MODELLING THE STAR FORMATION HISTORIES OF NEARBY ELLIPTICAL GALAXIES

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ABSTRACT

Since Lick indices were introduced in 1994, they have been used as a source of observational data against which computer models of galaxy evolution have been compared.

However, as this thesis demonstrates, observed Lick indices lead to mathematical ill-conditioning: small variations in observations can lead to very large differences in population synthesis models attempting to recreate the observed values. As such, limited reliance should be placed on any results currently or historically in the literature purporting to give the star formation history of a galaxy, or group of galaxies, where this is deduced from Lick observations taken from a single instrument, without separate verification from at least one other source.

Within these limitations, this thesis also constrains the star formation histories of 21 nearby elliptical galaxies, finding that they formed $13.26^{+0.09}_{-0.06}$ Gyrs ago, that all mergers are dry, and that galactic winds are formed from AGN activity (rather than being supernovae-driven). This thesis also finds evidence to support the established galaxy-formation theory of “downsizing”.

An existing galactic model from the literature is examined and evaluated, and the reasons for it being unable to establish star formation histories of individual galaxies are ascertained. A brand-new model is designed, developed, tested and used with two separate data sets, corroborated for 10 galaxies by data from a third source, and compared to results from a Single Stellar Population model from the literature, to model the star formation histories of nearby elliptical galaxies.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

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CHAPTER 1: MODELLING THE EVOLUTION OF ELLIPTICAL GALAXIES: FROM START TO STATE-OF-THE-ART

1.1 HOW DO ELLIPTICAL GALAXIES FORM?

1.1.1 Introduction

Establishing the formation mechanisms and evolutionary history of galaxies is an important aim of current astrophysics. Whilst data from high redshifts give a further look-back time and shows galaxies in the earlier stages of formation, the quality of the data is often poor, with low signal-to-noise ratios, making it difficult to conclusively determine galactic evolution directly from images of young galaxies from different passbands (e.g. Conselice et al. 2004, Reddy et al. 2008).

It is possible to draw conclusions about likely evolutionary processes based on models that successfully reproduce currently available data. Comparing models and observational data can enable the parameters defining galactic evolution to be constrained, and competing hypotheses can then be evaluated.

1.1.2 Observed galactic phenomena as constraints on galaxy evolution

Observations at a variety of wavelengths indicate astrophysical processes such as supernovae, new star formation and galaxy merging, which can be assumed to apply (for the purpose of modelling) universally in both space and time. Observations of distant objects show the Universe at earlier times and show, for example, the early Universe (at high redshifts), as with the later Universe, to be composed of spiral, irregular and elliptical galaxies (e.g. Driver et al. 1995, Elmegreen et al. 2005), albeit in different relative proportions.

Our local star, the Sun, has been extensively researched. Its chemical composition (e.g. Grevesse et al. 2010), layered structure (e.g. Basu et al. 2009) and the existence of stellar wind (Parker 1958) are all parameters that can be used in galactic modelling: if the Sun is taken as an average star in the middle of its life, its properties can be extrapolated to other stars within a model galaxy.

Observations of other nearby stars at different stages in their lifecycles (e.g. Kurtz et al. 2011, Arias et al. 2010), and of phenomena such as supernovae e.g. SN1987A (a type II event that took place in the nearby Large Magellanic Cloud), provide further data that galactic modellers can use.

Observational data on our Galaxy yield information on current physical processes within a barred spiral galaxy; some of these processes may have applied to elliptical galaxies during their formation epoch, for example, star formation processes observable in the Orion Nebula can be used to estimate star formation rates (e.g. Palla and Stuhler 1999). Observations of other galaxies may help to understand how various morphologies form and evolve with time: large scale evidence of merger events (e.g. Henriksen and Tittley 2002, Kitzbichler and White 2008), or evidence of historic mergers e.g. by tail remnants, such as in the Antennae galaxy (e.g. Read et al. 1995, Vigroux et al. 1996), or where the core of a galaxy is counter-rotating (e.g. Thomas et al. 2006) support hierarchical galaxy formation (the theory that large galaxies form by the merger of smaller galaxies and star clusters).

1.1.3 Chemical composition as a clue to galaxy evolution

α–elements (N, O, Mg, Ca, Na, Ne, S, Si, Ti) are formed by nuclear fusion of helium (α) with other light elements, and are mainly produced during SNII events (Thomas et al. 2004, Maeder 1992). SNII events are where a star with initial mass > ~10$M_\odot$ collapses and explodes (e.g. Burrows and Lattimer 1985) within ~0.03 Gyr of the star being formed (Wood 1992). Iron-peak elements (Cr, Mn, Fe, Co, Ni, Cu, Zn), which are formed by nuclear fusion are mainly formed during SNIa events where a CO white dwarf explodes several Gyrs after it initially formed, either by accretion of hydrogen from a companion binary star or by merging with another white dwarf (Truran 1972, Scannapieco and Bildsten 2005, Wood 1992). Hence, the chemical composition of the galaxy, as reflected in the ratio of α–elements to Fe peak elements, can indicate the star formation history (SFH) of that galaxy by indicating the relative number of SNIa and SNII events required (Matteucci and Greggio 1986) and hence the initial stellar populations, as the lifetimes and masses of stars that produce these events can be estimated from stellar luminosities and initial mass functions (IMF).

Whilst direct element abundances are available for our Galaxy (summarised in Goswami and Prantzos 2000) they are not yet generally available for more distant galaxies due to instrument limitations, and therefore must be inferred from integrated absorption indices from these unresolved populations.

1.1.4 Lick indices

Lick indices were introduced by Worthey et al. (1994). A single observed absorption index may not be sufficient to trace an individual element's abundance, due to blending of absorption lines from different elements at wavelengths covered by that index. However, each index is dominated by a small number of ions (Tripicco and Bell 1995, Korn et al. 2005), and, as each ion absorbs at various known wavelengths, these Lick indices can be used to indicate the underlying chemistry of the galaxy. In turn, this can be used to establish the star formation history, because the different proportions of elements formed can be traced back to the initial stellar masses of earlier populations within the galaxy. As individual stars cannot be resolved within distant galaxies, the integrated spectra from these galaxies, in the form of Lick indices, can be used to indicate the overall chemistry of that galaxy.

Increasing age reddens the population, because more of the stars are older, cooler, red giant branch stars.
Increasing metallicity also reddens the population, because metals preferentially absorb light in the blue region of the spectrum, mainly through the many blue-region photospheric absorption lines, but also possibly through a reddened continuum.
This gives rise to 'age-metallicity degeneracy', whereby a young, metal-rich galaxy will appear identical to an old, metal-poor galaxy. This degeneracy was broken by Worthey (1994) who identified that some Lick indices were more age-sensitive and others were more metallicity-sensitive: G4300, H_{β}, and higher-order Balmer-line indices are more age-sensitive, and C4668, Fe5015, Fe5709 and Fe5782 are more metallicity-sensitive (Worthey 1994, higher-order Balmer indices added in Worthey and Ottaviani 1997)). These indices can therefore be used to establish whether an observed galaxy is old and metal-poor or young and metal-rich. These models are based on the Revised Yale Isochrones (Green et al. 1987)

together with VandenBerg Isochrones (VandenBerg and Bell 1985), with extrapolations where required.

Many recent spectroscopic observations are at substantially higher spectral resolution than those used to compile the original Lick indices. Hence, some modern observations need to be degraded to the same resolution to enable comparisons to be made with the Lick reference stars, and therefore to other data sets of Lick indices from other authors. This enables different data sets composed of Lick indices to be compared on a like-for-like basis. Vazdekis et al. (2010) presented a new database of the Lick reference stars at a higher resolution and a mechanism for recalibrating existing data to this new system.

1.2 APPROACHES TO MODELLING GALAXY EVOLUTION

1.2.1 Introduction

Observational data can give information about chemical composition or astrophysical processes taking place. These data can be analysed to identify trends and relationships between parameters, and constrain the likely evolutionary processes. Additionally, computer models can be built, with variable initial parameters and physical processes, which then predict values against which the observed data can be compared. If a model can match the observations and be demonstrated to be a unique solution within the parameter space used by that model, then it can be inferred that the input parameters of the model may correctly describe the evolutionary processes that formed that galaxy.

Historically, elliptical galaxies were thought to have formed by either monolithic collapse of a gas cloud under gravity (e.g. Eggen et al. 1962, Larson 1974, Carlberg 1984, Kodama and Arimoto 1997, Chiosi and Carraro 2002), forming a population of stars that then evolved passively (e.g. Daddi et al. 2005, Johansson, et al. 2009, Cassata et al. 2010), or by hierarchical assembly from the merger of smaller systems (e.g. Côté et al. 2000, van Dokkum et al. 2008). More recently, additional processes have been proposed to try to explain the observed features of elliptical galaxies, which include the following:

- "Downsizing" (Cowie et al. 1996). This is the phenomenon whereby stars in more massive galaxies form earlier and over a shorter timescale (i.e. have older average ages) than those in smaller galaxies (e.g. Kodama et al. 2004, De Lucia et al. 2006). This cannot be explained by hierarchical galaxy formation theory, since that would be expected to show massive galaxies forming over a longer timescale, assuming galactic mergers trigger starbursts (e.g. Mihos and Hernquist 1994, Di Matteo et al. 2008a).

- "Dry mergers" are postulated to occur between two or more galaxies where there is no residual gas and hence no starburst when they merge. Dry mergers are necessary to explain the observed old populations of ellipticals whilst allowing them to merge hierarchically. Models including dry mergers are more successful at showing how slow rotating ellipticals could form (Naab et al. 2006), and can explain the formation of brightest cluster galaxies in line with observations of mass and luminosity of these structures (Liu et al. 2009).

- A mechanism is needed to 'turn off' star formation in elliptical galaxies, which are observed to consist largely of old populations. "Galactic winds" arising from active galactic nuclei (AGN) and/or supernovae (SN) may provide a mechanism to remove the gas from a galaxy so that star formation ceases (e.g. Gibson 1997). However, these or other processes will be required to continue to remove the gas that will be ejected from smaller stars undergoing SNIa or planetary nebulae after the timing of the galactic wind.

Croton and Farrar (2008) note that elliptical galaxies generally consist of old, "red and dead" populations – but what is not yet known conclusively is how these populations formed, and why these galaxies are no longer evolving. Graphical or statistical interpretation of observational data can be used in the first instance to constrain parameters; more advanced methods use a variety of computer modelling techniques.

1.2.2 Using large data sets to graphically and statistically constrain parameters of galactic evolution

In recent years, a number of major observational projects such as GOODS (60,000 galaxies), SDSS (930,000), COMBO-17 (40,000), and Gemini Deep Deep Survey (GDDS) (301 high-redshift galaxies) have provided the community with extensive data sets. These large data sets can be used to infer generalised characteristics of galaxies by simply plotting aspects of the observed data and noting correlations in order to suggest constraining parameters.

Trends are especially noticeable for elliptical galaxies, for example:

- The Faber-Jackson relationship (Faber and Jackson 1976) between luminosity L and central velocity dispersion σ:

$$L \propto \sigma^4 \qquad (1)$$

The Faber-Jackson relationship was originally calculated from a set of 25 galaxies, but has since been found to hold true with more recent larger surveys.

- The Fundamental Plane (Dressler et al. 1987, Djorgovski and Davis 1987), which expands the Faber-Jackson relationship to three dimensions by including the mean surface brightness Σ_e within the half-light radius:

  $$L \propto \sigma^{8/3} \Sigma_e^{-3/5} \quad (2)$$

  The existence of the fundamental plane suggests a common evolutionary history for elliptical galaxies, or that processes since formation have aligned these parameters.

- Tremonti et al. (2004) demonstrated a mass-metallicity relationship by plotting 53,400 local galaxies from the Sloan Digital Sky Survey (SDSS) survey, which showed metallicity increasing with galaxy mass. The mass-metallicity relationship was extended to more distant galaxies by Savaglio et al. (2005) using a sample of 69 galaxies from the GDDS at $0.4<z<1.0$. Savaglio et al. (2005)'s work demonstrated that more distant (younger) galaxies are less metal-rich than those of similar mass at lower redshifts – metallicity increases over time - and that metallicity as well as mass evolves more slowly for smaller galaxies than for more massive ones.

One ongoing area of research addresses whether there is evolution along the Hubble Sequence – do spirals merge to form ellipticals (e.g. Benson and Devereuax 2010), or do ellipticals merge and rotate in such a way that infall gas causes them to develop into spirals (e.g. Kauffmann 1996), or do spirals only develop from morphologically peculiar galaxies (e.g. Delgado-Serrano et al. 2010)? Counts of galaxies at different redshifts show that both elliptical and spiral morphologies existed in the early Universe (e.g. Volonteri et al. 2000). Of course, no single galaxy can be followed temporally, but Hubble Sequence evolution can be demonstrated with models that work physically and are supported by observations of galaxies mid-way between one morphology and another.

Kajisawa et al. (2009, 2010) used near-IR data from the GOODS survey (Cristiani et al. 2004) to estimate the variation of star formation rates over time, finding that the majority of the currently observed stellar mass formed at $1<z<3$, and that a bimodality of star formation rates exists, especially in smaller galaxies at higher redshifts, which was identified when plotted data was binned by galaxy mass and redshift, and can be explained as a consequence of starburst/high star formation rate (SFR) and continuous passive star formation (low SFR) in these galaxies.

Sánchez-Blázquez et al. (2006a) plotted the Balmer-index/central velocity dispersion of their sample of 98 elliptical galaxies. These suggest that the correlation of index/velocity-dispersion for galaxies in the high-density Coma cluster could be explained by truncated star formation/chemical enrichment histories when compared with galaxies in lower density environments.

As well as graphically plotting the data from these large surveys, more general statistical methods can be used to attempt to extract underlying patterns in the data. Examples include:

- Principal component analysis (data compression techniques using a model-independent statistical method to assess differences between data sets) were employed by Heavens et al. (2000) ('MOPED' code) to reduce a given data set to 23 parameters. They then applied this PCA model to SDSS DR1 (Heavens et al. 2004), which suggested (from plots of the reduced set of parameters) that the peak of star formation, irrespective of morphology, was 5 Gyr ago, and that galaxies with high stellar mass formed earlier than those with low stellar mass (i.e. downsizing).

- An adapted version of the same data compression software was used by Mathis et al. (2006) to re-assess the same set of SDSS data. However, in contrast to Heavens et al. (2004), they concluded that elliptical galaxies formed most of their stars 8 Gyr ago, with continued star formation up to 4 Gyr ago, and that late type galaxies have a broadly constant star formation rate. These different conclusions arose because Heavens et al. (2004) reviewed the parameters produced by the software against the entire large data set whereas Mathis et al. (2006) used these parameters in their separate star formation history (SFH) modelling software to attempt to recover the SFH for specific galaxies. Note that Mathis et al. (2006) assumed all galaxies have a constant metallicity over time and the published plots have fairly coarse time-bins for sampling, which make it difficult to distinguish peak SFH at either 8 or 5 Gyr ago, as these are both shown within the same sampling bin.

- The VESPA code of Tojeiro et al. (2007) uses a bounded-variable least squares method (Stark and Parker 1995) to parameterise star formation histories. The code was tested against 2,000 galaxies from SDSS data, and found that the number of parameters that could be uncovered depends upon the signal-to-noise ratio, the wavelength coverage and the presence or absence of a young population, and that the galaxies in the sample generally contained between two and five separate stellar populations.

- Ferreras et al. (2006) used principal component analysis to compare high signal-to-noise optical spectroscopic data for elliptical galaxies in Hickson Compact Groups to data from galaxies in looser groups, galaxies at the edge of compact groups, and galaxies in the field. They then used the single stellar population (SSP) models of Bruzual and Charlot (2003) to give a physical interpretation of the principal components identified. They concluded that the SFH for galaxies in compact groups is more complex than those of galaxies in other environments, as compact group galaxies showed more variation in the mass fraction of the galaxy held as younger stars, whereas the other ellipticals were more consistent with old stellar populations.

- Nolan et al. (2006) presented a method using Bayesian techniques to enable a search of a large data set to find galaxies meeting given selection criteria. A synthetic result is initially prepared, and the observational data then compared to that synthetic result, and tested statistically to extract just those observations which are likely to be a good fit to the selection criteria. This technique was tested to find young stellar populations within a sample of early-type galaxies, which are traditionally considered to be "red and dead" (Croton and Farrar 2008). Bayesian techniques were also used by Dye (2008) to recover star formation histories by setting idealised star formation rates for different epochs also using the SSP models of Bruzual and Charlot (2003), and then to find which combination of rates would produce the stellar masses observed. This method was subsequently applied (Dye et al. 2010) to 92 galaxies from the BLAST catalogue (Devlin et al. 2009) to infer that low mass systems form a large part of their mass in a dominant late burst of star formation, and high mass systems form the majority of their mass early on. These match findings of 'downsizing' from other modelling methods.

- Ocvirk et al. (2006a,b) used Singular Value Decomposition methods to factorise matrices in their STECMAP/STECKMAP models (the latter includes kinematics, hence the 'K') in order to find a least-squares solution, and then analyse the solution in terms of its singular vectors. STECKMAP was later used (Ocvirk 2010) to show how degeneracy effects from blue horizontal branch stars can distort results obtained from SSP models, as these stars appear to be younger than they are.

- Koleva et al. (2008) analysed results from statistical models including STECKMAP against known stellar populations. They found that the choice of input SSPs to set the idealised parameters was a significant factor, and although consistent results were obtained when the input SSPs were from either the ELODIE stellar library with Pegasus-HR SSPs (Prugniel and Soubiran 2001, Le Borgne et al. 2004) or the MILES stellar library with Vazdekis SSPs (Sánchez-Blázquez 2006c, Vazdekis 2010), limitations in the age, metallicity and surface gravity ranges in the stellar library STELIB used by the Bruzual and Charlot (2003) models led to systematic errors when used within SSPs. These systematic errors should be considered when reviewing results of models which use the Bruzual and Charlot (2003) SSPs as the source data set.

These methods (at least initially) ignore physics and what is known about galactic evolution, and just look at the data set purely as a mathematical and/or statistical problem. Interpretation of the results of these approaches generally requires use of computer models. Plotting and statistically analysing the data from these galactic surveys may indicate trends and relationships but cannot explain how they have arisen, and whether they exist coincidentally, or as a consequence of some underlying evolutionary or physical constraints.

1.2.3 Simple computer models

Early attempts to investigate galactic evolution concentrated on attempting to reproduce the integrated spectra by trial-and-error assembly of individual stellar spectra. Stellar spectra were combined in different proportions in order to try to recreate the observed spectra of a given galaxy.

Spinrad and Taylor (1971) and Faber (1972) were able to use this technique to successfully model M31 (but not M32 or M81), and O'Connell (1976) successfully modelled M31, NGC 4374, NGC 4472 and NGC 4552 using this method.

A 'classic' hydrodynamic model from first principles was derived by Larson (1974), who treated the gas and stars as two fluids, and tracked energy, star formation and total metal production within a closed-box spherical model of monolithic collapse. This model was able to reproduce the observed metallicity gradients of NGC 3379. There were long lists of assumptions that had to be made where the astrophysics at the time was simply not known, or the observational evidence was not available. However, Larson (1974) demonstrated that despite these limitations, computer modelling of galaxy formation could produce results that matched well with observations and could start to constrain parameters of galactic evolution.

1.2.4 More recent models

More recent models can be divided into four types, based on their approach, and what they are being used to explore:

- N-body and smooth-particle hydrodynamic models;
- semi-analytic models;
- single stellar populations; and
- integrated stellar population models.

These are discussed in more detail below.

1.2.5 N-body and smooth-particle hydrodynamic models

N-body simulations are used for tracking the movement of individual "particles", generally taken to represent matter (dark and visible) within a galaxy, or galaxies within a universe, or are used for cosmological simulations. They are useful tools for modelling galaxy formation and dynamics, particularly to enable understanding of formation of macro galactic structure such as bars, inner and outer haloes, and investigations into dark matter/visible matter distributions, and some specific examples are discussed in more detail below.

From the initial conditions, the gravitational, and, for some models (e.g. Roettiger and Stone1997), magnetic forces on all particles acting on all other particles are calculated, and used to update the particle positions and velocities. Energy is calculated and conserved within the system being modelled. These calculations are repeated either until the final structure being sought is modelled, or, if the model timesteps are equated to galaxy formation lifetimes, until the desired time has elapsed. The model keeps track of the particle's physical properties, such as position, velocity, mass, density and temperature, to facilitate analysis. Output is generally also presented as a two- or three- dimensional film which is run with the timesteps sufficiently sped-up to enable the observer to see the structures being developed. As the number of particles that can be modelled is considerably smaller than the number of stars within a galaxy (etc), the models produced must be considered as an approximation of the mass distribution in the system being modelled. For some models, N-body simulations are combined with smoothed particle hydrodynamics (SPH), where the effects of spatially very distant particles are ignored or smoothed, which in turn reduces computational intensity and speeds up processing time.

These N-body and SPH models are particularly useful for testing the type of cosmology within which the current Universe resides; with some cosmologies, the N-body/SPH models are unable to recreate the currently observed galaxy distribution, i.e. constrains cosmological parameters. For example, Davis et al. (1985) showed that a flat universe could not be modelled if it was assumed that galaxies were unbiased tracers of the overall mass distribution; for a Λ cold dark matter flat universe to model current galaxy distribution, galaxies had to form in pre-existing areas of high density.

N-body and SPH modelling processes are currently very CPU-intensive, both for the calculations and producing visual representation of the results, which in turn limits the number of particles that can be modelled and the number of time-steps undertaken. Processing time can be reduced by running the programme in parallel on several computers.

N-body and SPH models are ideal for establishing initial galaxy formation parameters, however, they are more limited in modelling the galaxy after it has assembled (i.e. modelling the impact of stellar evolution processes on galaxy evolution), as they are modelling large-scale processes in a very generalised way. N-body/SPH models therefore generally do not include aspects of galactic chemistry, although limited work by Tornatore et al. (2007) expanded the open-source SPH code GADGET-2 model (Springel 2005) to include the effects of contributions from SNIa, SNII, and planetary nebulae (results from Thielemann et al. 2003, Woosley and Weaver 1995 van den Hoek and Groenewegen 1997 respectively). GADGET-2 was then used to investigate chemical enrichment of the intra-cluster medium, and found that whilst a Salpeter (1955) IMF produced iron abundances in line with Chandra observations, the model was unable to reproduce any other observed element abundances. However, this showed that these models could be used in this way, and further developments using GADGET-2 were made by Oppenheimer and Davé (2008) who were able to model C, O, Si as well as Fe, by incorporating galactic winds into the model and finding that some material ejected by galactic winds is re-accreted by the original galaxy.

1.2.6 Semi-analytic models and numerical simulations

Semi-analytic models (SAMs) are provided with 'rules' that the galaxy model follows, using a combination of analytical approximations and empirical calculations. "SAMs" therefore technically include those models described above as N-body/SPH, but the term is generally taken to mean models that take the synthetic galaxy forward from initial collapse and merger of dark matter haloes to the present day by including phenomena such as gas inflow and outflow, supernovae, black hole formation, and AGN feedback. Numerical simulations generally model specific processes such as gas dynamics or disc momentum; these eventually become limited by the model resolution (Baugh 2006).

For some models, e.g. Helly et al. (2003), the model is a hybrid: the output of their N-body/SPH models form the input into a separate SAM (in this case, that of Cole et al. 2000) of the later evolution. The SPH model GASOLINE (Wadsley et al. 2004) was used by Feldmann et al. (2011) with outputs from the N-body code MHF (Gill et al. 2004), SSPs from Bruzual and Charlot (2003) and the two-dimensional fitting algorithm GALFIT (Peng et al. 2002) to show that elliptical galaxies in clusters appear to be formed by mergers occurring before the cluster itself is fully assembled, with quenching of star formation taking less than a Gyr to complete.

Most models aim as a minimum to compute the mass of stars and gas and the galaxy radius, morphology and rotation speed. The advantage of this hybrid approach over the "pure" N-body/SPH approach is that it is far less CPU-intensive, allowing for more rapid evaluation of parameter space. In addition these hybrid models aim to analyse galactic processes for the larger part (i.e. the post-formation period) of the galaxy's life. The disadvantage is that often large areas of physics have to be simplified, for example, using an instantaneous gas recycling assumption by ignoring the effects of SNIa and/or assuming increases to the ISM are immediately available for the next generation of stars, or modelling the galaxy as a single zone or a closed-box. With hybrid models, there is a risk that incompatible approximations and assumptions are used in the two parts, particularly if the two parts are from different research groups.

Semi-analytic approaches have been very successful, with models able to investigate aspects of galaxy formation such as galaxy colours and metallicities (e.g. Lanzoni et al. 2005), super-massive black hole formation and AGN feedback (e.g. Bower et al. 2006), and size/ mass evolution of galaxies (Somerville et al. 2008).

A successful model should be able to recreate as many features of observed galaxies as possible, and to that end, many SAMs have more recently been developed to incorporate chemical evolution. Such SAMs include the GCD+ model of Kawata and Gibson (2003), which demonstrated the importance of SNIa feedback, GRAPE-SPH (Kobayashi 2004), which showed that galaxies that form monolithically should have steeper radial metallicity gradients, GALFORM (Nagashima et al. 2005, adapted from Cole et al. 2000) was able to explain the observed α-element abundances in ellipticals and Calura and Menci's 2009 (unnamed) models were used to suggest that low-level starbursts, perhaps caused by fly-by 'harassments' (rather then full mergers), could explain the observed α/Fe ratio in ellipticals (Calura and Menci 2011).

However, the selection of simplifications used in these models may mean a model is able to successfully reproduce some but not all aspects of galaxy formation. Snaith et al. (2011) compared luminosity predictions for modelled galaxy groups produced by four semi-analytic models, and found that the differences in the underlying physics did result in output differences. For example, all four models yielded a different number of final galaxies despite starting from the same dark matter distribution, and no model was able to provide an overall good match to observations.

1.2.7 Evolutionary population synthesis I: single stellar population models

Stellar spectra from a given isochrone (i.e. stars of the same age and metallicity but different initial masses: a single stellar population or SSP) provide a model of a population formed in a single burst.

Synthetic spectra have to be used to create a complete data set for an isochrone due to the incompleteness of observational data, for example, a lack of nearby metal-rich or metal-poor stars. Standard model atmospheres may need to be physically inconsistent in order to obtain realistic results, for example the need to relax thermodynamic equilibrium requirements. Tests by Heiter and Eriksson (2006) and Gustafsson et al. (2007) have shown that even where the physics has had to be relaxed, the overall model results may be acceptable, although care should be used when extrapolating at the extreme ends of the data. The lack of observational data is why SSP models built from these spectra generally do not include very young or very metal rich populations: the extrapolation from the source data introduces too many uncertainties.

Entirely synthetic stellar data sets also exist; Martins and Coelho (2007) compared three synthetic and three empirical libraries and whilst they found that the comparison task was not easy, due to uncertainties in the atmospheric parameters of the observed stars, they concluded that either set was reasonable for indices, but the synthetic U-B colours were redder than the observations, and cool stars were less well modelled.

There are several empirical stellar libraries from which SSP models can be created. Isochrones for each age and metallicity are taken from these libraries to create a theoretical Hertzsprung-Russell diagram for individual stars, which are used to create the modelled absorption features and Lick indices by calibrating the age and metallicity with the spectral data from the stellar libraries. There are many uncertainties inherent in this process; the lifetimes, temperatures and luminosities of the stars, especially those which are not available observationally, can lead to incorrect calibration of the stars and their synthetic observables (Charlot et al. 1996, Percival and Salaris 2009). Some of the recent stellar libraries are compared in table 1 below:

	ELODIE	**STELIB**	**INDO-US (aka CFLIB)**	**MILES**
Published	Prugniel and Soubiran (2001)	Le Borgne et al. (2003)	Valdes et al., (2004)	Sánchez-Blázquez et al. (2006c)
Number of stellar spectra	709	249	1273	985
Source	Observatoire de Haute-Provence	Jacobus Kaptein Telescope in La Palma, Siding Spring Telescope, Australia, VLT-UT1 Antu Telescope	Coudé feed telescope at Kitt Peak National Observatory	Isaac Newton Telescope
Includes synthetic values	Not in this original set, although subsequently synthetic values included	Yes, although paper does not disclose how many	Yes, full values for 885 stars, rest have some synthetic values to complete gaps	Yes for some stars
Wavelength range	4100-6800 Å	3200 – 9500 Å	3460 – 9464 Å	3525-7500 Å
Metallicity range	[Fe/H] from -2.8 to +0.7	[Fe/H] from -1.9 to +0.47	[Fe/H] from -3.0 to +1.6	[Fe/H] from -2.7 to +1.0
Resolution	2 Å	3 Å	1 Å	2.3 Å

Table 1: Examples of recent empirical stellar libraries.

Recent libraries of SSP data include Bruzual and Charlot (2003), which they then expanded to give integrated population models, Thomas et al. (2003, 2004) which feature non-solar abundance ratios and are based on various theoretical stellar libraries, and the models of Vazdekis et al. (2010) which are based on the updated MILES stellar library of Sánchez-Blázquez et al. (2006c).

Observational data from globular clusters in both the Galaxy, and other nearby galaxies, suggest that globular clusters are probably formed from a single stellar population (e.g. Chaboyer et al. 1996, Fellhauer et al. 2006), and as such are a useful test of SSP models (e.g. Maraston 2005, Mendel et al. 2007, Lee et al. 2009).

Where SSP models are checked against galaxies rather than globular clusters, the method is generally to overlay the galaxy data from two indices (plotted as scatter points) on a grid from the SSP data (plotted as lines) which may show that the galaxy data is constrained within the SSP grid. For example, the left-hand grid in Figure 1 suggests that the early type galaxies in the sample from Proctor and Sansom (2002) are older and more iron-poor than the spiral bulges, but note that some cannot be modelled within the Vazdekis (1999) SSP grids.

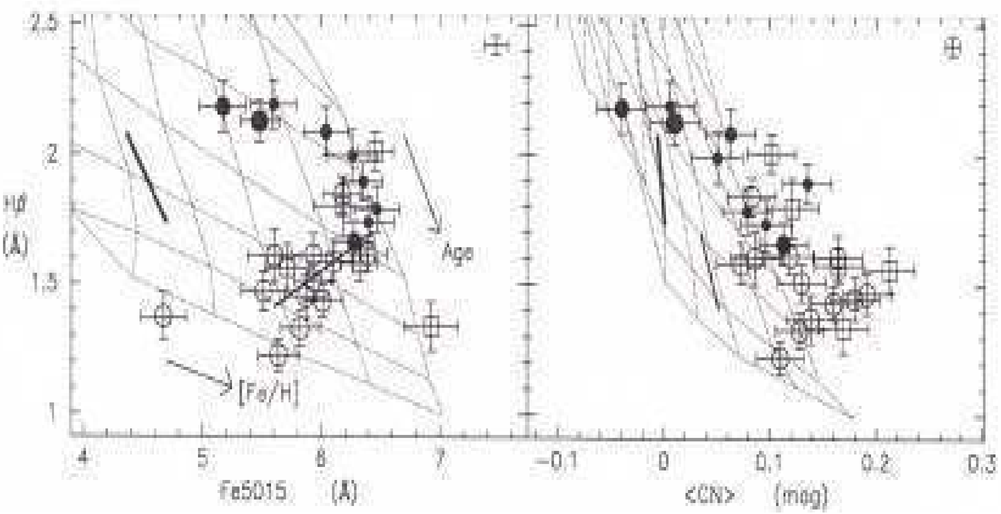


Figure 1: Extract from Proctor and Sansom (2002), showing their galaxy sample plotted against a grid taken from Vazdekis et al (1999) SSP models (open symbols for early-type galaxies and solid for spiral bulges). The left panel shows H_β against an iron-sensitive index, the right against an abundance-sensitive index.

Finding observational galaxy data that fits within an SSP grid does not mean that SSP models can successfully model galaxies: what it shows is trends within galaxies may map to trends within SSPs (such as a higher ratio of two indices being found within certain galaxy morphologies). In each instance only two parameters are being checked, and may indicate a good fit, but it does not necessarily follow that a single SSP model can successfully simultaneously reproduce the full set of Lick indices observed. Fitting observational data with SSPs is further explored in Chapter 6. Another difficulty arises from the lack of reference stars with extreme (high or low) metallicity, and with non-solar abundances of elements, as these are either not modelled within the SSP, or are based on synthetic spectra. The effect of non-solar abundances was modelled by Tripicco and Bell (1995), (updated by Korn et al 2005), by doubling the abundances of individual elements in their models and assessing the effect on the synthetic indices, showing which element(s) each index was particularly sensitive to. This resulted in Fe4668 being renamed as C4668, as it was found to be much more sensitive to carbon abundances than to iron.

1.2.8 Evolutionary population synthesis II: integrated stellar population models

A step forward from SSP models to evolutionary population synthesis models can be made if a galaxy is considered to be an integrated population of many SSPs. These integrated models attempt to recreate the colours, indices and spectra of observed galaxies, either by attempting to recreate the observables by a combination of SSPs (which is referred to in this thesis as a "top-down" approach), or by evolving a model galaxy and combining the SSPs of its component populations and then comparing them to the observed data (which is referred to in this thesis as a "bottom-up" approach).

The "top-down" approach is used by Bruzual and Charlot (2003), who create their model galaxy (GALAXEV) by Monte Carlo sampling of their SSPs until the galaxy mass required is created, and then comparing the resultant indices to the early-release SDSS data. Their models do not make any adjustments for α-enhancement; they consider α-enhancement to mainly affect galaxies with large velocity dispersions, and only use the Lick indices which are not greatly affected by non-solar abundances, such as the Balmer indices and D4000, noting that their underlying SSPs are at fixed metallicity and chemical composition.

This publicly-available code has been widely integrated into other models, or used to assess observational data, to the point of almost becoming “industry standard” (e.g. Yan and Thompson 2003, Stanford et al. 2004, Mei et al. 2005, Metcalfe et al. 2006, Wolf et al. 2007, Coelho et al. 2007, Tortora et al. 2009). Maraston et al. (2006) and, independently, van der Wel (2006) both found better results from the Maraston et al. (2005) models when compared to the Bruzual and Charlot (2003) models, although Conroy and Gunn (2010a) find the Maraston et al. (2005) models to be too red, and both Maraston et al. (2005) and Bruzual and Charlot (2003) to fail in the far-UV compared to the observational data. As noted above in 1.2.2, Koleva et al. (2008) found that limitations in the stellar library used by Bruzual and Charlot (2003) led to systematic errors, which may explain some of these findings.

The STARLIGHT (Cid Fernandes et al. 2005) code also takes a “top-down” approach by breaking down an observed spectrum into a sum of SSPs. Source SSPs from Bruzual and Charlot (2003) are combined with the 1994 Padova isochrones (Bertelli et al. 1994) and the STELIB library (Le Borgne et al. 2003); this code has subsequently been updated with the MILES library (Sánchez-Blázquez et al. 2006c) and the Vazdekis et al. (2010) SSPs. A recent review (Cid Fernandes and González Delgado 2010) compares the updated version of STARLIGHT to the Vazdekis (2010) models. This review finds better spectral fits with the newer stellar libraries, but note that metallicities correlate poorly, due to the limitations of the spectral range available, and to the coarseness in the metallicity grids.

The code works by testing different combinations of SSPs against the observational data, finding local minima in calculations of χ^2 and then, through an algorithm, traps the most likely region of parameter space where the solution would be found. The code may find multiple solutions, although the inclusion of the entire spectrum is expected to minimise the instances where this arises from the intrinsic age-metallicity degeneracy of stellar populations (1.1.4), as different parts of the spectrum are age- or abundance-sensitive. Note, however, that the

results are a list of the individual SSPs that can together reproduce the observed spectrum; it does not take the enhanced ejecta products of one population to form the next generation of stars, and so should be considered as a hierarchical merging of several populations without any population affecting any other population, and without consideration of how those individual populations came to exist in the first place.
The original STARLIGHT model (Cid Fernandes et al. 2005) was applied to a volume-limited sample of 50,362 galaxies from SDSS DR2, and was able to recover properties such as mean stellar ages and galaxy masses comparable to those plotted by Kauffmann et al. (2003).

Chen et al. (2010) compared six sets of SSP models by applying them to the STARLIGHT code to attempt to establish the SFH of "representative galaxies", created by combining spectra from several observed galaxies. As expected, younger populations were found to be more important when modelling star-forming galaxies than early-type quiescent galaxies, but this work also showed that different input SSP sets did generate different SFH. Selection of SSP age and metallicity was shown to be more important than the underlying stellar evolution tracks used in the SSP.

A "bottom-up" approach evolves the model galaxy from the initial gas cloud using physical principles, and at any given point replicates the integrated spectrum by summing the SSP-equivalent values for all the stars then present in the model. This "bottom-up" approach enables the models to be chemically consistent, with each new generation of stars inheriting the metallicity and chemical composition of the ISM at the point of formation. The initial mass function (IMF), which defines how each new population is distributed over different stellar masses, is important in these models because the IMF determines the evolutionary paths for these individual stars, and consequently the yields and recycled material for the next generation.

The "bottom-up" approach was pioneered by Larson and Tinsley (1978), who modelled synthetic integrated colours and showed that later bursts of star formation were better able to replicate the observed colours in peculiar galaxies (as defined by Arp 1966), whereas non-interacting galaxies were better modelled with older populations.

GALEV models (Schulz et al. 2002), summarised in Kotulla et al. (2009), have been able to successfully model E+A galaxies (blue galaxies without emission lines), seen as an intermediate stage of evolution between late- and early-type galaxy morphologies (Falkenberg et al. 2009 a, b). GALEV models have been mainly used to investigate star cluster evolution, aspects of spiral galaxies and the significance of non-solar abundances particularly at high redshift.

Mollá and Díaz (2005) used their multiphase chemical evolution model (CEM) to model radial distribution of elements in spiral and irregular galaxies, and then used this to find that nitrogen and oxygen abundances were influenced by both the star formation rate and the IMF (Mollá et al. 2006).

"Bottom-up" integrated evolution population synthesis method is the basis of the GCE and Phoenix models, described extensively in the remainder of this thesis.

1.2.9 Comparison of model approaches

Different models as discussed above have individual advantages and limitations (table 2). These determine the questions they are best suited to answer. For example, single stellar population and integrated stellar population models are both limited by available spectral data but the former can successfully model small globular clusters whereas the latter can recreate star formation histories of more complex populations.

Galaxy modelling enables parameters for galaxy formation to be constrained, and by comparison of theoretical physical phenomena may be able to indicate preference of one hypothesis over another, for example, which method of gas loss in elliptical galaxies is more likely.

Models that are open-source, or have a user-friendly web interface, are obviously more widely tested and used than those kept within an individual research group. The risk is that other users are not fully aware of the code limitations or assumptions within the model, and the impact these limitations may have when applying the code to a new problem.

In addition, very few models are built entirely from first principles: galactic modellers take results from stellar modellers, stellar modellers use extrapolated data from stellar libraries etc. There is a risk therefore of assumptions not being compatible.

Model category	**Model successes**	**Model limitations**
Reviews and statistical modelling of large data sets	Establishment of correlations between physical properties of galaxies.	Cannot necessarily explain the reasons for the trends noted. Cannot explain processes in individual galaxies.
N-body simulations/ smoothed-particle hydrodynamics	Establishing routes for initial formation of structure	Not suited to modelling post-formation evolution. Very CPU-intensive. Limited by sub-grid physics i.e. the selected resolution of the model
Semi-analytic models	Establishing physical properties of, and processes within, galaxies: individually and within clusters.	Requires extensive assumptions, simplifications and approximations of the source 'rules'. Non-linear processes may have to be interpreted linearly. Cannot predict internal properties such as metallicity gradients.
Single stellar population models	Can successfully model star clusters	Limited by the quality of underlying spectral libraries, which may not be observationally (i.e. empirically) complete. Do not include cosmological effects.
Integrated stellar population models	Able to recreate star formation histories of unresolved complex populations	Limited by the quality of underlying spectral libraries, which may not be observationally (i.e. empirically) complete. Do not include cosmological effects.

Table 2: Comparison of different modelling approaches.

1.3 OVERVIEW OF THIS THESIS

1.3.1 Overview

Chapter 2 gives a detailed review of an existing integrated "bottom-up" evolutionary population synthesis model ("GCE" model), together with a discussion of several new code enhancements which were written and tested with the intention of using this model to propose the star formation histories of individual galaxies. Chapter 3 discusses the remaining limitations of this code, and as a result of this work, a new model and code, Phoenix, was written. This is described in Chapter 4, and its testing, including against other models from the literature, is discussed in Chapter 5.

In Chapter 6 the new code is used to propose, for the first time, the star formation histories from two data sets, each of eleven nearby elliptical galaxies, taken from different telescopes. Results are compared to those found using the Single Stellar Population models of Thomas et al. (2004), and results for 10 galaxies (five from each data set) are verified using observational data from a third data set, also from a separate telescope. Finally, Chapter 7 draws together a brief general discussion and the main conclusions from this project, together with some suggestions for future related work.

CHAPTER 2: OVERVIEW AND ENHANCEMENT OF AN EXISTING POPULATION SYNTHESIS MODEL FROM THE LITERATURE

2.1 THE GALACTIC CHEMICAL EVOLUTION MODEL

2.1.1 The GCE model

The Galactic Chemical Evolution (GCE) model reviewed here was developed by Dr Anne Sansom from 1996 onwards, with additions and modifications by Dr Robert Proctor, Dr Pierre Ocvirk, Mr N Gjshchkhmyj and the present author (section 2.2 below). The model evolves a hypothetical spherical stellar population of mass 10^6 $M_\odot$ from initial conditions, using various stellar yield and ejecta tables from the literature, to select appropriate synthetic Lick indices from SSP models, also from the literature, which can then be compared to those of observational data i.e. the GCE model is a "bottom-up" integrated stellar population model.

