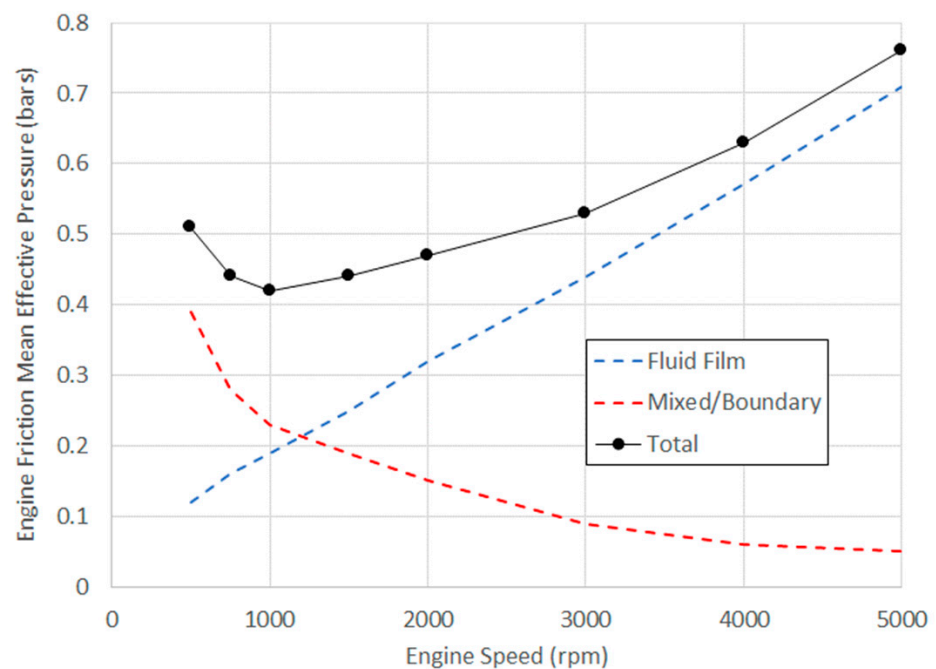


Figure 3. Experimental motored friction data (adapted from [34]) for Friction Mean Effective Pressure (FMEP) (bars) of a fully warmed up 2.0 L gasoline engine (sump oil temperature was 93 °C), lubricated with an SAE 0W-8 lubricant.

It is also worth noting from Figure 3 that the minimum friction in this instance occurs at a speed of just over 1000 rpm, and at the minimum friction position, there is still a substantial amount of mixed/boundary friction. It is clearly of great interest to estimate how much fuel is used to overcome mixed/boundary friction since knowing the split between hydrodynamic friction losses and mixed/boundary losses is essential when deciding how to reduce friction further. If friction is dominated by fluid–film friction, then the approach would be to simply reduce fluid viscosity. However, if friction due to mixed/boundary lubrication is already a significant proportion of friction leading to power loss, then the use of a lower viscosity fluid may not be a good idea (as it would cause further friction losses), and friction modifier additives and/or alternative materials would need to be considered. The amount of fuel used to overcome friction (and other losses) will depend on the driving cycle, and standard driving cycles are used to determine the fuel economy and emissions of modern passenger cars [35]. Two typical driving cycles that have been used in Europe are shown in Figure 4.