The model allows the user to select some of the variables via an input file '*values.in*', including defining two changes in star formation rate (through an arbitrary constant related to star formation efficiency, which can be set to zero to halt star formation) and two changes to gas inflow rate (gas outflow is not modelled). These are listed in table 3.

Overall life of the galaxy in Gyrs	Initial constant in the Schmidt (1959) star formation rate equation	Initial gas inflow rate in $M_\odot$/Gyr
Time change 1: Gyrs after start of galaxy when star formation rate and gas inflow changes	Constant in the Schmidt (1959) star formation equation after Time Change 1	Gas inflow rate in $M_\odot$/Gyr after Time Change 1
Time change 2: Gyrs after start of galaxy when star formation rate and gas inflow changes	Constant in the Schmidt (1959) star formation equation after Time Change 2	Gas inflow rate in $M_\odot$/Gyr after Time Change 2
Mass of CO core for black hole formation ($M_\odot$)		
Maximum mass of stars that undergo SNII events ($M_\odot$)		
SNIa rate (events $M_\odot^{-1}$ Gyr^{-1})		

Table 3: User-set model variables (12 parameters) ('*values.in*').

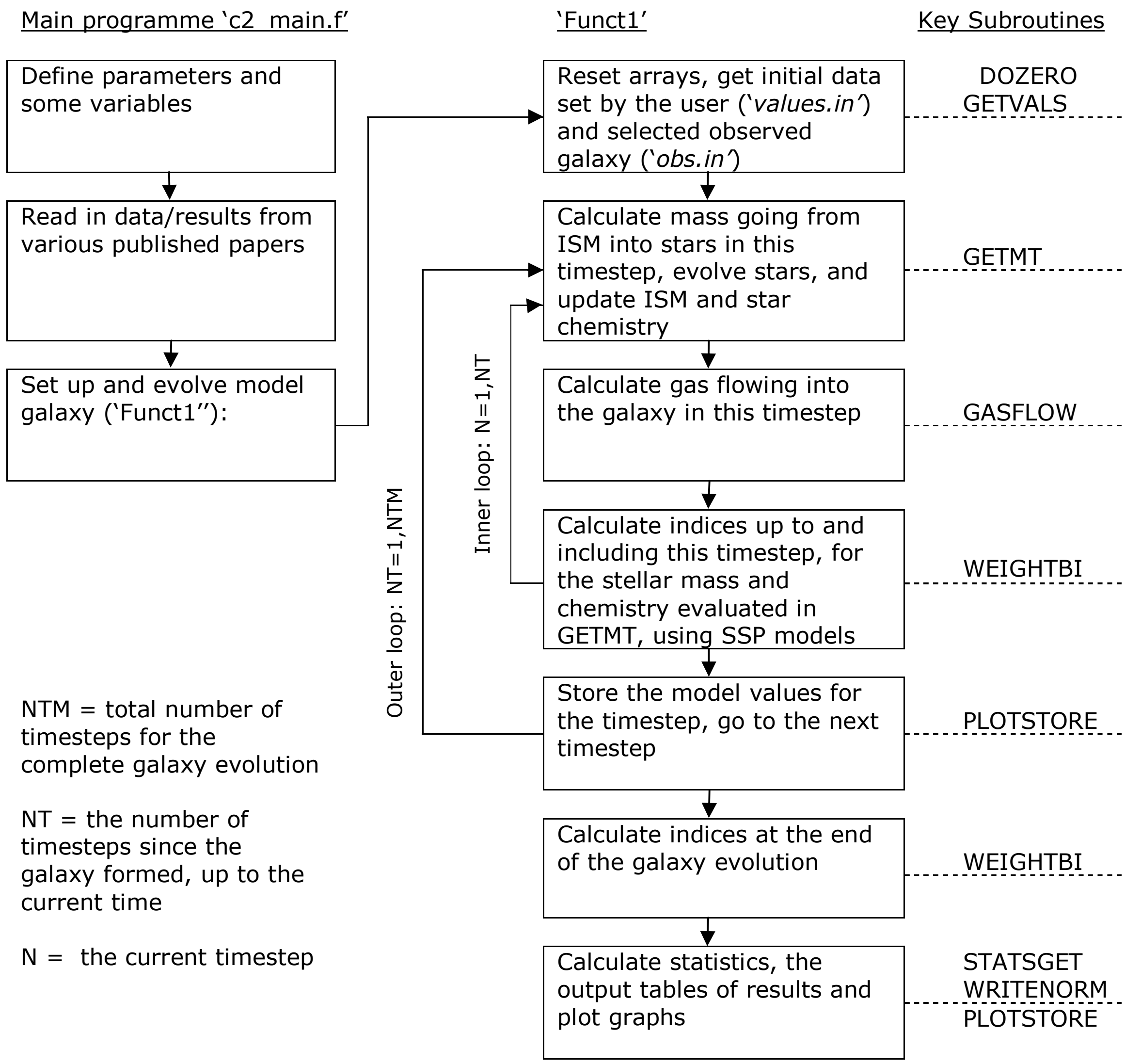


Figure 2: Summary of the main GCE model subroutines and activity.

Until the enhancements by the present author were added, the main external data sources for the GGE model were:

- planetary nebula yields from Renzini and Voli (1981) (hereafter RV81);
- SNIa ejecta from Nomoto et al. (1984);
- SNII ejecta from Woolsey and Weaver (1995) (hereafter WW95) or yields from Maeder (1992) (M92), modified with the more reasonable results from Meynet and Maeder (2002) (MM02) for stars > 40 $M_\odot$, or a weighted combination of both; and

- SSP data from either Worthey et al. 1994 (hereafter W94) or Vazdekis et al. (1999) (V99)

The GCE model was originally written to use W94 SSPs, which were based on single-burst models with a Salpeter (1955) IMF and 10^6 $M_\odot$ stars. W94 noted that this enabled users of his SSP models to scale the mass to their own purpose, however, the GCE model uses this stellar population mass unscaled as the total mass of the model galaxy, stored as a hard-coded parameter.

The galaxy evolution process can be summarised as:

$$dM_{star}/dt = M_{\text{stars formed from gas in timestep}} - M_{\text{stars exploding at end of life in timestep}} \quad (3)$$

$$dM_{gas}/dt = M_{\text{gas inflow in timestep}} + M_{\text{stars exploding at end of life in timestep}} - M_{\text{stars formed from gas in timestep}} \quad (4)$$

$$dM_{galaxy}/dt = dM_{star}/dt + dM_{gas}/dt \quad (5)$$

where:

- M = mass in $M_\odot$
- the mass of stars exploding at end of life in the timestep is calculated using stellar ejecta data from the literature
- the mass of stars formed from gas is calculated using the Schmidt (1959) star formation rate equation SFR = C $\rho^{1.3}$ (6)
  (C is a constant, in units of $kpc^3\ Gyr^{-1}$, ρ is gas density, 1.3 from Kennicutt 1989)
- the mass of stars in different mass ranges (and hence different evolutionary ends) is defined using the Salpeter (1955) initial mass function

As the model galaxy evolves, the yields/ejecta of elements produced by supernovae and planetary nebulae in each timestep are collated, and the metallicity of the galaxy (assumed to be the cumulative mass of the metal elements as a percentage of the total mass of the galaxy) is calculated. This metallicity selects the appropriate SSP by interpolation of W94 or V99 (as selected by the user). The SSPs for each timestep up to and including the current timestep are totalled and then weighted in proportion to the amount of light expected in three sections (B, V, I) of the spectrum, to give the overall synthetic indices for the galaxy at the end of that timestep.

The GCE programme also allows for non-solar abundance ratios, by modifying the interpolated SSP, using results from Tripicco and Bell (1995) (hereafter TB95), Weiss et al. (1995) and Barbuy (1994).

Tabular output gives the synthetic indices produced by the model, the observed indices from the user-selected galaxy, and computes the value it refers to as χ_ν^2 for each index and for the overall model where

$$\chi^2 = \sum \left(\frac{observed - synthetic}{error} \right)^2 \tag{7}$$

$$\chi_\nu^2 = \frac{\chi^2}{degrees\ of\ freedom} \tag{8}$$

'Degrees of freedom' is taken as the number of radial ranges included in the model, and a successful model is taken where $\chi_\nu^2 = 1$. The suitability/correct implementation of this statistical measure are discussed further in section 3.4.

It is important to note that the synthetic Lick indices output by the model are not "built up" from the elements created by nucleosynthesis; the 'chemistry' in the model is just a track of yield/ejecta results and is only used to calculate the value of metallicity for the appropriate selection of SSP data, and to check if the abundances are not solar in order to apply TB95 weightings to these synthetic indices. Luminosity-weighting is based on the colour data provided in the SSP data sets of the indices and not on the proportions of different stars in the model galaxy.

The model can also be run with separate "stepping software", which processes 23,040 runs of the GCE model, in series, storing the lowest χ_ν^2 value of each run and the parameters used to obtain this. The "stepping software" cycles through four parameters, with pre-set combinations of

- the star formation rate after the first starburst (C1) and
- the time (T1), duration (D1) and inflow rate (F1) of the second starburst.

The other eight model parameters are constant for all 23,040 runs and are set by the user as with the single-run model. These are given in table 4.

Variable name	Definition	Stepping values where applicable
TCHANGE1 (T1)	Time in Gyrs after start of model where flow and star formation rate are altered to the values R00C1 and FLOWRATE1 (also varied)	30 values from 0.0 to 14.0 Gyrs
ROOC1 (C1)	Revised constant in the Schmidt (1959) star formation rate equation after TCHANGE1	8 values from 0.03125 to 4.0
FLOWRATE1 (F1)	Revised flow rate of gas into the galaxy, in $M_\odot$ per Gyr after TCHANGE1	0.0 then 7 values from 5 x 10^4 to 5 x 10^7 $M_\odot$ Gyr^{-1}.
TCHANGE2 (D1)	Time in Gyrs after TCHANGE1 where flow and star formation rate are altered to the values R00C2 and FLOWRATE2 (which are not varied)	12 values from 0.0 to 15.081 Gyrs.
TIME	Overall life of the galaxy in Gyrs	These variables are set to a single parameter by the user in the file *values.in*, and are used consistently for all 23,040 runs of the stepping software.
ROOC0	Initial constant in the Schmidt (1959) star formation rate equation	
FLOWRATE0	Initial gas inflow rate in $M_\odot$/Gyr	
ROOC2	Constant in the Schmidt (1959) star formation equation after Time Change 2	
FLOWRATE2	Gas inflow rate in $M_\odot$/Gyr after Time Change 2	
BHMASS	Mass of CO core for black hole formation ($M_\odot$)	
SNH	Maximum mass of stars that undergo SNII events ($M_\odot$)	
SNIA_RATE	SNIa rate (events $M_\odot^{-1}$ Gyr^{-1})	

Table 4: "Stepping software" variables and parameters.

The "stepping software" outputs the value of the parameters C1, T1, D1 and F1 of the model with the lowest χ_ν^2 value from the 23,040 models processed. The results of all 23,040 runs enable 4-dimensional contour plots (represented on a 2-dimensional plane) to be produced and examples of these plots are given in figures 7 and 8 below. Analysis of these contour plots indicates whether the model finds a solution within these four parameters, and the closeness of the contours indicates the size of the uncertainty on the result. As the steps within the arrays C1, T1, D1 and F1 are relatively coarse, further work is required, using manual iterations with the single-run software, to find the actual best-fit model. Note that the GCE model operates in 12-parameter space and the "stepping software" only operates in four of these parameters; for a solution, the model must be fitted within *all* the parameters of the model and therefore a unique solution, if one exists, cannot be found with the "stepping software" alone.

2.2 UPDATES TO THE GCE MODEL

2.2.1 Introduction

The GCE model had previously been used to form general conclusions about star formation mechanisms, by comparing 'toy' galaxies (i.e. 'best guess' generalised input parameters for a given galaxy morphology) to overall observed datasets e.g. Sansom and Proctor (1998) (hereafter SP98), Proctor, Sansom and Reid (2000) and Proctor and Sansom (2002) (hereafter PS02). As described below, the model was enhanced by incorporating more recent data from the literature, to see if this enabled star formation histories of individual observed galaxies to be proposed.

2.2.2 Solar abundances

The solar metal mass fraction $Z_{\odot}$ was originally hard-coded within several subroutines, but not consistently. These were replaced with a single parameter (so that future updates can be made in one place and will then apply across the entire code). $Z_{\odot}$ used by other authors whose results are incorporated in the GCE model were checked, and where the source data used fixed solar mass fractions rather than relative values, the GCE code was updated so that it would adjust the source data appropriately if $Z_{\odot}$ was updated.

Source	**Solar metal mass fraction ($Z_{\odot}$)**
Anders and Grevesse (1989)	0.0189
Grevesse, Noels and Sauval (1996)	0.0174
Grevesse and Sauval (1998)	0.0170
Grevesse and Sauval (2005)	0.0165
Asplund, Grevesse and Sauval (2005)	0.0122
Grevesse, Asplund and Sauval (2007)	0.0120
Asplund, Grevesse, Sauval and Scott (2009)	0.0134
Grevesse, Asplund, Sauval and Scott (2010)	0.0142

Table 5: Updates to generally accepted value of metal mass fraction $Z_{\odot}$.

2.2.3 Planetary nebula yields using Gavilán et al. (2005) and van den Hoek & Groenewegen (1997) results

Intermediate mass stars (initial masses in the range 1-8 $M_{\odot}$) produce carbon, nitrogen and oxygen, released into the ISM via stellar winds and planetary nebula. The GCE model used yields from Renzini and Voli (1981) (hereafter RV81), but more recent models of intermediate star yields are now available and a graphical review of some of the more up-to-date yields with appropriate ranges of mass and metallicity suggested they may provide alternatives to RV81 (figure 3).

Ventura et al. (2002) only gave yields for low metallicity stars, Dray et al. (2003) only gave yields for solar metallicity, and Marigo et al. (1996 and 1998) only covered a small initial stellar mass range (up to $5M_{\odot}$), so these were rejected. Izzard et al. (2004) included the effect of binaries, but these are now known not to have any significant effect on yields (Zhang et al. 2005, Li and Han 2008, Sansom et al. 2009); the effects may have been overstated in the results presented, so were rejected. The GCE code was therefore updated with new subroutines so that results from Gavilán et al. (2005) (hereafter G05) or van den Hoek and Groenewegen (1997) (vdH&G97) could be selected by the user via the '*values.in*' file, as an alternative to RV81.

G05 models, especially at lower metallicities, have smaller relative radii, and hence higher surface gravity for stars of the same mass as those of vdH&G97. This in turn reduces the mass loss experienced by the G05 models due to stellar wind, which will extend their asymptotic giant branch lifetime and consequently these models experience more third dredge-up events, mixing more carbon into the outer envelope, resulting in higher carbon yields for stars < 4 $M_{\odot}$.

The GCE model was run with the two 'toy' galaxies from SP98 and a 'best fit' model of NGC 3226 (PS02) (2.4.3 below); the differences between the results are noted as not material (table 6).

Planetary nebula yields	**'toy' monolithic collapse model from SP98**	**'toy' hierarchical model from SP98**	**Best fit model** **(high star formation and gas inflow for 4Gyrs, then quiescence)**
RV81	220.45	91.22	12.02
vdH&G97	225.20	96.69	11.70
G05	239.04	84.95	11.52

Table 6: χ_{ν}^2 results from the GCE model run with different planetary nebulae yields.

Matteucci et al. (2006) used vdH&G97 in their models of SNIa events and obtained results in agreement with observations from the Galactic Halo, as did

Calura and Menci (2009) with their chemical evolution models. Mattsson (2010) tested models with both vdH&G97 or G05 yields, to investigate carbon production and found using vdh&G97 leads to an overproduction of C/Fe, and that G05 produced better results (although not perfect) for the solar neighbourhood.

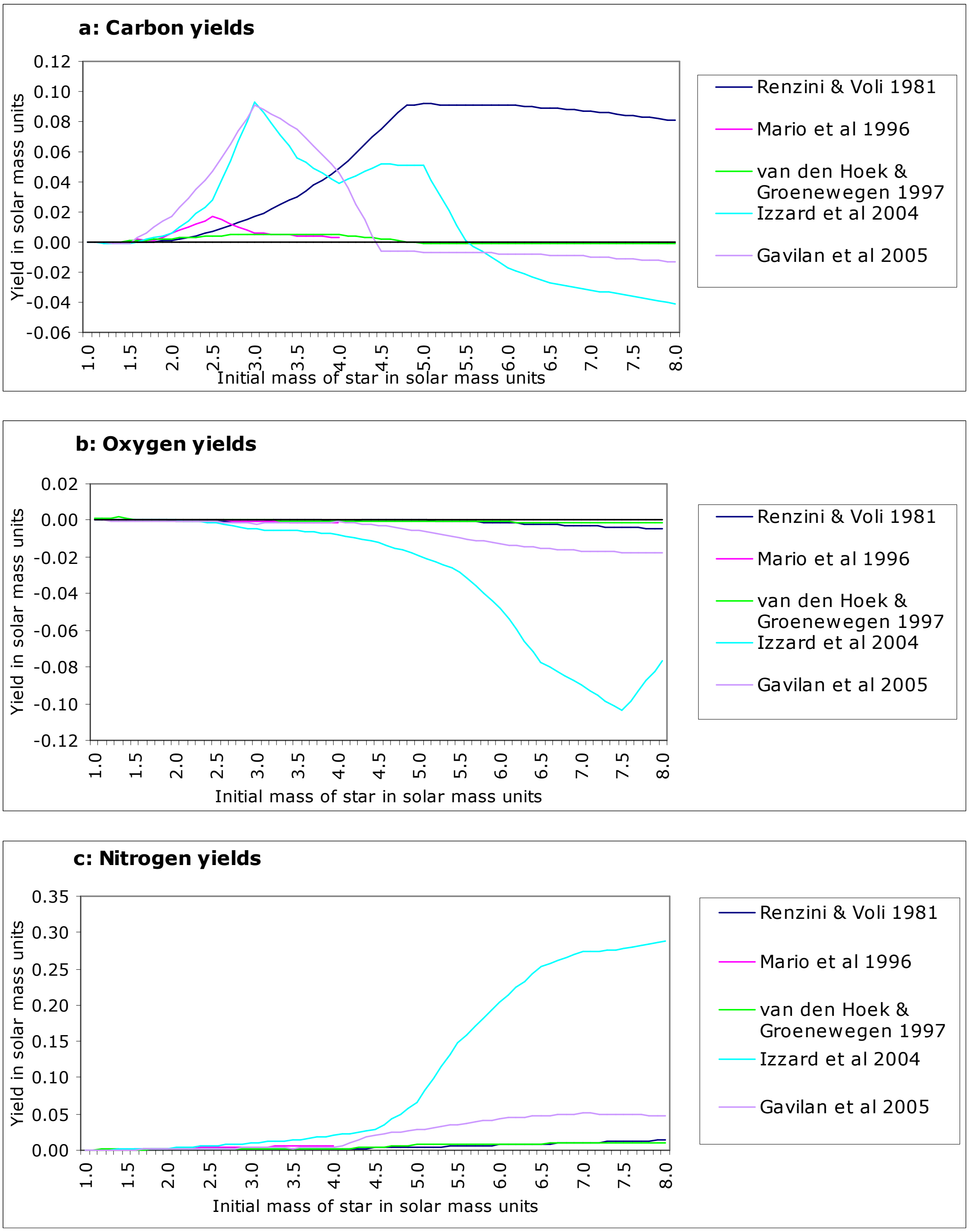


Figure 3: Planetary nebulae yields from different authors at $Z_{\odot}$.

2.2.4 SSP options using Thomas et al. (2004) results

The GCE model had options to use either W94 or V99 SSPs. Thomas, Maraston and Korn (2004) (hereafter T04) give synthetic Lick indices for SSPs at each combination of:

- 20 ages in the range 0.1 to 15 Gyrs;
- 6 metallicities [Z/H] in the range -2.250 to 0.670; and
- 4 values of [α/Fe]: -0.3, 0.0, 0.3 and 0.5.

A new subroutine was written to provide T04 SSPs as an alternative to W94/ V99. The results are read in by the code as a 4-dimensional array, which is then collapsed by successive interpolations to a 1-dimensional array as required for the appropriate age/metallicity/[α/Fe] (see figure 4 below).

The updated GCE model was then tested using the two 'toy' galaxies from table 1 of SP98, running with each SSP option (W95, V99 and T04). A sample of the results for one index, Fe5105, is given in figure 5 below, using non α− enhanced SSPs from T04 but correcting all SSPs for non-solar abundances using TB95, in order to compare like-with-like. These graphs indicate that the T04 SSPs are an acceptable alternative to W94 and V99 SSPs, although from figure 5 it can be seen that W94 give the best fit for this sample index using this model set-up. Note also that this graph supports the findings of PS02 with the GCE model, i.e. that the 'toy' monolithic collapse models considerably under-produce the synthetic indices compared to 'toy' hierarchical models.

Pierce et al. (2005) used T04 SSPs and found enhanced α-element abundances modelled NGC 1052 successfully. Beasley et al. (2005) also updated their SSP models with the results from T04 and found that globular clusters within the Galaxy and M31 were better matched to α-enhanced models, with [α/Fe] ~ 0.4. Gallazzi et al. (2005) compared the T04 results to 3000 models from their library and found that including enhanced α-element introduced systematic errors, overestimating metallicity and underestimating age. Smith (2005) found that the T04 results under predicted the observed slope in plots of Hα:velocity dispersion when compared to 410 galaxies from the observational data of Nelan et al. (2005).

The literature therefore suggests that whilst some individual galaxies may be better modelled as α-enhanced, generally, when averaged over a large sample, galaxies are probably not α-enhanced.

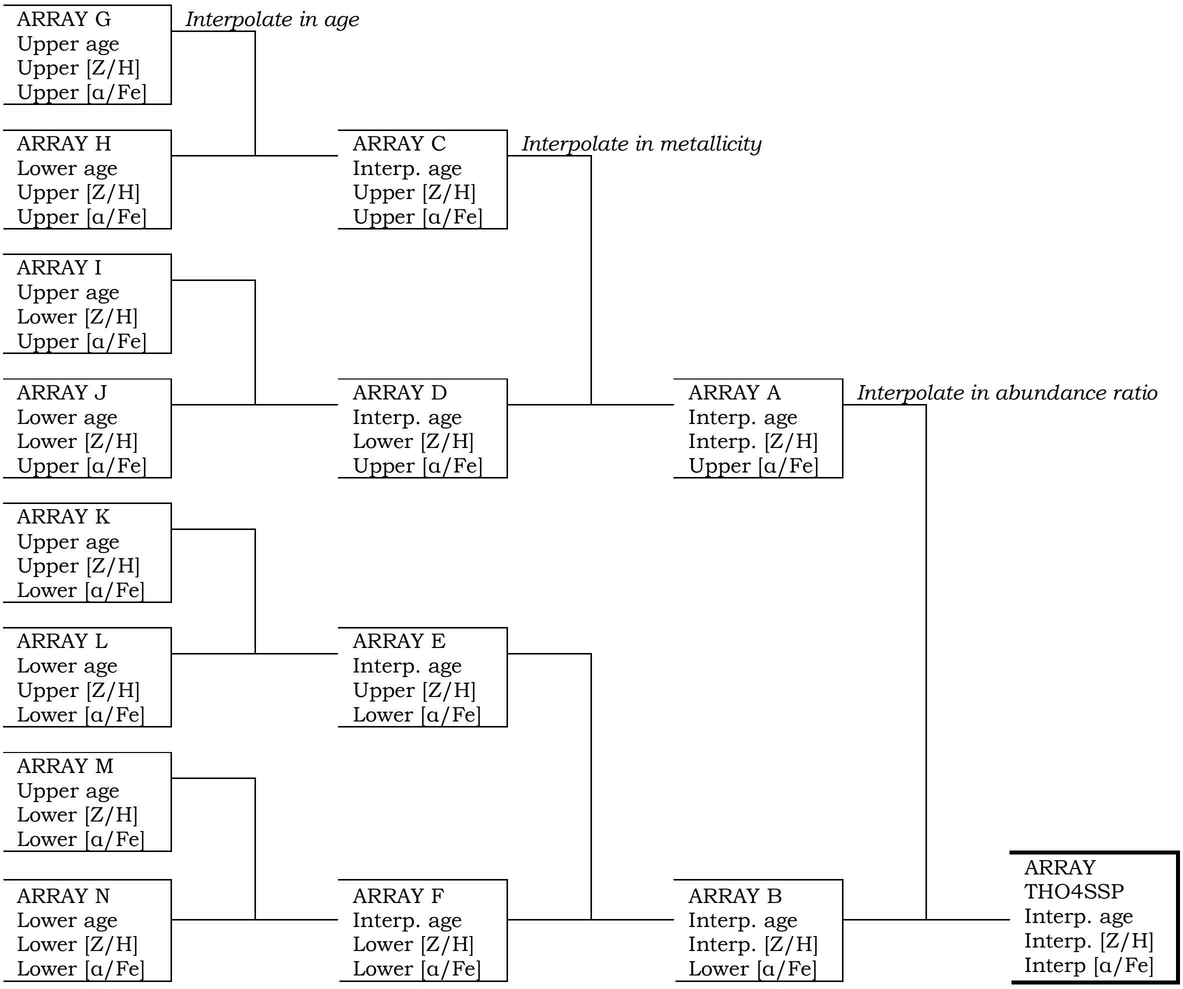


Figure 4: Diagram to show successive interpolations within the Thomas subroutine.

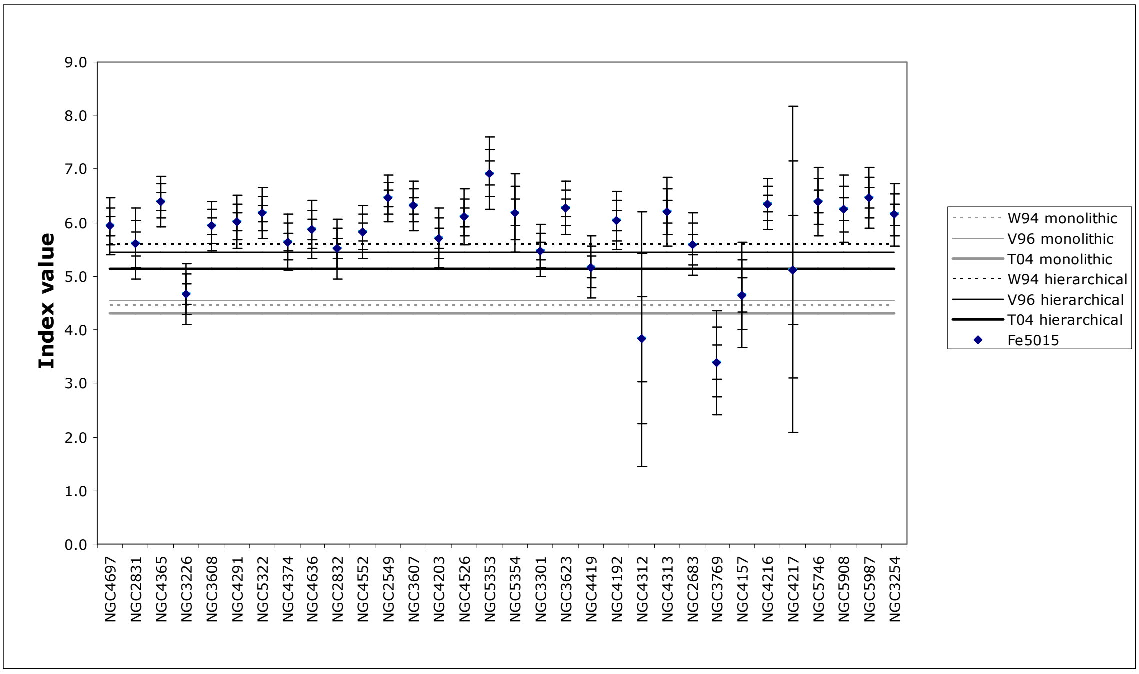


Figure 5: A sample index (Fe 5015) showing the GCE model's results for 6 runs of the GCE model compared with observational data (shown with uncertainties at 1, 2 and 3 standard deviations) from PS02 for that index. Galaxies are in T-type order from left (-5) to right (+4). The GCE model was run using the monolithic and hierarchical 'toy' galaxy parameters from SP98. These two 'toy' galaxies were each run with the three different options for SSPs from W94, V99 and T04.

2.2.5 Lick index responses using Korn et al 2005 results

Korn et al (2005) (hereafter K05) tabulated the effect on each Lick index when individual element abundances were doubled within model stellar atmospheres. This provides an update to Tripicco and Bell (1995) (hereafter TB95), who only investigated the response functions when the abundances were doubled from solar, on a single 5Gyr isochrone, as K05 investigate the effects on a number of different base metallicities and isochrone ages. K05 note that their solar metallicity results are similar to those of TB95. Additionally, K05 included the higher-order Balmer indices Hδ and Hγ. The GCE model uses the TB95 results in the subroutine EMODS, and a new subroutine was written to incorporate the more up-to-date K05 results.

The new subroutine to incorporate the K05 model results into the GCE model consists of three parts:

- Results tables from K05 are read in as 4-dimensional array: star type (cool giants, cool dwarfs and turnoff stars), elements, Lick indices and metallicity.
- The three stellar types at each metallicity are combined using the (fixed) proportions of stars in a galaxy as suggested by Trager et al. 2000 (53% cool giants, 3% cool dwarfs and 44% turnoff stars), to give a 3-dimensional array.
- When called by the programme, this 3-dimensional array is interpolated to give a 2-dimensional array of response functions for each element at the metallicity of the modelled galaxy at the time of the call.

As this is a wider set of results than TB95, it is expected that this enhancement to the GCE model would assist in achieving more accurate results for non-solar metallicities and ratios, in the form of lower χ_{ν}^{2} values, although work by Mendel et al. (2007) did not find this when they tested K05 at very low and very high metallicities with their models. Note that if results from the Geneva Group (e.g. M92, MM02) are used for large and/or massive stars, element abundances will be understated (because the Geneva Group results are only given for carbon and oxygen) – so any TB95/K05 adjustment to indices will be based on incomplete element abundances.

2.3 PROGRAMMING LANGUAGE UPDATE

2.3.1 Fortran 77

The GCE code was written in Fortran 77, which is now out of date.
Fortran 77 uses implicit variables i.e. any variable whose name begins with I, J, K, L, M, or N is automatically defined as an integer, and any other variable automatically defined as real, with a maximum length of 8 characters. The programmer does not need to define the variables but can just start using them within the body of the code, provided the above rules are followed.

This leads to two potential problems for the GCE code:
Firstly: some of the variables have names that are not obvious in their use, either because the "obvious" name would start with the "wrong" initial letter for the variable type, or that severe compaction of the name to fit the maximum number of letters renders it unreadable. It is also perfectly legitimate to use different variable names for the same variable in different parts of the code, or indeed using the same variable name for different variables, but this does risk leading to coding errors (e.g. 3.2.2 where this occurs with volume, mass and density).

Secondly, because this convention does away with the need to formally identify and list variables, typographical mistakes can occur which are not picked up by the compiler, because the mis-typed variable name is just accepted under the implicit naming convention. For example, in the subroutine SIMLOSS, the number of stars is SNSEQC and the average star mass is SMSEQC; not only are the variable names difficult to interpret when reading the code, a typographical mistake can easily occur if these variables are used elsewhere.

2.3.2 Fortran 90/95

The GCE code was converted into Fortran 90/95, which required all the variables to be collated in a separate programme file ('*shared.f90*'), which in turn enabled their uniqueness to be checked, and also provides a convenient "dictionary" for the code. The code instruction IMPLICIT NONE was added to each subroutine, which instructs Fortran to only use variables that are formally defined. Some naming conventions were updated in order to improve the readability of the code.

There are still some precision limitations with Fortran 90/95, due to the maximum length of a number that can be held by the programme. Where the number is too long, Fortran truncates it. For example, on a 32-bit machine, if the model calculates the mass of gas to be 1,234,567,891,234 $M_{\odot}$, it stores this value as 1.23456E12, and then uses 1,234,500,000,000 for any subsequent calculations. This issue means the model galaxy has (in this example) effectively "lost" 678,912 $M_{\odot}$ (in the order of 10^{-5} %).

Syntax identified as obsolete, or likely to become obsolete in future versions of Fortran, was removed.

2.4 USING THE ENHANCED GCE MODEL TO PROPOSE STAR FORMATION HISTORIES OF NEARBY ELLIPTICAL GALAXIES

2.4.1 Introduction

The updated model was tested against two galaxies, NGC 4217 (a spiral bulge) and NGC 3226 (an elliptical galaxy), from the PS02 sample, initially using the "stepping software" version of the GCE model to see whether, within the 4-dimensional parameter space modelled, there was a unique solution. If a solution was found, the single-run GCE model would then be used to find the best fit iteratively.

The GCE model operates in 12-parameter space; these parameters are entered by the user in a file '*values.in*' and are detailed in table 7 below. The "stepping software" version of the model sequentially overwrites 4 of these values (see table 4 above) and runs the model with different combinations of these four parameters, measuring the χ_ν^2 for each combination and reporting back the minimum value found. In addition, the χ_ν^2 results can be plotted to indicate whether this was a single minimum over the parameter space searched, or just the lowest of a number of minima.

Input model choices (with the variable name from the GCE model)	Value
Number of radial ranges (NRR)	1
Index in Schmidt star formation rate equation (AL)	1.0
Index in Salpeter initial mass function equation (AM) (negative sign added within the code)	1.35
Critical density above which stars can form (RCRIT)	0.0
Massive star data weighting (1.0 = WW94, 0.0 = M92 (FLOSSLIM)	1.0
Selected IMF (S=Salpeter, M=Modified) (TYPEIMF)	S
Source of SSP data (W=94, V=V09, T=T04) (SSP DATA)	T
Source of planetary nebula data (RV=RV81, GA=GA05, VG=VG97 (DATAIMS)	GA
Initial mass fraction of hydrogen (X0)	0.7718
Initial mass fraction of helium (Y0)	0.2280
Initial mass fraction of metals (Z0)	0.0002
Is inflow enriched (Y) (= same composition as ISM) or primordial (N) (RICH)	Y
Minimum timestep used by model in Gyrs (DTMIN)	0.10

Table 7: Model set up in '*values.in*' to find the SFH of NGC 4217 and NGC 3226.

2.4.2 Spiral bulge NGC 4217

Spiral bulge NGC 4217 had a χ_{ν}^{2} value of 1.20 when the 'toy' hierarchical galaxy of SP98 was run with W94 SSPs and RV81 yields for planetary nebulae. Testing with the "stepping software", using the code updates of T04 SSPs and G05 yields for planetary nebula found that the solution, whilst still good, was certainly not unique, as figure 7 shows, and figure 6 indicates that the large uncertainties in the data would be the reason for the low χ_{ν}^{2} found because a number of different solutions to the 4 parameters would be expected to fit within these large uncertainties.

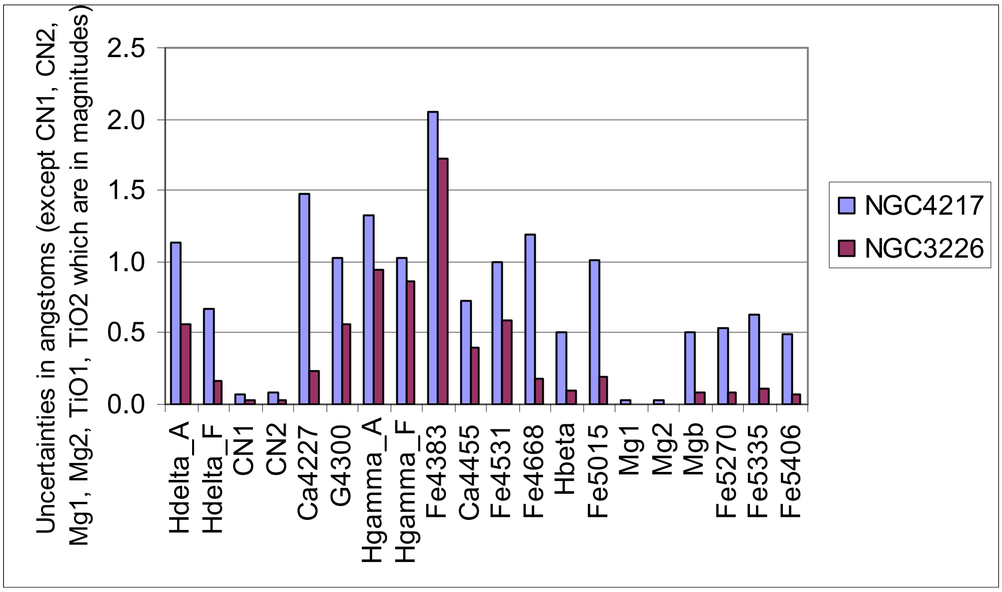


Figure 6: A comparison of the size of the uncertainties for NGC 4217 and NGC 3226, showing the comparatively larger uncertainties (and consequently the large number of solutions to the SFH in four dimensions) for NGC 4217.

Note the extremely small uncertainties on the Mg1 and Mg2 indices, which may make it difficult to find *any* acceptable fit to this data set.

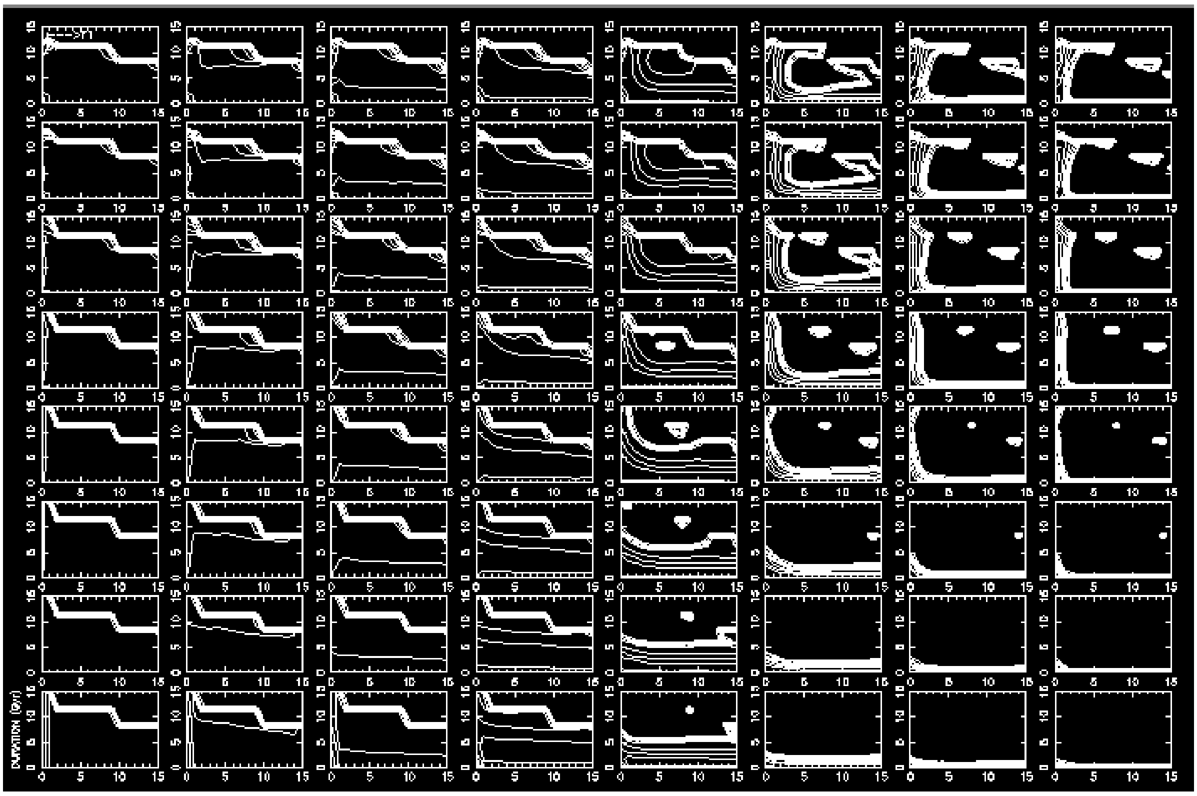


Figure 7: Four dimensional parameter space, represented in two dimensions, for the "stepping software" version of the GCE model, run with NGC 4217, T04 SSPs and G05 yields for intermediate mass stars.

Each small graph plots the time in Gyrs after TCHANGE1 (D1), where flow and star formation rate are altered to the values R00C2 and FLOWRATE2 (which are not varied) (12 values from 0.0 to 15.081 Gyrs) (y axis) against the time in Gyrs after start of model where flow and star formation rate are altered to the values R00C1 and FLOWRATE1 (T1) (30 values from 0.0 to 14.0 Gyrs) (x axis).

The graphs form a larger grid, with the revised flow rate of gas into the galaxy, in $M_\odot$ per Gyr after TCHANGE1 (F1) increasing along the horizontal with 8 values (from 5 x 10^4 to 5 x 10^7 $M_\odot$ Gyr^{-1}) = 8 graphs, and the revised constant in the Schmidt star formation rate equation after TCHANGE1 (C1) increasing down the vertical with 8 values (in the range 0.03125 to 4.0) = 8 graphs.

If the solution found for a model run with the "stepping software" is within 3 standard deviations, a point is plotted on the appropriate graph, so any points plotted indicate a region where a solution may be found. Large or multiple areas plotted indicate many solutions have been found (compare to figure 8 where a solution is shown to exist within a small area of the parameter space plotted).

2.4.3 Elliptical galaxy NGC 3226

The GCE model was set to run the "stepping software" using the T04 and G05 yields as above, but testing the output against the elliptical galaxy NGC 3226.
The parameter space that resulted is as plotted in figure 8, indicating that, within the four parameters being searched, there *was* a unique solution (compare to the output for NGC 4217 in figure 7 above). The actual minima found with the "stepping software" was 124.76 see table 8 column 2 below. These results then gave a framework against which more detailed iterative searching using the single run software could be carried out (the steps in the "stepping software" code are quite coarse). With the iterative single runs, the lowest χ_{ν}^{2} value was 14.66 (table 8 column 3 below). Uncertainties on this galaxy are smaller than those of NGC 4217, as shown in figure 6 above. Note however that the parameter space search is confined to four dimensions, so this result does not prove a unique solution, as the GCE model has 12 parameters.