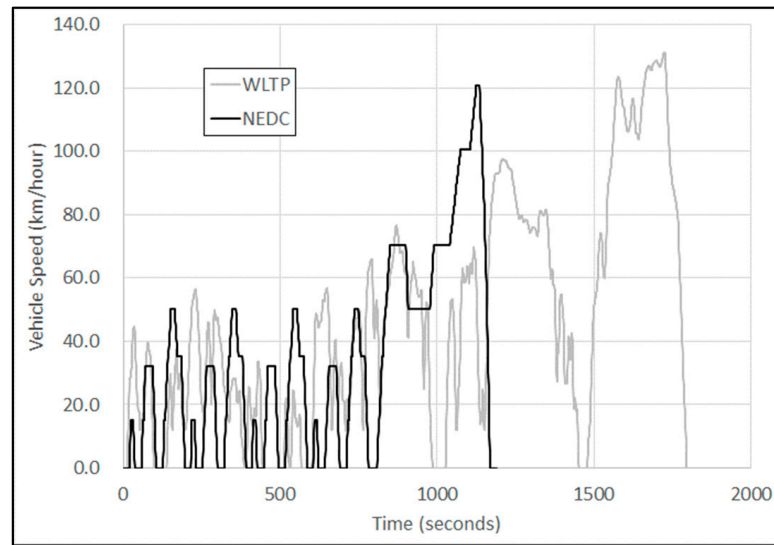


Figure 4. The older NEDC driving cycle used for vehicle fuel economy tests in Europe up until 2020. The newer driving cycle currently used in Europe (WLTP) is also shown (NEDC stands for “New European Driving Cycle”, and WLTP is the “Worldwide Harmonized Light Vehicle Test Procedure”).

In conventional gasoline fueled vehicles, the amount of fuel used can be predicted using a basic power balance approach (power in from the fuel equals the power needed to drive and accelerate the car and overcome all the losses due to aerodynamics, tyre/road friction, and engine/transmission friction) [36]. In such an approach, the basic equation used, for a vehicle travelling on a level road, is

$$\eta P_{in} = \frac{P_{wheels}}{\varepsilon} + P_{engine} + P_{auxiliaries} \quad (19)$$

where P_{in} is the power from the fuel (Watts), η is the engine’s thermal efficiency (approximately 40%), P_{wheels} is the power (Watts) required at the wheels (to drive and accelerate the car and to overcome aerodynamic and tyre/road losses), ε is the transmission efficiency (typically around 90% when the vehicle is fully warmed up), P_{engine} is the power loss due to engine friction (Watts) and $P_{auxiliaries}$ is the power loss due to vehicle auxiliaries (lighting, heating, air conditioning, etc.). The power required at the wheels is:

$$P_{wheels} = Mav + \frac{1}{2}C_D A \rho v^3 + C_{RR} M g v \quad (20)$$

where M is the mass of the vehicle and driver (kg), a is vehicle acceleration (m/s^2), v is vehicle speed (m/s), C_D is the drag coefficient (dimensionless), A is the frontal area of the vehicle (m^2), ρ is air density (kg/m^3), C_{RR} is the rolling resistance (dimensionless), and g is the acceleration due to gravity (roughly 9.81 m/s^2).

In a typical driving cycle, all the auxiliaries will be switched off, so $P_{auxiliaries}$ can be taken to be zero. When the vehicle is stationary, and the engine idling, $v = 0$, so all of the power available from the fuel is used to overcome engine friction, and P_{engine}/P_{in} is simply equal to η , which is approximately 40%.

Figure 5 shows the NEDC driving cycle again, with both vehicle speed and engine rpm plotted against time. In this particular driving cycle, the vehicle is stationary, with the engine idling at 700 rpm, for roughly 30% of the total cycle time. The total time for the driving cycle is 1190 s, so engine idling occurs for 357 s (almost 6 min).

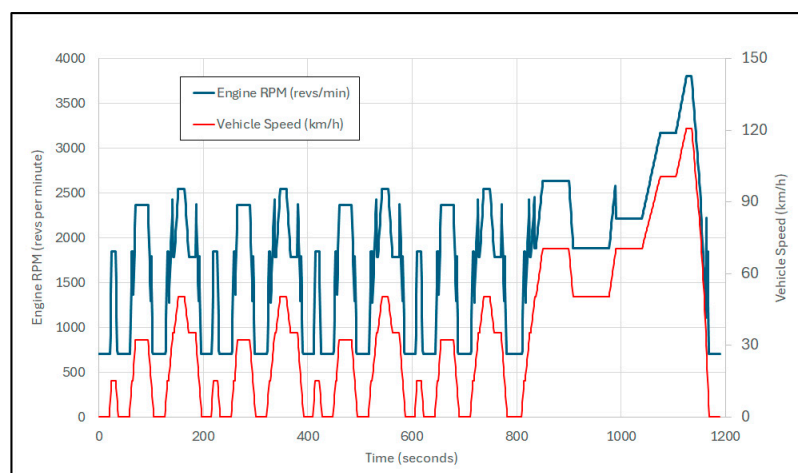


Figure 5. Vehicle speed (km/h) and engine speed (rpm) versus time for the NEDC driving cycle.