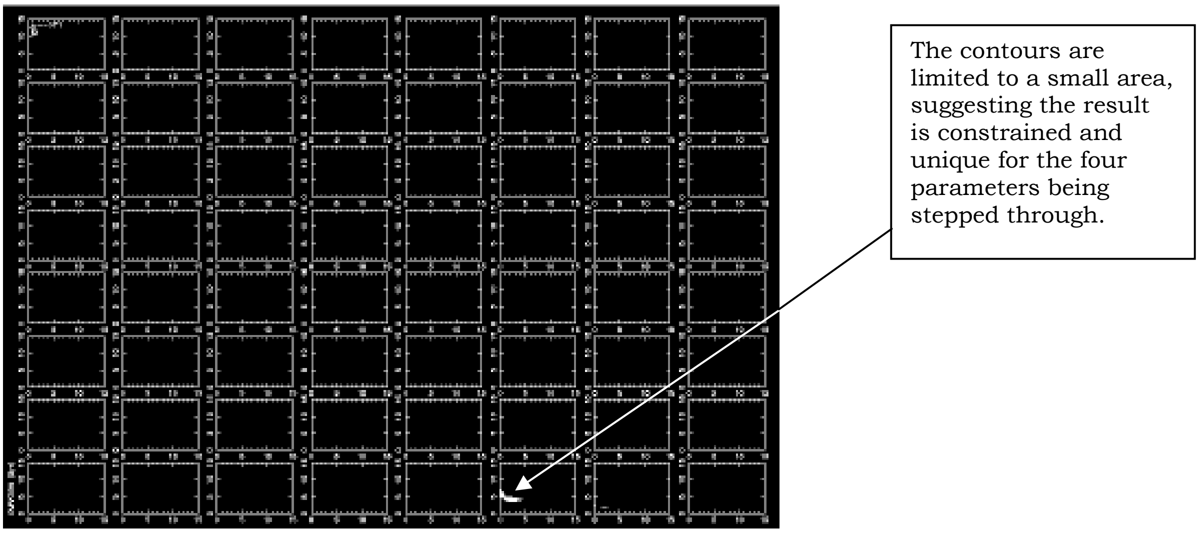


Figure 8: Parameter searching for NGC 3226 with parameters as in table 8 column 2. Axes as given in figure 7 above.

The results from figure 8 enabled further searching using the single–run model as detailed in table 8 column 3, giving an overall χ_{ν}^{2} of 14.66. These results suggest that the star formation history of NGC 3226 is as shown in figure 9 and can be described as:

The SFR "efficiency parameter" – the constant in the Schmidt star formation equation (equation 6) – starts at 5.0 and is reduced to 4.5 after 0.5 Gyrs (note 1): initially very efficient star formation, reducing slightly after a short period of time. At this point, gas starts flowing into the galaxy, at a rate of 10^6 $M_\odot$ Gyr^{-1} for 8 Gyrs (note 2): a long period of merger with enriched gas (gas with the same chemical composition as the galaxy being modelled). When the galaxy is 8.5 Gyrs old, the merger event ceases and the star formation rate falls to zero (note 3). The overall life of the galaxy is 12 Gyrs.

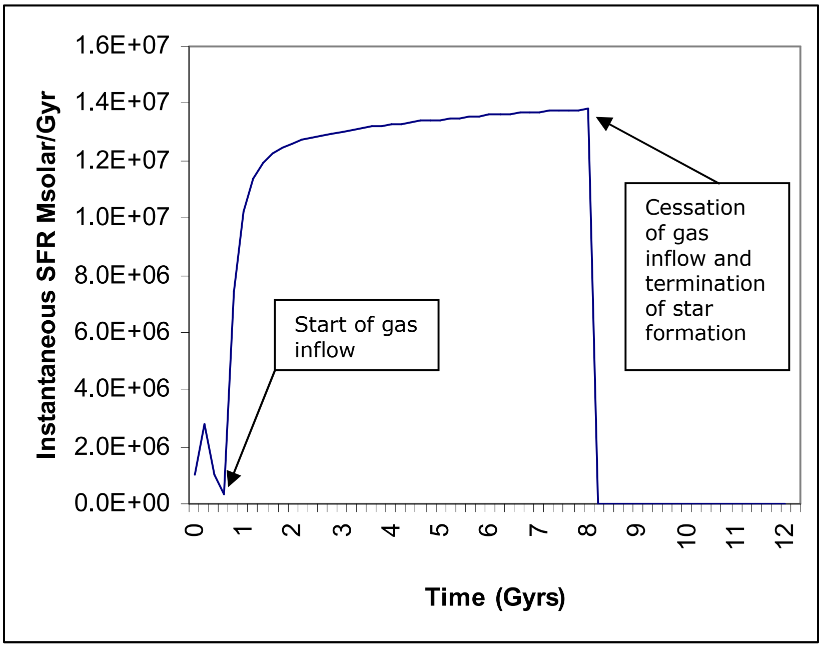


Figure 9: Star formation history of NGC 3226 as modelled by the GCE.

Note 1: The first change in the SFR is between 0 and 4 Gyrs (small x-axis) – the actual minima found by the "stepping software" is at 0.5 Gyrs, and this is also the best fit value found using iterative searching with the single-run GCE model.
The SFR "efficiency parameter" at this time is found to be 4.0 with the "stepping software" version of the model (the contours are in the lowest vertical grid (i.e. large y-axis) and 4.5 with the single run version.

Note 2: the contours are in the 6th horizontal grid (i.e. large x-axis); the 6th bin for FLOWRATE1 = 10^6 $M_\odot$ Gyr^{-1}. Gas flow lasts at this rate for 5.003 Gyrs (small y-axis), i.e. stops after (5.003 + 0.5) = 5.503 Gyrs. With the single-run software, and with the overall lifetime of the galaxy reduced to 12 Gyrs, the actual minimum was found to be at 8 Gyrs.

Note 3: the GCE does not model gas flowing out of the galaxy (further discussed in section 3.2.8), but instead the user can set the star formation efficiency parameter ROOC2) to zero at TCHANGE2, mimicking the point at which star formation ceases.

Grey shading in table 8 indicates the parameters that were notably different between the "stepping" and single run versions. This suggests that the 4 parameters selected for searching with the "stepping software" are not necessarily the most critical, as varying the parameter TIME (the life of the galaxy) was found to have the largest effect on the value of χ_ν^2.

Input model variables (12 parameters) set in '*values.in*'. Grey highlights mark those that particularly varied between the "stepping software" and the single-run iterative searches.	**"stepping software" best fit results**	**Iterative test with single run GCE model**
Constant rate of SNIa formation in events per $M_\odot$ per Gyr (SN1A_RATE)	3.8E-05	3.8E-05
Upper limit of CO core for black hole formation in $M_\odot$ (BHMASS)	20.0	20.0
Maximum mass of stars in $M_\odot$ that undergo SNII events (SNH)	70.0	70.0
Total life for the galaxy in Gyrs (TIME)	17.0	12.0
Initial constant in the Schmidt star formation rate equation (R00C0)	5.0	5.0
Initial flow rate of gas into galaxy, in $M_\odot$ per Gyr (FLOWRATE0)	0.0	0.0
Time in Gyrs after start of model (TCHANGE1) where flow and star formation rate are altered to the values set below R00C1 and FLOWRATE1	Step: minima at 0.5	0.5
Revised constant in the Schmidt star formation rate equation (R00C1) after TCHANGE1	Step: minima at 4.0	4.5
Revised flow rate of gas into the galaxy, in $M_\odot$ per (FLOWRATE1) after TCHANGE1	Step: minima at 10^6	10^6
Time in Gyrs after start of model (TCHANGE2) where flow and star formation rate are altered to the values set below R00C2 and FLOWRATE2 (on "stepping software", this is the duration of the starburst_	Step: minima at 5.003, so TCHANGE2= 5.003 + 0.5 = 5.503	8.0
Revised constant in the Schmidt star formation rate equation (R00C2) after TCHANGE2	0.0	0.0
Revised flow rate of gas into the galaxy, in $M_\odot$ per (FLOWRATE2) after TCHANGE2	0.0	0.0
χ_ν^2 results when compared to NGC 3226	124.76	14.66

Table 8: Model selection and best-fit parameters used together with χ_ν^2 results when testing alternatives for NGC 3226.

2.4.4 Conclusions

At first glance, it seems that a well-constrained model of the star formation history of a single galaxy can be obtained using the GCE model, *provided* the uncertainties on the observational data are neither too large (otherwise many models can be fitted) nor too small (difficult to simultaneously fit all indices), as a small single area of contours are found on the output plot from the "stepping software" version of the GCE model when used with NGC 3226 (figure 8), whereas the larger uncertainties on the observational data for NGC 4217 (as shown in figure 6) allow many solutions to be found (figure 7).

Further work with the single-run version of the GCE model suggests that NGC 3226 had exceptionally efficient star formation for the first 8 Gyr, followed by 4 Gyr of no star formation. Pre-enriched gas (i.e. with the same chemical composition as the ISM of the model galaxy), assumed to be associated with mergers, infalling at a rate of 10^6 $M_\odot$/Gyr, started when the galaxy was 0.5 Gyrs old and lasted for 7.5 Gyrs.

However, this solution *cannot* be considered unique, because only 4 parameters were stepped through, whereas the GCE has 12 parameters, so there may be other areas within parameter space where the model produces reasonable results.

This solution also *cannot* be defined as good, because the value of χ_ν^2 found by the "stepping software" is still high, and even when a lower value can be obtained by manually varying some of the other parameters using the single-run model, it is not sufficiently close to unity, as defined as a 'good model' for this statistical measure (equation 8).

No test was undertaken to see if this was a unique model in 12-parameter space.

2.5 DISCUSSION

This Chapter has discussed the updating of the data and the Fortran coding used by the GCE model, followed by use of this enhanced model to propose the star formation histories for two galaxies from the PS02 sample. It was found that where the uncertainties on the observational data were large, the GCE model was able to find many well-fitting models (low χ_ν^2). In addition, the extremely small uncertainties on the Mg1 and Mg2 indices, which are not representative of the expected uncertainty in the data due to instrumentation at the WHT (William Herschel Telescope) (see discussion in section 6.1.2), may make it difficult to find any acceptably-fitting model to this data.

Major code enhancements using more recent results from the literature were written. These now give the user the ability to compare the effect on the results of the alternative data sets. As the GCE outputs were not significantly altered when using the updated literature, it can be concluded that any uncertainties within these input data are minor; giving important reassurance in this area.

The GCE model is not set up to test whether the solutions found are unique within the 12-parameter space in which it operates. A star formation history may be found that is unique within the 4-parameter space used by the "stepping software" version of the model, but this does not confirm a unique model. Testing in this area showed that a better result might be obtained by altering other parameters, notably the overall age of the galaxy (TIME).

As this work developed, it became apparent that there were a number of errors and limitations within the GCE code, and the difficulty of proposing individual star formation histories of galaxies may therefore not be a consequence of the model using out-of-date data from the literature, particularly as updating the literature references in the model did not notably alter the model outputs.

Generally, code errors and limitations fall into a number of categories:

1. Compile-time, run time or Fortran syntax errors (programming errors). These have to be cleared by the programmer before the code can be run.

2. Inaccurate use of mathematics, physics or astrophysics, or typographical mistakes.
3. Limitations due to out of date external data sources, or data sources incorrectly applied.
4. Poor assumptions or over simplification of the processes used to evolve the model galaxy.

In addition, the observational data against which the synthetic indices produced by model are compared may be wrong:

5. Mis-reported observational data, for example, the datum and uncertainty on the datum being exchanged, missing minus signs and/or having the decimal point in the wrong place.
6. Uncertainties on observational data incorrectly calculated, or not adjusted to include both systematic and equipment errors.

2.6 CONCLUSIONS

Updates to data sources used by the GCE model using new results within the literature would be expected to enable the GCE model to produce more accurate star formation histories for individual galaxies. Updates to yields for planetary nebulae, to SSP results and to response functions for non-solar abundances were incorporated and tested. These updates were then used to check whether unique star formation histories for two galaxies from the PS02 sample could be obtained.

It was found that the influence of the size of the uncertainties on the observational data prevented a unique solution from being found for the spiral bulge NGC 4217. Whilst the 4-parameter space being searched using the "stepping software" version of the model found a unique solution for NGC 3226, it was noted that the "stepping software" does not check whether the solution is unique across all 12 parameters used by the GCE model.

The overall life of the galaxy, which is not a variable checked by the "stepping software", was found to be a significant factor in the fitting of the GCE model to an observed galaxy. The duration of the gas inflow was also found for the test galaxy to be considerably longer than the pre-set values that the "stepping software" uses, at 7.5 Gyrs rather than 5.0 Gyrs.

The GCE model could be amended to step through all 12 parameters, which could be set to cover a larger range of values, or, more ideally, reprogrammed to undertake non-linear parameter optimisation. However, this was not considered to be a practical solution.

Following the above work, the GCE model was critically reviewed in detail, analysing errors and limitations, and this discussion is presented in Chapter 3.

Some of the new subroutines developed in the present Chapter for the GCE code were re-used within a new model, Phoenix, which is described in Chapter 4, tested in Chapter 5 and used to model star formation histories of nearby elliptical galaxies in Chapter 6.

CHAPTER 3: DETAILED CRITIQUE OF THE GALACTIC CHEMICAL EVOLUTION MODEL

3.1 INTRODUCTION

This Chapter gives a detailed review of the assumptions in the Galactic Chemical Evolution model, identifies some errors and establishes whether these are significant. All computer models will include some degree of simplification and assumptions, and these are reviewed to determine whether they impact on the results. A review of the papers published using this model is also given.

3.2 REVIEW OF PHYSICS AND ASTROPHYSICS USED IN THE GCE MODEL

3.2.1 Introduction

Simplifications and assumptions in the physics and astrophysics within the GCE model are examined, and assessed for whether they affect the ability to model the star formation of individual galaxies. Limitations and inaccuracies are identified and the impact of these is assessed.

3.2.2 Model galactic mass and density

The value 10^6 is initially hard-coded against the variable ROO, noted to represent the "initial density in solar masses per unit volume", and was probably used because this is the value W95 use in their models, and is typical for a large globular cluster. However, this variable is used interchangeably within the GCE code as

- mass of the gas;
- mass of the stars; and
- density of gas in the galaxy.

This variable is then updated when any of these values are updated. This means that when gas is flowed into the model galaxy, the value of ROO (if it should be density) increases incorrectly, as $M_{\odot}$ Gyr^{-1} is added to $M_{\odot}$ unit volume^{-1}, without any amendment to volume. For example, if a total of 10^6 $M_{\odot}$ of gas flows in, the code treats this as the density doubling (10^6 initial + 10^6 additional) so when the value of ROO is later used in the Schmidt (1959) star formation equation doubling the mass of the galaxy more than doubles the star formation rate (2.6 times if the Schmidt index is 1.4).

Because the Schmidt (1959) star formation equation in the code uses the variable ROO as the density of the galaxy, and because this erroneously becomes excessively high by being updated when the galaxy mass increases, far too many stars are produced in each timestep. A Salpeter (1955) IMF gives 26% of the total mass of stars produced in each timestep as between 8 and 70 $M_\odot$, which are then exploded as SNII in the same timestep; a problem if there are too many stars being produced, as this leads to excessive enrichment ($Z_{final} \approx 8\ Z_\odot$). This was not noticed because the overall metallicity of the galaxy is not a model output and as the code uses the nearest value (generally solar or slightly super-solar) when taking data from yield/ejecta tables and SSPs (see 3.3.3 below), it is possible to produce apparently reasonable synthetic indices from an unreasonable model galaxy.

It is of course possible within Fortran to use the same variable name for different variables in different subroutines, although this risks leading to confusion when working with the code, as has happened here. This oversight has lead inadvertently to unrealistic physical parameters, which are hidden in the model output, because the code uses the nearest values from data tables (see 3.3.3 below) – so excessively high metallicity is "pulled back" to use (generally) solar data - and does not output the values it is holding for mass, density or metallicity which could have indicated the problem.

3.2.3 Critical density set as zero

The value RCRIT, the critical gas density below which stars would not be formed, is generally set to zero within the GCE parameters when the model is run. Within the GCE model, coding exists to only make stars if the galaxy gas density (ROO) is above the critical density – which, if critical density is zero, and because ROO is erroneously updated with increases in mass, stars will always be made.

The user can set the star formation rate equal to zero at some point in the galaxy's lifetime, to mimic the move from active to quiescent galaxy evolution.

However, the model does not remove any remaining gas (to mimic galactic winds) and as the gas density at that time could be above the critical density (and indeed will be if the critical density is set to zero), this is physically incorrect, although this would not affect the synthetic Lick indices the model produces.

3.2.4 Calculation of main sequence lifetimes

In the subroutine SEJECT, which calculates the yields/ejecta from SNII events, the equation from Wood (1992) is used to calculate main sequence lifetimes, but this equation was only intended for low and intermediate initial-mass stars (up to a limit of $8M_\odot$).

Pre-white dwarf lifetime $t_{MS} = 10\ (M/\ M_\odot)^{-2.5}$ Gyr (9)

It is therefore not strictly valid to use it in a subroutine dealing with more massive stars, but the equation, when used for these more massive stars, gives lifetimes of less than one timestep provided the timestep is >~ 0.06 Gyr; for the instantaneous mixing assumption to hold (3.3.7 below), the minimum timestep should be no lower than ~0.1 Gyrs, and hence using this equation for larger stars is acceptable.

3.2.5 Modelled initial conditions

The GCE model starts as gas with no stars, but the initial density of that gas at model time T = 0 is considerably above the critical density – which means stars must have started forming at T < 0. Stars would have formed and evolved prior to the point at which the model starts and the gas would have been enriched by these earlier generations of stars. This means the metallicity of the galaxy will be understated, and the time set for the overall galaxy life will be shorter than the actual lifespan of the galaxy.

The subroutine which models SN1a takes star mass from the timestep 0.3 GYrs previously, and as the model starts with just gas, there are no modelled SN1a events until T = 0.3 Gyrs. However, as the critical density would have been exceeded at T < 0, and stars would have been formed at these earlier periods, SNIa would also be expected to occur in what the model considers the first 0.3 Gyrs. This means that elements produced in SNIa events will be understated.

3.2.6 Variable timesteps

Within the subroutine GETMT, which evolves the model galaxy, there is a section which ensures that the timestep, set by the user as DTMIN, aligns to the points at which there are changes in star formation rates and/or gas inflow rates (TCHANGE1 or TCHANGE2). For example, if DTMIN is set at 0.3 Gyrs, and TCHANGE1 is set at 1 Gyr, then TCHANGE1 will occur part-way through a timestep. The code deals with this by introducing shorter "partial timesteps". Whilst some of the code adjusts for this partial timestep (for example, fewer stars are made), other parts of the code are not adjusted - the new stars that will evolve as SNII are fully evolved in this partial timestep, and return the enriched material to the modelled ISM. This enriched material is used to form the stars in the next partial timestep – which means that a model with 2 x 0.1 Gyr timesteps will have a different chemistry from one with 1 x 0.2 Gyr timesteps and consequently different indices will be produced. These partial timesteps may potentially also invalidate the instantaneous mixing assumption (3.3.7 below).

Additionally, the value of timestep DTMIN is overwritten as 1.0 Gyr if the star formation rate is set to zero (as an alternative to modelling gas outflow, see 3.2.8 below). This saves computer time, but means that the model loops for a different number of timesteps depending on when the star formation rate becomes zero. Each loop creates additional chemistry since even if the galaxy is quiescent, it will still undergo SNIa and planetary nebulae events; these are calculated per timestep irrespective of the length of that timestep. There is no adjustment elsewhere in the code for the varying timesteps, so the final metallicity will be lower than if the timesteps had been constant at DTMIN.

3.2.7 Luminosity weighting of the SSPs

The subroutine WEIGHTBI collates the SSP results for all previous timesteps up to and including the current one, adjusts them for non-solar abundance ratios, and normalises these to the total luminosity. The code does not make adjustments to remove results for stars that no longer exist but did exist in the earlier timesteps. Larger stars will have greater luminosity than smaller stars and so should dominate the overall integrated indices observed, but because isochrones are not used to weight the indices, this effect is not accounted for.

Instead, a notional weighting is applied, based on the proportion of indices in each of the blue, visible and red areas of the spectrum. This will result in the indices in the red area of the spectrum dominating, as there are larger stars from earlier timesteps, which shouldn't be included but are, mitigated to some extent by their greater luminosity not being accounted for.

Note that the GCE model does not calculate the indices directly from the elements produced by the evolutionary processes, but uses the metallicity of the ISM to indicate the appropriate SSP. Where the metallicity has become excessive, see 3.2.2 above and 3.3.3 below, the nearest value (solar) is used. The tracked elements are only used to correct the SSPs for non-solar abundances using TB05 results.

3.2.8 Gas inflow and outflow

The GCE model simulates a galaxy merger as an inflow of gas, however, the model takes no account of gas lost from the galaxy due to galactic winds. Galactic winds, removing the gas from an elliptical galaxy, is the mechanism thought to "turn off" star formation (Gibson 1997). As the GCE does not model gas outflow, the "turning off" of star formation is achieved by the user setting the star formation rate to zero, irrespective of whether there is sufficient gas in the galaxy for stars to continue to form. This simplification would be acceptable, as it should not affect the modelled indices, provided yields from events that take place after the star formation process stops are not used to alter the overall metallicity of the galaxy (used to select SSP data) or affect the adjustment for non-solar abundances. Unfortunately, the model does not "switch off" these updates when the star formation ceases, so this simplification is not reasonable.

3.2.9 Equation used for supernovae Ia rate

Supernovae Ia arise from white dwarf stars interacting with a companion star: either accreting material from a larger binary companion, or merging with the companion, reaching a critical mass, and exploding (Branch et al. 1995, Scannapieco and Bildsten 2005).

The GCE model allows the user to set a single constant rate for SN1a, with the default value of 3.8×10^{-5} events Gyr^{-1} $M_{\odot}^{-1}$, with a time-lag of 0.3 Gyrs, quoted as being from Timmes et al. (1995), who give an observed present day value of 0.53 events century^{-1}, for the Galaxy (mass 1.7 x 10^{11} $M_{\odot}$); the calculated value should therefore be 3.1 x 10^{-5} events Gyr^{-1} $M_{\odot}^{-1}$. With more up-to-date values for the mass of the Galaxy (e.g. 6.43 x 10^{10} $M_{\odot}$: McMillan 2011) the value would be 8.83 x 10^{-5} events Gyr^{-1} $M_{\odot}^{-1}$. An alternative value for elliptical galaxies is 0.12 events per century per 10^{10} $L_{\odot}$ (Turatto et al. 1994). The impact of these differences is not significant.

3.2.10 Correction of mass fractions

When the model has made the new stars from the ISM, and evolved them, there is a short section of code to update the mass fractions of X (hydrogen), Y (helium) and Z (metals) to their new values. Calculation of the mass fraction of X is incorrect, using a mixture of both masses and mass fractions. This error went unobserved because this section is followed by consistency check to ensure that X+Y+Z is always =1, by adjusting X as the balancing number. No warning is given to the user if any non-trivial adjustment to X is made: by using mass rather than mass fraction, the adjustment to X is material each time.

3.2.11 Adjusting the Mg indices

The subroutine DFACT always returns a null result; removing the call to this subroutine does not alter the model output. This subroutine has been written to adjust the Mg2 index using results from Barbuy (1994) in an attempt to deal with the poor modelling of the magnesium indices (which actually arise from the source observational data, not problems with the GCE model: see 6.1.2). DFACT was also found to have the following coding/typographical mistakes, which were corrected, which ensured that when the call to the subroutine was made, the returned result was not zero:

- LOG (natural logarithm) used instead of LOG10 (logarithm to base 10);
- hard-coded value of [Mg/Fe] not updated when updated solar values for Mg and Fe abundances were updated (see 2.2.2 above); and
- the IF loop to check whether the value of [Fe/H] was within the tabulated range was missing.

3.2.12 Evolution of stars

The GCE model takes the material held in stars calculated using a Salpeter IMF, and evolves them, depending on the initial mass of the stars as shown in table 9.

Stars	Process	Data	Subroutines in the GCE code	Notes/issues
All material held as stars	SNIa	Nomoto et al. (1984)	Ejecta hard-coded within GETMT (the main subroutine calculating the exchange of mass between stars and the ISM in each timestep).	Delayed evolution (i.e. stars from a prior timestep) evolved at a constant rate set by the user, generally set as 3.8 x 10^{-5} events Gyr^{-1} $M_{\odot}^{-1}$. Ignores whether this material would have followed another evolutionary path.
0.6-8 $M_{\odot}$	Planetary nebulae	Renzini and Voli (1981)	READRV – to read in the yields data EJECT – interpolates the yields data for the current metallicity for all masses SIMLOSS – select the yields based on the median star mass from this range (NOT weighted by the IMF), from the tables interpolated by metallicity.	Only uses the median value for the range of 0.8-8.0 $M_{\odot}$ – so not utilising the RV81 data in full.
8-40 $M_{\odot}$	SNII (large stars)	Either Woolsey and Weaver (1995) or Maeder (1992), or a combination of them both.	READSNII – to read in the ejecta from Woolsey and Weaver (1995). READIF – to read in the yields from Maeder (1992). AMODIFY – combines the data from WW95 and M92, weighted by FLOSSLIM to one table. EJECT – interpolate the yields data for the current metallicity for all tabulated masses SEJECT– select the yields based on the IMF weighted mass fraction from the tables interpolated by metallicity.	If the user sets the variable FLOSSLIM to 0.0, the model selects yields from Maeder (1992), if 1.0, ejecta from Woosley and Weaver (1995), if a value between the two, takes a weighted mixture of both data sets, even though the assumptions and results in these papers are very different. SEJECT uses the equation from Wood (1992) to calculate the mass range of stars ending their main sequence lifetime.
40- limit set by user (max 120 $M_{\odot}$).	SNII (massive stars)	Maeder (1992), Meynet and Maeder (2002)	READIF – the upper 4 values of data from Maeder (1992) were overtyped with data estimated from graph 19 of Meynet and Maeder 2002 EJECT and SEJECT as for 8-40 $M_{\odot}$ SNII	Maeder (1992) yields for very massive stars were identified by his group as being too high in the Meynet and Maeder (2002) and the revised figures were overtyped into READIF.

Table 9: Processes to calculate updates to elements from different initial star masses in the GCE model.

3.2.13 Yields and ejecta

Within the GCE model, no adjustment is made for the data from Nomoto et al. (1984) and WW95 being ejecta and RV81 and M92 being yields, indeed, WW95 and M92 data can be combined using a weighting factor FLOSSLIM (discussed further in 3.3.4 below).

SNII are a major source of, for example, magnesium, which should be a key measure for the accuracy of the model (as expected values of [Mg/Fe] are known), however, as M92 only provide details of carbon and oxygen yields, data on elements other than these two cannot be updated and any weighting for non-solar abundances using elements tracked by the code will be inaccurate.

Additionally, a typographical mistake in the subroutine SIMLOSS, which calculates the amount of material returned to the ISM from intermediate mass stars undergoing planetary nebulae, sets the upper mass limit for these events to the variable SNH which is the upper limit for SNII events. This typographical mistake resulted in excessive yields of carbon, nitrogen and oxygen, which went unnoticed because the model does not include control checks on these values.

3.3 REVIEW OF 'RANGE EXCEEDED' PROBLEMS, EXTRAPOLATION/ INTERPOLATION ASSUMPTIONS, AND MODEL SIMPLIFICATIONS

3.3.1 Introduction

The GCE model uses published results from other authors for yields, indices, etc. Where the GCE model requires data that is outside the range published, the model takes the nearest value that is within the published results (i.e. the lowest or highest value in the provided table, as applicable). This assumption is also used by Kotulla et al. (2009) for their GALEV models. This coding assumption has led, however, to significant errors in the GCE model going unnoticed (3.3.3 below), whereby the model can produce reasonable results from unreasonable models. This section also includes a discussion of the use of extrapolation and interpolation, and a review of some of the simplifications used by the GCE model.

3.3.2 Interpolation and extrapolation assumptions

When the GCE model requires data that is not exactly matched within the published data tables, linear interpolation is used between adjacent results, using the highest or lowest value if the data point required is outside the data available (rather than extrapolating the data). Linear interpolation within a data table would not be expected to lead to significant errors, as the authors of these papers are aware that this is how their results will be used and generally provide more data points around areas where linear interpolation is not valid. Note that many data sets are only given for solar or sub-solar metallicity results, in which case if the model generates Z>solar, solar results are automatically used. This means that if the model becomes excessively metal-rich, it can go unobserved, because the code will default back to using solar values for yields/ejecta and SSPs, which do not reflect the actual galaxy produced by the code (see below).

3.3.3 Metallicity out of range

If the GCE model calculates metallicity in the model to be 15% and needs e.g. yield data at that metallicity, because those data do not exist in the data tables it uses the nearest value, which is likely to be solar, and returns results based on that solar metallicity. This is reasonable and is in fact a general limitation of models built on this basis. However, because the GCE model does not warn the user that the metallicity has become so high, reasonable outputs (SSPs based on solar) are taken from an unreasonable model (metallicity at 15%).

Metallicity can become excessively high in the GCE model; too many stars are produced in each timestep (see 3.2.2 above for the cause of this problem), and as any stars > 8 $M_{\odot}$ are evolved as SNII in the same timestep as they are formed, the overall metallicity of the galaxy is promptly and excessively increased. Some of this enriched ISM material is then re-formed as highly metal-rich stars in the next timestep – some of which will explode in the same timestep as SNII - and so on.

Metallicity in the GCE model increases until the star formation rate is set to zero (to model the move to quiescence), and then starts to slowly decrease. This is clearly physically incorrect as decreasing metallicity would only be expected if the galaxy merges with gas at a lower metallicity than the ISM. The cause of this has not been investigated further, other than to note:

- Metallicity increases dramatically in the subroutine SIMLOSS which calculates the mass loss from intermediate mass stars due to planetary nebulae, and calculates the increase in Z into the ISM. A change in Z would be expected from this subroutine, but not the extent noted.
- Metallicity decreases dramatically in the subroutine GASFLOW. This subroutine deals with gas inflow (i.e. modelled galaxy merger), however, this decrease in Z occurred even when the model was set so that the gas inflow has the same chemical composition as the existing galaxy, where no change to Z would be expected.

3.3.4 Massive stars

The GCE model allows the user to select massive star element results from:

- Geneva group results from M92 (modified with the more reasonable MM02) results for masses >= 40 $M_{\odot}$); or
- WW95, extrapolated with MM02 for masses > 40 M⊙ $M_{\odot}$; or
- a combination of both by using the variable FLOSSLIM in the *'values.in'* table to set a weighted proportion of data from each of the above two datasets.

Assumptions made by the Geneva Group and WW95 are not the same; it is therefore not appropriate to combine them. However, until there are comprehensive models for massive star evolution, there is little alternative. The impact of this is more significant for higher-metallicity massive stars, for which

- stellar wind effects become significant (not included in WW95);
- using yield data as a proxy for ejecta data will be inaccurate as the initial star will have some metals ejected unaltered which will affect the metallicity of the ISM;
- yield data for C and O is much higher for the Geneva Group than the ejecta data from WW95; and
- data for Mg and Fe, used as tracers of SN events, are not given by the Geneva group. This also means model adjustments for non-solar abundances will be inaccurate as element abundances other than C and O will be understated.

	Geneva Group	**WW95**
Range of stars modelled	1-120$M_\odot$	11 - 40$M_\odot$
Range of metallicity of stars modelled	Z = 0.001 and 0.020	Z = 0, 10^{-4} $Z_\odot$, 0.01 $Z_\odot$, 0.1 $Z_\odot$ and $Z_\odot$
Effects of wind included?	Yes	No
Effects of rotation included?	Yes in MM02	No
Data relates to before or after the SNII event?	Before	After
Model a range of elements?	No: He, C, O and Z only	Yes
Yields or ejecta?	Yields	Ejecta

Table 10: Comparison of data for stars undergoing SNII; $Z_\odot$ in WW95 is 0.0189

Figure 10 below shows the results available in the literature from various papers, weighted by a Salpeter IMF and highlighting the data used by the GCE – a heavy dotted line for WW95 extrapolated with MM02, and a heavy dashed line for M92 updated with MM02 modification at higher masses. WW95 results are for total ejecta (new and recycled material transferred into the ISM), whereas MM02 are for yields (newly synthesised material transferred into the ISM only). However, the GCE model does not make any recycled material adjustment when using the Geneva group data, and just treats these results as ejecta.

Figure 10 suggests that the extrapolation of WW95 with MM02, and data combination by using a percentage of each authors' results is *not* reasonable – although it is difficult to provide an alternative.

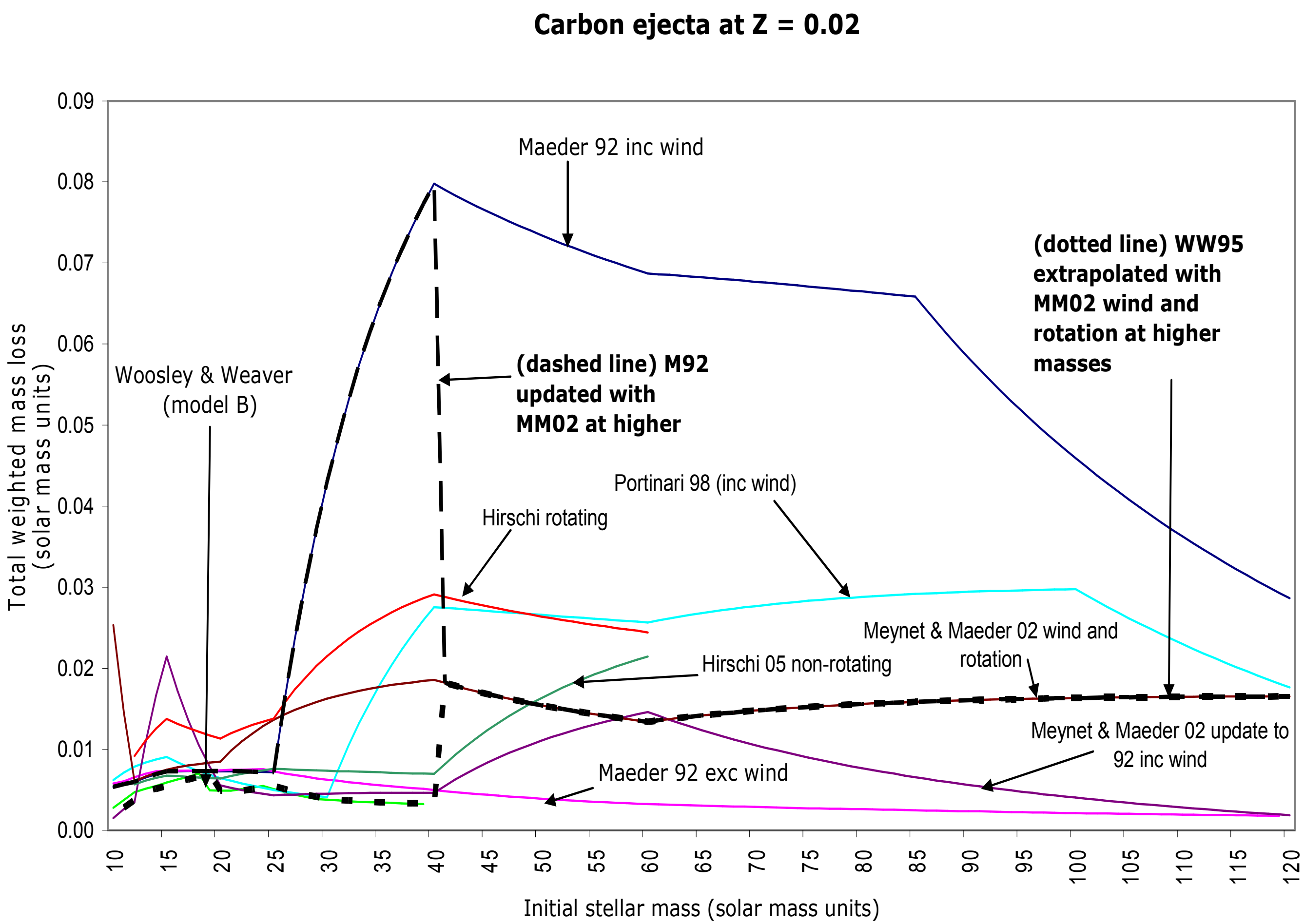


Figure 10: Salpeter-IMF weighted carbon and oxygen ejecta at different metallicities for different initial stellar masses from the literature. Data options for the GCE model are also shown with bold dotted/dashed lines. These graphs show it is not reasonable to extrapolate WW95 results with those from the Geneva Group particularly at higher metallicities.

Figure 10/continued

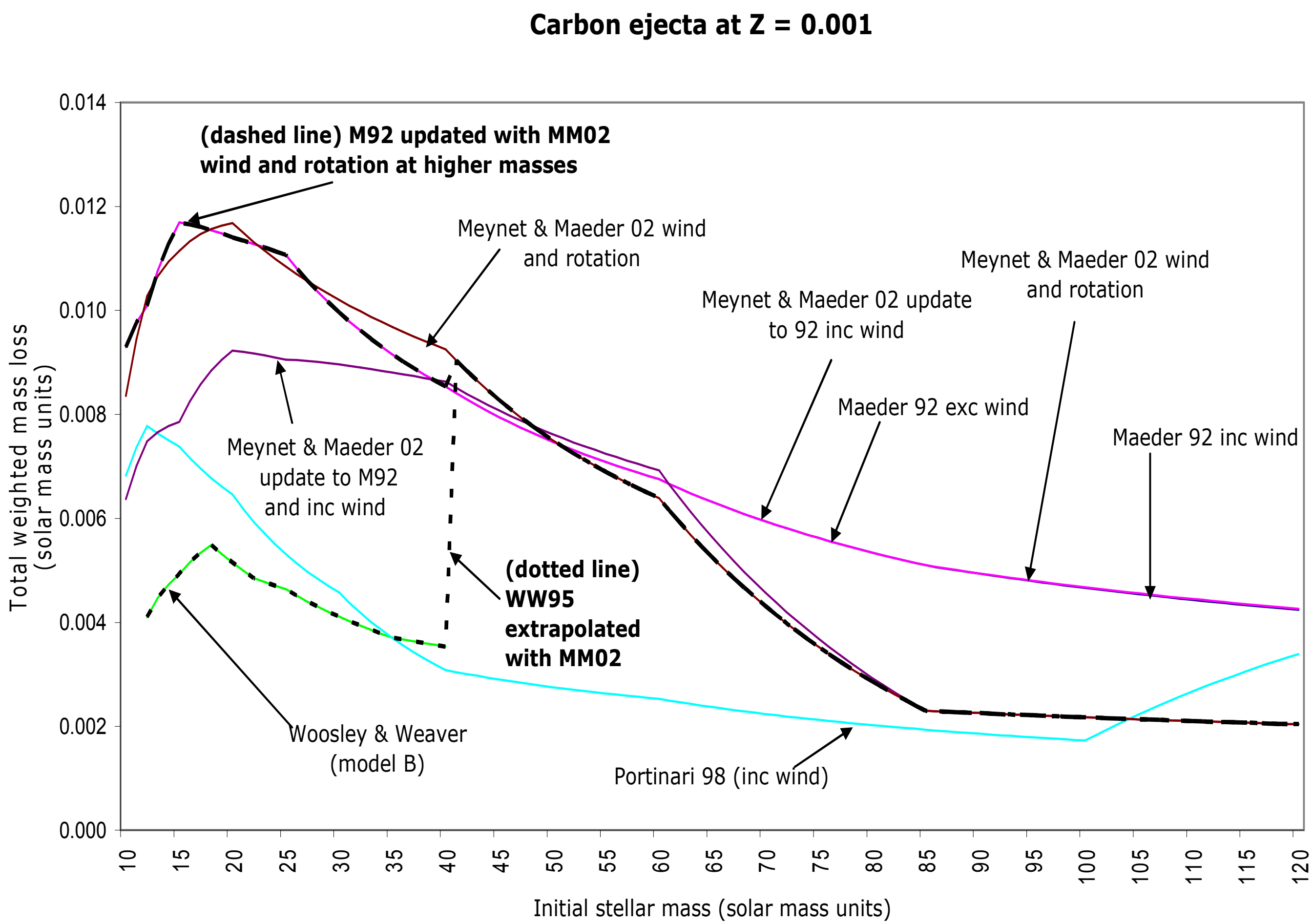
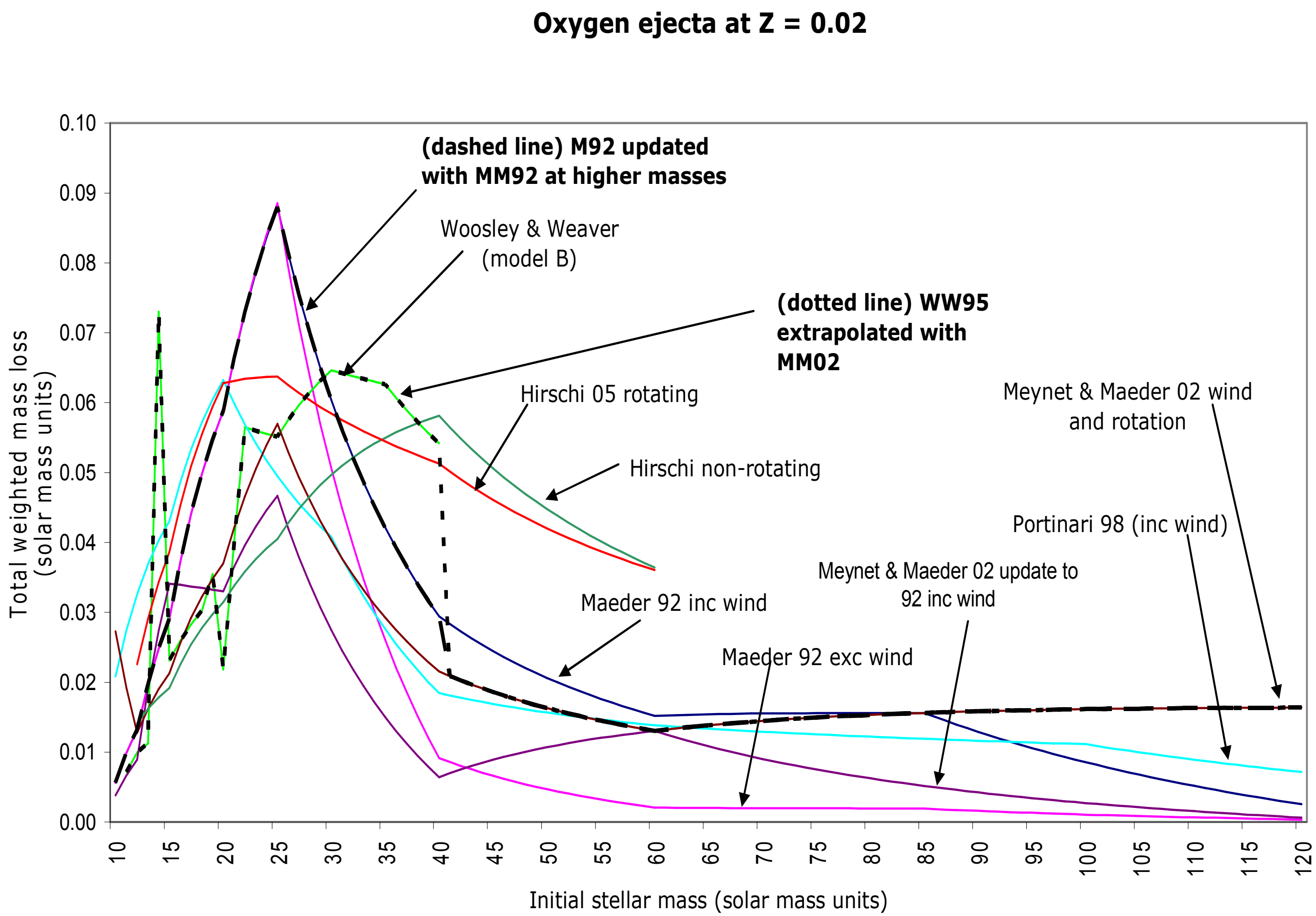


Figure 10/continued



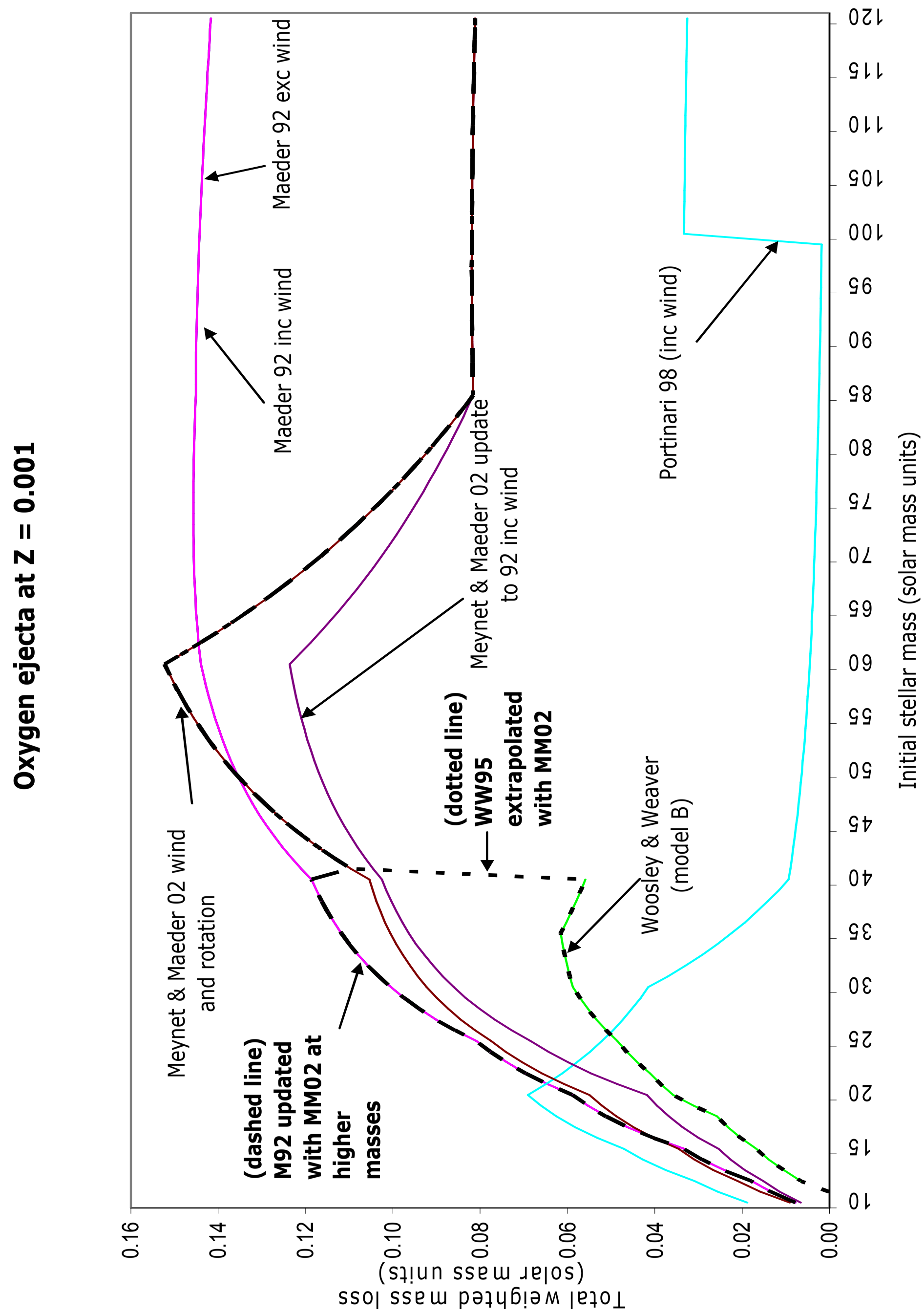


Figure 10/continued

3.3.5 Transition between intermediate and massive stars

The upper limit for the initial mass of stars becoming planetary nebulae from RV81, G05 or vdH&G97 is $8M_{\odot}$, whereas the lower limit for initial mass of massive star evolution by SNII is 9 $M_{\odot}$ (M92) or 11 $M_{\odot}$ (WW95). This means that there is no data available for the evolutionary end to stars initially between 8 and 9 $M_{\odot}$ (or 8 and 11 $M_{\odot}$ if using WW95 data). The GCE model treats the lower limit for SNII equal to the upper limit for planetary nebulae (hard-coded as 8$M_{\odot}$, which is the generally accepted value for solar-metallicity stars). Stars formed above this limit are evolved by the model as SNII taking the nearest values tabulated. Although M92 and WW95 results are given in absolute terms, the code does not scale these down for the smaller initial star sizes when using this extrapolation, so yields/ejecta will be overstated for stars in this range. Stars of exactly 8$M_{\odot}$ are missed by both subroutines. Stars at metallicities lower than solar would be expected to be larger at the end of their lives, due to less mass lost through stellar winds, so the transition point between intermediate and massive stars would be expected to be lower. This is not adjusted for in the model.

3.3.6 One model for ellipticals and spiral bulges?

The GCE model is intended to be used to model elliptical and spiral bulges. Steeper IMFs (such as those of Scalo 1986 or Kroupa 2001) are considered to be more appropriate for spiral galaxies; the GCE model does offer a modified IMF as an alternative although this is not automatically selected by the model.

3.3.7 Instantaneous mixing assumption

The GCE model assumes instantaneous mixing: elements ejected in one timestep are assumed to be uniformly available to the generation of stars formed in the next timestep. This is a reasonable assumption provided the timesteps are fairly coarse. Malinie et al. (1993) discussed observed variation in metallicities of stars of a given age within nearby clusters and groups, and found that mixing took around 10^{8-9} years. As stars can form in around 10^5 years (McKee and Tan 2003), stars could form before the mixing has completed, suggesting that instantaneous mixing may not be a valid assumption unless the time steps in the model are > ~0.1 Gyr. Timesteps in the GCE model are generally set to 0.03 Gyrs, which may invalidate the instantaneous mixing assumption: some of the next generation of stars would be formed from the ISM at its previous composition.

3.3.8 Single/multiple zone modelling

The GCE model has been written to allow different physical parameters to apply at up to 20 concentric radial ranges from the centre of the modelled galaxy. The nature of these shells is not defined - as either shells of equal thickness or as shells of equal volume. The subroutine GASFLOW deals with the ISM moving between these different radial ranges, however the model's instantaneous mixing assumption assumes any changes in metallicity in the ISM applies immediately across the entire galaxy, rendering this redundant. No adjustment is made to the radial ranges if there is gas inflow, which would be expected to enlarge the galaxy and should therefore either add radial ranges, or alter their volumes.

Ideally, to run a multiple-zone model, the total number of radial ranges modelled should be n+1, where n is the number of radial ranges modelled within the galaxy, and a further zone is used for "outside the galaxy" (as it is not a closed-box model). The GCE model does not include a zone "outside the galaxy"; as the model does not include galactic winds, and gas inflow is just added to the outermost zone, there is no impact from not having this extra "outside the galaxy" zone. Note that the actual location of the gas is irrelevant, as the output of synthetic indices does not depend on gas or stellar locations, so this feature of the model is not required.

3.3.9 Galaxy mass

The model is hard-coded at a mass of 10^6 $M_\odot$, but is compared to observational data from much more massive galaxies. Physics and astrophysics which would be valid for this small globular-cluster sized model may not necessarily hold true for the galaxies with which it is being compared.

Physical dimensions of the galaxy being modelled, other than mass, are not defined, and overall dimensions of the galaxy are not altered to take into account modelled mergers, which might be expected to increase the dimensions as well as the mass of the galaxy. For example, the successful model of NGC 3226 discussed in 2.4.3 above starts with the hard-coded mass of 1.0×10^6 $M_\odot$ but the modelled gas inflow increases the mass to 8.5×10^6 $M_\odot$. An appropriate adjustment to physical dimensions would be expected, in order to correctly calculate the gas density and consequently the star formation rate.

3.4 REVIEW OF THE STATISTICAL MEASURES USED TO ASSESS THE GCE MODEL

3.4.1 χ_v^2 as used within the GCE model

Recall from section 2.1 above that the GCE model computes the value it refers to as χ_v^2 for each index and for the overall model by comparing the observed and synthetic values using equations 7 and 8 where the 'error' is the uncertainty on the observed index, and the degrees of freedom is taken as the number of radial ranges in the model:

$$\chi^2 = \sum \left(\frac{observed - synthetic}{error} \right)^2 \tag{7}$$

$$\chi_v^2 = \frac{\chi^2}{degrees\ of\ freedom} \tag{8}$$

The user aims to get this value as close to unity as possible. However, a perfect model, where synthetic value = observed value (provided there was still an uncertainty on the observed value for the denominator to prevent the calculation $\rightarrow \infty$) would have this as zero rather than unity using equation 8.

Standard formulation for χ_v^2 is:

$$\chi^2 = \sum \frac{(observed - synthetic)^2}{error^2} \tag{10}$$

$$\chi_v^2 = \frac{\chi^2}{Number\ of\ observations - number\ of\ fitted\ parameters\ -\ 1} \tag{11}$$

The denominator in the χ_v^2 equation should therefore relate to the number of indices observed and the number of parameters being fitted; 12 in the case of the GCE, not the number of radial ranges modelled.

3.4.2 Use of χ_v^2 parameter space in four dimensions

The advantage of using a statistical technique that gives a measurement in parameter space is that plots of this can be used to establish whether there is a unique solution or not. This is what the GCE "stepping software" is designed to do, and examples of the contour plots obtained are given in figures 7 and 8 above.

However, this is limited to the number of dimensions that can be visualised; the GCE has 12 parameters (table 3 above) but the "stepping software" and resultant output plots only model 4-parameter space. Note that these are contour plots based on the above definition of the χ_{ν}^{2} equation (8). The "stepping software" outputs the lowest value it finds from a coarse grid of input parameters, but the contour plots indicate if there are localised minima within the results.

3.4.3 An alternative measure of model accuracy

It is important that the method chosen to measure the accuracy of the model is as robust to any underestimated uncertainties as possible; the uncertainties given in this thesis are those reported by the authors of the observational data, and are known to be, in some instances, possibly understated (i.e. exclude instrumentation or systematic errors, see, for example, discussion on Mg indices taken using the WHT in section 6.1.2).

The measure of a "good" model is one where the overall difference between the model and the observation is minimised. The nature of the data and the model used in this case may mean that a model could be "good", apart from one or two outliers, so the mechanism for measuring the "goodness-of-fit" of the model must not be excessively distorted by the presence of any outliers (Ke and Kanade 2003).

χ^{2} is a statistical method generally suitable where the data includes a measure of the frequency of events such that the data can be binned; this is not therefore a logically suitable statistical measure for assessing the accuracy of this type of model.

As the uncertainties on the observational data are all at one standard deviation, a measure of the model in terms of number of standard deviations, and the average number of standard deviations between the model and the observed is more appropriate. For this thesis, a goodness-of-fit criterion of this type is referred to as β.

$$\beta = \frac{|\textit{observed} - \textit{model}|}{\textit{standard error on observed}} \qquad (12)$$

$$\beta_{ave} = \frac{\sum \beta}{\textit{number of observed indices}} \qquad (13)$$

A perfect model would have the value of β_{ave} as zero: model = observed irrespective of uncertainty, provided the uncertainty is not equal to zero. As the uncertainty is calculated real observations, it will never be zero.

In this method, the difference between the model and the observed is weighted by 1/(the size of the error). β is therefore consistently in units of one standard deviation; this means that those measurements of Lick indices taken in angstroms and those taken in magnitudes can be safely combined in β_{ave}.

The distribution of the variation between the model and the observation is not expected to follow a Normal or Gaussian distribution because very high values of β clearly occur during the current analysis in this thesis, i.e. where an index within a model is very different from that observed. On the other hand, a Laplace distribution (e.g. Kotz et al 2001), which appears as back-to-back exponential distributions curves (and therefore has a logarithmic singularity at zero, which would also be expected from equation 12 if the error were zero), allows for these very high values of β, and has a theoretical maximum of ∞. It should be noted, however, that β values greater than ~5 correspond to very low likelihoods of occurrence. A model with a "good fit" would have $\beta < 2$ (94% confidence) and a model with a "reasonable fit" would have $\beta < 3$ (98% confidence) (Kotz et al 2001).

For this reason, a Least-Squares method (also known as L2-norm, and which is described by a Normal or Gaussian distribution) was rejected and a Least Absolute method (L1-norm, which is described by a Laplacian distribution), was selected.

Although the choice of a Least-Absolute method deals with the expected non-Gaussian distribution of the results, and the possibility of distortion by outliers, a "good result" as measured by this mechanism can still be achieved if the uncertainties on the data are large, as previously identified in 2.4, and can be difficult to achieve at all if the uncertainties are small.

Results are quoted in terms of β_{ave} (where a low value indicates an overall well-fit model) and β_{max}, being the largest value of $|\beta|$, which gives an indication of the spread of the results.

3.5 WORK DONE BY OTHER AUTHORS USING THE GCE MODEL

3.5.1 Introduction

A number of papers have been published using the GCE model, and these are reviewed below.

3.5.2 Sansom and Proctor 1998 (SP98)

This used 2-dimensional χ_{ν}^{2} space to identify good fits between two 'toy' galaxies (input files representing generalised (1) monolithic collapse and (2) hierarchical merger formation) and observational data from 10 elliptical galaxies taken from Davies et al. (1993) and Fisher et al. (1995) with the GCE model. Best fits were obtained with super-solar abundance SSPs from W94 modelling a pre-enriched galaxy undergoing a single merger event.

Both 'toy' galaxies start with a mass of 10^6 $M_{\odot}$, with the 'toy' monolithic collapse model receiving gas inflow of 10^7 $M_{\odot}$/Gyr over the first 0.3 Gyrs (so total final galaxy mass = 4.0 x 10^6 $M_{\odot}$). The 'toy' hierarchical merger model receives a gas inflow of 10^7 $M_{\odot}$/Gyr for 0.1 Gyrs (after 12 Gyrs of evolution) (so total final galaxy mass = 2.0 x 10^6 $M_{\odot}$). Not only are these final galaxy masses representative of dwarf galaxies (whereas the observational data is that of more massive objects), due to the confusion between mass and density (see 3.2.2 above), it is likely that the model will treat differently these two models just on the basis that one has twice the final mass of the other. The GCE model is written to utilise timesteps of different lengths (see section 3.2.6 above); the monolithic collapse model runs for 83 timesteps and the hierarchical merger model runs for 127 timesteps, although both models have the same galaxy lifetime. The additional timesteps (and consequently additional evolution loops) in the hierarchical model could be the factor that enables this model to produce the higher synthetic line strengths shown in Figure 2 of SP98, upon which some of the conclusions of the paper are drawn.

The conclusion of this paper was that elliptical galaxies must form from pre-enriched and not primordial material. As the nature of the model set-up is that the start point of the model *has* to already be part-way into the galaxy's evolution (see section 3.2.5), *a priori* the material present at the start point of the galaxy is "pre-enriched".

3.5.3 Proctor, Sansom and Reid (2000) (hereafter PSR00)

New observational data from the central bulges of four spiral galaxies was used with the GCE model (and the two 'toy' galaxies from SP98) to further support the hierarchical galaxy formation theory. Observational data points were found to be closer to/contained within the contours plotted for the 'toy' hierarchical merger model and further from/not contained within the contours plotted for the 'toy' monolithic collapse model. Comments on SP98 (above) regarding additional evolution inadvertently processed for the hierarchical model possibly leading to higher synthetic indices would also apply here.

3.5.4 Proctor and Sansom (2002) (hereafter PS02)

A new observational data set of 32 nearby galaxies was modelled with the same two 'toy' galaxies, with the GCE model updated to use V99 SSPs. The GCE model with the 'toy' hierarchical input gave a reasonable match to index-index scatter plots from the observational data, whereas when run with the 'toy' galaxy representing monolithic collapse, it did not. Some of the data points were excluded from the scatter plots in this paper where they were felt to be outliers, although the paper does not draw attention to this. Code errors, including an unnoticed corruption of the data file for V99 may also have affected the results, but it is difficult to quantify this.

3.5.5 Gjshchkhmyj (2006)

The MPhys project of Gjshchkhmyj (2006) investigated the use of commercial software offering 3-D representations of four-dimensional parameter searches, using the GCE model and observational data from one galaxy from the PS02 sample (NGC 3623). He found that the GCE model was giving particularly poor results for Mg_1, Mg_2 and Mgb. These poor χ^2 calculations for the magnesium indices distorted the overall χ_v^2 value, where other indices appeared to be well modelled. However, he did not note that the PS02 observational data having been taken from the WHT, which is known to poorly calculate magnesium indices (see 6.1.2 below), and the uncertainties on these data had not been adjusted to allow for this. The problem here, therefore, was with the uncertainties on the observational data and not the GCE model.

3.5.6 Sansom, Izzard and Ocvirk 2009

The GCE model, using the new subroutines from the present author described in Chapter 2 for planetary nebulae yields and T04 SSPs, was combined with results from the models of Izzard (2006) to assess the importance of yields from binary stars other than via SNIa. They concluded that these additional yields were not important, which follows similar results from Zhang et al. (2005) and Li and Han (2008).

3.6 DISCUSSION AND CONCLUSIONS

This Chapter has discussed a number of limitations and coding issues with the GCE model. The main concern is the use of the variable ROO for a number of different physical properties, which leads to calculation of excessively high metallicities in the modelled galaxy. This was hidden because the code uses the nearest available data for yields/ejecta and indices when the required data is outside the range available. This means the model generates very high metallicities but reverts to using (generally) solar values for yields/ejecta and the synthetic indices, giving reasonable results from a physically unrealistic model galaxy. This overshadows the effects of other limitations such as using yield data as ejecta data, not weighting luminosity in proportion to the masses of the stellar population, or setting timesteps too short to have a valid instantaneous mixing assumption.

In addition to the enhancements to the GCE code described in Chapter 2, and following the review of its limitations described in this Chapter, a new model was developed, incorporating the learning from this work. This model, Phoenix, is described in detail in the next Chapter, is tested in Chapter 5 and used to propose star formation histories of nearby elliptical galaxies in Chapter 6.

CHAPTER 4: THE PHOENIX MODEL

4.1 OVERVIEW OF THE MODEL

4.1.1 Introduction

Phoenix is a self-consistent, open-box integrated stellar population model of an homogenous spherical elliptical galaxy. It tracks the lifecycles of stars over small mass ranges formed at the same time, calculating the indices such stars would produce from SSP data and luminosity-weighting them to give the expected integrated spectra for comparison to one or more observed galaxies. The model can be used in two different ways. First, a ‘single run’ can be used, to make comparisons with one observed galaxy, with the user setting the free parameters. Second, the entire set of free parameters can be systematically worked through by the code to produce a large number of different models to which the observational galaxy can be compared simultaneously i.e. parameter space can be searched to find the best-fit model. The user selects “single” or “search” when the model is run; either option runs the same model but the output report formats are different. The structure of the model is given in figure 11 below, and further details of the main subroutines are given in section 4.3 and outputs in section 4.4. The full code is presented in Appendix B.

4.1.2 Outline of the Phoenix model

The Phoenix code is written in Fortran 90/95, and uses the subroutines written by the present author for the GCE model as outlined in Chapter 2 for planetary nebulae options and T04 SSPs. Data sources for the model, which can be selected by the user where there is a choice, are given in table 11. Modified and simplified versions of the GCE’s data-reading subroutines were also incorporated into this new model. The remainder of the Phoenix model is entirely new and independent.

This model uses the following structure of galactic evolution:

$$dM_{star} /dt = SFR - E \qquad (14)$$

$$dM_{gas} /dt = -SFR + E + f \qquad (15)$$

Where
SFR = the star formation rate, given by the Schmidt (1959) equation (equation 6)
E= mass ejected by stars as gas due to supernova or planetary nebula events
f = gas flowing into (+) or out of (-) the galaxy

Free parameters in the Phoenix model:	
1	Initial mass of galaxy, in $M_\odot$
2	Overall duration of the galaxy lifetime, in Gyrs
3	Constant C in the Schmidt (1959) equation with Kennicutt (1989) index $SFR=C\rho^{1.3}$ where ρ = density of gas in galaxy
4	Proportion of initial gas forming Population III stars
5	Time in Gyrs after start of galaxy of the galactic wind OR multiple of stellar mass expelled as galactic wind (gas loading)
6	Rate of gas inflow, in $M_\odot$/Gyr
7	If applicable: time in Gyrs after start of galaxy when gas inflow starts
8	If applicable: duration of gas inflow in Gyrs.

Table 11: Free parameters in the Phoenix model.

Parameters (table 11) can either be set by the user, or the model can run several times, with the model varying these parameters systematically in each run. The Phoenix model uses a number of data sources from the literature. In some instances, there is a choice which the user can make before running the model.

Process/information required	Data source
SNIa ejecta	• Nomoto et al. (1984)
SNII yields (adjusted to ejecta)/ejecta (large stars up to 40 $M_\odot$)	• Woosley and Weaver (1995) (ejecta) • Maeder (1992) (yields)
SNII yields (adjusted to ejecta) (massive stars over 40 $M_\odot$)	• Meynet and Maeder (2002)
Planetary nebulae yields (adjusted to ejecta)	• Renzini and Voli (1981) • Van den Hoek and Groenewegen (1997) • Gavilán et al. (2005)
SNIa rates	• Timmes et al. (1995) • Scannapieco and Bildsten (2005)
Gas inflow composition	• Primordial • Same as current gas composition • Solar • Twice solar
Isochrones	• Bertelli et al. (1994)
Initial Mass Function	• Salpeter (1955)

Table 12: Data sources used by Phoenix model.

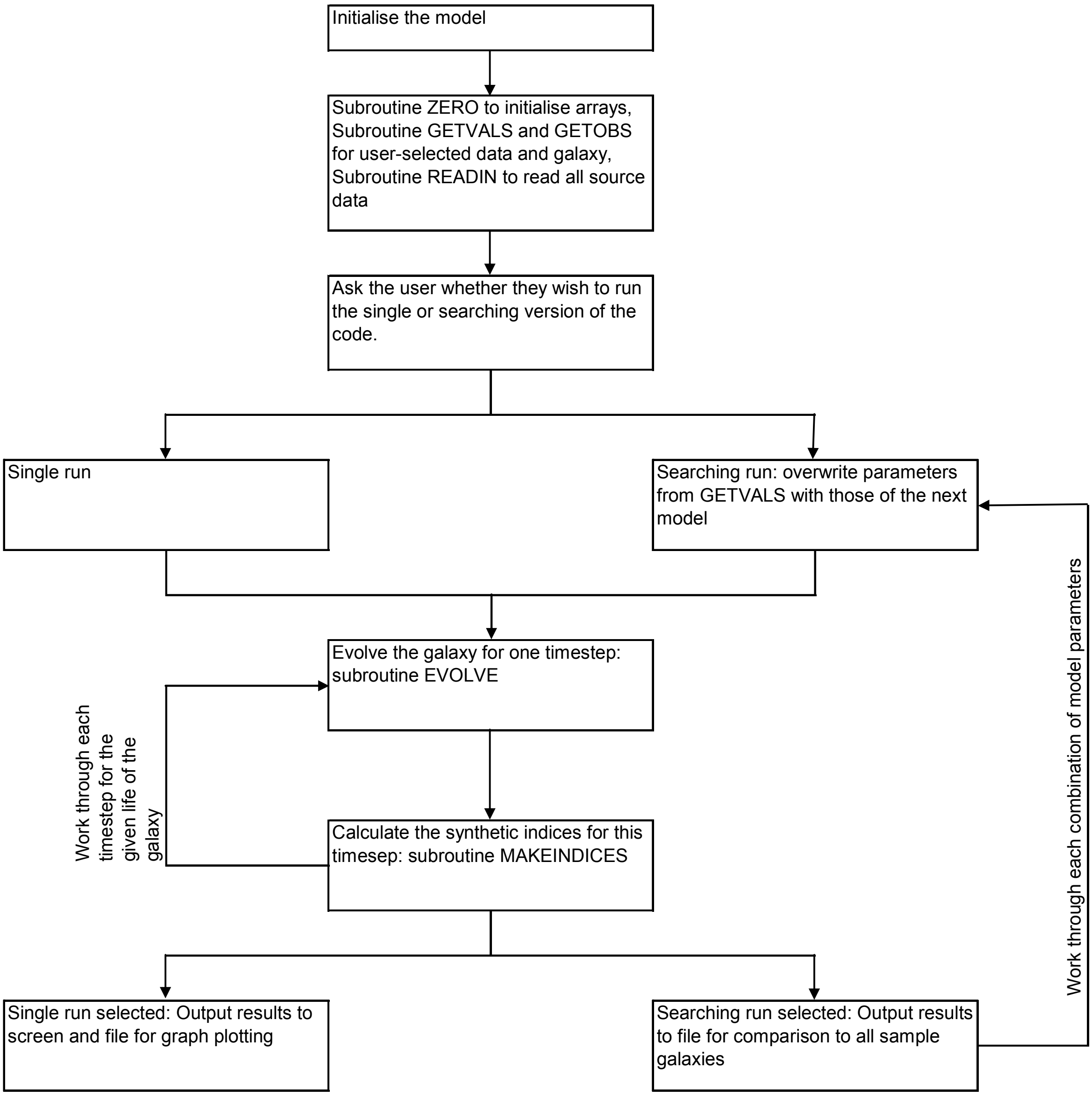


Figure 11: Overview of Phoenix model.

4.1.3 Brief comparison of Phoenix and GCE

As noted in Chapter 1, integrated evolutionary population synthesis models may work on a "top-down" approach, in that they attempt to fit existing SSPs to the observed data, or a "bottom-up" approach of tracking the formation of a modelled galaxy and assessing whether the indices it would produce match the observed data or not. The Phoenix and GCE models both follow a "bottom-up" approach.

As with the GCE, the Phoenix model is only 'chemical' insofar as it keeps a track of ejecta to give the overall metallicity, and, where required, a value for [α/Fe], which in turn selects the appropriate SSP. Neither model builds up the indices from the component elements. The main differences between these two models is that Phoenix, as well as taking note of the issues raised in Chapter 3, tracks the lifetimes of individual stars, enabling the model to use isochrones to calculate the luminosity of each mass bin (and hence enable luminosity-weighting of the indices). The other differences are summarised in table 13.

	GCE Model	**Phoenix**
Individual stars modelled?	No	Yes
Isochrones used to calculate the luminosity weighting?	No	Yes
Number of free parameters/number of parameters searched	12/4	8/8
Galaxy volume varies with mass?	No	Yes
Number of evolutionary processes leading to SNIa	1	2
Single-run and "stepping" runs from same model?	Partially: separate codes are run but call same set of subroutines.	Yes
Options for planetary nebula	Was 1, updated to 3 by present author	3
Options for SSPs	Was 2, one of which used corrupted data, updated to 3 by current author	3, including clearing corruption in V99 data
Non-solar abundance corrections to SSPs options	TB95 (only solar) K05 incorporated as an option	None
Radial ranges modelled	Yes, but not accurately: not updated if mass, volume or density are altered	No; simple open box model
Chemical composition of inflow	Primordial, same as current galaxy or solar	Primordial, same as current galaxy, solar or 2 x solar

Table 13: Comparison of the GCE and Phoenix models.

4.1.4 Checks built into the model

To check whether the 'range exceeded' errors discussed section 3.3 occur, warnings are written into the model. These appeared on screen during testing and have since been diverted to an output file, '*warnings.out*'. This allows the user to view each instance where the data required by the model is not available and the nearest value has been used instead. The impact on the final output can then be evaluated in its proper context. It is important to note that results from Phoenix reported in this thesis as successful did not generate any warnings.

Limitations in ejecta data from the literature are an inherent problem with this type of model; by giving a range of options for the yield/ejecta data, the importance (or otherwise) of these limitations can be assessed. For example, the results using each of the three options for planetary nebulae yields do not vary much, despite the different approaches used in each of the models of RV81, vdH&G97 and G05. Limitations in yield/ejecta data are discussed in more detail below (4.2.13).

The model runs self-consistency checks to verify how much, if any, is "lost" due to Fortran precision limitations (2.3.2), and makes corrections by adding rounding values to the largest component (for example, if the gas is mostly hydrogen, then the calculated adjustment is made to hydrogen), and self-consistency checks are also output to the results file. Consequently, there will be a slight alteration to the overall proportions held as hydrogen/helium/metals, or held as gas/stars (etc) but these are not significant and should not affect the overall results produced.

As the code was written, each section was tested in isolation. For some parts of the code, this was done by overwriting parameters, running that section of the code and then verifying the output against the source data (for example, fixing the model's metallicity to test that the correct data is picked up from the table). For other parts, the output was compared to values separately computed with a calculator or on spreadsheets.

4.2 ASSUMPTIONS, SIMPLIFICATIONS AND LIMITATIONS IN THE MODEL

The Phoenix model uses the following assumptions and simplifications, which, in some instances, limit the model:

4.2.1 Starting point of model

The model begins at time T=0 with primordial chemical composition, from which a percentage, set by the user (as a free parameter) or the "searching software" will instantaneously form Population III stars with zero metallicity. Whilst this distorts the initial star formation rate compared to that given by the Schmidt (1959) equation, Mii and Totani (2005) find that very efficient Population III star formation supports observational evidence of ultra-luminous X-ray sources if these are assumed to be intermediate mass black holes and suggest 10% of the cosmic baryons form Population III stars. Using the Phoenix model, the initial formation was found to be between 37% and 54%; it was found that no model was successful if the percentage of initial Population III stars was set to 55% or more of the original primordial gas. There is no physical reason for these percentages; they are merely the values that enable the models to work, as it is presently difficult to establish the population of primordial stars in elliptical galaxies, as the stars are unable to be resolved with current observational equipment. Population III stars will have a limited effect on the final modelled indices, even if a standard Salpeter (1955) IMF is used and the final galaxy contains a high proportion of these stars, because the low metallicity will result in low-luminosity weighting. In practice, a different IMF might apply to these stars (as modelled by e.g. Nakamura and Umemura 2001, and Omukai and Yoshii 2003).

4.2.2 Salpeter IMF

The model is set to work with a Salpeter (1955) IMF; work by Pipino and Matteucci (2004) and Calura et al.(2007) shows that models of ellipticals and S0 galaxies are more accurately reproduced using this IMF rather than e.g. a Scalo (1986) IMF. Chiappini et al. (2003) models show the steeper Scalo (1986) IMF is more appropriate to spiral galaxies. The IMF is assumed to be constant over time.

	Salpeter (1955) IMF	Miller-Scalo (1979) IMF	Kroupa (2001) IMF
Percentage of stars M < 1 $M_{\odot}$	96%	78%	90%
Percentage of star mass held in stars < 1 $M_{\odot}$	60%	31%	45%
Mean mass of stars in this range	0.35	0.89	0.80

Table 14: Effect of different IMFs on proportion of stars at lower masses, taken from Hillenbrand (2004), given an overall range of 0.1-120 $M_{\odot}$.

As shown in table 14, using a Kroupa (2001) IMF would have the effect of decreasing the proportion of stars that form brown dwarves (from which no indices are modelled), have minimal effect on the intermediate mass stars undergoing planetary nebulae and increase the proportion of stars that will evolve as SNII. This would increase ratios such as [Mg/Fe], as yields from SNII would be increased, but this cannot be properly tested in evolutionary population models due to the limited yield data for stars > 40 $M_{\odot}$ (Geneva Group, the main source for massive star yields, only give data for carbon and oxygen).

4.2.3 Galaxy dimensions

A review of equations in the literature relating galaxy mass to diameter was undertaken (table 15), and the Shen et al. (2003, amended 2007) equation was deemed to be the most reasonable over a wide range of galaxy masses, and had been formulated based on observational data.

Mass	Aizu (1980)	Gibson (1997) eqn 16	Shen et al. (2003) eqn 17	Shen et al. (2003) eqn17 corrected 2007
1.E+06	0.001	0.013	0.079	0.007
1.E+07	0.003	0.046	0.289	0.024
1.E+08	0.014	0.164	1.048	0.087
1.E+09	0.063	0.582	3.805	0.316
1.E+10	0.275	2.065	13.814	1.147
1.E+11	1.202	7.328	50.157	4.163
1.E+12	5.248	26.000	182.108	15.114
1.E+13	22.909	92.251	661.195	54.877

Table 15: Half-light radii of galaxies in kpc from different equations in the literature.

Equation 12, taken from Shen et al. (2003 amended 2007) is used in Phoenix to calculate the dimensions of the galaxy (assumed to be spherical). In order for the modelled galaxy not to significantly change dimensions when gas flows in or out, the galaxy mass is taken as M_{stars}; inflow of gas will form stars in the next timestep and this will change the dimensions appropriately at that point.

$$\text{Half light radius (kpc)} = 2.88 \times 10^{-6} (M/M_{\odot})^{0.56} \qquad (16)$$

This equation gives the half-light radius; what is needed for the model is the radius of the galaxy. The galaxy is assumed to be an homogenous sphere, so the light from a sphere of volume $4/3\ \pi\ A^3$ can be assumed to be half that of a sphere of volume $4/3\ \pi\ B^3$ if:

$$2(4/3\ \pi\ A^3) = 4/3\ \pi\ B^3 \qquad (17)$$

It can then be shown that $B = \sqrt[3]{2}\,A$, so the equation to calculate the radius of the galaxy from the mass using the corrected equation from Shen 2007 becomes:

$$\text{Radius (kpc)} = \sqrt[3]{2} \times (2.88 \times 10^{-6} (M/M_{\odot})^{0.56}) \qquad (18)$$

and this is the equation used by Phoenix.

4.2.4 Critical density and star formation rates

Critical density of the gas is the point at which stars are able to form. Dunham et al. 2010 suggest a mean critical density for star formation of 6.2×10^3 particles per cubic centimetre – given the further data of average masses of these particles (2.37 x the mass of a proton), the critical density can be calculated at 2.45×10^{-29} kgm^{-3}, which is $0.3625\ M_{\odot}kpc^{-3}$. The model therefore checks that the gas density exceeds this before calculating any star formation, which it does using the Schmidt (1959) equation (equation 8).

The model only allows stars to form if either the galactic wind has not taken place (or not removed all of the gas), or if gas is flowing into the galaxy, as ejecta from planetary nebula and SNIa taking place after the galactic wind would be very enriched; stars formed from gas enriched to this level are not observed.

The SFR equation is not varied over time, although of course the actual rate of star formation will, because gas density varies over time.

Density is calculated by the model based on the equation given by Shen et al. (2003, revised 2007) to relate mass of the galaxy to its radius and hence volume. Work by Kennicutt (e.g. 1998, 2007) supports this as a universal law.

Gas mass (and hence gas density) calculated by the model may become understated, because both the Geneva Group and WW95 data are limited to sub-solar and solar metallicities: at higher metallicities, higher stellar winds would be expected, and also more material would be expected to be lost as a consequence of the supernovae event (leaving smaller remnants), although this would not be expected to be significant and consequently not alter the overall results.

4.2.5 Black holes, brown dwarfs and remnants

The model has an upper stellar mass (set at 120 $M_\odot$); any stars formed above this are assumed to collapse directly to a black hole and not participate in integrated spectra. The model also has a lower stellar mass (set at 0.1 $M_\odot$); any stars formed below this are assumed to form brown dwarf stars and not participate in integrated spectra. Remnants from planetary nebula and SNII events are also assumed to not participate in integrated spectra, but form material from which SNIa may arise.

It is assumed that any central massive black hole would remove matter non-discriminately and hence not affect mass fractions or observed indices.

4.2.6 Binary stars

Binary stars are not included in the Phoenix model, other than being noted as a formation method for SNIa, as they have been shown to have only insignificant effects on derived Lick indices (Zhang et al. (2005), Li and Han (2008), Sansom et al. 2009).

4.2.7 Dust

The Phoenix model assumes no dust is present; dust might lead to differences in the modelled outputs, as it may redden the Colours observed. Generally, elliptical galaxies have minimal dust, making this a reasonable assumption.

4.2.8 Dark matter

Dark matter is assumed to be outside the visible modelled galaxy (Matteucci 1992, Oñorbe et al. 2007) and is therefore assumed to have no effect on the synthetic Lick indices produced; hence it is ignored.

4.2.9 Modelling of merger events

A merger is modelled as gas inflow only (rather than gas + stars). A merger with another galaxy with a different stellar population would alter the observed integrated indices, however, it would be possible recreate the final observed integrated galaxy required by simply adding large numbers of stars of the appropriate age/metallicity/size to sufficiently influence the selected weighted SSPs, irrespective of whether these stars would in practice exist. This approach would convert this from a "bottom-up" to a "top-down" model. Gas inflow enables the model to produce a further burst of younger stars, without the need to introduce a number of additional parameters to describe a merging stellar population.

In the Phoenix model, the rate of gas inflow is modelled as a free parameter for which the chemical composition can be selected; if the rate is non-zero, then the timing and duration of the inflow are also modelled as free parameters.

There has previously been some speculation as to whether elliptical galaxies have formed from spiral galaxies that have merged and lost their structure (e.g. Vedel and Sommer-Larsen 1990, Rothberg and Joseph 2004), and/or whether spiral galaxies have formed from the effects of stellar orbital velocities in elliptical galaxies (e.g. Kauffmann 1996, Pavlov and Pavlova 2003). Certainly, the observational evidence from counter-rotating cores in elliptical galaxies suggests that elliptical/spiral mergers do take place and affect the morphology of the resultant galaxy (e.g. Mirabel et al. 1999, Di Matteo 2008b). Where a model is not able to propose the star formation history of a galaxy, it could be that this level of complex formation lies in its history, which is beyond the scope of the Phoenix model.

4.2.10 Galactic winds

The model gives two options for galactic winds: TIME, where the galactic wind occurs at a particular number of Gyrs after the initial formation of the galaxy, at which point all the gas is removed, to model AGN as the source of the wind, or LOAD, where the gas loss depends on the mass of stars being formed in that timestep (Strickland and Heckman 2009), to model SN as the source.

It is assumed that

- the galactic wind removes all the gas (and hence all the tracked elements), in order to "switch off" star formation;
- the reduction in all elements is in proportion to their abundance, and is not weighted towards any individual elements; and
- that any gas subsequently produced by SNIa and planetary nebulae events after the galactic wind is immediately removed from the zone of the galaxy.

Without this last assumption, the metallicity of the ISM would become excessively enriched, as the products of evolution are all very high % metals. This is reasonable, as elliptical galaxies are generally observed as being gas-free.

The model does not contain any dynamics or energy calculations; if the user selects TIME as the method for processing the galactic wind, then the timing of that wind is a user-set free parameter, rather than being calculated by the model as the point where the thermal energy of the outflow (AGN) exceeds the binding energy of the galaxy (e.g. Gibson 1997); all the gas in the ISM is expelled, rather than just sufficient gas to bring the system back to equilibrium.

Whilst galaxies are dynamic systems, elliptical galaxies, which Phoenix is attempting to model, are less affected by dynamics than spiral galaxies, as their star formation is thought to be minimal after initial formation and once the gas is expelled. The main disadvantage the model has from not modelling dynamics is that the gas outflow must remain as a free parameter.