Both vehicle speed (km/h) and engine rpm (revs/minute) are available on a second-by-second basis for this driving cycle and so P_{in} can be calculated at every second of the driving cycle. In this calculation, the data of Figure 3 were used to calculate P_{engine} versus rpm. In addition, the data in Figure 3 were split into the amount of engine friction due to fluid film lubrication and the amount due to mixed/boundary friction, so the effect of these separate contributions to overall fuel consumption could be separated out.

A simulation of the fuel consumption of a typical European vehicle driving on the NEDC driving cycle was performed, assuming that $M = 1400$ kg, $C_D = 0.3$, $A = 2.0$ m², $\rho = 1.225$ kg/m³, $C_{RR} = 0.01$, and $\varepsilon = 0.9$. The speed, v (m/s), and engine rpm (revs/minute) were available from the NEDC driving cycle on a second-by-second basis, and the acceleration, a (m/s²), could be calculated from the speed variation during the cycle. For simplicity, the vehicle was assumed to be fully warmed up for the whole driving cycle. To calculate the fuel flow rate (in g/s), the power from the fuel, P_{in} , was divided by the Lower Heating Value (LHV) of the fuel, which has units of MJ/kg. For gasoline, it was assumed that the LHV was 44 MJ/kg. It is also worth noting that 1 kWh is equivalent to 3.6 MJ. From the simulation and the various conversion factors, it was found that the estimated fuel consumption for the whole NEDC driving cycle was 401 g. Given that the NEDC driving cycle is 10.932 km in length, this is equivalent to approximately 4.94 Ls per 100 km (where the density of gasoline was taken to be 0.7429 g/cm³), which is a reasonable estimate considering that most modern gasoline cars are claimed by their manufacturers to have fuel consumption values in the range of 5 to 7 L/100 km. For the simulation performed, of the 401 g of fuel estimated to be consumed during the cycle, approximately 97 g of fuel was consumed to overcome engine friction, of which 30 g was consumed due to mixed/boundary friction. Therefore, for this simulation, fuel consumption due to engine friction is approximately 24% of total fuel consumption, and mixed/boundary friction accounted for 7.5% of total fuel consumption (and in addition, mixed/boundary friction accounted for, on average, in this driving cycle, for this engine, 31% of total engine friction).

It should be mentioned that on the NEDC driving cycle, there is a large amount of engine idling (idling occurs for more than 25% of the cycle time). Modern vehicles that are equipped with stop–start systems would switch the engine off when the vehicle is stationary, as opposed to keeping the engine idling. If a stop–start system had been fitted to this particular engine, the amount of fuel used to overcome mixed/boundary friction would be lowered to around 23 g, which would represent 5.7% of total fuel consumption. In addition, there is evidence that the engine considered here (a 1990s 2.0 L Mercedes Benz M111 gasoline engine) had more mixed/boundary friction than typical engines, since it had heavy valves with stiff springs and was thought to have particularly starved top piston rings. More modern engines with lighter valves, softer springs, and different piston assembly designs are likely to have less mixed/boundary friction than the engine studied

here. It is also worth noting that the simulation above assumed a fully warmed-up engine for the whole driving cycle. In practice, vehicles are run from cold on such cycles, with initial oil sump temperatures in the range of 20–25 °C, and the vehicle warms up as the cycle progresses. To simulate this situation, the transmission efficiency would need to be changed over time, with lower values initially, and in addition, engine friction would need to be increased at lower sump temperatures, with the proportion of mixed/boundary friction also decreasing at lower sump temperatures. Data are not readily available on how these various parameters vary with temperature. It would be expected that the total fuel consumption would increase if a full cold-start simulation were to have been performed and also that the proportion of mixed/boundary friction would be less than predicted for a fully warmed-up fuel consumption simulation.