4.2.11 Stellar evolution

The Phoenix model separately holds data for different initial stellar masses produced in each timestep, and, using the equation from Wood (1992) (equation 9), calculates which timestep the stars in that mass bin will evolve from the main sequence.

SNIb and SNIc, which have helium in their spectra, are associated with young stellar populations (Pagel 1997) and are noted to only constitute about 1% of Galactic supernova (Higdon et al. 2004). As such, these are not included within the model. In addition, yield data are not available for these events within the literature, making inclusion difficult were it to be appropriate.

Stellar evolution models in the literature generally assume a maximum initial mass of 8 $M_\odot$ (RV81, vdH&G97, G05) for planetary nebula but a minimum initial mass of 11 $M_\odot$ (WW95) for the minimum mass for SNII, leaving the evolutionary fate of stars in the range 8 – 11 $M_\odot$ undetermined. The Phoenix model treats stars below or equal to 10 $M_\odot$ as linear extrapolations of planetary nebulae data, and stars above 10 $M_\odot$ as linear extrapolations of SNII data.

Following work by Mannucci et al. (2005) on supernova rates, which hinted at a the existence of 'old' and 'young' progenitors, Scannapieco and Bildsten (2005) gave a two-component model of SNIa: a prompt component, which depends on the instantaneous star formation rate, and which represents SNIa as a consequence of merger of two white dwarf stars, and a delayed or extended component, which is depends on the mass of the galaxy and which represents SNIa as a consequence of accreting binaries. The text notes a delay to the 'prompt' component of 0.7 Gyrs. However, in their plots the time delay is plotted instead against the extended component.

As the delay represents the time taken for the binary to accrete matter from the companion star, the delay should be calculated on the extended component, i.e. the graph is correct and the text not, so the equation (19), is amended to reflect that correction.

$$\text{SNR } (100\text{yr})^{-1}\ 10^{-10}\ M_\odot = \quad (19)$$
$$0.044 \text{ x mass in galaxy 0.7 Gyrs ago (extended component) } (M_\odot)$$
$$+$$
$$2.6 \text{ x instantaneous SFR } (10^{-10}\ M_\odot \text{ Gyr}^{-1} \text{ (prompt component)}$$

4.2.12 Instantaneous mixing

It is assumed that the next generation of stars will form from gas including ejecta from evolutionary processes that took place in the previous timestep, and that this gas is homogenous throughout the galaxy. The timestep used in the model is 0.1 Gyrs, as discussed in 3.3.7 above.

4.2.13 Yields and ejecta

The model, as with other models of this type, does not calculate the indices directly from the elements produced by the evolutionary processes, but instead keeps track of the elements in order to calculate the metallicity of the ISM and the chemical composition of next generation of stars formed, as well as the [α/Fe] ratio. These calculated metallicities ensure the correct data set is selected when each star reaches the end of its main sequence life, and that the appropriate SSP and isochrone is selected (which gives the final weighted indices for that sub-population) if the star is still on the main sequence.

As discussed above in 3.2.13, some of the data in the literature is given as yields (material newly synthesised and ejected into the ISM) rather than as ejecta; ejecta are required to give the chemical composition of the next generation of stars. Metal yields and ejecta are related as given in equation 20:

Ejecta + remnant = yields + unaltered material + stellar nucleosynthesis (20)

where:

Ejecta =	total material released into the ISM for the next generation of stars.
Remnant =	(where applicable): degenerate star left at site of original star.
Stellar nucleo-synthesis =	new material produced by nucleosynthesis within the star, between initial formation from the ISM and the evolutionary end, and either • ejected during the final star disruption without being further altered • forming part of the remnant
Yields =	new material produced by nucleosynthesis and ejected from the star, either • stellar nucleosynthesis products ejected by winds before the end point of evolution, or • produced by the supernova or planetary nebula and ejected at that point.
Unaltered material =	material chemically unaltered from the time the star formed.

For the processes within Phoenix, where the information is not available, stellar metals are assumed to be negligible, and any metals in the main sequence star are assumed to be ejected unaltered. Details of the processing for each evolutionary process is given in table 16 below.

Evolutionary process	**Data source**	**Issues**	**Processing by Phoenix**
Planetary nebula (stars < 10 $M_{\odot}$)	van den Hoek and Groenewegen (1997) or Gavilán et al. (2005) or Renzini and Voli (1981)	Problem: only gives yield data. No information about the composition of the remnant.	The model adds the metals in the original star from the time it was formed to the yield data, to give an approximation of total ejecta. This means the ejecta will be understated by any elements created within the star during the main sequence stage. Remnant assumed to be CO dwarf.
SNIa	Nomoto et al. (1984)	Star destroyed so no remnant, and data is all for ejecta. Data assumes all SNIa stars have initial mass of 1.378 $M_{\odot}$	Model ignores chemical composition of original star. Model takes number of SNIa events from Timmes et al. (1995) or Scannapieco and Bildsten (2005) as selected and multiplies the ejecta data by this to give the total ejected in that timestep by this process.
Large stars: SNII (stars between 11 $M_{\odot}$ and 40 $M_{\odot}$)	Woosley and Weaver (1995)	All data is for ejecta. Detailed chemical composition of the remnant is not given.	Remnant star (all metal) assumed to be made entirely from hydrogen in the original star.
Large and massive stars: SNII (stars between 9 $M_{\odot}$ and 120 $M_{\odot}$)	Maeder (1992), modified with results from Meynet and Maeder (2002) for stars between 40 and 120 $M_{\odot}$	Data is quoted as yields but noted as being the pre-supernova composition i.e. ignores any elements created as a consequence of the explosion. Only data on carbon and oxygen provided.	Model ejects the yield data as given, plus all the metals from the original star (= metals in ISM at time the star was formed). Remnant star (all metal) assumed to be made entirely from hydrogen in the original star. As only provided with data for carbon and oxygen, the mass of other tracked elements will be very understated.

Table 16: Phoenix processing of yield and ejecta data from different evolutionary processes.

Main sequence products of stellar nucleosynthesis, either ejected or retained within the remnant are not included in the data provided in the literature. This means it is not possible to accurately track the overall chemical composition of the galaxy, because the detailed chemical composition of the stars at any given time cannot be known. The main problem, however, for tracking individual element abundances in the model galaxy is due to the limited data from the Geneva Group. SNII are a major source of magnesium, for example, which should be a key measure for the accuracy of the model (as expected values of [Mg/Fe] are known), however, this measure cannot be used to test Phoenix because the yields of these elements are not given by the Geneva group, who only provide details of carbon and oxygen yields.

Because the overall ejecta in a timestep will be understated, the gas for the next generation will have a slightly lower metallicity in the model than it would be expected to have in practice. This in turn will mean

- lower metallicity data is selected when these next-generation stars reach their evolutionary end;
- SSPs selected to provide the indices against which the observed data is compared will be those of a lower metallicity; and
- calculated luminosity by which these indices are weighted will be those of lower metallicity.

It is also difficult to use these data sources to compare *yields* against other references in the literature, because the WW95 data does not discriminate between new material and recycled material, and arguably some of the material given by Nomoto et al. (1984) will be recycled from the original star rather than all new.

Total elements in a galaxy are not the same as the abundance of elements observed because some of the material, weighted towards the heavier elements, will be inside stars and of course can't be observed. This needs to be accounted for if, in the future, observational abundance data are compared to the models, or if indices are calculated directly from abundances rather than being taken from SSP tables.

4.2.14 Chemical composition and effect on synthetic indices

The SSP selected is based on the metallicity and age of the stars in each mass bin. Metallicity selected is that of the ISM at the time the stars were formed; the model does not make any adjustment for chemical evolution taking place during the main sequence life of the star, nor for stellar winds which might remove the outer layers of the star, as these are considered to be negligible for the majority of the stars that are included at the end of the timestep. This value for metallicity, whilst not accurate, should be acceptable as the enrichment during the main sequence lifetime takes place largely in the core, and stellar winds only affect large, high metallicity stars. This assumption could mean that the SSP, and the isochrone used to weight that SSP, have marginally lower metallicity than should be used.

TB95 and K05 produced tables to show the impact on individual indices if the abundance of an individual element was doubled with respect to solar. As the detailed abundances of the individual elements in the stars cannot be produced by the model from the yield and ejecta results available in the literature, these results cannot yet be successfully incorporated into Phoenix. α/Fe values are calculated by Phoenix, based on the computed abundances, but will be understated, as the α-elements produced in massive star evolution are missing from the Geneva group. These calculated α/Fe values are used when the T04 SSPs are selected.

The ability to adjust for non-solar enhancements is one of the incentives for tracking the chemistry in these models, as well as having additional results against which observational data can be compared. However, the currently limited data means that these tweaks to the indices cannot be correctly assessed. Inclusion of this is therefore left as a planned model enhancement. The Phoenix model should therefore be considered as an evolutionary stellar population model, and not a chemical evolution model, although the storing of the element yields insofar as they are available may enable the model to be developed into a chemical evolution model in the future.

4.2.15 Massive stars at the end of a timestep

The model identifies the timestep when each star will be fully evolved through the path identified by its initial mass, as indicated in table 16 above. For larger stars, they will be formed and fully evolved within one timestep, and thus not contribute to the integrated indices calculated at the end of that timestep. In practice of course, there would be some of these stars that form just before the end of one timestep and explode just after the start of the next, and as such should be part of the integrated stellar population recorded at the end of the timestep, but are not. For later timesteps, when the modelled galaxy consists only of a population of smaller, older stars, this simplification is reasonable, but it will mean that the overall luminosity, and the strength of the individual indices will be understated at earlier times, and this should be considered when graphs of these values over time are evaluated.

4.2.16 Galactic environment

The Phoenix model does not consider the effects of the galaxy being in a group or in the field. Sánchez-Blázquez et al. (2006b) showed that elliptical galaxies in low-density environments appear to be on average 1.5 Gyrs younger than those in higher-density cluster environments when modelled with the MILES SSP models of Vazdekis et al. (2010). Bregman et al. (2006) find an average galactic age of 10 Gyrs with no effect from environment. The galaxy's life is a free parameter within the model; the results of the Phoenix modelling do not find any difference in total galactic age with environment (table 40 below), and have an overall average age for the final populations of $13.05^{+0.1}_{-0.6}$ Gyrs for those galaxies with a prompt galactic wind (0.65-0.765 Gyrs after galaxy formed), and 12.68 $12.68^{+0.37}_{-3.68}$ Gyrs for those galaxies with a more delayed galactic wind (4.0-4.2 Gyrs after galaxy formed); the overall average galaxy age from these models is 13.26 Gyrs (6.5 below).

4.3 DETAILS OF MAJOR SUBROUTINES WRITTEN

4.3.1 Code written for GCE used in Phoenix

Phoenix incorporates the subroutines to read in and use data for planetary nebula options from G05 and vdH&G97, and SSP results from T04 that were originally written by the present author for the GCE model. For further details on these subroutines, see 2.2.3 and 2.2.4 above.

4.3.2 Evolve the galaxy

Details of the subroutine EVOLVE are given in figure 12. The user can select which yield options to use within the subroutines PNYIELDS (RV81, vdH&G97 or G05, whether to use results from WW95 or the Geneva group for large stars (between 8 and 40 $M_\odot$), and the rates to use for SNIa evolution (Timmes 1995 or Scannapieco and Bildsten 2005) via the file '*values.in*'. The model creates massbins (in steps of 0.1 $M_\odot$ up to 10 $M_\odot$, thereafter in steps of 1 $M_\odot$) of the new stars formed, calculating the mass held in that bin, the average star size, the chemical content of these stars, their main sequence lifetime, and, when calculated, the indices, weighted and unweighted, of these stars at the end of each timestep between them being formed and being fully evolved.

At each stage during this subroutine, the total mass and the mass fractions of hydrogen, helium and metals in stars and in gas, together with the masses of 14 selected elements, are updated. The evolutionary steps are calculated in series but of course in practice would occur in parallel. As planetary nebula events for higher metallicity stars can result in a reduction in oxygen, the model may in early timesteps temporarily appear to have "negative oxygen" or "negative carbon", because this process is calculated before the oxygen and carbon-enriching processes of SNII. A check is built in to ensure that by the end of the timestep, this has been corrected to a net positive figure.

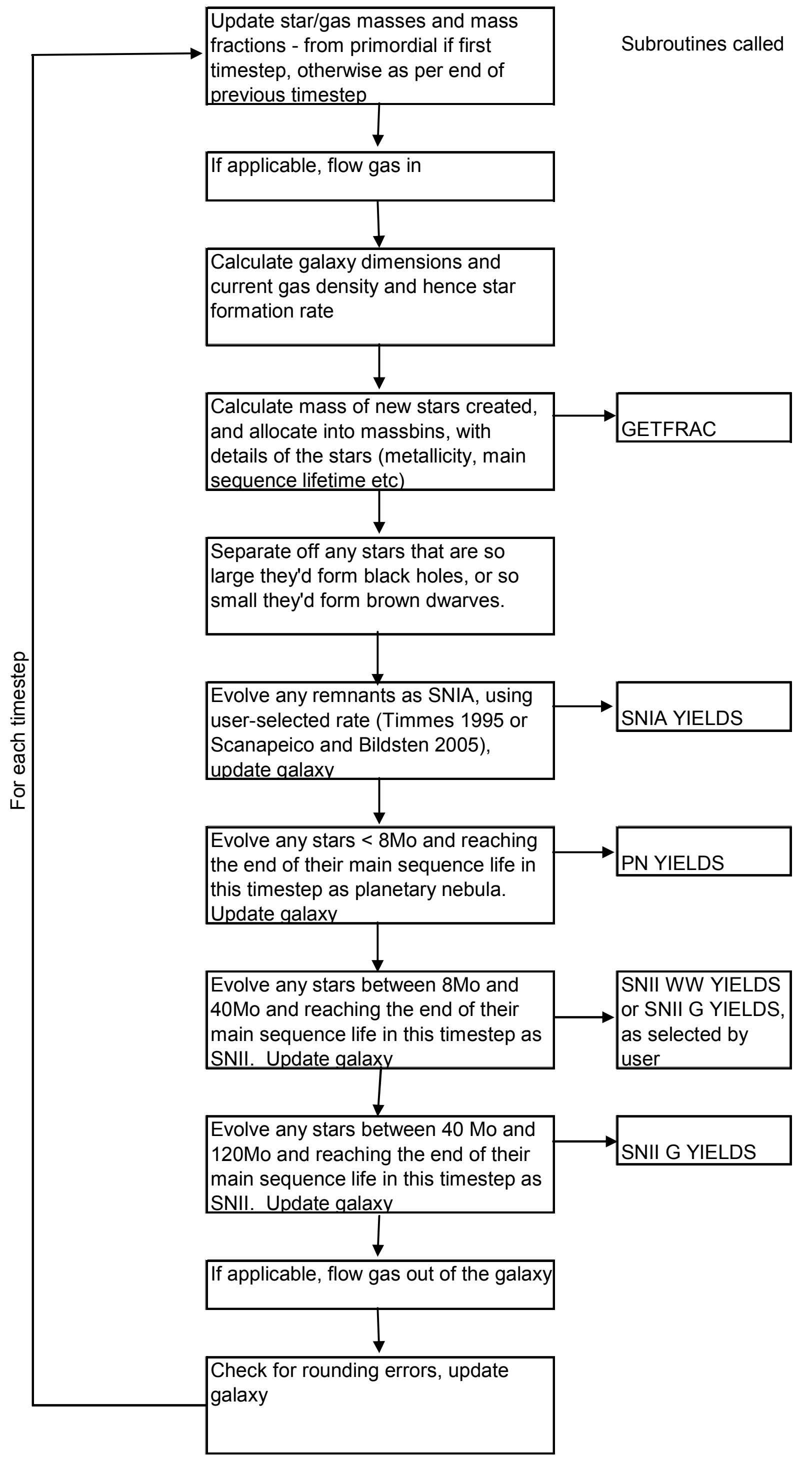


Figure 12: Flowchart for the subroutines EVOLVE.

4.3.3 Produce synthetic indices and colours

The process for creating the synthetic Lick indices and colours at the end of each timestep is given in figure 13 below.

Lick indices for each stellar combination of age and metallicity at the end of each timestep can be obtained by looking up (and interpolating where necessary) this information from the SSP selected by the user. However, the mass of the individual stars is important because larger stars will be more luminous and consequently the indices from these stars are more important when calculating the overall integrated indices of the modelled galaxy; the luminosity of the stars in each mass bin is used to appropriately weight the synthetic indices.

Isochrones give the luminosity for a given age, mass and metallicity of a star. Isochrones from Bertelli et al. (1994) (also known as the Padova isochrones) (hereafter B94) were chosen as they cover a wide range of ages, masses and metallicities, and in addition to the luminosity give values for the colours, which can be used where the SSP data set does not include this information.

The source data first needs to be sorted, as the interpolation subroutine within Phoenix requires the data to be monotonically increasing, however, the data within each table was presented in order of reducing age, and within each age broadly, but not consistently in order of increasing mass. Code within the READBERTELLI subroutine therefore re-orders the data within each table to have increasing order of age and within each age, increasing order of mass. The READBERTELLI subroutine also converts [age] to actual age, M_{bol} to luminosity using the relation (Ridpath 1997):

$$M_{bol} - 4.72 = 2.5 \log(L/ L_{\odot}) \quad (21)$$

The Phoenix model does not distinguish between stars of different temperatures, so where several isochrones are provided for one stellar mass at a given age and metallicity, the average is taken. The isochrone tables are of different lengths, which the code adjusts for, and, as elsewhere, where the data required is outside the range available, the nearest value is used and a warning sent to file.

The massbin is then updated with the absolute luminosity of those stars in that timestep, and the colours from the appropriate interpolated isochrone. Once all the massbins for that timestep have this data, the total luminosity for the galaxy can be obtained, enabling the luminosity contribution of the stars in that mass bin to the overall luminosity be calculated, and hence the indices can be weighted, enabling the total integrated Lick indices and total integrated colours of the galaxy at the end of that timestep to be output.

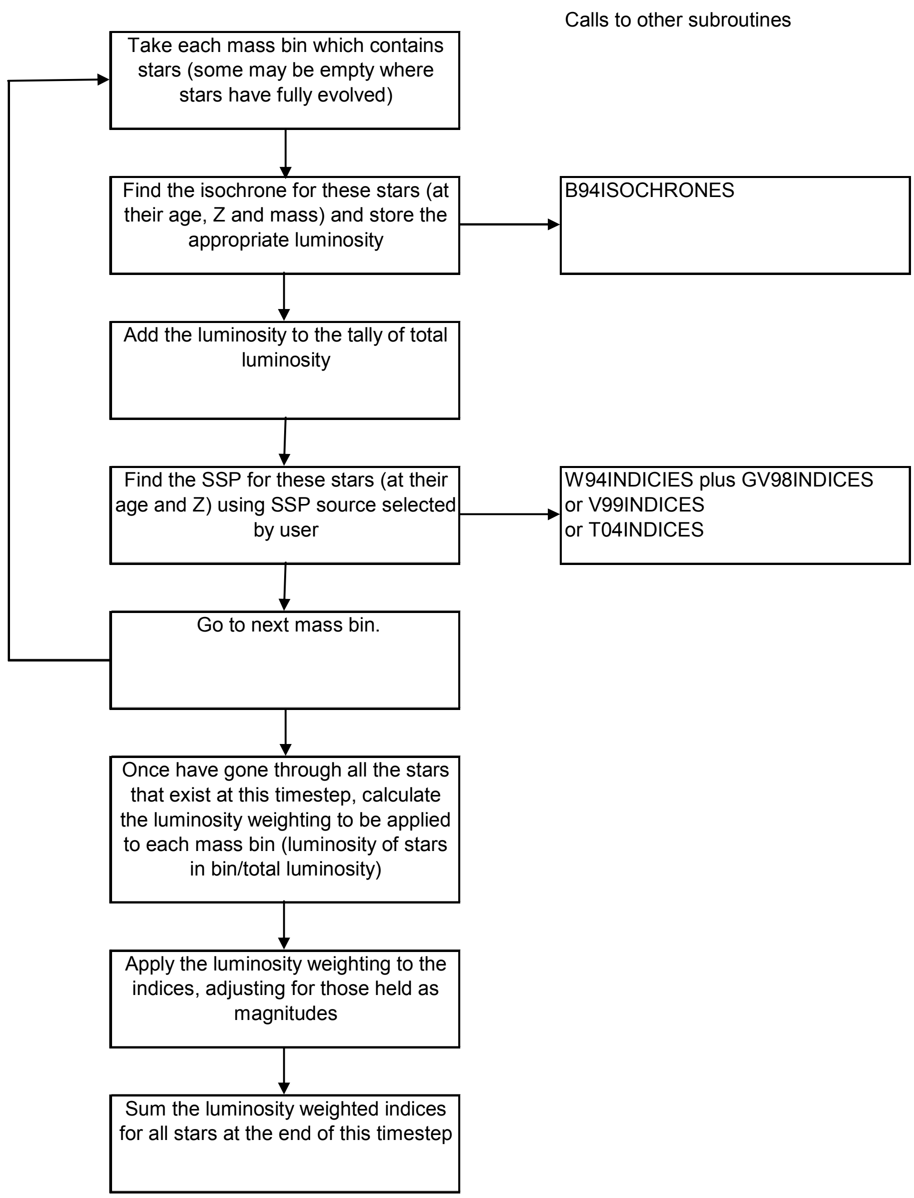


Figure 13: Subroutine MAKEINDICES within the Phoenix model.

4.4 MODEL OUTPUTS

4.4.1 Output of warning messages

During testing, warning messages were output to screen; once the model was working these were instead sent to a file which can be reviewed separately by the user. Warnings highlight where the model is using the nearest value because the data required is outside the range of data available, and where any values are becoming inappropriate (e.g. metallicity becoming unrealistically high). Warning messages are only generated on the single-run software; all Phoenix models reported within this thesis as successful were tested on the single-run software to ensure that they had not generated warning messages.

4.4.2 Output from single run model to Excel

The single-run software produces two outputs: one to screen and one to file. Screen outputs are a summary of the model run, compared to the selected observed galaxy, and are given as five tables. The statistical measure used is that outlined in 3.4.3.

1	Confirmation of the user-selected options for the model being run
2	Masses held as stars, gas, remnants (a sub-set of stars) and mass flowing in and out, in the first and last timesteps, and at the end of each Gyr.
3	Ejecta from different evolutionary processes, again in the first and last timestep, and at the end of each Gyr, together with the metallicity of the gas, and the overall galaxy in the galaxy at that time, and the luminosity of the galaxy.
4	Some anticipated outputs from the literature, such as final SNIa rate, final galaxy metallicity, final colours, final galaxy mass and luminosity, and those found by the model
5	A table of Lick indices and colours, giving the model value, the observed value and the uncertainty on the observed value, together with the calculated β for each index and an overall β_{ave} and a note of β_{msc}

Table 17: screen outputs from single run model.

The data output to file is at the end of every timestep, and has a column for each of the following (note: one box in this table may represent several columns in the data file):

Timestep, and Time since start of galaxy (Gyrs)
% hydrogen, helium and metals in the stars, gas and galaxy
Stars formed in this timestep, and star formation rate
Mass evolved as planetary nebulae, SNIa and SNII
Number of planetary nebulae, SNIa and SNII events
Mass held in stars, gas and galaxy
Mass of gas flowing in and out
Luminosity of the galaxy ($L_\odot$)
Radius of galaxy
Mass of each of 14 selected elements in the galaxy, and [α/Fe]
Luminosity-weighted lick indices and colours at the end of this timestep

Table 18: File outputs from single run software.

This data file can be output into a template Excel spreadsheet, which plots graphs of various functions over time. The template also allows a single model's outputs to be compared at one time to all the observational data from the two data sets. The following graphs and tables are produced within the template file:

- Metallicity of ISM over time
- Individual elements ejected by different evolutionary processes over time
- Mass held in stars and gas, and total galaxy mass, over time
- Luminosity over time
- SFR over time
- Mg/Fe and O/Fe relationships
- Synthetic indices plotted against the observed data sets (example given in figure 5 above)

Examples of some of these outputs are given in Chapter 5, where they are compared to results from the literature.

4.4.3 Output to Excel from "stepping software" model, for comparison of synthetic indices to observed data sets

The "stepping software" can be selected when the model is run, to work through a variety of values for the 8 free parameters, and to run the model for each combination. The final synthetic indices produced by each model, together with details of the final stellar population, are output into a file which can be exported into an Excel spreadsheet.

This Excel template takes the synthetic indices from all the models run by the stepping software, and compares them to all the observational data, calculating β for each index and β_{ave} and β_{max} for each galaxy. It identifies the model(s) with the lowest β_{ave} for each galaxy, and summarises these in a table. Results for each galaxy can then be checked to see if this is a unique model, or not. Where a unique model is identified as existing, either

- the searching parameters can be refined and the "stepping software" re-run; or
- the single run model can be used iteratively to find the star formation history of that galaxy;

in order to establish the star formation history of that galaxy, as proposed by the model.

4.5 CONCLUSIONS

The Phoenix model is a new, independent evolutionary population synthesis model based on the “bottom-up” approach, i.e. it evolves a galaxy based on stellar lifetimes and masses, and calculates synthetic luminosity-weighted Lick indices, which can be compared to observational data.

Chemical data is stored and can be used as a check on the accuracy of the model, but is not considered detailed enough to enable the Lick indices to be adjusted for non-solar abundances, or used for comparison to element data from the literature. As such, the model is not currently considered as a chemical evolution model, but could perhaps be developed into one in the future when there are better resources for yield/ejecta data in the literature.

As with any model, simplifications and assumptions are needed, either due to limitations within the literature, or in order for the model to be practical in terms of its complexity.

The model is tested in Chapter 5 and used to propose the star formation histories of nearby elliptical galaxies from two data sets in Chapter 6.

CHAPTER 5: TESTING PHOENIX

5.1 TESTING THE PHYSICS OF THE MODEL GALAXY

5.1.1 Introduction

The validity of any model of galactic evolution is found in its ability to reproduce successfully, and simultaneously, a variety of parameters from data sets of actual observations. This is the subject of Chapter 6; this Chapter deals with other tests of the model. This is achieved through three processes:

- comparing results when different input parameters are used, by varying the user-defined options and comparing results when changing each option, to optimise the model set-up;
- reviewing and identifying which parameters the model is most sensitive to, by temporarily amending some parameters with extreme values and assessing the impact; and
- testing the model against other models from the literature by making adjustments to the code (such as altering the star formation rate equation, or fixing the galactic radius rather than allowing it to be a calculated value) in order to align it to the comparison model, and then plotting results to see if similar outputs were obtained.

In practice, these tests were undertaken in parallel and iteratively with the testing of the model against actual observations.

The necessary temporary amendments to Phoenix are noted here where applicable.

5.2 TESTING USER OPTIONS

5.2.1 Introduction

The Phoenix model has a number of user options, mainly to select data sets to be used by the model, and these options can be set by the user in the file '*values.in*'. Parameters which are not expected to vary are listed within the file '*shared.f90*' (see Appendix B), where they can be amended if required.

To test the user options, two well-modelled galaxies NGC 2831 and NGC 3608 (as identified in Chapter 6) from the PS02 sample were taken, and the model was run with one single user option varied in turn. These two galaxies were selected as being well modelled but having very different timing of the galactic wind (0.75 Gyrs and 4.0 Gyrs respectively) in their best-fit model (Chapter 6).

The base Phoenix model set-up is as follows:

Parameter	**Value/star formation history**	
Galaxy mass	6 x 10^{10} $M_{\odot}$	
Proportion of initial gas in Population III stars	50%	
Galaxy life	13.26 Gyrs	
SFR constant	0.65	
Time of galactic wind	After 0.65 Gyrs	
Gas infall	none	
Model set-up	**Option**	
Planetary nebula yields from	Van den Hoek and Groenewegen 1997	
Large star ejecta from	Woosley and Weaver 1995	
Massive star yields from	Meynet and Maeder 2002 with rotation and wind	
SNIa ejecta from	Nomoto et al. 1984	
SNIa rates from	Scannapieco and Bildsten 2005	
SSP data from	Thomas et al. 2004	
Isochrone data from	Bertelli et al. 1994	
Model result	**NGC 2831**	**NGC 3608**
β_{ave} with this model	1.63	2.88
β_{max} with this best-fit model	5.595 (Mgb)	19.35 (Mgb)

Table 19: Parameters used to model NGC 2831 and NGC 3608 from the PS02 dataset. Note that values of β greater than ~5 correspond to very low likelihoods of occurrence and therefore that relatively small differences between two β values which are higher than ~5 are not statistically significant differences (see section 3.4.3). This applies to all uses of β in this thesis.

5.2.2 Varying input options

Not all model set-up options listed above can be varied with the current version of Phoenix, due to data limitations in the literature. Where there are user options, these were varied one at a time and the results are presented in table 20 below. Where the result is a better model than the base set up, the result is highlighted.

Data	Option selected	β_{ave}		β_{max}	
		NGC 2831	**NGC 3608**	**NGC 2831**	**NGC 3608**
Planetary nebula	Gavilán et al. (2005)	1.622	3.29	5.589 (Mgb)	20.20 (Mgb)
Planetary nebula	Renzini and Voli (1981)	1.619	2.83	5.548 (Mgb)	19.24 (Mgb)
Large star yields	Meynet and Maeder (2002) with rotation and wind	2.151	4.81	7.741 (Mgb)	24.24 (Mgb)
Large and massive star yields	Maeder (1992) with Meynet and Maeder (2002) correction for stars > 40 $M_{\odot}$	2.160	5.25	7.708 (Mgb)	24.92 (Mgb)
SNIa rates	Timmes et al. (1995)	2.301	3.87	7.141 (Fe5335)	16.08 (Mg1)
SSP data	Worthey (1994)	1.870	3.69	7.163 (Mg2)	19.42 (Mg2)
SSP data	Vazdekis (1999)	9.978	10.67	25.409 (Mg2)	32.07 (Mg2)

Table 20: β_{ave} and β_{max} for NGC 2831 and NGC 3608 from the PS02 data set obtained when available options selected. Where results are better than those of the selected "best fit" models, these are highlighted.

Results from RV81 can be seen to give better results for both galaxies, and results from G05 give better results for NGC 2831 than the selected set from vdH&G97. However, it was decided to continue to use the vdH&G97 results for two reasons: firstly, from the literature, the majority of models appear to use results from the vdH&G97 models, and secondly, the variation in the results from the test above shows the impact of changing source data is minimal.

As the better results were obtained using large star ejecta from WW95, massive star yields (converted to ejecta) from MM02, SNIa rates from Scannapieco and

Bildsten (2005) and SSPs from T04 (as given in the base model), these remained as the data sources for other runs of the model.

5.2.3 Testing gas inflow: timing, rate, duration and chemical composition

The Phoenix model is set up to allow the user to choose the following parameters:

- Time in Gyrs after the start of the galaxy when the gas inflow begins
- Duration of gas inflow in Gyrs
- Rate of gas inflow in $M_\odot$/Gyr
- Composition of gas inflow from
    - Primordial
    - Same as current composition of ISM
    - Solar
    - Enhanced (= twice solar)

If "same as current composition" is selected, but the gas outflow has taken place (i.e. there is no current ISM), the model uses solar composition. The first three of these parameters are also set as searching options within the parameter-space "stepping software" option of the model.

Gas inflow would enable the model galaxy to produce a new generation of stars from the combined chemical composition of the galactic ISM at the time of the inflow, and the inflowing gas (instantaneous mixing is assumed). Primordial gas inflow, therefore, would "dilute" any enriched ISM, lowering the metallicity for the next generation of stars, whereas "enhanced" inflow would be expected to increase the enrichment of the ISM and consequently increase the metallicity of the next generation of stars. This was tested using two galaxies from the SB07 data set: NGC 3384 and NGC 4472.

Galaxy	**NGC 3384**	**NGC 4472**
Population III percentage	33%	45%
Galaxy mass ($M_\odot$)	1 x 10^{11}	5 x 10^{10}
Galaxy age (Gyrs)	9	9
SFR constant	0.5	0.5
Time of gas leaving galaxy (Gyrs)	4.4	4.4
Gas inflow rate ($M_\odot$/Gyr)	10^9	10^{11}
Gas inflow composition	Primordial	Primordial
Gas inflow start time (Gyrs after start of galaxy)	2	2
Gas inflow duration (Gyrs)	2	0.5
β_{ave} of Lick indices with this model	35.50	36.42

Table 21: Parameters for galaxies from the SB07 sample which were initially modelled with gas inflow, from coarse-grid parameter-space searches.

The impact of amending the composition of the gas inflow was tested, keeping all other parameters the same:

Composition	**NGC 3384**	**NGC 4472**
Primordial	35.50	36.42
Same	37.93	51.04
Solar	37.37	38.98
Twice solar	37.42	39.66

Table 22: β_{ave} for two galaxies within the SB07 data set where initial coarse-grid parameter searching indicated gas inflow may be required for a well-fit model, showing effect of different chemical composition of inflow. Best option in each instance is highlighted.

This suggests that *if* gas inflow is required by the model, the composition should be primordial. This was also found by Pipino and Matteucci (2004), who model accreted primordial gas to moderate the star formation in their models. However, further testing of the Phoenix model with the galaxies from the PS02 and SB07 data sets indicated that a better-fit model was obtained if there was *no* gas inflow, irrespective of its composition.

5.2.4 Testing gas outflow: timing

Whilst the process of removing gas from elliptical galaxies is needed in order to quench star formation, the actual method by which this happens is not yet known (e.g. Gabor et al. 2011) although thought to be as a result of AGN and/or SNII wind energy being sufficient to expel the gas from the galaxy's gravitational effects. The Phoenix model has been written to explore these two methods: an instantaneous loss of gas, at a given time, followed by any residual gas (produced by subsequent stellar evolution) being immediately ejected to mimic AGN effects, or gas loss dependant upon star formation - 'mass loading' – to mimic SNII driven feedback.

This was tested using the data sets from PS02 and SB07 and the "stepping software", enabling parameter space to be searched for the best-fit models, measured by β_{ave} (table 23). From the results discussed in Chapter 6, the galaxy life was set to 13.26 Gyrs and gas inflow set to zero.

Galaxy	Data set	Galactic wind method	Best fit value for time of galactic wind (Gyr after start)/ loading factor (multiple of stars formed)	Best fit model: galaxy mass (x 10^{10} Gyr)	Best fit model: SFR constant	Best fit model: % of initial gas forming Pop. III stars	Lowest β_{ave}
NGC 2831	PS02	Time	0.75	6.0	0.65	44%	1.57
		Load	1.5	3.1	0.70	43%	2.91
NGC 2832	PS02	Time	4.1	3.7	0.45	53%	2.86
		Load	1.0	3.7	0.65	43%	4.06
NGC 3226	PS02	Time	4.0	5.7	0.43	39%	3.50
		Load	1.5	3.1	0.70	39%	4.30
NGC 3608	PS02	Time	4.0	5.7	0.43	39%	2.88
		Load	1.5	4.0	0.70	39%	3.79
NGC 4291	PS02	Time	4.0	5.7	0.45	37%	3.38
		Load	1.5	3.5	0.70	39%	4.29
NGC 4365	PS02	Time	4.2	4.0	0.55	54%	3.24
		Load	1.0	3.7	0.65	43%	3.83
NGC 4374	PS02	Time	4.0	5.7	0.53	39%	2.95
		Load	1.5	3.1	0.60	39%	4.00
NGC 4552	PS02	Time	4.0	3.7	0.45	53%	3.35
		Load	1.5	3.1	0.60	39%	4.22
NGC 4636	PS02	Time	4.1	3.7	0.45	53%	2.60
		Load	1.5	4.0	0.70	39%	3.46
NGC 4697	PS02	Time	4.2	6.0	0.65	42%	2.74
		Load	0.85	3.1	0.70	43%	3.20
NGC 5322	PS02	Time	0.765	5.6	0.65	53%	2.58
		Load	1.5	4.0	0.60	39%	3.43
NGC 1600	SB07	Time	4.00	3.7	0.45	53%	19.07
		Load	1.5	3.5	0.70	39%	20.13
NGC 1700	SB07	Time	0.765	5.5	0.57	47%	18.42
		Load	1.5	3.1	0.70	39%	27.75
NGC 3377	SB07	Time	0.65	6.0	0.65	46%	25.36
		Load	1.5	3.1	0.70	39%	31.81
NGC 3379	SB07	Time	4.00	3.7	0.45	53%	29.44
		Load	1.5	3.5	0.70	39%	41.10
NGC 3384	SB07	Time	0.765	6.7	0.45	43%	32.67
		Load	1.5	3.5	0.70	39%	35.90
NGC 4387	SB07	Time	0.765	5.5	0.45	41%	9.52
		Load	1.5	3.5	0.60	43%	22.16
NGC 4458	SB07	Time	0.65	6.0	0.65	46%	10.23
		Load	1.5	3.1	0.70	39%	22.73
NGC 4464	SB07	Time	0.765	6.5	0.67	43%	15.39
		Load	1.5	3.1	0.70	39%	22.68
NGC 4472	SB07	Time	4.00	3.7	0.45	53%	27.08
		Load	1.5	3.5	0.70	39%	37.00
NGC 4551	SB07	Time	0.765	5.5	0.57	47%	11.31
		Load	1.5	3.5	0.60	43%	20.92

Table 23: Comparing best-fit models with different methods for gas removal. Better option in each instance highlighted.

As well as the timing of the galactic wind and the mass loading factor, galaxy mass, percentage of initial gas that immediately forms Population III stars and the constant in the star formation rate equation were searched find the best fit model.

In each instance, a better-fitting model was obtained using the galactic wind at a specific time method, rather than the mass-loading method, suggesting that the ISM is lost due to AGN. Note that the best-fit models for the gas-loading method generally have lower initial galaxy masses and higher initial Population III percentages and star formation rates.
Other modelling tests were therefore only carried out using the "galactic wind at a specific time" method.

5.3 TESTING MODEL SENSITIVITY

5.3.1 What makes the model fail?

During the building of the Phoenix model, various parameters were tested to see whether the model could withstand extreme values, including those outside the expected ranges. The model sends warnings to a file (to the screen during testing) indicating if it is failing (for example, if it is forced to use values outside the range of data available).

5.3.2 Galactic radius

It was found that the results were critically dependent upon the star formation rate (compared with any other factor), which in turn was dependent upon the value used for the galaxy radius, as this gives the volume and hence density of the galaxy. Using a fixed value for the radius or using an equation that did not hold over a wide range of galactic masses resulted in star formation histories that were inconsistent with expected results. For example, setting the radius too large reduces density and results in too few SNII events for adequate galactic enrichment (see table 15 above for the literature sources originally tested with Phoenix).

The Phoenix model calculates the galaxy radius from the half-light equation for elliptical galaxies given by Shen et al. (2003, corrected 2007) (equations 15-17). This (from their paper) is only valid over the mass range 4 x 10^8 to 1 x 10^{12} $M_\odot$, Phoenix uses the relationship irrespective of these limits. If this valid mass range is contravened, for example, when gas inflow is modelled and new stars are formed, a warning appears on screen.

5.3.3 Population III stars forming from initial gas cloud

The initial model set-up is a gas cloud, consisting of (to two decimal places) 75.23% hydrogen, 24.77% helium and 0.00% metals (Peimbert 2008). Some of this gas is assumed to form Population III stars during the first timestep. Rather than using the SFR equations for this first timestep, a user-defined percentage of this initial gas is converted to stars. This was found to only be valid between 37% and 54%.

5.3.4 Other tests

Other tests were performed by manipulating values normally taken as fixed parameters, as shown in table 24 below.

Parameter	"standard" value	Comments
SFR index in Schmidt (1959) equation	1.3 (Kennicutt (1989)	Model fails if set to 1.0 (i.e. setting SFR simply proportional to gas density). Schmidt (1959) and Bothwell et al. (2011) suggest higher values of 1.4 and 1.51 respectively. These values were tested, but it was impossible to model the output SFR in a way that was comparable to others in the literature (e.g. Calura et al. 2009 see 5.4.1 below)
IMF index	-1.35 (Salpeter 1955)	Model fails if set to -1.0 as this results in a divide-by zero error in the equation. If set to above -1.35, fewer low mass stars are produced, and more high mass stars. This increases the ISM enrichment (as more stars evolve as SNII) and reduces the final population (fewer small stars with long lives). The inverse is true if set to below -1.35.
Critical density	0.3625 $M_\odot$ kpc^{-3}.	If set to absolute minimum i.e. zero, model still works but is physically incorrect. If set to higher values, model works but fewer stars form so fail to generate adequate chemical enrichment.
Maximum star size	120 $M_\odot$	Stars above 120 $M_\odot$ use scaled-up values of yields from the Geneva Group; the IMF means that these large stars are rare and consequently have minimal impact on the overall enrichment of the galaxy.
Minimum star size	0.05 $M_\odot$	Maximum size for brown dwarf is 0.08 $M_\odot$; stars this size and smaller do not contribute to the luminosity of the galaxy and hence not to the indices. Increasing the minimum value above 0.08 $M_\odot$ removes from the model some of the long-life stars, reducing the population of the final galaxy.
Minimum size for black hole	130 $M_\odot$ (i.e. no black holes formed)	The code "removes" stars that form above this threshold; they do not participate in integrated spectra nor contribute to nucleosynthesis. Therefore, reducing this value reduces the chemical enrichment of the galaxy, as fewer stars undergo SNII.

Table 24: Effect of varying parameters within the Phoenix model.

5.4 TESTING PHOENIX BY COMPARISON WITH OTHER MODELS IN THE LITERATURE

5.4.1 Basic galaxy parameters

Models of Calura et al. (2009) were recreated using Phoenix. This required setting the initial galaxy mass at 1.5 x 10^{10}, 5 x 10^{11} and 5x10^{12} so that after the gas has left the galaxy, the residual galaxy mass was respectively 10^{10}, 10^{11} and 10^{12} $M_{\odot}$. The star formation constant, primordial gas inflow and timing of gas outflow were set as in the Calura et al. (2009) models. The percentage of initial galactic mass forming Population III stars was set at 40%; this is not a parameter noted within the Calura models but this was selected as being representative from the final results of the Phoenix model (Chapter 6).

Figure 14 below shows that the Phoenix model produces similar star formation and supernovae rates to those of Calura et al. (2009). As the galactic wind modelled by Phoenix removes all of the interstellar gas, the mass of oxygen in the ISM after the time of the wind is zero in the Phoenix models, whereas the graphs of the Calura et al. (2009) models indicate a continuation of ISM after the galactic wind. The main differences are in values during the initial timesteps of the models, which are distorted for the Phoenix models due to the Population III stars which are input rather than modelled; it would appear from the graphs that the Calura et al. (2009) models take all outputs from standard star formation equations. These Population III stars in the Phoenix model give rise to the distorting early peaks of star formation and consequently SNII rates. It is noted by Pagel (1997) that some evolution models do have 'prompt initial enrichment' or 'initial nucleosynthesis spike' representing hypothetical pre-galactic or proto-galactic processes, perhaps involving high-mass objects, or prior enrichment by products from a neighbouring more evolved system.

The models are similar enough for post-initial stages to provide reassurance that the Phoenix model is physically similar to those of Calura et al. (2009) for these parameters over this set of galactic masses.

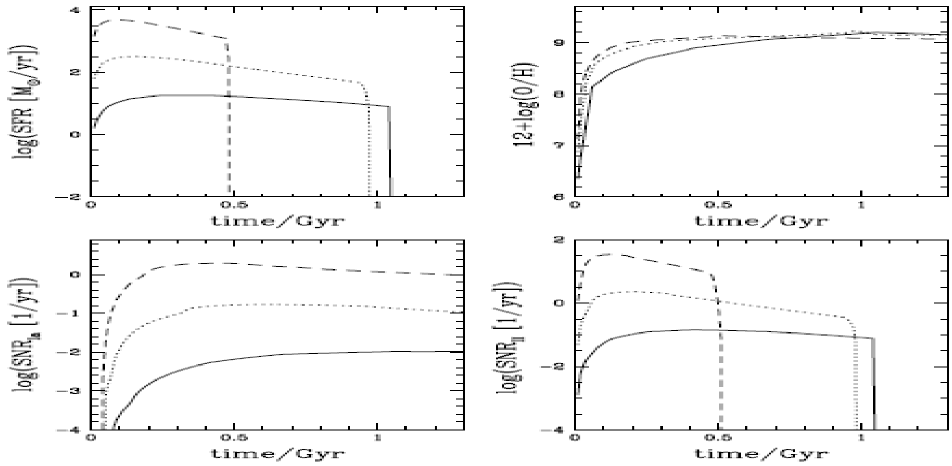


Fig. 1. From top left corner, clockwise: SFR, interstellar O abundance (in units log(O/H)+12), Type II SNR, and Type Ia SNR vs time for the three elliptical galaxy models used in this paper. The solid, dotted and dashed lines are the predictions for the models with total baryonic mass $10^{10}M_\odot$, $10^{11}M_\odot$, and $10^{12}M_\odot$, respectively.

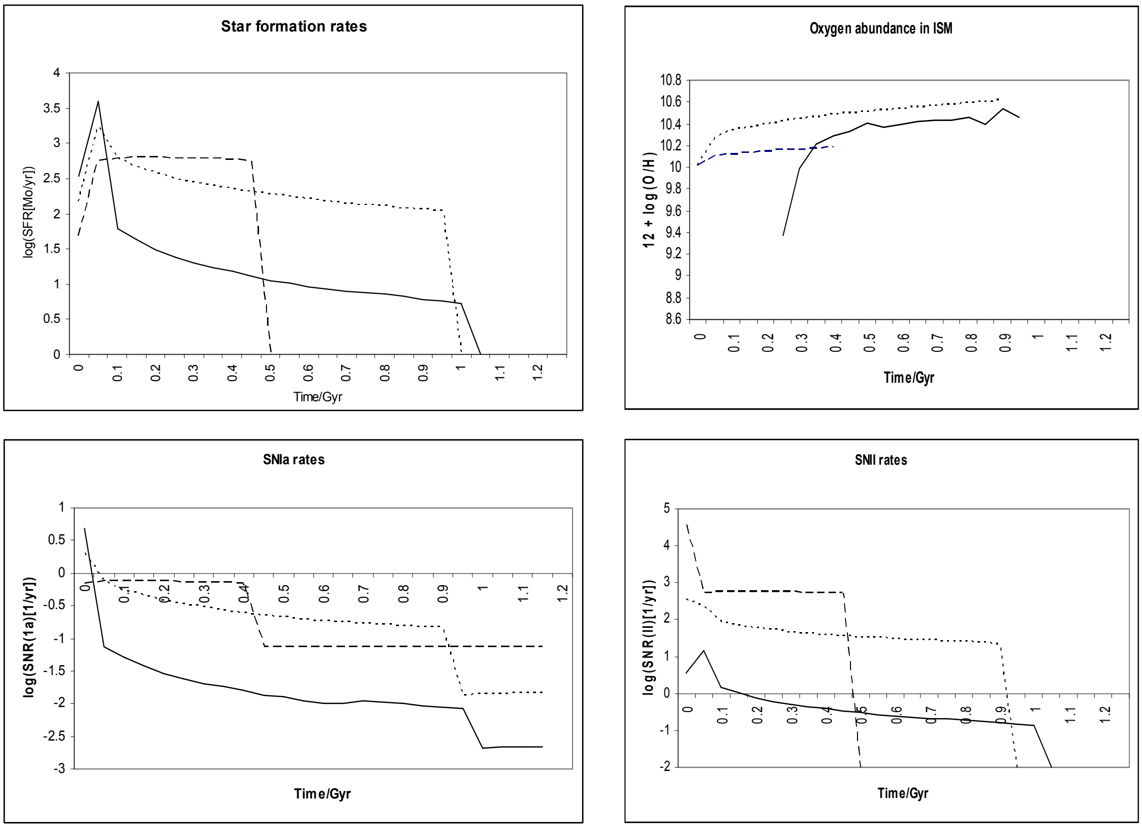


Figure 14: Top: extract from Calura et al. (2009), bottom: same graphs created from Phoenix model, using same key as Calura et al. (2009).

5.4.2 Supernova rates

A bi-modal SNIa rate was proposed by Scannapieco and Bildsten (2005) to reflect the two mechanisms by which these can form: from collisions between white dwarf stars (double degenerate) or where the hydrogen envelope is lost from a star to its smaller companion in a binary system (single degenerate).

A test was performed to see if Phoenix could recreate the graph from Scannapieco and Bildsten (2005) (figure 15 below). The only change to the Phoenix code was to make the star formation rate equation proportional to $e^{-t/2Gyr}$ (as an alternative to the Schmidt (1959) equation normally used by Phoenix) The constant of proportionality used was 10^{10}; this forced the final stellar mass of the Phoenix galaxy to be 10^{10} $M_\odot$, which recreates the SFR equation and final stellar mass of the models used by Scannapieco and Bildsten (2005).

Variable	**Option selected**
Planetary nebula yields from	Van den Hoek & Groenewegen 1997
Large star ejecta from	Woosley and Weaver 1995
Massive star yields from	Meynet and Maeder 2002
SSP data from	Thomas et al. 2004
SNIa rate equation from	Scannapieco and Bildsten 2005
Galaxy mass	10^{12} $M_\odot$
Galaxy lifetime	13 Gyrs
Gas inflow/outflow	none

Table 25: Model set up for testing supernovae rates over time.

Scannapieco and Bildsten (2005) fixed their SNII rate as 3 x the SNIa rate, rather than modelling it separately; for the Phoenix output, the actual SNII rates were plotted, and can be seen to be comparable.

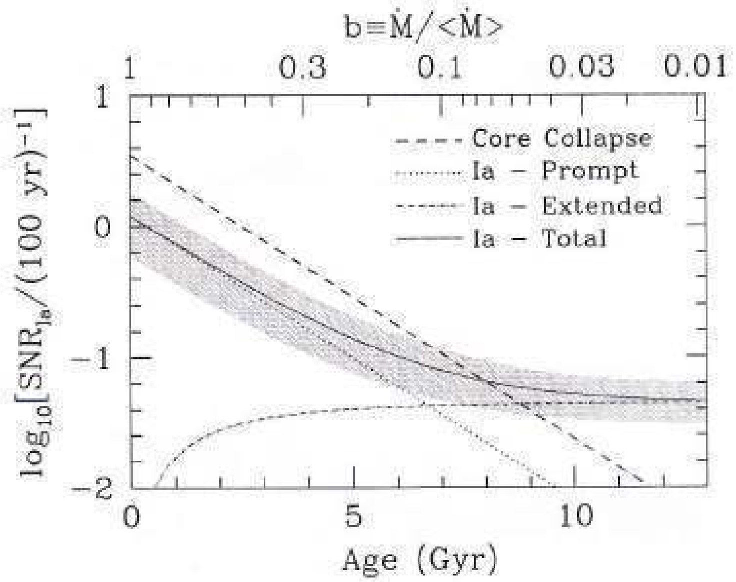


FIG. 1.— The supernova rate in a galaxy with a final stellar mass of $10^{10} M_\odot$. The solid line gives our model predictions for the Type-Ia SN rate (bracketed by 1 sigma errors), which is made up of the prompt (dotted) and extended (dot-dashed) components. The dashed line gives the core-collapse SN rate. In all cases we assume a star formation rate $\propto e^{-t/2\mathrm{Gyr}}$. Choosing a different characteristic star-formation decay time would rescale the time axis, while leaving the Scalo b values unchanged.

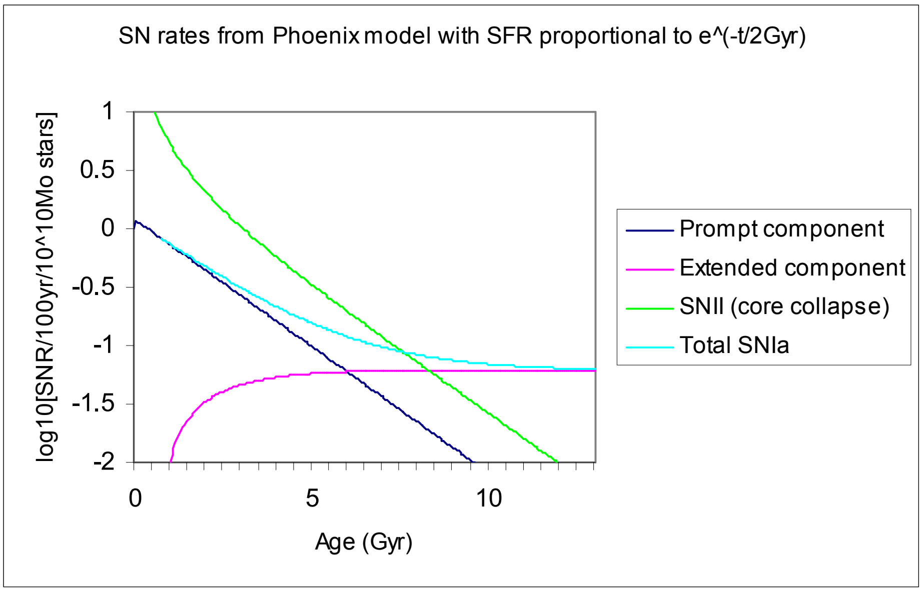
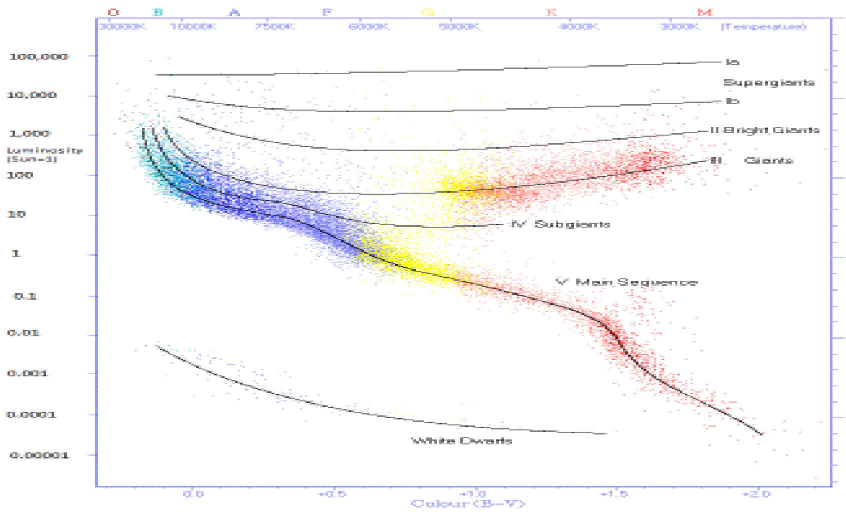


Figure 15: Top: extract from Scannapieco and Bildsten (2005), bottom: modelled by the Phoenix with a final stellar mass of 10^{10} $M_\odot$. The star formation rate is taken as α $e^{-t/2Gyr}$.

5.4.3 H-R diagram

The stars that exist in the final timestep (i.e. model of present-day population) for all the best-fit models to the data from PS02 and SB07 (Chapter 6) are plotted on a single Hertzsprung-Russell diagram (Figure 16). This plot does not distinguish between the different galaxies modelled, nor between individual stars if they have the same B-V/L co-ordinates (the plot is actually of some 10^{13} stars).

This shows overall that the final population stars are as expected for evolved elliptical galaxies: a mixture of white dwarf and evolved lower-main sequence, and no hot blue stars.



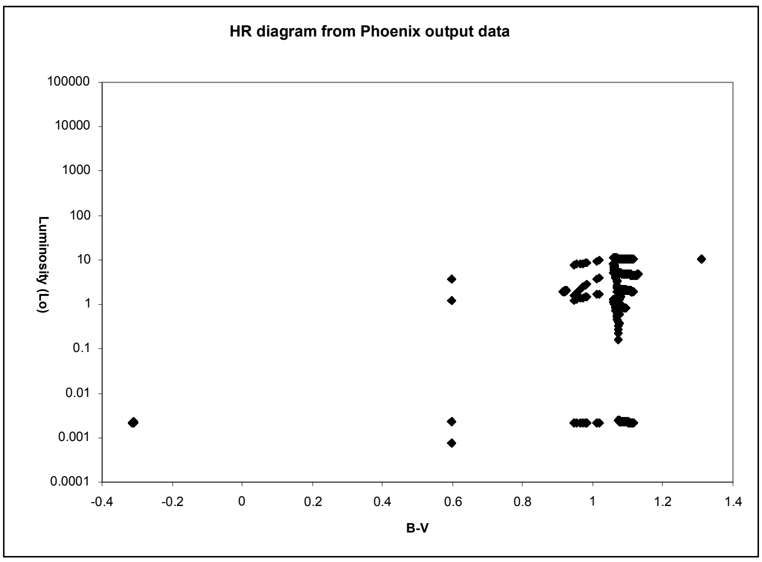


Figure 16: Top: HR diagram adapted from www.atlasoftheuniverse.com/hr, bottom: HR diagram from Phoenix results for elliptical galaxies.

5.4.4 Element production

The Phoenix model, whilst tracking individual elements as far as it is able to, would not be expected to produce accurate element abundances due to limitations of data used from the literature.

- WW95 (for SNII up to 40 $M_{\odot}$) and Nomoto et al. (1984) (for SNIa) results are for ejecta of a wide range of elements;
- However, models from the Geneva Group (e.g. M92, MM02, Hirschi et al. 2005), used for massive stars, only give details of carbon and oxygen, and results are for yields not ejecta; and
- Models for planetary nebulae from RV81, G05 and vdH&G97 are yield rather than ejecta data.

Where only yield data is provided, the Phoenix model calculates material that would be recycled from the star, to give the expected ejecta.

The Phoenix model ignores the element production that takes place within stars during their lifetimes, using the chemical composition of the ISM to calculate the initial composition of new stars as they are formed, and recycling these elements without further evolution into the ISM where the evolutionary end data are for yields rather than for ejecta. No allowance is made for the other elements ejected by massive stars where the Geneva Group data is limited to carbon and oxygen. Elements tracked by the Phoenix model are not linked to the synthetic Lick indices produced; these are taken from tables of SSPs and based on the overall metallicity, rather than individual element abundances. The current Phoenix model is, therefore, NOT a chemical evolution model, but is a galactic evolution model that may in the future be developed into a chemical evolution model (when there are more comprehensive results for supernova and planetary nebula ejecta available in the literature).

If it is assumed that a galactic wind removes the entire ISM and at that point star formation ceases (having no material from which to form new stars), then a short time later, all SNII events will cease, as all the large stars formed prior to the galactic wind will have fully evolved. In addition, the Phoenix model assumes the ISM will be negligible from that point forward, and any enriched ejecta from SNIa or planetary nebulae are immediately ejected from the galaxy .

For the following test, Phoenix was compared to the elliptical galaxy model of Pipino and Matteucci (2004).

Parameter/ data	**Pipino and Matteucci (2004)**	**Phoenix**
Galactic radius	Fixed at 3.0 kpc	Calculated by the model as 2.47 kpc
Galactic mass	10^{11} $M_{\odot}$	10^{11} $M_{\odot}$
SFR	= 10 ρ	= 70 $\rho^{1.12}$ (Set to this value in order to replicate Pipino and Matteucci outputs)
Initial mass function	Salpeter (1955)	Salpeter (1955)
SNIa rate	Fixed at 0.18 century^{-1}	Fixed at 0.18 century^{-1}
Gas inflow	Primordial for 0.709 Gyrs	Primordial for 0.709 Gyrs
Timing of galactic wind	At 0.709 Gyrs	At 0.709 Gyrs
Ejecta for SNIa	Nomoto et al. (1984)	Nomoto et al. (1984)
Ejecta for stars < 8 $M_{\odot}$	Van den Hoek and Groenewegen (1997)	Van den Hoek and Groenewegen (1997)
Ejecta for stars > 8 $M_{\odot}$	Thielemann et al. (1996) or scaled from this (source data goes up to 25 $M_{\odot}$)	Woosley and Weaver 1995) or scaled from this (source data goes up to 40 $M_{\odot}$)
Manual adjustments to ejecta data	Mg increased by factor 10 in mass range 11-22 $M_{\odot}$ and reduced by factor 10 for >22 $M_{\odot}$	none
Solar values	Anders and Grevesse (1989) but Holweger (2001) for oxygen	Anders and Grevesse (1989) but Holweger (2001) for oxygen

Table 26: Comparison of models set up to compare abundance ratios in the ISM.

Pipino and Matteucci (2004) note that the SNII yields they use (Thielemann et al. 1996) are systematically higher than those of WW95, which are used by Phoenix. Hence the abundance ratio data in Pipino and Matteucci (2004) figures 1 and 3 (reproduced on Figure 17 below) will be expected to be higher than the results from Phoenix. These graphs shows that α–elements are enhanced in the ISM at low metallicities i.e. SNII events dominate element production in the early stages of the galaxy's life. In order to replicate the Pipino and Matteucci (2004) data, the star formation rate equation used in Phoenix had to be set at a higher rate than that used by Pipino and Matteucci. Note also that the Phoenix model ceases to have any ISM elements after the galactic wind unlike the Pipino and Matteucci (2004) models.

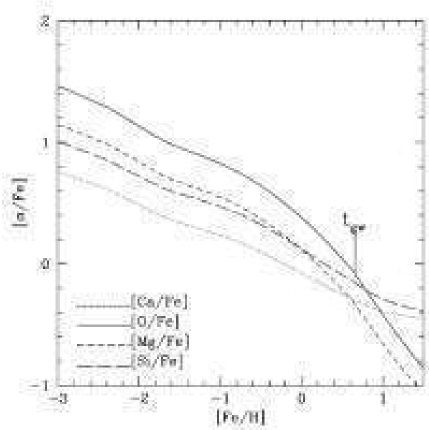


Figure 1. Theoretical [O, Mg, Si, Ca/Fe] abundance ratios in the ISM as functions of [Fe/H] predicted by Model II, for the core of a 10^{11}-$M_\odot$ galaxy. Solar value for O by Holweger (2001). The time for the occurrence of the galactic wind, t_{gw}, is indicated.

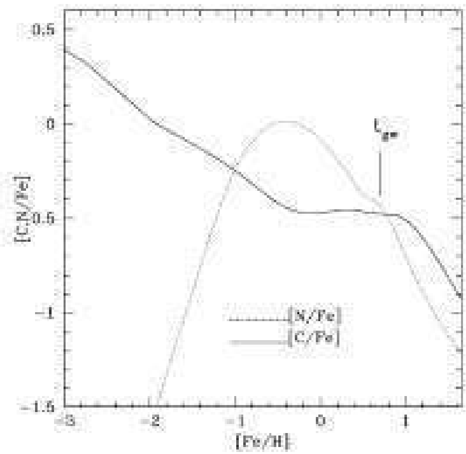
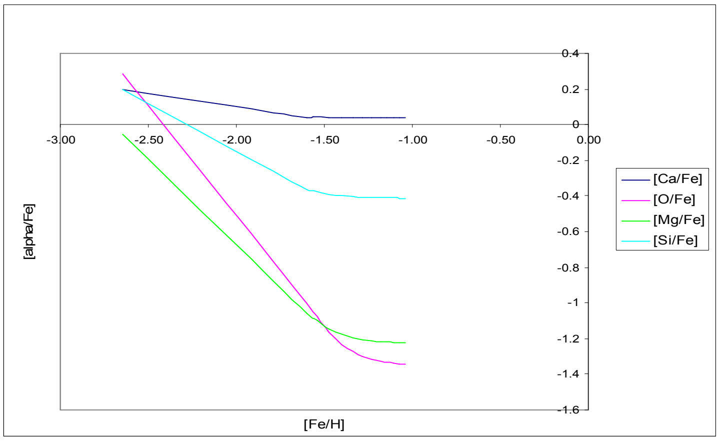


Figure 3. Theoretical [C,N/Fe] abundance ratios in the ISM as functions of [Fe/H] predicted by Model II for the core of a 10^{11}-$M_\odot$ galaxy.



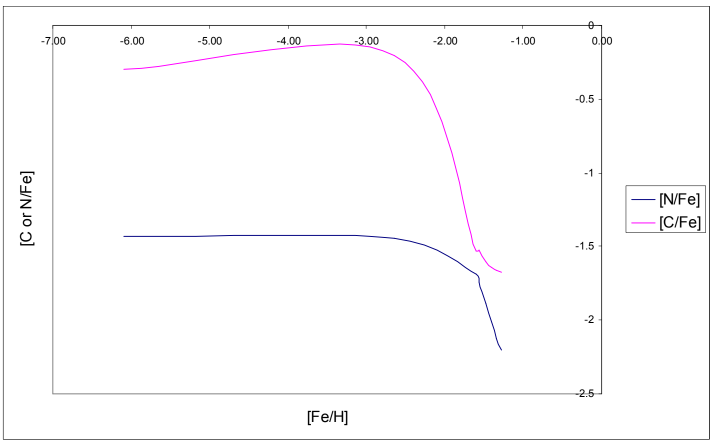


Figure 17: Top: Figures 1 and 3 from Pipino and Matteucci (2004). Note that this model continues to have gas in the ISM after the galactic wind, whereas Phoenix does not. Bottom: abundance ratios in the ISM as functions of [Fe/H] from the Phoenix model.

5.5 ERROR ESTIMATES FOR THE SYNTHETIC INDICES OUTPUT BY THE PHOENIX MODEL

5.5.1 Introduction

The Phoenix model results do not include uncertainties with the synthetic indices, however, that is not to say that there are no uncertainties in the model.

Estimating uncertainties on the Phoenix model is not straightforward. To estimate these uncertainties, the various input parameters could be individually changed to their maximum and minimum values, the model re-run and the maximum and minimum effect of these cumulative changes identified. However, the input values used by Phoenix come from two sources: observed (which generally include uncertainties), and other models (which generally don't), making it virtually impossible to estimate the overall uncertainty consequently arising within the Phoenix model. Detailed testing would also require varying different combinations of uncertainties, as well as testing them one at a time, because uncertainties may be negatively correlated: the maximum of one uncertainty combined with the minimum of another may make an overall larger uncertainty than each of these individually.

5.5.2 Intrinsic coding limits

Due to the nature of Fortran, π and e, where required, have to be hard-coded by the programmer; to minimise errors, these values have been given in Phoenix to 9 decimal places.

In addition, Fortran has precision limits – the number of digits that the computer can hold for any given number - which can affect a code of this nature, dealing as it does over the wide range of data from the very small (gas densities) to very large (galaxy dimensions) (discussed in more detail in 2.3.2 above). The code could not deal in one line, for example, with the equation from Gibson (1997) for the radius, which required the galaxy mass to be converted to units of 10^{12} $M_{\odot}$; the mass had to be divided by 10^6 twice.

5.5.3 Source data and rounding errors

Values from the literature used within the model are used without the uncertainties given in the original source, because calculating the cumulative effect of these errors would require the model to be run with each parameter tested at each extremity, and then the results combined to give an overall estimate of the error. Note that as some sources do not have any estimate of uncertainties, although some uncertainties would be expected to be significant, such an exercise could only be complete as far as source uncertainties have been provided by the original authors.

<u>SFR index</u>

The value of the Schmidt (1959) SFR index used by the Phoenix model for this thesis was 1.3 $M_{\odot}$ pc^{-2} Gyr^{-1} (Kennicutt 1989). Schmidt (1959) gave 1.4 for the current rate of gas consumption and noted this would vary amongst different objects. Kennicutt (1989) calculated the index to be 1.3 ± 0.3 provided the density was above a critical threshold, and higher close to that threshold density. His calculations of this value were derived from relatively small galaxy samples (15 galaxies, all spirals, as he required present-day star-forming regions for the data); he assumed that the SFR in a current spiral is the same as in an elliptical during its star-forming period. Current large-survey data sets enable this value to be further refined, with the latest value being 1.51 ± 0.08 (Bothwell et al. 2011), although it was found that using these alternative values gave model outputs that were not comparable to others in the literature (5.3.4 above).

<u>IMF index</u>

The power-law index in the Salpeter (1955) initial mass function was given as "approximately" -1.35, but without any associated uncertainty. Work since then has focused on the extreme values of stellar masses - at low masses, the function is flatter (Miller and Scalo 1979, Kroupa et al. 1990, but neither of these papers gave estimates of uncertainties on the index. Scalo (1986), reviewing the high-mass end of the range suggests that the index for higher masses is between -1.3 and -2.3 ± 0.5. The Phoenix model uses the standard Salpeter IMF.

<u>SNIa rates</u>

Scannapieco and Bildsten (2005) give an equation for a two-component model for SNIa rates; each component includes a constant whose value has an uncertainty associated with it: the delayed component, which tracks the galactic mass, has a constant A = $4.4^{+1.6}_{-1.4}$ x 10^{-2}, and the prompt component, which tracks the instantaneous star formation rate, has a constant B = 2.6 ± 1.1. These uncertainties will increase/decrease the number of SNIa events, and therefore increase/decrease both the metallicity of the galaxy, and decrease/increase the α/Fe ratio. The equation is used within Phoenix without the uncertainties because, when tested, the effect on the final galaxy was minimal.

As an alternative to Scannapieco and Bildsten (2005), the user of the Phoenix model can select the SNIa rate from Timmes et al. (1995); this is from their model which does not include any uncertainties so is a single constant value for the rate.

<u>Main sequence lifetimes</u>

Wood (1992) gives a relationship between mass and main sequence lifetime of stars below 8 $M_{\odot}$ (equation 8); there are no uncertainties in either the final equation or the assumptions used to derive it.

Errors estimates in the source data are not always provided, or, when available and tested in isolation, do not significantly alter the output from the Phoenix model.

5.5.4 Yield/ejecta, SSP and isochrone uncertainties

SSP data sets from W94, V99 and T04 provide synthetic indices for different sub-populations of the Phoenix model galaxy. These data sets, constructed from underlying isochrones, are not published with uncertainties. Together with isochrones (used to luminosity-weight these model indices) they produce the final model with which the observed data are compared. As there are (albeit minor) differences in the SSP sets (see an example in figure 5) there must be underlying systematic errors, arising from different assumptions or different input data.

Yield/ejecta data used by the Phoenix model are also based on theoretical stellar models. These models are also presented in the literature without reference to uncertainties, and so the impact of uncertainties in these yields/ejecta, when used in another model such as Phoenix, cannot be easily estimated.

In addition, and in particular, theoretical calculations of the yields/ejecta of Mg in SN models are known to have a large uncertainty factor of ~ 3 (Timmes et al. 1995), so a large uncertainty on magnesium indices produced in SSP models would be expected.

Conroy et al. (2009, 2010a, 2010b) have also reviewed the uncertainties within SSP models. This series of papers reviews the different areas where errors in SSP models (and consequently in models such as Phoenix, which rely on SSP data) exist. Conroy et al. note that the main areas of weakness in the SSP models are:

- inadequate modelling of the metallicity-dependence of the thermally-pulsating asymptotic giant branch phase of stars;
- a lack of appropriate star cluster data that can be used for calibration of simple models to more extensive systems – there are not many old, metal rich star clusters to use to calibrate results for old, metal rich galaxies;
- stellar libraries do not have complete sets of data for the key stellar parameters of effective temperature (T_{eff}), metallicity and surface gravity (*g*);
- the general issue of a poor understanding of detailed stellar evolution of high mass stars; and
- uncertainties in the IMF, both its slope and whether it varies spatially or temporally.

5.6 DISCUSSION

Data sets used by Phoenix are all limited in some way: they generally only give results for solar metallicities and lower, or only include stellar masses in a certain range, or are based on limited data or on other models which in turn may have limitations. The ability to model the star formation history of nearby elliptical galaxies accurately may well therefore be correspondingly limited due to these constraints; Phoenix will use the nearest available value when the actual value it requires is not available and cannot be reasonably extrapolated from the data. The model outputs a warning to a file whenever the "nearest value" is used; it is important to note that the successful models reported in this Chapter and Chapter 6 were checked and did not need to make use of these nearest-value estimates.

The model is very sensitive to changes in the radius of the galaxy, and the consequential impact on density and hence star formation rate. There are rather limited data within the literature correlating mass and radius, or indeed radius with any other parameter.

The Phoenix model is also sensitive to the proportion of the initial gas cloud which forms zero metallicity (Population III) stars in the first timestep. These stars are not well understood, and there is little in the literature to give physical support to any assumption about the percentage of stars that may be formed in this way. The "G-dwarf problem" (van den Bergh 1962), whereby in the solar neighbourhood there appears to be inadequate low-metallicity stars compared to models which include them may be a function only of spiral galaxies, or of limited observations, and may in any case be resolved with modelled gas inflow (Lynden-Bell 1975, Clayton 1988, Martinelli and Matteucci 2000) – and as such, may not be a relevant criticism of the proportion of zero-metallicity stars within these models of elliptical galaxies.

The model is not particularly sensitive to the choice of yields for planetary nebulae; the more recent results of vdH&G97 are used in preference to those of RV81.

Options to optimise the model have been here tested individually in isolation; it may be that a different combination of the input options provides a better result in terms of lower β_{ave} between the model and the observed.

5.7 CONCLUSIONS

The Phoenix model is a relatively straightforward galactic evolutionary model using a "bottom-up" approach, i.e. starting with a gas cloud and evolving a galaxy over a number of timesteps, then using luminosity-weighted SSP data to give the synthetic indices, rather than a "top-down" approach of combining different SSPs to match the observable data.

Various options within the model have been reviewed and tested in order to achieve optimisation, and the parameters to which the model is most sensitive (namely radius and percentage of stars forming Population III from the initial gas cloud) have been reviewed in this section. A discussion of uncertainties concluded that these are difficult to quantify but are unlikely to be significant as the model can be used to compare and test different yield/ejecta data, SSP tables and other parameters from the literature.

The Phoenix model is able to successfully reproduce results from other models within the literature.

Test results using the Phoenix model suggest that AGN are the principal source of galactic winds and the "switching off" of star formation, rather than supernovae winds.

CHAPTER 6: STAR FORMATION HISTORIES OF NEARBY ELLIPTICAL GALAXIES

6.1 DATA SET OF NEARBY ELLIPTICAL GALAXIES

6.1.1 Details of observational data sets

In this Chapter, two separate data sets of local elliptical galaxies are used to compare the Phoenix model with the single stellar population models (SSPs) of T04. These data sets have been obtained from different telescopes and instruments at different times, and have used different data reduction techniques. These were originally published in PS02 and Sánchez-Blázquez et al. (2007) (hereafter SB07). In addition, a third data set, published in Denicoló et al. (2005) (D05) contains 10 galaxies which overlap with those in the PS02 and SB07 samples (PS02 and SB07 do not have any overlap). Data in this third set are taken from a different telescope to those used by PS02 and SB07. These 10 D05 galaxies are evaluated using the same two models, in order to check the results found for PS02 and SB07, as the same star formation history would be expected when the same computer model is used to analyse separate observations of the same galaxy.

6.1.2 Comparison of the datasets

The telescopes used and data collected are summarised in table 27 below. All three data sets include uncertainties set at one standard deviation.

SB07 data set is provided at an extremely high signal-to-noise ratio; the uncertainties on the data are consequently relatively small as can be seen in figure 18 below. As the robustness of the models being tested is given by comparing the model datum to the equivalent observed datum, and quoting the difference as a multiple of the uncertainty (which is equal to one standard deviation), it is clear that it will be harder to model the SB07 data accurately. Neither PS02 nor D05 include the D4000 index, whereas this is in the SB07 data set. Fe5406 is in PS02 and D05 but not in SB07. Neither PS02 nor SB07 include Fe5709, Fe5782, NaD, Ti01 or TiO2, which are included in the D05 data.

	PS02 dataset	**SB07 dataset**	**D05 dataset**
Telescope	William Herschel telescope in La Palma with double-beam ISIS spectroscope	Keck II telescope in Hawaii with Low Resolution Imaging Spectrograph	Observatorio Astrofísico Guillermo Haro in Cananea, Mexico with Boller and Chivens spectrograph
Dates of observations	1998 Feb 28-Mar 03	2005 Feb 08-09	30 dates between 2000 Mar 25 and 2002 Apr 08
Spectra taken	Long-slit: length 4 arcmin, width 1.25 arcsec	Long-slit: length 3 arcmin, width 1.5 arcsec	Long-slit: length 3 arcmin, width 1.5 arcsec
Galaxies observed	15 spiral 6 lenticular 11 elliptical	11 elliptical	52 elliptical 34 lenticular
Reference stars observed for data calibration	24	5	27
Number of Lick indices observed for each galaxy	20	20	25
Data reduction tool used	CCDPACK, FIGARO, KAPPA Starlink packages.	REDUCEME (Cardiel 1999)	IRAF, CRMEDIAN, APALL packages.
Signal-to-noise ratio index errors	20 at $r_e/2$	11 at 2 r_e	14 at $r_e/8$
Elliptical galaxies		NGC 1600	NGC 1600
observed		NGC 1700	NGC 1700
	NGC 2831		
(for D05 sample,	NGC 2832		
only the 10 galaxies		NGC 2865	
that also appear in	NGC 3226		NGC 3226
either PS02 or SB07		NGC 3377	NGC 3377
are selected)		NGC 3379	NGC 3379
		NGC 3384	NGC 3384
	NGC 3608		NGC 3608
	NGC 4291		
	NGC 4365		NGC4365
	NGC 4374		NGC4374
		NGC 4387	
		NCG 4458	
		NGC 4464	
		NGC 4472	
		NGC 4551	
	NGC 4552		
	NGC 4363		
	NGC 4697		
	NGC 5322		NGC 5322

Table 27: Comparison of observations taken by PS02, SB07 and D05.

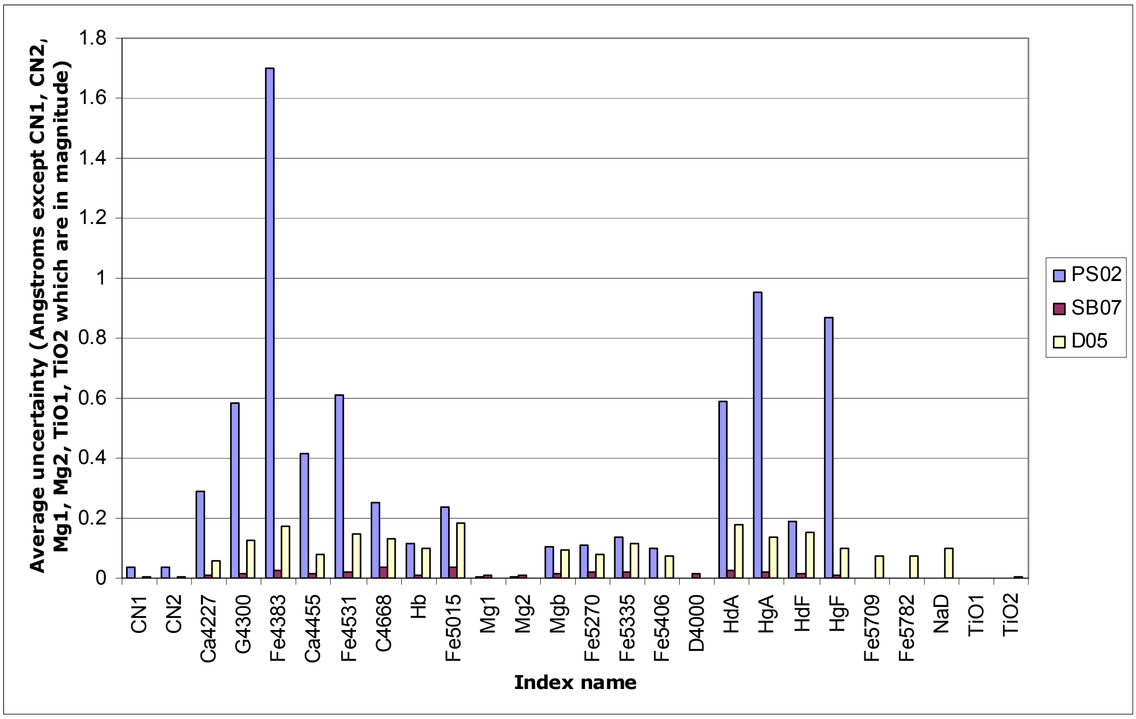


Figure 18: Comparison of average uncertainty on Lick index values for elliptical galaxy data from PS02, SB07 and D05.

PS02 observational data were obtained with the William Herschel Telescope (WHT) and double-beam ISIS spectrograph. This uses a dichroic mirror which splits the incident light into blue and red spectra. The instrument has low sensitivity at the end of the blue spectrum and at the beginning of the red spectrum – which is just at the wavelengths where the magnesium indices are found. The reported WHT Mg indices would therefore be expected to have relatively large uncertainties. However, the PS02 results only include systematic and data reduction errors, which gives the Mg indices the smallest uncertainties of all their observations. The smaller the uncertainty, the harder to successfully fit a model to the observational data; if there is uncertainty in the observational data point, and it is not incorporated in the error bar, a model that is actually reasonable may be discarded as unsuccessful. Note that within the GCE code, three subroutines (DFACT,WEIGHTBI and QFEATURE) included 'tweaks' to the synthetic magnesium indices in isolation, i.e. were trying to adjust the output from the GCE model to fit to noisy observational data.

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Figure 19 below shows an extract from Appendix A, plotting, for 3 selected indices, the full set of galaxies for PS02 (blue diamonds) together with the elliptical galaxies (red squares) from SB07. The vertical lines separate (from left to right) elliptical, lenticular, spiral morphologies, and the galaxies are ordered left to right by increasing T-type (de Vaucouleurs et al. 1991).

As can be seen from the graphs in figure 19 and Appendix A, the data relating to NGC 2865 in the SB07 sample appear to be an outlier from the data set (marked with a green * symbol) and as such, NGC 2865 is removed from the sample The complete set of figures covering 21 indices is given in Appendix A.