For these various reasons, for the purposes of estimating the cost and CO₂ emissions of mixed/boundary, we propose using a figure of 5% for the amount of fuel needed to overcome mixed/boundary friction in passenger car engines. Such a figure is also consistent with a “back of the envelope calculation”, where it is assumed that the fuel needed to overcome friction in a passenger car engine is in the range of 20–25% and that the “average” amount of mixed/boundary friction in a fully warmed up passenger car engine is in the range of 20–30%. This simple approach would lead to an estimate of the amount of fuel needed to overcome mixed/boundary friction to be between 4% and 7.5%

4. Mixed/Boundary Friction Estimates for Heavy-Duty Vehicles

Perhaps somewhat surprisingly, large diesel engines used in heavy-duty vehicles appear to be significantly more hydrodynamic than small gasoline engines. Motored engine friction tests [37] on large diesel engines have not found any evidence of an upturn in friction at low speeds (in contrast to typical motored friction tests on passenger cars), and it is also worth noting that friction modifier additives are not usually used in heavy-duty diesel engine lubricants. A useful explanation for this difference is to consider a small 1 L gasoline engine, which may have up to 50% mixed/boundary friction at idle, and then scale it up to become a 10 L engine. This “scaling up” makes the hydrodynamic components of the engine (the journal bearings and piston assembly) much more important compared to the valve train. In addition, if the base circle radius of the cams increases, the valve trains become easier to lubricate. These changes will generally result in a substantial lowering of the overall proportion of mixed/boundary friction in the larger 10 L engine, compared to the smaller 1 L engine. In addition, the larger size of the 10 L engine means that rolling elements can be used in the valve train design (whereas there is not enough space in smaller engines for rolling element valve trains, and sliding valve train designs are usually used). It has been reported that the use of rolling elements in valve trains can result in 50% lower friction compared to an equivalent sliding valve train [38].

In addition, because of the greater weight of highly loaded heavy-duty vehicles (up to 40 tonnes for a fully loaded truck, compared to only 1–2 tonnes for a typical passenger car), their typical operation at higher speeds, and the higher aerodynamic losses, the proportion of fuel used to overcome friction in a heavy-duty truck is usually in the range of 4–15% [39], compared to 20–25% for passenger car engines.

If it is assumed that, on average, engine friction in a large diesel engine is responsible for around 10% of overall truck fuel consumption, and that, on average, mixed/boundary friction (mainly coming from the valve train) comprises 10% of overall engine friction, then this would suggest that only 1% of the fuel used by a heavy-duty truck would be used to overcome mixed/boundary friction.

On the other hand, however, the fuel consumption of a heavy-duty truck is considerably larger than that of a typical passenger car, and the annual mileages are also significantly greater. In Europe, a typical heavily loaded truck would have a fuel consumption of 30 L/100 km (compared to usual values of 6 or 7 L/100 km for modern passenger cars) and an annual mileage of around 100,000 km. Therefore, a typical heavy-duty truck’s

annual fuel consumption is approximately 30,000 L, whereas a typical passenger car may only use 1000 L or so per year.

5. The Financial and Environmental impact of Mixed/Boundary Friction

In the UK, at the time of writing, a realistic fuel-consumption figure for an average modern gasoline engine vehicle is approximately 7 L/100 km. If it is assumed that an average UK car is driven 16,000 km per year, then the amount of fuel used per car per year is roughly 1100 L. Currently, in the UK, there are approximately 19 million gasoline-engine vehicles. Therefore, the total annual fuel consumption for gasoline-fueled passenger cars will be almost 21 billion L. In Section 3, it was found that 5% of fuel is used to overcome mixed/boundary friction, so for gasoline-fueled passenger cars, this will equate to just over 1 billion L of fuel, with a price of GBP 1.5 billion (assuming the current UK fuel price of GBP 1.50 per litre).