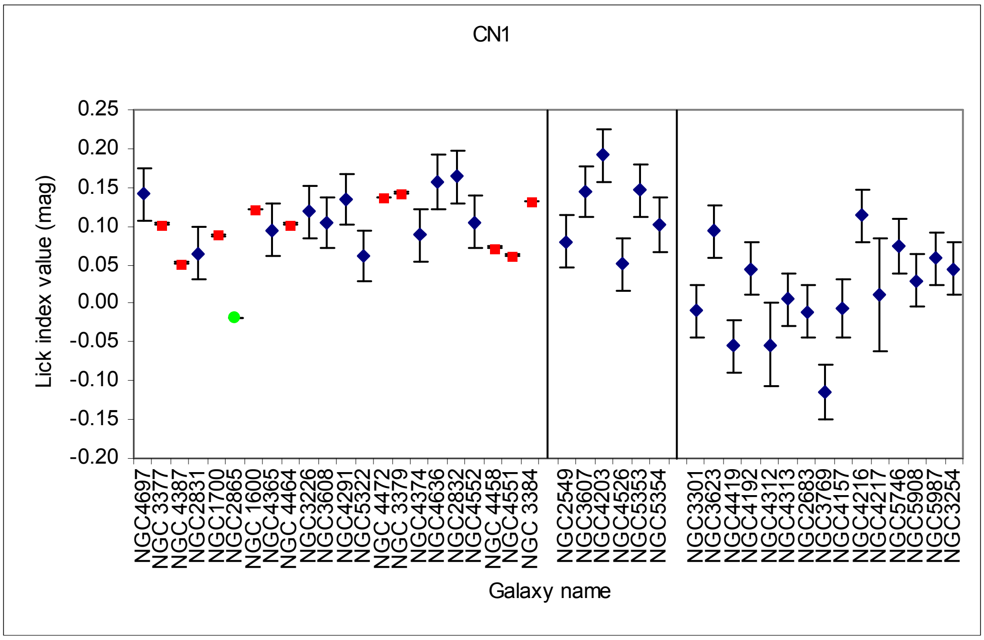
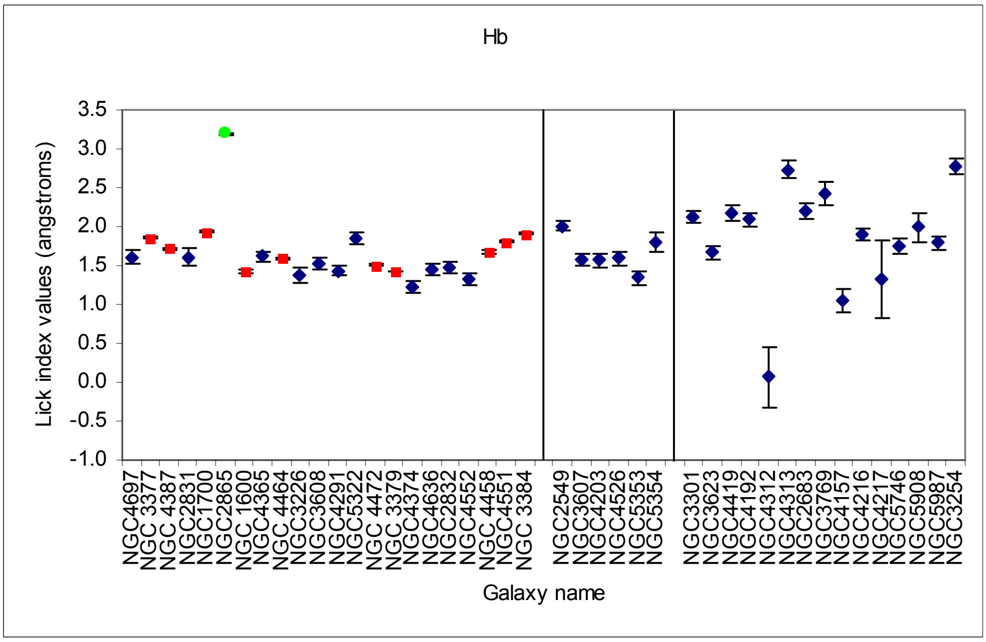


Figure 19: Sample Lick index data from the PS02 (blue diamonds) and SB07 (red squares) data sets plotted (from left to right) in order of increasing T-type (de Vaucouleurs et al. 1991). The outlier galaxy NGC 2865 from SB07 is marked with a green star. The three delineated sections are (from left to right) ellipticals, lenticulars, spirals. The full set of 21 indices is given in Appendix A.



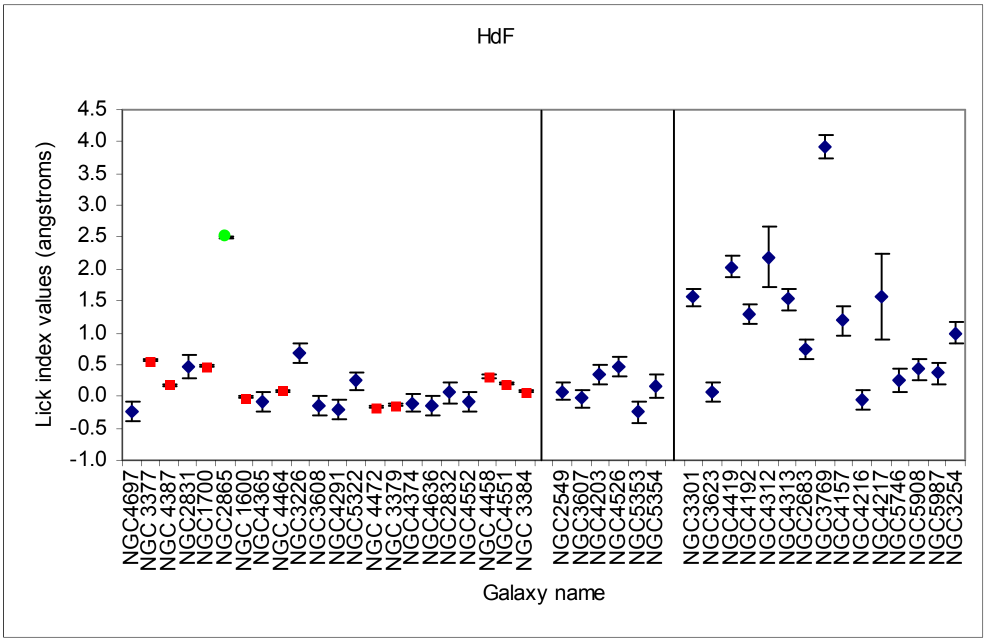


Figure 19/continued

It can be seen from figure 19 and Appendix A that there is more variation in the observed indices within the spirals and lenticulars than within the ellipticals. Indices for the elliptical galaxies from both samples show little variation across a wide range of galaxies: in sizes from dwarf to cD; in environment from field to within clusters; and in morphology from E0 to E7. This suggests that elliptical galaxies must have had similar star formation histories, irrespective of other factors, and/or must have since undergone evolutionary development/processes which have removed any initial differences, to result in similar currently-observable indices, and consequently similar current chemical compositions. For example, if the galactic winds are discriminatory with regard to the elements removed (Arimoto and Yoshii 1987), this could be a process that removes the chemical differences that are observed between active galaxies, leaving them as passive and chemically similar. The more pronounced variation seen here within spirals suggests that either these galaxies have different star formation histories, both relative to other morphologies and to each other, or that processes which would moderate this variation have not yet taken place. Recall that the galactic models reviewed in Chapter 1 ignore this possibility, and instead seek to reproduce the observed parameters of the galaxy from initial conditions.

Data from D05 provide a check to the results obtained using PS02 and SB07 for those galaxies which are in both data sets. Observed indices for these coincident galaxies are compared in figure 20 below. In each case these are observations of the same galaxy, and so the index values would be expected to be the same, within the observational uncertainties.

One galaxy, NGC 1600 (figure 20a), shows some significant differences between SB07 and D05 in the HdA, Fe4383, C4668 and Mgb indices. However, for the other four galaxies that are in both SB07 and D05 (NGC 1700, NGC 3377, NGC 3379 and NGC 3384: figures 20b, d, e and f respectively), there are very similar results for the two sets of observations.

On the other hand, the PS02 data are not so well replicated by the D05 observations, particularly for the HdA, HgA, Fe4383, Fe5015 and Mgb indices.

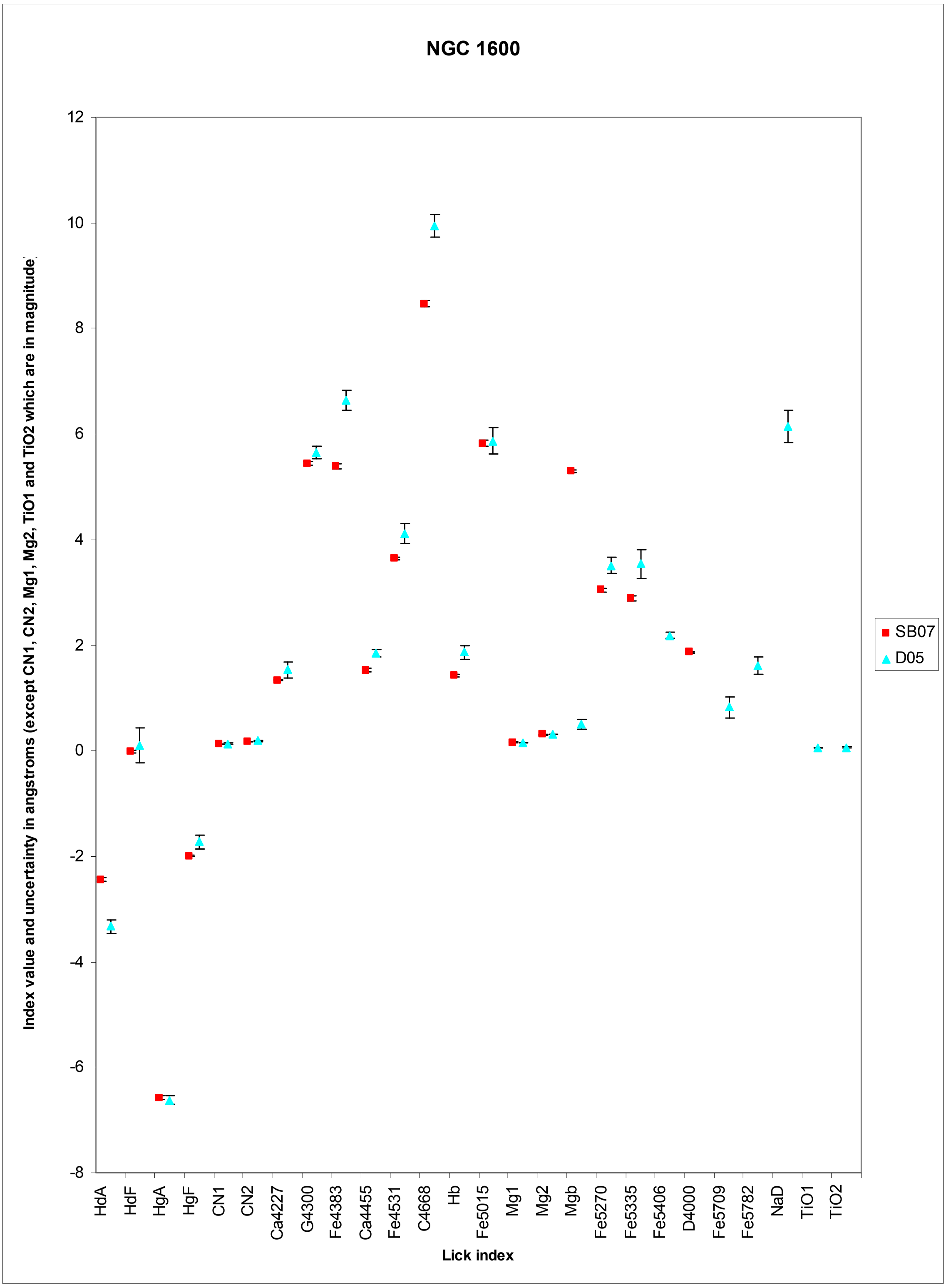


Figure 20a: Comparison of observed Lick indices for NGC 1600 from SB07 (red squares) with D05 (turquoise triangles).

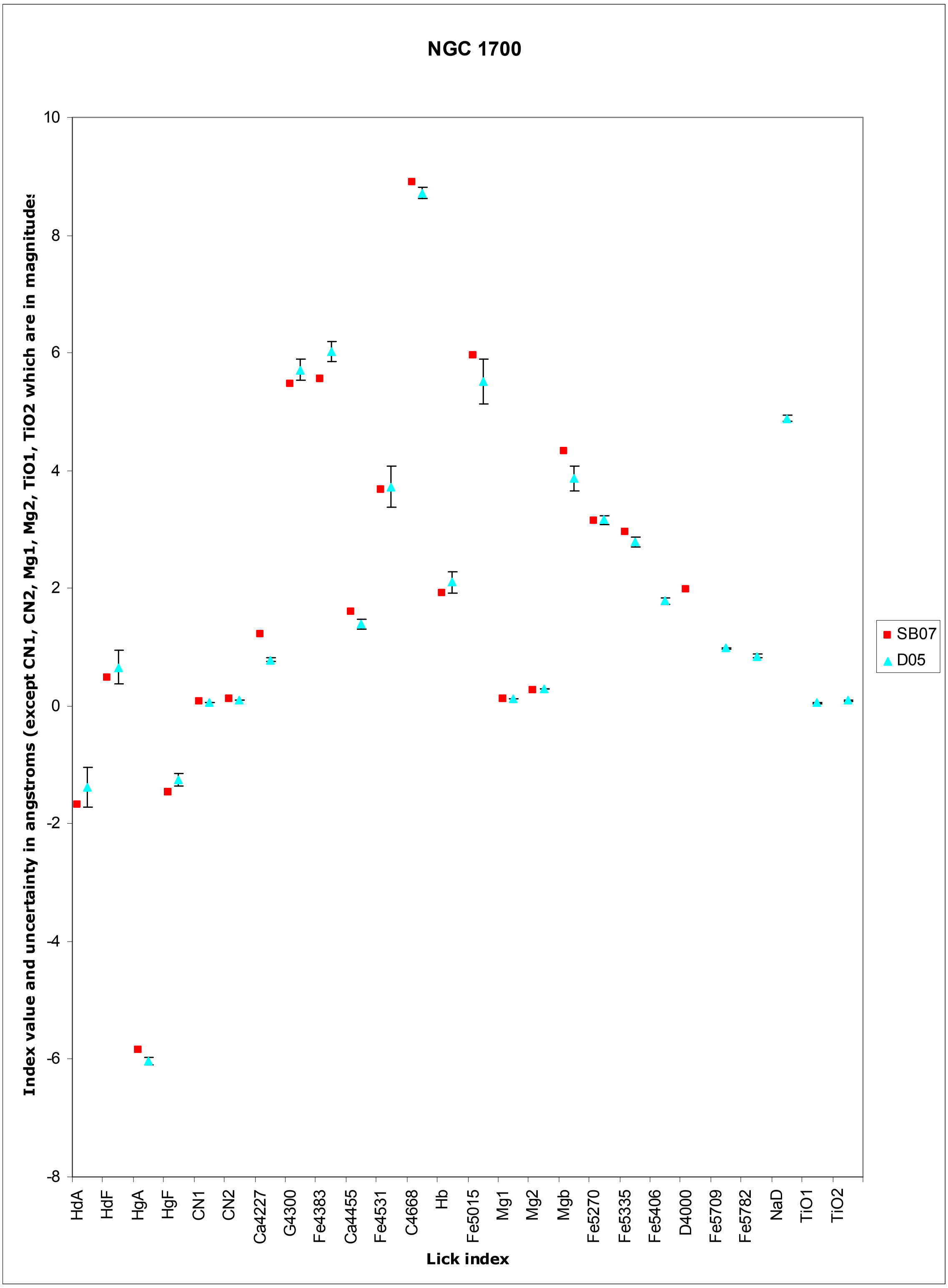


Figure 20b: Comparison of observed Lick indices for NGC 1700 from SB07 (red squares) with D05 (turquoise triangles).

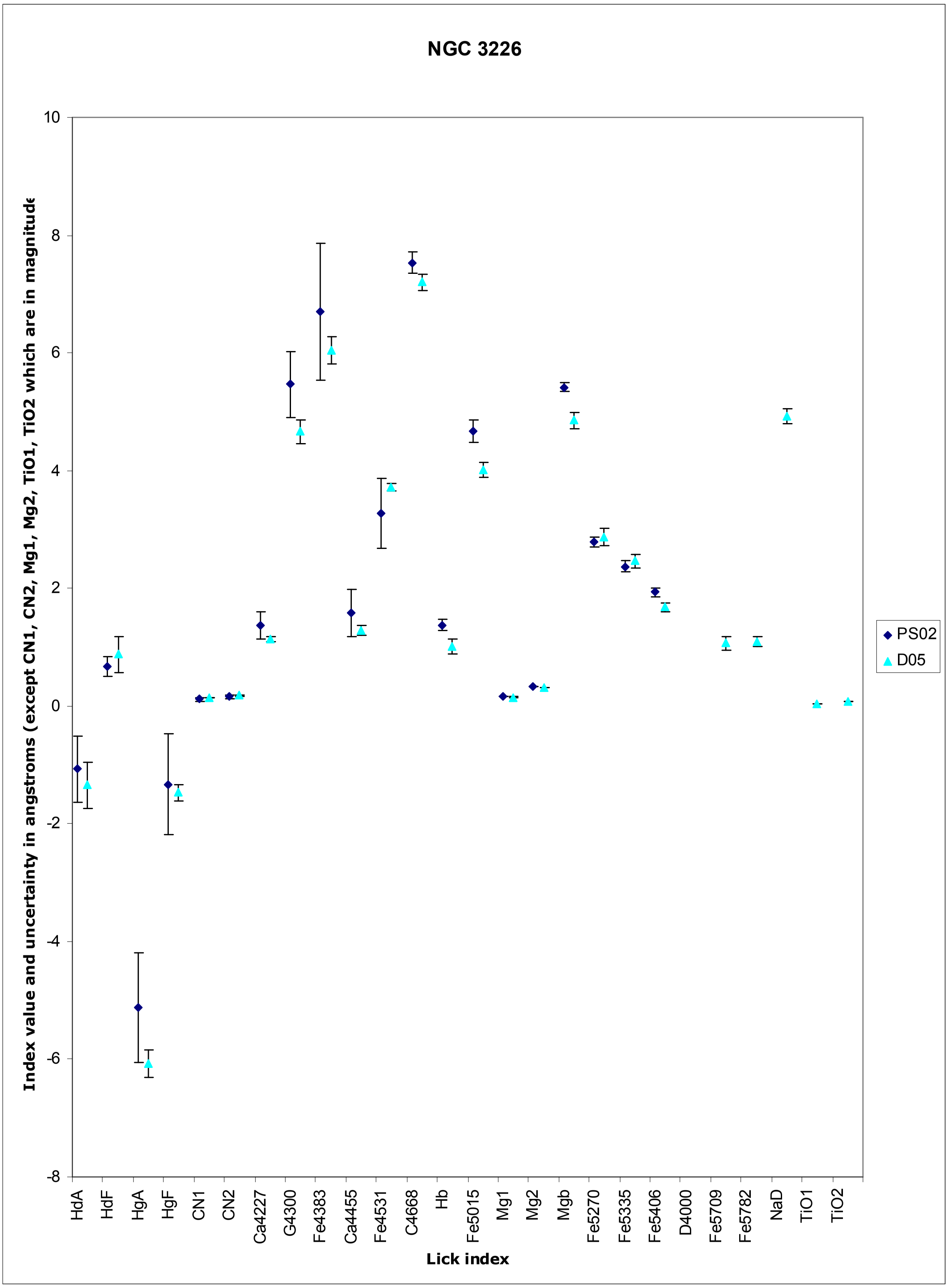


Figure 20c: Comparison of observed Lick indices for NGC 3226 from PS02 (blue diamonds) with D05 (turquoise triangles).

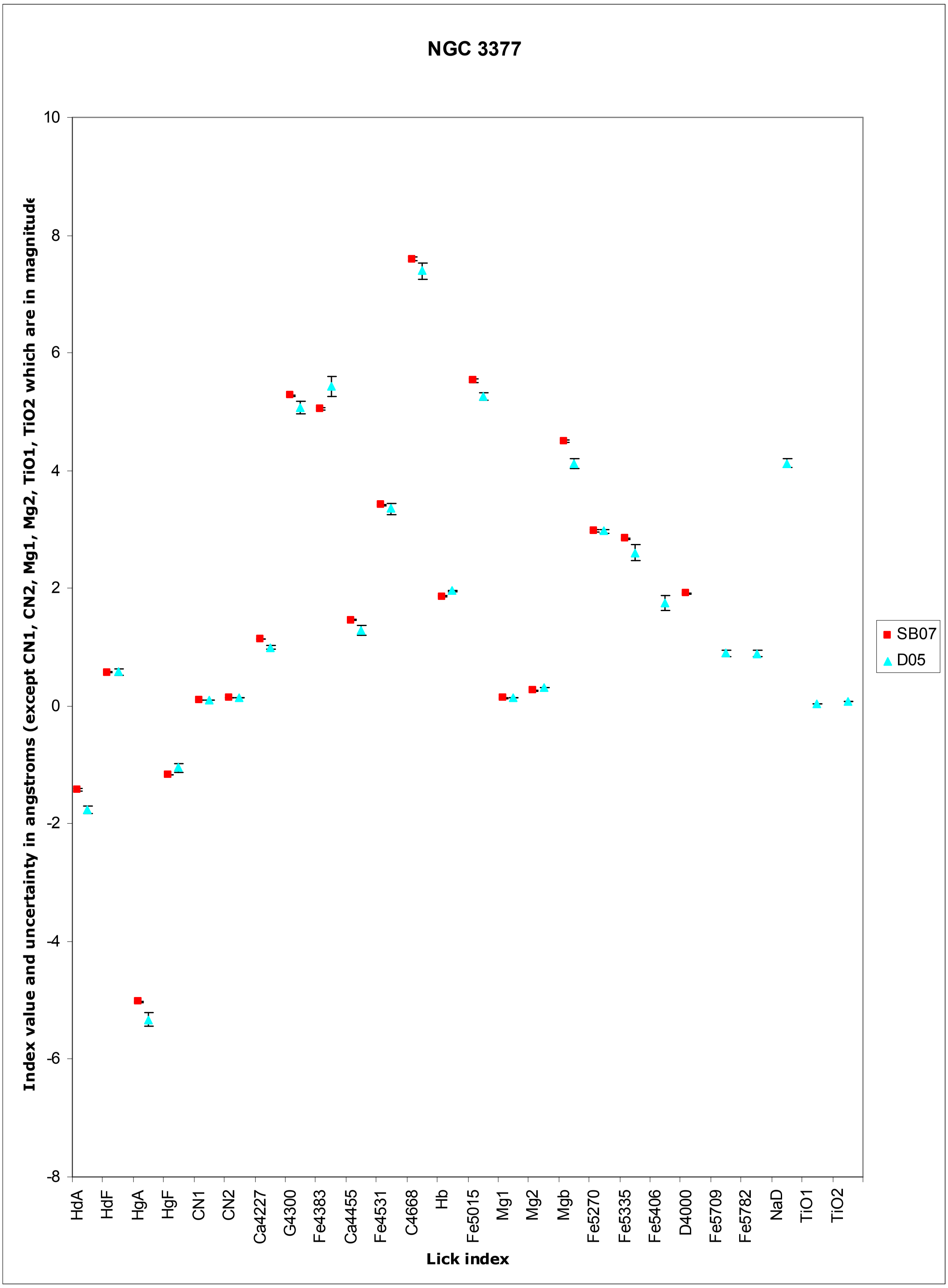


Figure 20d: Comparison of observed Lick indices for NGC 3377 from SB07 (red squares) with D05 (turquoise triangles).

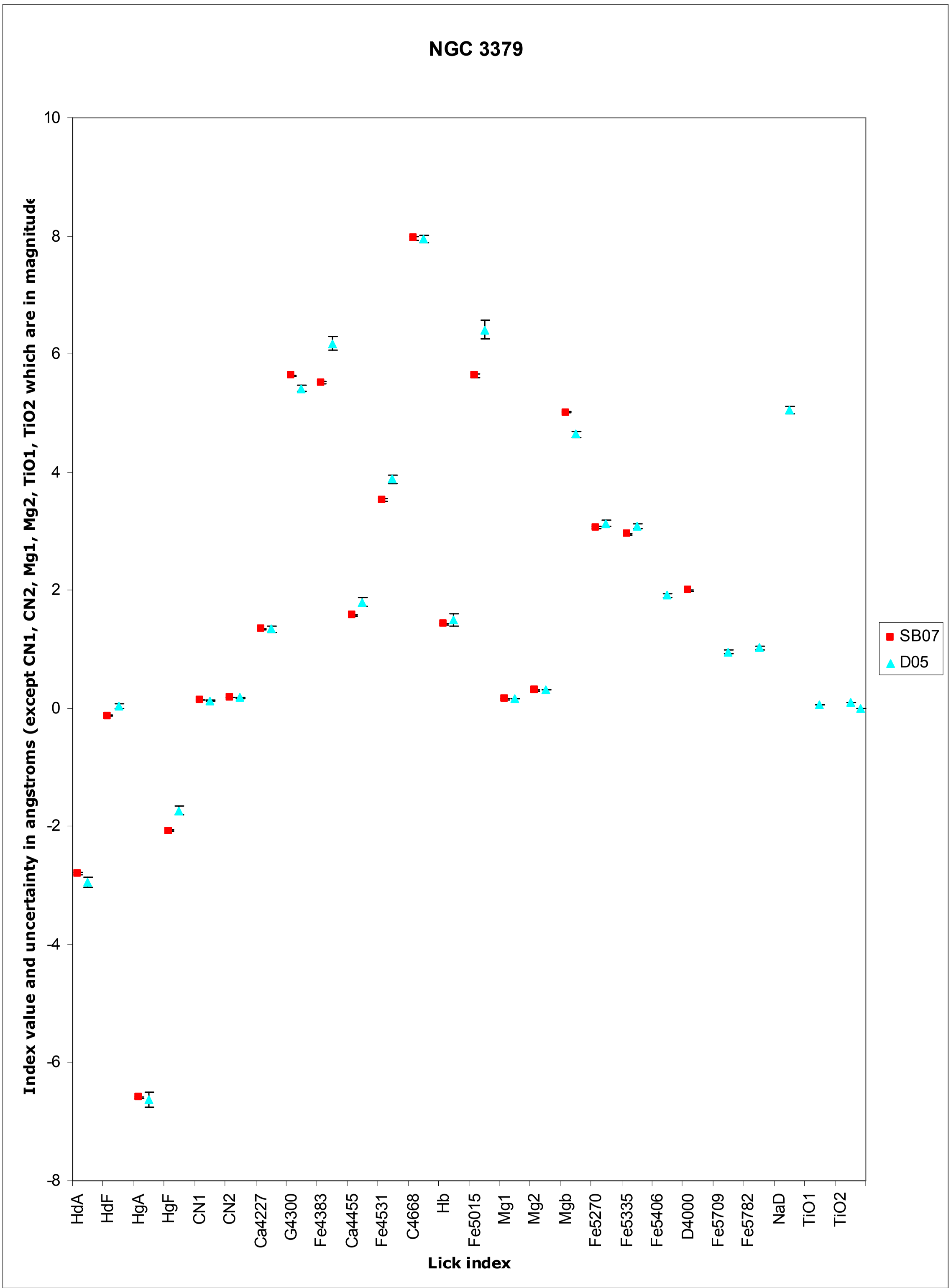


Figure 20e: Comparison of observed Lick indices for NGC 3379 from SB07 (red squares) with D05 (turquoise triangles).

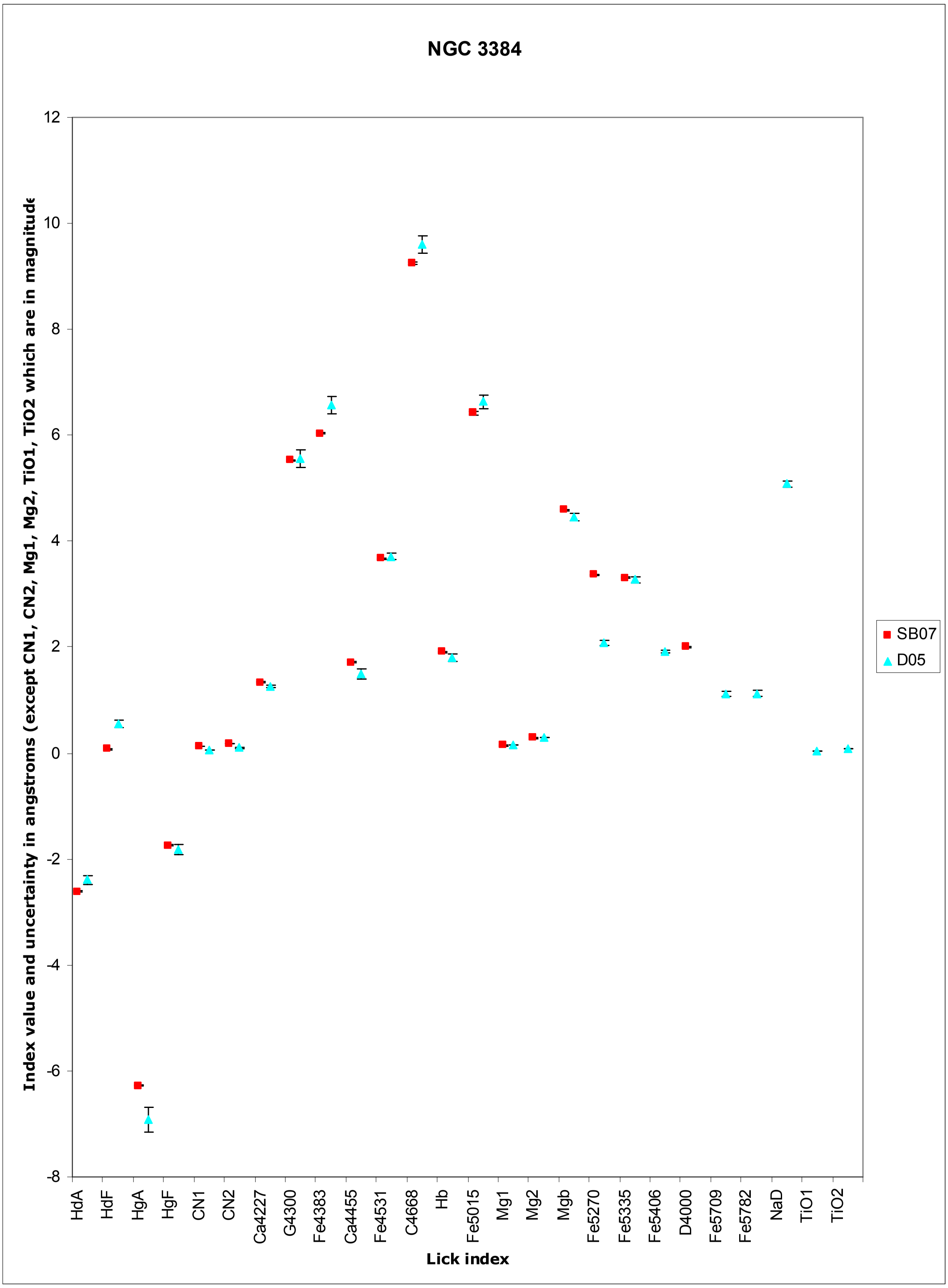


Figure 20f: Comparison of observed Lick indices for NGC 3384 from SB07 (red squares) with D05 (turquoise triangles).

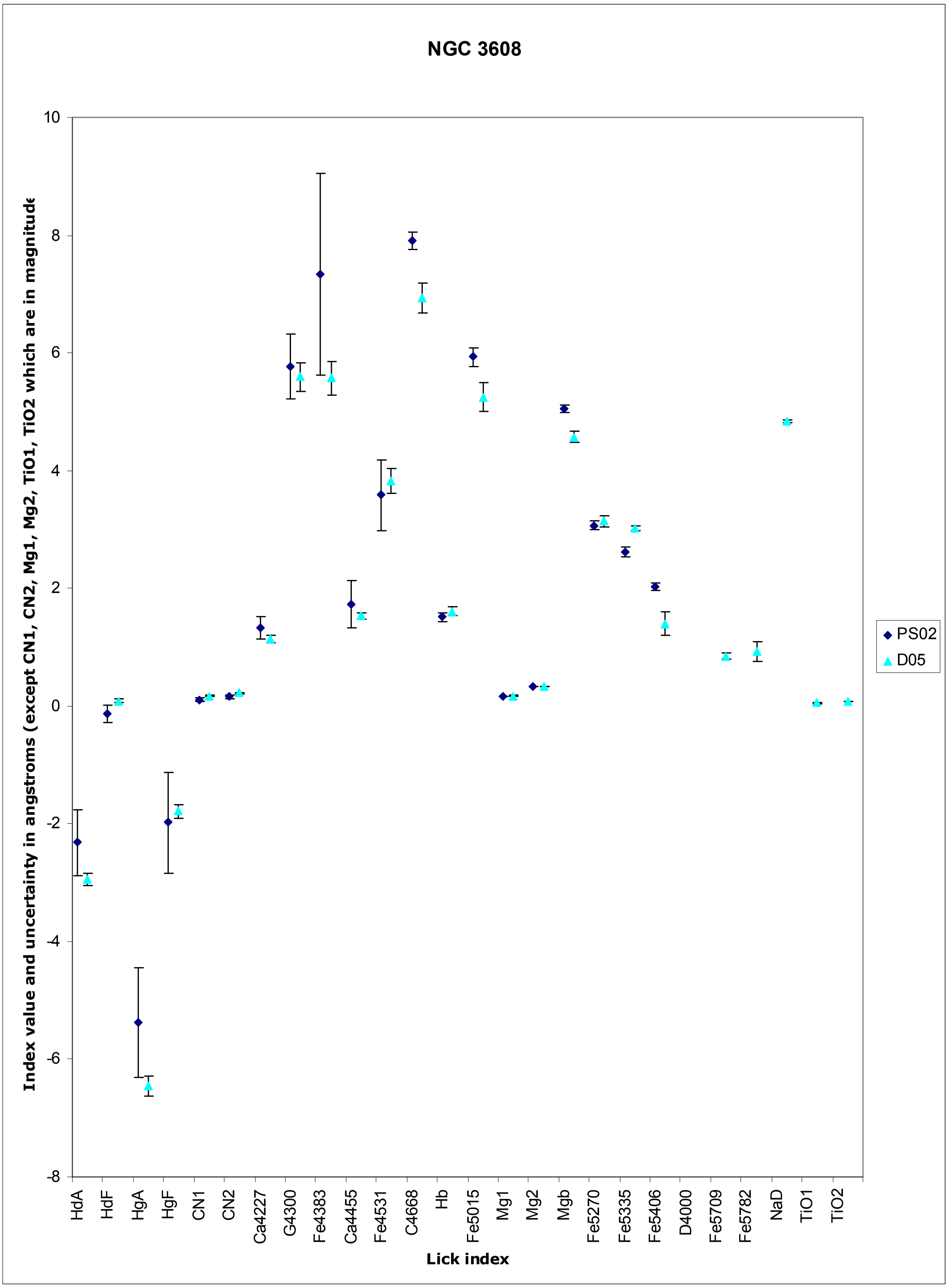


Figure 20g: Comparison of observed Lick indices for NGC 3608 from PS02 (blue diamonds) with D05 (turquoise triangles).

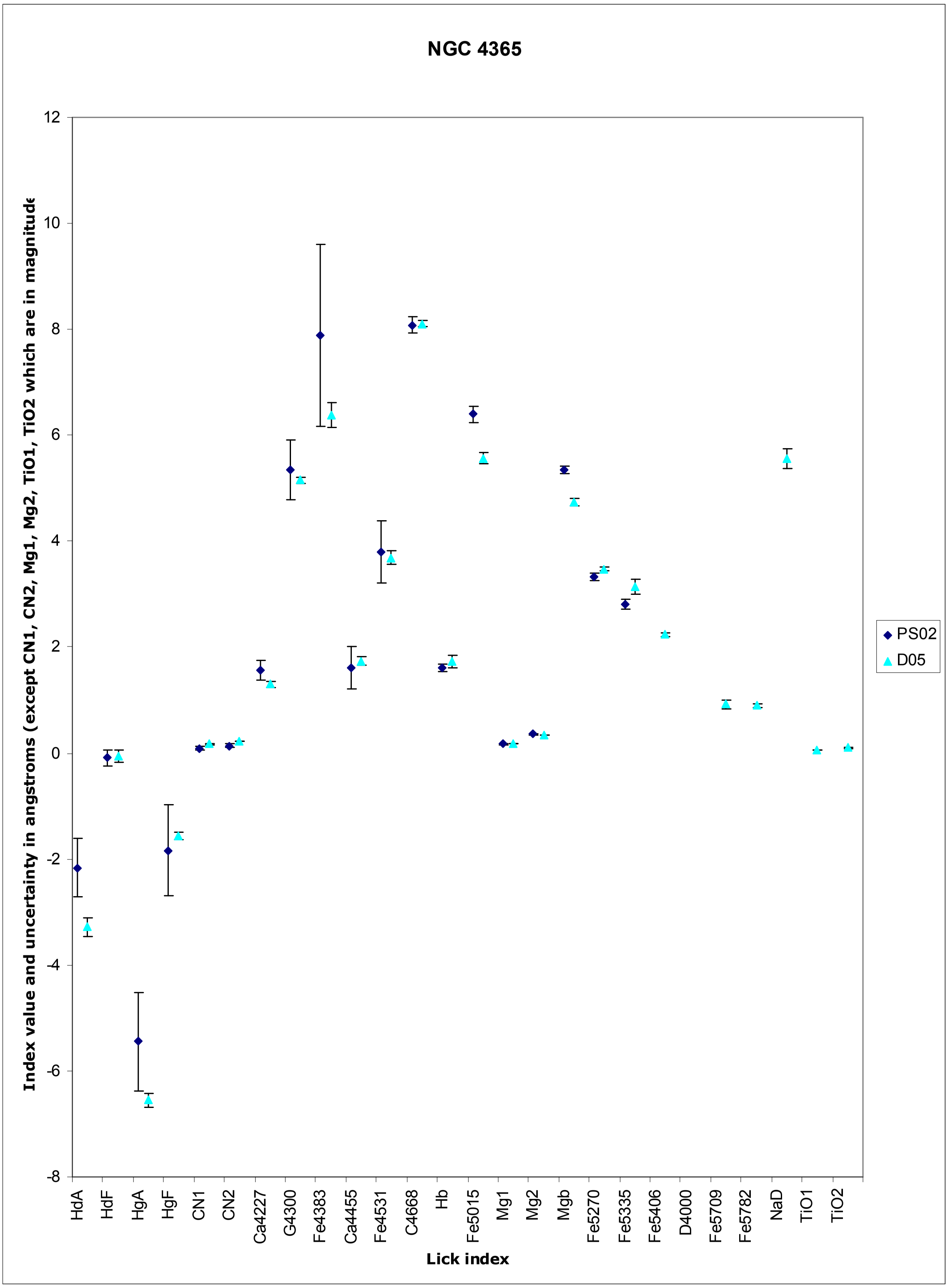


Figure 20h: Comparison of observed Lick indices for NGC 4365 from PS02 (blue diamonds) with D05 (turquoise triangles).

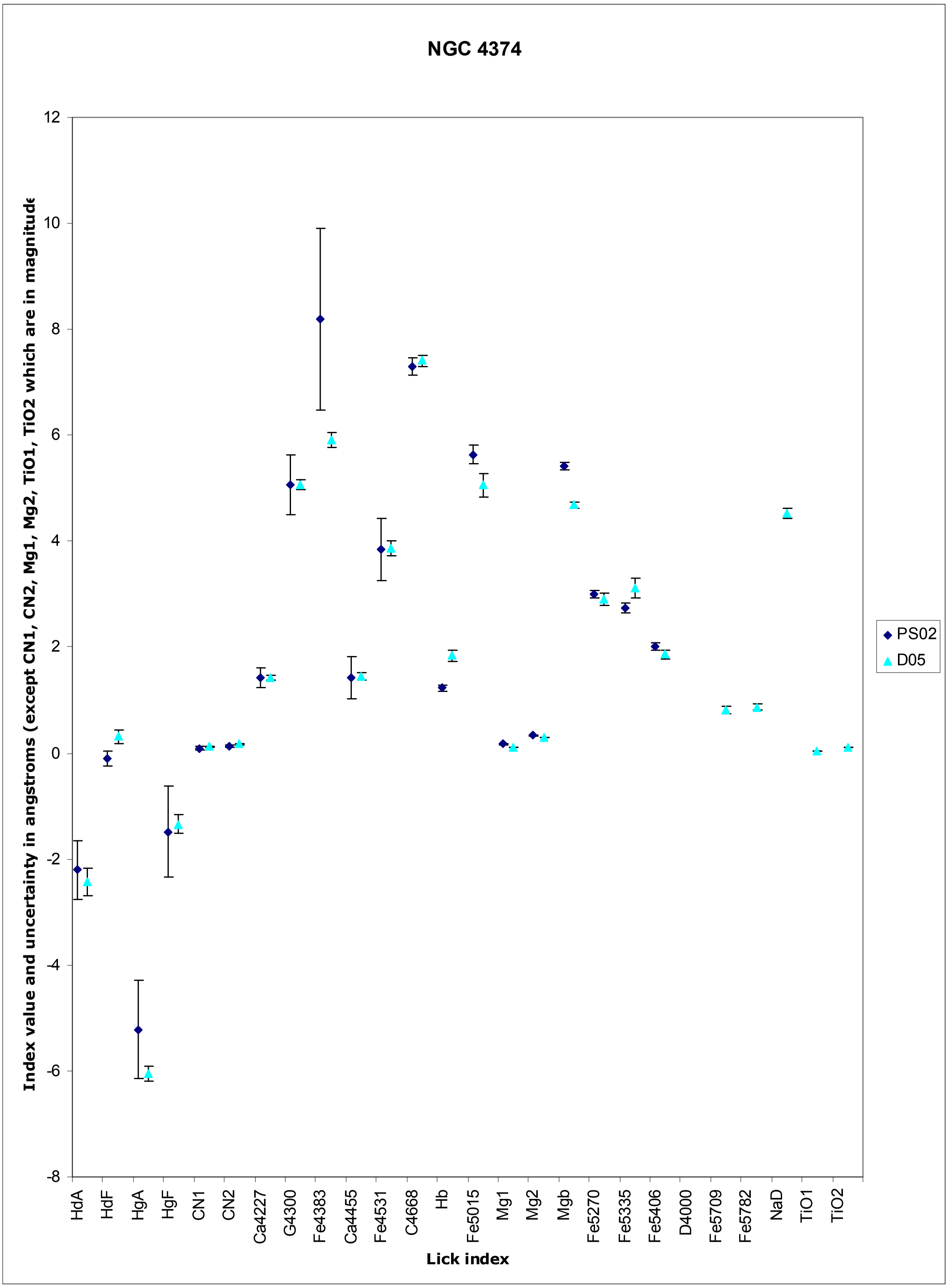


Figure 20i: Comparison of observed Lick indices for NGC 4374 from PS02 (blue diamonds) with D05 (turquoise triangles).