In addition, in terms of CO₂ emissions, it has been estimated that 2.4 kg of CO₂ is generated from the combustion of one litre of gasoline, and in addition, approximately 0.7 kg of CO₂ is generated during the manufacturing of gasoline and transporting it to where it is needed [40]. Therefore, for each litre of gasoline used, approximately 3.1 kg of CO₂ is emitted. Since the annual average fuel consumption is 1100 L, the amount of fuel burnt to overcome mixed/boundary friction will be 55 L, and so each car will generate, on average, 170 kg of CO₂ per year, to overcome mixed/boundary friction. By scaling this up to all gasoline-fueled passenger cars in the UK, this will result in approximately 3.2 million tonnes of CO₂ released each year by the UK's gasoline-fueled passenger car fleet to overcome mixed/boundary friction.

Worldwide, there are about 1.4 billion passenger cars, of which the vast majority will be gasoline-fueled vehicles (in Europe, there are a significant number of diesel fueled passenger cars, but the market share of these vehicles has been declining since around 2015, and at most, there are only probably around 50 million diesel passenger car vehicles in Europe). In addition, there will be electric and electric hybrid vehicles on the road as well. To account for these alternative-fueled vehicles, it will be assumed that there are 1.2 billion gasoline-fueled vehicles worldwide. If the above figures for costs and CO₂ emissions estimated for the UK are typical, then the total amount of fuel used annually worldwide to overcome mixed/boundary friction in gasoline-fueled passenger cars will be approximately 63 billion L, at a cost of USD 120 billion, and in addition, there will be CO₂ emissions of around 200 million tonnes per year associated with mixed/boundary friction in gasoline-fueled passenger cars.

For heavy-duty diesel trucks, it was concluded in Section 5 that approximately 1% of total fuel consumption is used to overcome mixed/boundary friction in the diesel engine. Assuming an annual fuel consumption for the truck of 30,000 L, this will result in 300 L of fuel per truck being used to overcome mixed/boundary friction at an approximate cost of GBP 450 per truck, with annual CO₂ emissions of approximately 930 kg per truck. Worldwide, there are approximately 300 million commercial vehicles in use. Therefore, the total amount of fuel used to overcome mixed/boundary in heavy-duty trucks is approximately 90 billion L, at a cost of about USD 170 billion, with associated CO₂ emissions of 280 million tonnes per year.

Despite the lower proportion of mixed/boundary friction in heavy-duty trucks compared to passenger cars (1% versus 5%), the impact of mixed/boundary friction on costs and CO₂ emissions are greater for heavy-duty trucks, primarily due to the much larger amount of fuel used in these vehicles (approximately 30 times more fuel is used annually in these vehicles compared to passenger car fuel usage).

In summary, the total amount of fuel needed to overcome mixed/boundary friction in gasoline fueled passenger cars and heavy-duty trucks has been estimated to be 150 billion L, at a cost of USD 290 billion, with associated CO₂ emissions of 480 million tonnes.

It would be instructive to extend the above analysis to other sectors (ships, trains, buses, etc.) and to other machine elements (such as industrial and automotive gears, hydraulic

pumps, rolling element bearings, etc.) to estimate the importance of mixed/boundary friction in these sectors/machine elements to obtain better estimates of the overall importance of mixed/boundary friction across all applications.

6. Discussion

Section 4 has highlighted the large financial and environmental impacts of mixed/boundary friction in gasoline-fueled passenger cars and heavy-duty trucks. Worldwide, many billions of litres of fuel are consumed annually simply to overcome mixed/boundary friction in these vehicles at a cost of hundreds of billions of dollars. In addition, many hundreds of millions of tonnes of CO₂ are released.

As lubricant viscosities decrease, to reduce the impact of viscous shear and improve machine energy efficiency, it is likely that mixed/boundary friction will become more important, so measures aimed at reducing mixed/boundary friction will become of greater and greater interest. Examples of some of the measures that have been, or are being, taken to reduce the impact of mixed/boundary friction include:

- The use of friction modifier additives in passenger car lubricants (mainly aimed at reducing friction in valve trains and near the top dead centre of piston ring travel);
- The use of start–stop systems to minimize the amount of time engines idle;
- The use of coatings to reduce friction and wear (examples include ceramic coatings and diamond-like carbon (DLC) coatings);
- Superfinishing of components to reduce surface roughness;
- Texturing of surfaces to increase the thickness of fluid film-lubricated contacts.

From a quantitative perspective, when predictions of oil film thickness at the component level for vehicle drivetrain are available from simulations or empirical models, it will be possible to predict boundary and mixed friction conveniently and accurately using the lambda ratio model (Equation (18)) described in Section 2 of this paper. Such investigations will lead to helpful insights into how modifications to component design influence the efficiency of a specific vehicle. For example, quantifying how component surface roughness, as well as other design elements, will affect overall friction and vehicle power loss. Using these data, a fleet analysis, as presented in this paper, can then be applied to predict the impact on fuel requirements and CO₂ emissions for given vehicles.

7. Conclusions

One route to reducing CO₂ emissions and energy usage is to improve the energy efficiency of machines, such as internal combustion engines. Lubricant viscosities have been decreasing steadily since the 1990s in order to reduce engine friction, and this approach has been effective in substantially reducing hydrodynamic friction. However, this approach also leads to a decreased lubricant film thickness separating the rough-moving surfaces and will increase the levels of mixed/boundary lubrication where there is interface contact).

It is thus becoming increasingly important to be able to accurately predict the amount of mixed/boundary friction losses. The accurate prediction of mixed/boundary friction requires a knowledge of how the proportion of mixed/boundary friction (X) in a contact varies with λ . There are a wide range of mixed/boundary friction models that can be used, some of which assume elastic deformation of the asperities and some which assume plastic deformation. The choice of model impacts how X varies with λ . Recent work [25] has used experimental data [7,20,21] to derive an equation for X versus λ that is relatively easy to apply in modelling. This experimental work [20], along with other research [22], indicates that the Greenwood and Tripp model, which is still a widely used model for predicting mixed/boundary friction losses, can significantly underestimate the amount of mixed/boundary friction under certain conditions. There will also be uncertainties in such modelling as to the surface roughness to use (as this can change as “running-in” progresses) and how the asperity friction coefficient changes with operating conditions (speeds, loads, and temperatures).

Studies reported here have found that approximately 5% of the total fuel consumption of a gasoline-fueled passenger car is used to overcome mixed/boundary friction in the engine. In contrast, for heavy-duty trucks, only about 1% of total fuel consumption is used to overcome mixed/boundary friction in these large diesel engines. Using these figures, and extrapolating to the worldwide vehicle population, the total amount of fuel needed to overcome mixed/boundary friction in these vehicles is roughly 150 billion L per year, with an estimated cost of USD 290 billion and associated annual CO₂ emissions of 480 million tonnes.

Internal combustion engines will remain in use across many applications for decades to come (especially for transport and power generation). Even small improvements in efficiency can be valuable in cases where an efficiency improvement is realized over many millions of newly produced machines. It is imperative to reduce mixed/boundary friction in newly produced engines (and other machines), either by reducing friction coefficient directly (using friction-modified lubricants and/or low-friction coatings) or by reducing the amount of time the machine spends in mixed/boundary lubrication (such as the use of start–stop systems in cars to reduce the amount of engine idling).

The use of the boundary/mixed friction (λ) model (Equation (18)) in conjunction with oil film thickness data will allow researchers to make accurate predictions of vehicle-specific predictions of power loss due to contacts operating in different friction regimes. This will permit relatively straightforward investigations into how modifications to specific component designs influence the efficiency of a specific vehicle, particularly where it may involve significant boundary/mixed friction. Using these data, a fleet analysis, similar to that presented in this paper, can then be used to predict the impact on fuel requirements and CO₂ emissions for given vehicles. The authors believe that an approach of this type is critical to understanding future operational and climate impacts of vehicle powertrain designs.

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