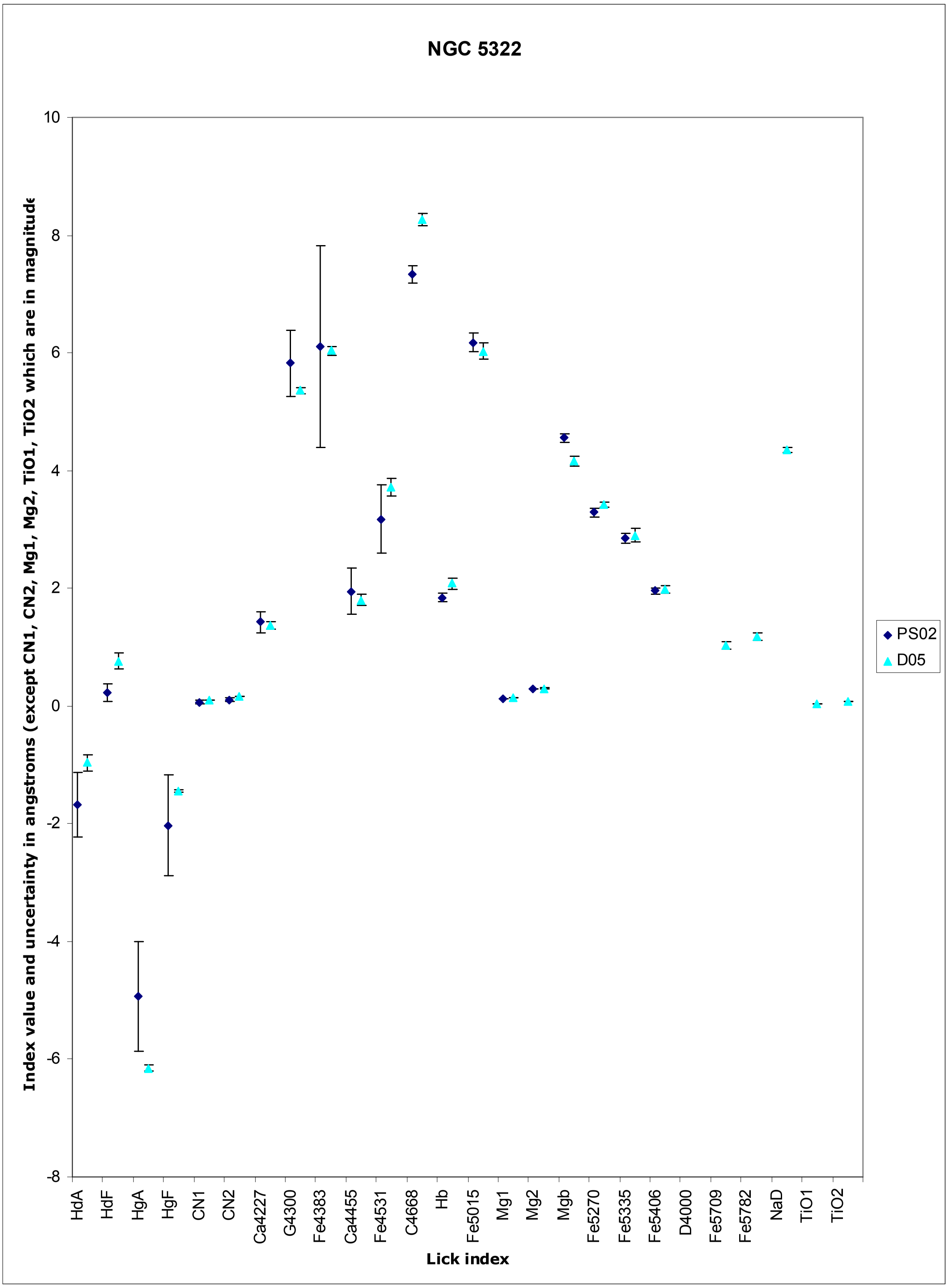


Figure 20j: Comparison of observed Lick indices for NGC 5322 from PS02 (blue diamonds) with D05 (turquoise triangles).

6.2 CAN THE THOMAS ET AL. (2004) SSP MODELS PROPOSE STAR FORMATION HISTORIES OF NEARBY ELLIPTICAL GALAXIES?

6.2.1 Introduction

There are a number of different models discussed within the literature (see Chapter 1); a selection of these were reviewed to see if they would be able to provide an holistic comparator to the Phoenix model by modelling the observational Lick index results from PS02 and SB07. This is in addition to the tests of specific outputs carried out in Chapter 5.

As discussed in Chapter 1, integrated stellar population models are built using either a “top-down” or “bottom-up” approach. The “top-down” approach is a less-than-ideal comparator, as the evolutionary steps that have formed the galaxy are ignored. “Top-down” models that were reviewed (and rejected) as possible comparator models included:

- The GALEXEV models of Bruzual and Charlot (2003). Although publically available and widely used, GALEXEV was rejected as it has been found by Koleva et al. (2008) to have systematic errors (section 1.2.2 above).
- STECMAP and STECKMAP (Ocvirk et al. 2006 a and b) is also publically available, but the code fits entire spectra (rather than Lick indices), so is not suitable for the observational data from PS02 and SB07.
- The STARLIGHT code of Cid Fernandes et al (2005) also requires the full spectrum data rather than Lick indices, which it breaks down into a sum of SSPs. Chen et al (2010) tested this code using different SSP inputs, finding the code sensitive to the source SSP data used.

“Bottom-up” models, i.e. models that use a similar approach to Phoenix by evolving the galaxy over time and then comparing model outputs to observational data, would make a more appropriate comparator, provided they model Lick indices, as that is the format of the observational data, and the code is publically available (or at least, available to this author), so that the sources of similarities and differences can be analysed.

- The GCE model of Sansom and Proctor (1998) meets these criteria and has been discussed extensively in this thesis. Notwithstanding the issues uncovered, a test against Phoenix was carried out and is discussed in Section 6.4.3 below.

- The GALEV models of Kotulla et al. (2009) are chemically consistent single-zone models without dynamics, with a publically-available front-end. Whilst it has a number of similarities to Phoenix, such as using Bertelli et al (1994) isochrones, and yield/ejecta data from vdH&G97 and WW95, it only allows single runs of a proposed star formation history, so cannot be used to search parameter space for the best-fit model, which is a requirement for comparison to Phoenix outputs. The GALEV model is a closed box, with gas inflow modelled simply as an increase to the SFR, and galactic winds (as with the GCE model) as a decrease to the SFR, but with no resultant change to overall galaxy mass. The SFR is modelled as an exponential decay-curve proportional to total galaxy mass (rather than using the Schmidt (1959) formula. The code itself is written in a mixture of Fortran, C and C++; the detailed code is not publically available, preventing similarities/differences to Phoenix from being fully assessed.

Therefore, there are limitations to working with any of the above models as a comparator to Phoenix, as the ideal model would be a “bottom-up” model which works with Lick indices and where the code could be directly compared to Phoenix so that the similarities and differences in the results could be properly assessed. Due to lack of alternatives, it was decided to use a simple SSP model as the comparator.

This section therefore investigates whether a simple galaxy model (SSP) can give reliable star formation histories for individual local elliptical galaxies.

The majority of the verification work done with SSP models in the literature can be split into two broad categories. One group of investigations compares graphically pairs of indices for several galaxies to the same two indices within the model, and establishes whether there are any trends. For example, Greggio (1997) and PS02 both use V99 SSPs, Sánchez-Blázquez et al. (2009) use Vazdekis (2010) SSPs, and model self-tests within Thomas et al. (2003) use this method. An example of this is given in figure 1 above.

The second group of investigations study an individual galaxy in more detail but may selectively remove any Lick index data from the observational sample or the SSP model for which there is not a good fit. For example, SP98 selectively modelled NGC 4472 against W94 SSPs using 6 out of 19 indices and Loubser et al. (2009) disregard data on 6 indices because the observational uncertainties on these 6 indices were considered too large.

6.2.2 Thomas et al. (2004) SSP models

T04 give tables of 24 synthetic Lick indices for SSPs, with each model having different values of three parameters: age (20 ages in range 0.1-15Gyr), metallicity (6 values: see table 28 below), and [α/Fe] ratio (4 values -0.3, 0.0, 0.3, 0.5); i.e. 480 different models. An SSP assumes that all the stars in the galaxy were formed in a single starburst, giving the component stars different masses but identical values of other physical properties such as age and metallicity.

[Z/H]	Z/H	
-2.250	1.1×10^{-4}	Approx 1/200th solar
-1.135	8.9×10^{-4}	Approx 1/20th solar
-0.330	0.009	Approx half solar
0.000	0.020	Solar
0.35	0.045	Approx twice solar
0.67	0.093	Approx 3.5 times solar

Table 28: Metallicity parameters for T04 SSPs.

Thomas and Davies (2006) evaluate the PS02 sample against the T04 models. They took the same approach as in PS02, i.e. compared the total observational sample to a grid of the SSP models, rather than considering individual observed galaxies against individual modelled SSPs. This work indicated that the ellipticals in the sample are generally older and have higher metallicity than the spiral bulges. This was the same conclusion as that reached by PS02 when comparing this observational sample with the V99 SSPs (figure 1 above), suggesting that the T04 SSPs are not significantly different from the V99 SSPs when comparing of bulk data sets. This was supported by the work in this thesis, when the GCE model was updated to include T04 SSPs (figure 5).

6.2.3 Using T04 SSPs to investigate and constrain the SFHs for individual PS02 elliptical galaxies

Lick indices of individual elliptical galaxies in the PS02 sample were compared to those given by the T04 modelled SSPs. The same comparison was done for those galaxies in the D05 sample that are also in the PS02 sample. Each observed galaxy was compared on an index-by-index basis to 432 of the 480 SSP models in T04 (the 48 SSP models where the galaxy age is 14 or 15 Gyrs were disregarded as these are older than the currently accepted age of the Universe).

The success or otherwise of the models compared to the observation were measured in terms of β_{ave} (as defined in 3.4.3 equation 13), together with β_{max}, being the largest value of β found, to show the spread of the results.

For the present analysis, the SSP model was considered to be a good fit to the observed data if $\beta_{ave} < 2$ and $\beta_{max} < 3$. The best fit model(s) is/are presented in table 29.

Galaxy (dataset)	Good fit with T04 SSP models?	Lowest β_{ave}	β_{max}	Best fit parameter 1: Age (Gyrs)	Best fit parameter 2: metallicity	Best fit parameter 3: [α /Fe]
NGC 2831 (PS02)	Yes	1.37	2.84	5.0	Twice solar	0.3
NGC 2832 (PS02)	Yes	1.19	2.95	13.0	Twice solar	0.3
NGC 3226 (PS02)	Two SSPs with β_{ave} <2 but cannot model Fe4668, Hβ, Fe5105, Mg1 or Fe5406 at β < 3					
	Solution 1	1.86	7.97	9.0	Twice solar	0.5
		1.85	6.77	10.0	Twice solar	0.5
NGC 3226 (D05)	No, best fit model is β_{ave} of 4.12			8.0	Twice solar	0.3
NGC 3608 (PS02)	Five SSPs have β_{ave} <2 but cannot model Mg1 or Fe5406 at β < 3					
	Solution 1 (5 models)	1.89	10.23	9.0	Twice solar	0.3
		1.54	9.03	10.0	Twice solar	0.3
		1.40	7.60	11.0	Twice solar	0.3
		1.35	6.23	12.0	Twice solar	0.3
		1.54	4.87	13.0	Twice solar	0.3
NGC 3608 (D05)	No, best fit model is β_{ave} of 4.36			11.0	Twice solar	0.3

Table 29: Best solutions for elliptical galaxies from the PS02 sample, and, where applicable, the D05 sample, modelled with T04 SSPs.

Galaxy (dataset)	Good fit with T04 SSP models?	Lowest β_{ave}	β_{max}	Best fit parameter 1: Age (Gyrs)	Best fit parameter 2: metallicity	Best fit parameter 3: [α /Fe]
NGC 4291 (PS02)	Four SSPs β_{ave} <2 but cannot model Ca4227 or Fe4668 at β < 3					
	Solution 1	1.94	7.79	10.0	Twice solar	0.3
		1.81	8.63	11.0	Twice solar	0.3
		1.86	9.31	12.0	Twice solar	0.3
		1.93	10.02	13.0	Twice solar	0.3
NGC 4365 (PS02)	Three SSPs β_{ave} <2 but cannot model Mg1 or Mg2 at β < 3					
	Solution 1	2.00	6.57	12.0	Twice solar	0.3
		1.83	5.20	13.0	Twice solar	0.3
	Solution 2	1.71	12.22	4.0	3.5 x solar	0.3
NGC 4365 (D05)	No, best fit model is β_{ave} of 5.96			4.0	3.5 x solar	0.3
NGC 4374 (PS02)	No SSP β_{ave} <2, 12 β_{ave} <3. Cannot model Fe4668, Hβ, Mg1, Mgb or Fe5406 < 3 β					
	Solution 1 (7 models)	2.08-2.93	11.57	All 7 models in range 7.0-13.0	All 7 models at twice solar	All 7 models at 0.3
	Solution 2 (5 models)	2.77-2.94	8.28	All 5 models in range 9.0-13.0	All 5 models at twice solar	All 5 models at 0.5
NGC 4374 (D05)	No, best fit model is β_{ave} of 4.18			7.0	Twice solar	0.3
NGC 4552 (PS02)	No SSP β_{ave} <2. Cannot model Ca4227 or Mg1< 3 β, solution 2 also cannot model Ca4455, Fe4668, Fe5105					
	Solution 1	2.75	14.23	12.0	Twice solar	0.3
		2.49	12.87	13.0	Twice solar	0.3
	Solution 2	2.59	8.04	4.0	3.5 x solar	0.3
		2.61	8.99	5.0	3.5 x solar	0.3
		2.74	9.81	6.0	3.5 x solar	0.3
		2.97	10.69	7.0	3.5 x solar	0.3
NGC 4636 (PS02)	Three SSPs β_{ave} <2 but cannot model Mg1 or Fe5406 at β < 3					
	Solution 1	1.76	6.93	11.0	Twice solar	0.3
		1.42	5.57	12.0	Twice solar	0.3
		1.21	4.20	13.0	Twice solar	0.3
NGC 4697 (PS02)	Five SSPs β_{ave} <2 but cannot model Fe5270, Fe5335 or Fe5406 at β < 3					
	Solution 1	1.91	4.74	10.0	Twice solar	0.3
		1.79	4.18	11.0	Twice solar	0.3
		1.78	4.07	12.0	Twice solar	0.3
		1.82	5.70	13.0	Twice solar	0.3
	Solution 2	1.85	5.03	3.0	3.5 x solar	0.3

Table 29/continued

Galaxy (dataset)	Good fit with T04 SSP models?	Lowest β_{ave}	β_{max}	Best fit parameter 1: Age (Gyrs)	Best fit parameter 2: metallicity	Best fit parameter 3: [α/Fe]
NGC 5322 (PS02)	Can model β_{ave} <2 but cannot model Mg1,Mg2, Mgb or Fe3553 at β < 3 for solution 1, and cannot model Fe5270, Fe5335 or Fe5406 at β < 3 for solution 2					
	Solution 1	1.95	8.19	6.0	Twice solar	0.0
		1.96	7.18	7.0	Twice solar	0.0
		1.97	5.94	8.0	Twice solar	0.0
	Solution 2	1.48	5.28	5.0	Twice solar	0.3
		1.40	4.61	6.0	Twice solar	0.3
		1.58	4.27	7.0	Twice solar	0.3
		1.82	4.63	8.0	Twice solar	0.3
		1.99	5.90	9.0	Twice solar	0.3
NGC 5322 (D05)	No, best fit model is β_{ave} of 4.99			3.0	3.5 x solar	0.3

Table 29/continued

Results

Other than NGC 2381 and 2382, where a single SSP was well-matched to the observational data, the galaxies in the PS02 sample are matched by a number of SSPs. These only vary within one parameter, galaxy age, although there may be two or more regions of parameter space which bound a potential solution.

Hence, the following possibilities could be considered:

- a single solution exists in the region of parameter space bound by the solutions found; or
- the star formation was spread over a number of Gyrs (therefore not an SSP by definition); or
- the SSP model's results do not vary significantly over this timescale allowing several models to fit the data.

An elliptical galaxy could be expected to be modelled by an SSP provided there is no independent evidence of separate star forming episodes. Mergers, if evidenced, might indicate an SSP is not a suitable model: whilst a merger can be 'dry' (gasless) and thus not trigger a starburst at the point of merger, for a post-merger galaxy to be accurately modelled by an SSP, the merger components would also have to be composed of identical populations of stars.

As discussed in Chapter 1, historic mergers can be identified by decoupled cores, tidal tail remnants or sub-structures which must have had separate origins indicated by different dynamical or other observational features. Individual galaxies are reviewed for observational evidence of historic mergers:

- NGC 2831 and 2832 are each modelled successfully by a single SSP, suggesting sudden monolithic collapse and no subsequent contamination through merger with another population with a different history. Both galaxies are identified as weak X-ray sources (Dahlem and Stuhrmann 1998), as would be expected for old, quiescent elliptical galaxies.
  These two galaxies are close companions. NGC 2831 is a small satellite galaxy to NGC 2832 and there is nothing in the literature to indicate it is anything other than a single population. NGC 2832 is a cD galaxy whose surface brightness is described by Naab and Burkert 2003 as 'boxy', which their N-body simulations suggest can arise from historical tidal interaction with a nearby massive companion. Jordán et al. (2004) show the metallicity distributions suggest it to have developed through cannibalisation of smaller galaxies and remnants and Moss (2006) notes that NGC 2832 and NGC 2831 are currently undergoing a relatively fast ($\Delta\upsilon$ ~900 km s^{-1}) encounter with the spiral galaxy NGC 2830. Taken overall, this detailed analysis of these specific galaxies suggests that NGC 2832 was *not* formed by a sudden monolithic collapse. It is therefore likely that the successful modelling here by an SSP is in fact coincidental.

- NGC 3226 – This may be a single population, however it is observed to be merging with spiral NGC 3227 (Martel et al. 2004) and this merger is generating a starburst (Mundell et al. 2004).

- NGC 3608 – Has a kinematically decoupled core (Halliday et al. 2001).

- NGC 4291 – There is nothing in the current literature to indicate anything other than a single population.

- NGC 4365 – Has a kinematically decoupled core (van den Bosch et al. 2007).

- NGC 4374 – The uneven metallicity distribution in this gas-rich elliptical suggests AGN activity (and hence extended star formation) or mergers (Xu et al., 2010).

- NGC 4552 - PS02 data can be modelled as an old population (12-13 Gyrs) with metallicity of twice solar or as a younger population (4-7Gyrs) with a metallicity of 3.5 times solar. Data from D05, are not as well-modelled; the best fit model has a younger age and higher metallicity than those of the PS02 data.

  There is no significant difference in the 'goodness of fit' of the age-sensitive indices, but the metallicity-sensitive indices are better modelled with the older population models.

  Renzini et al. (1995) report an ultraviolet flare from the centre of this galaxy which is identified as otherwise (optically) quiescent. Machacek et al. (2006) identify optical features indicative of ram pressure stripping the galaxy of gas as it moves through the Virgo cluster. Neither of these papers on dynamic processes suggests there is associated starburst activity (which would result in multiple populations and thus render an SSP model invalid). The literature does not give an independent age estimate for this galaxy. Therefore, this galaxy may be able to be reasonably modelled by an SSP, although as stated above there is not a unique solution from the T04 models.

- NGC 4636 – This contains a varied population of blue and red globular clusters (Lee et al. 2010) and the metallicity distribution in this gas-rich elliptical suggests AGN activity (and hence extended star formation) or mergers (Xu et al., 2010).

- NGC 4697 can be modelled as an old population (age between 10 and 13 Gyrs) with metallicity of twice solar, or as a younger population (age = 3 Gyrs) at a higher metallicity of 3.5 times solar; there is no unique solution modelled for this galaxy. The specific indices that are poorly fitted with both solutions are neither age-sensitive (G4300 and the Balmer lines) nor metallicity-sensitive (Fe4668, Fe5015, Fe5709 and Fe5782). However, the metallicity-sensitive lines (Fe4668 and Fe5015) are more closely modelled with the older population than the younger (β_{ave} of 2.35 v. 4.36, and 0.57 v. 2.62 respectively). The age-sensitive lines do not favour one solution over another.

Maccarone (2005) notes that the observed flaring X-ray binary star population in this galaxy can only be modelled if the ages of the pulsars are ~4 Gyrs; this would suggest that the younger age model is the less likely result (as it gives a galaxy age of 3 Gyrs). Zezas et al. (2003) identify the age of this galaxy as 9-13 Gyrs with no recent merging activity but with X-ray evidence associated with young stellar populations, which they attribute to a rejuvenating fallback of material, or shock-induced star formation from the tidal tail giving this old elliptical galaxy a sub-population of much younger stars, of the order of 0.1 Gyrs old.

This galaxy therefore probably consists of at least two populations with very differing ages (~10 Gyrs and ~0.1 Gyrs). This is of course inconsistent with any SSP modelling (which only allows for a single population), although the fraction of optical flux from the young, X-ray emitting 0.1 Gyr population may be relatively small (Zezas et al. 2003).

- NGC 5322 – Has a kinematically decoupled core (Rix and White 1992).

<u>Comparison of PS02 and D05 results</u>

Five of the PS02 galaxies are also observed by D05. None of the D05 data can be modelled with a β_{ave} of less than 2 when compared to the T04 SSPs; the age, metallicity and [α/Fe] of the best-fit models are included in table 29.

For three of the galaxies, NGC 3608, NGC 4365 and NGC 4374, the best fit model from the D05 data is also one of the best-fit models of the PS02 data. This may support the SFH proposed. However, the best-fit models of NGC 3226 and NGC 5332 are different for the PS02 and D05 data sets suggesting a difference between the observations of these galaxies has led to the different proposed SFHs.

6.2.4 Using T04 SSPs to investigate and constrain the SFHs for individual SB07 elliptical galaxies

The same methodology as outlined in 6.2.3 was used to compare the elliptical galaxies of the SB07 dataset with T04 SSPs, together with those galaxies in the D05 sample that are also in the SB07 sample. NGC 2865 is not included here as it has been identified as an outlier in the graphical review of the SB07 data (Appendix A).

It is noted that the uncertainties on the SB07 Lick indices are smaller than those of the PS02 data, and as such, any fitting measured in terms of β is therefore anticipated to be more difficult. Best fit models found are given in table 30. Other localised minima were found, but no results where β_{ave} is less than 2, and within this modelling, in some instances, values of β_{max} reach several hundred.

Galaxy (dataset)	Lowest β_{ave}	Parameter 1: Age (Gyrs)	Parameter 2: metallicity	Parameter 3: [α /Fe]
NGC 1600 (SB07)	20.73	12.0	Twice solar	0.3
NGC 1600 (D05)	5.61	3.0	3.5 x solar	0.0
NGC 1700 (SB07)	13.26	7.0	Twice solar	0.3
NGC 1700 (D05)	5.19	6.0	Twice solar	0.3
NGC 3377 (SB07)	15.39	5.0	Twice solar	0.3
NGC 3377 (D05)	4.52	6.0	Twice solar	0.3
NGC 3379 (SB07)	19.80	13.0	Twice solar	0.3
NGC 3379 (D05)	4.69	12.0	Twice solar	0.3
NGC 3384 (SB07)	22.14	5.0	Twice solar	0.0
NGC 3384 (D05)	5.33	3.0	3.5 x solar	0.3
NGC 4387 (SB07)	9.78	9.0	Solar	0.0
NGC 4458 (SB07)	7.08	11.0	Solar	0.3
NGC 4464 (SB07)	11.74	13.0	Solar	0.3
NGC 4472 (SB07)	16.59	12.0	Twice solar	0.3
NGC 4551 (SB07)	9.54	5.0	Twice solar	0.0

Table 30: Best fit T04 SSP models for the SB07 ellipticals and overlapping D05 data sets.

From table 30 it can be seen that T04 SSPs cannot reasonably model any of the 11 galaxies in the SB07 data set. As expected, better-fit models (when measured by β) are obtained for the D05 galaxies, which have greater uncertainties (6.1.2 above) but again no model fit is within 2 β_{ave}. Some individual indices are well modelled but there is no single T04 SSP that can simultaneously provide a good match to all 19 indices. This suggests that none of the galaxies in this data set have been formed as a single burst of stars, and that their star formation histories are more complex.

In the SB07 paper, the data were also compared to T04 models. Only three indices, Fe4383, Hβ and Mgb, were tested to find the best fit parameters. A χ^2 minimisation test was used with all indices, but discarding any index data which exceeded 3σ. Whilst this was able to obtain apparently better-constrained results than achieved here, here the full set of 19 indices are being simultaneously modelled.

Other observational data indicate that some of these galaxies have not formed as a single stellar population:

- NGC 1600 – Has an anisotropic structure indicating merger origin (Matthias and Gerhard 1999).

- NGC 1700 – Has a counter-rotating core (Statler et al. 1996).

- NGC 2865 – Has a kinematically decoupled core (SB07).

- NGC 3377 – The surface brightness is observed as 'disky' in the inner regions but 'boxy' in the outer regions; this unevenness indicates historical disruption probably from merger (Peletier et al. 1990).

- NGC 3384 – There are observed asymmetries interpreted as a relic of the Spitzer-Baade collision event 0.5 Gya between NGC 3384 and NGC 3368 (Busarello et al. 1996).

- NGC 4387 – There is evidence of an equal-mass merger of two spirals (Bendo and Barnes 2000).

- NGC 4458 – Has a kinematically decoupled core (SB07).

- NGC 4472 – Has a kinematically decoupled core (SB07).

There is nothing in the literature to date to indicate the remaining galaxies (NGC 3379, NGC 4464 and NGC 4551) are anything other than single populations. Of course this does not mean that they *are* single populations, merely that their structure has not been analysed in detail within the literature, or that current observational limitations provide an absence of evidence that they have more complex histories.

Comparison of SB07 and D05 results

Star formation histories found using the D05 observational data are better modelled (in terms of a lower β_{ave}) than those of SB07; this would be expected from the relative size of the observational uncertainties (figure 18 above).

Very different results are obtained for NGC 1600 and NGC 3384 when SB07 and D05 are modelled with T04. Hence, no confidence can be assigned to either result.

Results for NGC 1700, NGC 3377 and NGC 3379 are in each case similar for both data sets, which suggests that these galaxies could have been formed by the SSP indicated.

6.2.5 Discussion

At first glance, it would appear that the galaxies within the PS02 sample, and three of the galaxies in the SB07 sample (NGC 1700, NGC 3377 and NGC 3379, where support for the results is given by the D05 data), can be modelled using the T04 SSPs. Galaxies within the SB07 sample taken alone cannot be successfully modelled by T04 SSPs, although the SB07 paper indicates that this can in fact be done, provided that any data that do not demonstrate a good fit are discarded.

However, a closer examination using other data and observations reported in the literature indicates that even well-modelled solutions are not necessarily giving the correct SFH of those galaxies, because these galaxies may have other features which indicate they *must* be more complex than a single stellar population. Only six galaxies from the sample (NGC 2831, NGC 3226 and NGC 4291 from PS02, and NGC 3379, NGC 4464 and NGC 4551 from SB07) have no evidence (yet) in the literature to indicate anything other than a single population.

SSP models have been shown to give reasonable matches to globular clusters (e.g. Beasley et al. 2002, Maraston et al. 2003). The physical properties of the SSP are an indication of the properties of the gas cloud which formed it. Thus, for example, for a globular cluster to be modelled by a high-metallicity, α-enhanced SSP suggests there had been a previous population of massive stars undergoing SNII to provide that enrichment. This in turn would indicate not a single burst of star formation but *at least* two separate generations. If the first generation *only* consisted of high mass stars and these were fully evolved to leave only an enriched gas cloud, this *could* lead to a high-metallicity SSP. However, such enriched gas clouds have not been observed and the physics of initial mass functions shows that low-mass stars are always produced along with higher-mass stars.

Observations of currently merging structures and evidence of historic merging both indicate the presence of more than one population. Clearly, this could only be modelled correctly by an SSP if the merging galaxies had the same chemical composition/star formation history, *and* if the merger event itself did not trigger renewed star formation.

Conclusions from this exercise are that, if the 21 elliptical galaxies selected are representative of elliptical galaxies in the local Universe, then elliptical galaxy formation is generally more complex than that of globular clusters, and data that can successfully reproduce the indices of a simple globular cluster cannot be assumed to successfully model larger systems. In addition, any galaxy which appears to have been successfully modelled by an SSP should be reviewed to assess whether it would in fact require an earlier stellar generation to provide appropriately pre-enriched material. In addition, a separate, wider review of observations (not just relying on Lick indices to compare with the SSP model) is necessary in order to establish whether the galaxy is likely to have been formed as a single event.

6.3 CAN THE PHOENIX MODEL PROPOSE STAR FORMATION HISTORIES OF NEARBY ELLIPTICAL GALAXIES?

6.3.1 Introduction

In this section, the Phoenix model was used to search parameter space to find best fits (identified by low values of β_{ave} and β_{max}) using data from PS02 and SB07, together with observational data from D05 for galaxies that are also in either the PS02 or the SB07 samples.

A parameter space search was done using the values given in table 31 with the data sources listed in table 32. This was originally conducted as a coarse search, with additional values added to closely investigate areas of parameter space where well-modelled results were apparent.

Parameter	Coarse-grid values	Fine-grid values
Galaxy mass ($M_{\odot}$)	5 x 10^{10}, 1 x 10^{11}, 5 x 10^{11}, 1 x 10^{12}	All x 10^{10}: 3.5, 3.7, 3.8, 3.9, 4.0, 4.1, 4.2, 4.5, 4.7, 5.5, 5.7, 6.5, 6.7
Galaxy life (Gyrs)	9, 12, 13 Gyrs	12.8, 12.9, 13.1, 13.15, 13.17, 13.2,13.23, 13.25, 13.27, 13.3
SFR constant	0.1, 0.5	0.45, 0.53, 0.55, 0.57, 0.60, 0.63, 0.65, 0.67
Timing of galactic wind (Gyrs after start of galaxy)	0.0, 0.44, 0.7, 4.4, 6.0	0.65, 0.74, 0.75, 0.76, 0.77, 1.4, 2.0, 2.4, 3.4, 4.0, 4.1, 4.2, 4.3, 4.6
Gas inflow ($M_{\odot}$/Gyr)	0, 10^9, 10^{12}	Not tested further, as coarse search showed that gas inflow was not required for a well-modelled result.
Gas inflow start time (Gyr after start of galaxy)	0, 2, 4, 8	
Gas inflow duration (Gyr)	0, 0.5, 2, 4	
Percentage Population III from initial primordial gas (%)	5, 10, 33, 45, 50, 55, 75	Even values between 12 and 54

Table 31: Searching grids used with Phoenix model.

Model set-up	Source data/parameter setting
Planetary nebulae yields from	Van den Hoek and Groenewegen (1997)
Large star ejecta from	Woosley and Weaver (1995)
Massive star yields from	Meynet and Maeder (2002)
SNIa ejecta from	Nomoto et al. (1984)
SNIa rates from	Scannapieco and Bildsten (2005)
SSP data from	Thomas et al. (2004)
Gas inflow composition	Primordial
Galactic wind mechanism based on	Fixed time rather than loaded to star formation

Table 32: Data sources used, selected following the model tests discussed in Chapter 5.

6.3.2 Star formation histories: PS02 data

Best-fit models were found using the "stepping software" option of the Phoenix code. In some instances, more than one minimum was given by the model. Best-fit results are shown in table 33 below.

The final galaxies produced by the individual best-fit models were checked to ensure they produced reasonable results; model (b) for NGC 2831 was rejected as the final SNIa rate was outside the expected range (marked in grey) (table 34) of 0.03-0.08 SNu (Turatto et al. 1994). This "expected range" of SNu is supported by Sand et al. 2012 who suggest a value of 0.041 ± 0.015 SNu for ellipticals, and a range 0.056-0.096 SNu found by Graham et al. 2008. Where D05 data were available for galaxies in the PS02 sample, these were also modelled by Phoenix, and the best-fit model is included in table 33 for comparison.

Star formation histories of the PS02 sample are plotted in figure 21. As discussed in Chapter 5, the initial peak is due to Population III stars being formed in the first time step. Star formation thereafter follows the standard Schmidt (1959) equation with the constant in the equation being found by the "stepping software" and the density of the gas being calculated by the model.

Galaxy (dataset)	Model	β_{ave} of best-fit model	Initial galaxy mass ($M_{\odot}$)	Galaxy age (Gyr)	SFR constant	Timing of galactic wind (Gyr after start)	Percentage of initial gas forming Population III stars in first timestep
NGC 2831 (PS02)	a	1.57	6.0 x 10^{10}	13.20	0.65	0.75	44%
	b	2.36	5.0 x 10^{10}	9.00	0.5	4.4	45%
NGC 2832 (PS02)	a	2.86	3.7 x 10^{10}	13.26	0.45	4.1	53%
NGC 3226 (PS02)	a	3.39	5.7 x 10^{10}	13.26	0.53	4.0	39%
	b	3.91	6.0 x 10^{10}	13.20	0.65	0.75	42%
NGC 3226 (D05)		6.21	5.6 x 10^{10}	13.26	0.65	0.765	53%
NGC 3608 (PS02)	a	2.88	5.7 x 10^{10}	13.26	0.53	4.0	39%
NGC 3608 (D05)		11.67	5.5 x 10^{10}	13.26	0.63	0.765	53%
NGC 4291 (PS02)	a	3.38	5.7 x 10^{10}	13.29	0.45	4.0	37%
	b	3.81	6.2 x 10^{10}	13.25	0.55	0.75	49%
NGC 4365 (PS02)	a	3.24	4.0 x 10^{10}	13.20	0.55	4.2	54%
NGC 4365 (D05)		7.81	4.0 x 10^{10}	13.20	0.55	4.5	54%
NGC 4374 (PS02)	a	2.95	5.7 x 10^{10}	13.26	0.53	4.0	39%
NGC 4374 (D05)		7.13	5.5 x 10^{10}	13.26	0.67	0.675	53%
NGC 4552 (PS02)	a	3.35	3.7 x 10^{10}	13.27	0.45	4.0	53%
NGC 4636 (PS02)	a	2.60	3.7 x 10^{10}	13.26	0.45	4.1	53%
	b	2.60	3.7 x 10^{10}	13.26	0.53	4.0	39%
NGC 4697 (PS02)	a	2.74	6.0 x 10^{10}	13.20	0.65	4.2	42%
NGC 5322 (PS02)	a	2.58	5.6 x 10^{10}	13.26	0.65	0.765	53%
	b	3.25	4.5 x 10^{10}	13.26	0.57	4.0	53%
NGC 5322 (D05)		7.96	5.7 x 10^{10}	13.26	0.63	0.765	53%

Table 33: Best fit results for parameter-space searches for the elliptical galaxies in the PS02 sample, together with those for D05 data where available for the PS02 galaxies. One model for NGC 2831 (shaded in grey) is rejected as the final SNIa rates were outside the range given by Turatto et al. 1994 (table 34). Where there are two well-fit models, these are (arbitrarily) marked as 'a' and 'b'.

Galaxy	Final SNu (events/ century/ 10^{10} $L_\odot$) (model a)	Final SNu (events/ century/ 10^{10} $L_\odot$) (model b) (where applicable)
NGC 2831	0.0406	0.0012
NGC 2832	0.0361	N/A
NGC 3226	0.0386	0.0413
NGC 3608	0.0386	N/A
NGC 4291	0.0381	0.0395
NGC 4365	0.0325	N/A
NGC 4374	0.0386	N/A
NGC 4552	0.0388	N/A
NGC 4636	0.0411	0.0361
NGC 4697	0.0317	N/A
NGC 5322	0.0404	0.0384

Table 34: Present day SNIa rates for galaxies in PS02 sample as modelled by Phoenix; expected result is in the range 0.03-0.08 (Turatto et al. 1994). As two best fit models were found for NGC 2831, NGC 3226, NGC 4291, NGC 4636 and NGC 5322, the final SNu results for each model are given. The second model for NGC 2831 is rejected as the value for SNu is well outside the expected range.

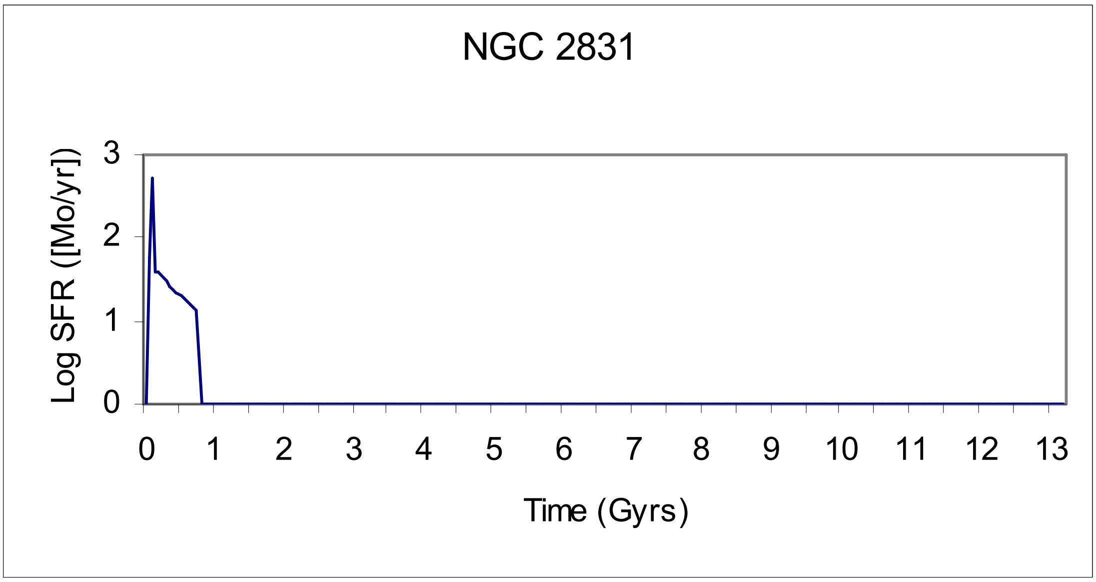
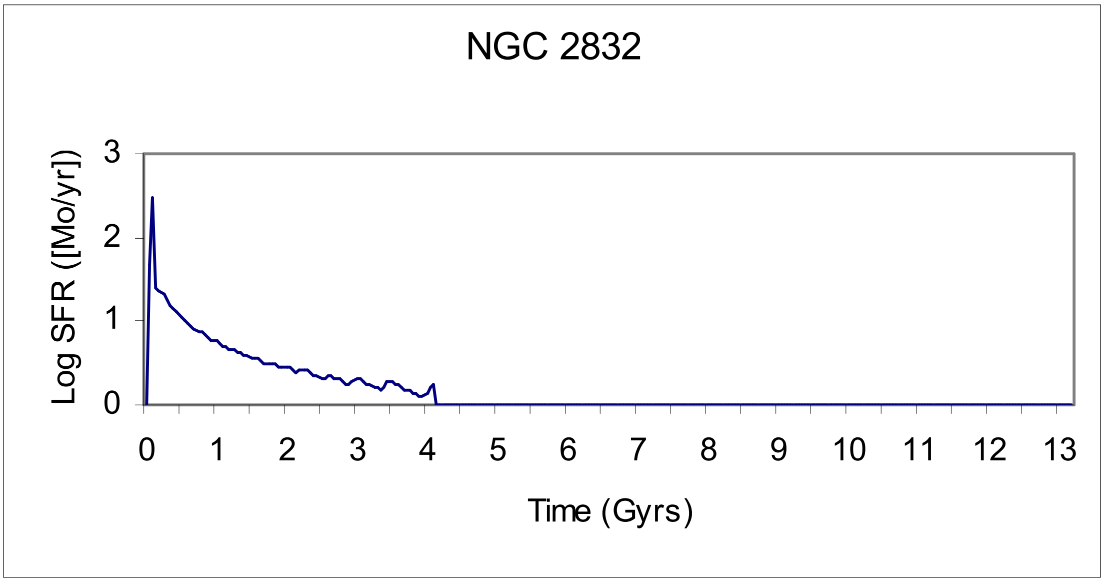


Figure 21: Star formation histories of the PS02 sample, derived using Phoenix model. Model (a) is shown in blue, and where applicable, model (b) is shown in pink. D05 results, where applicable, shown in green.



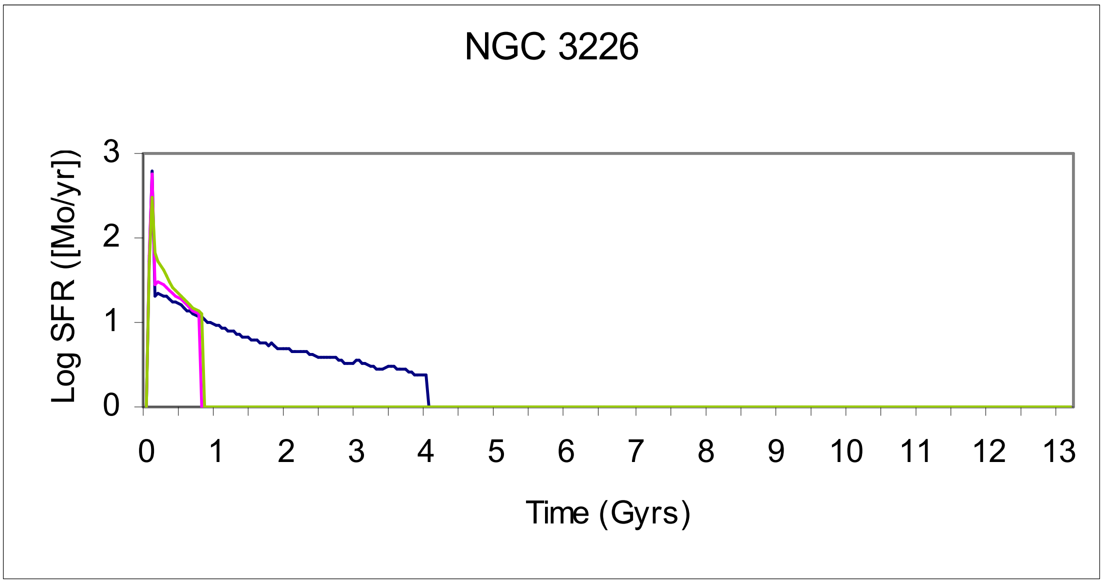
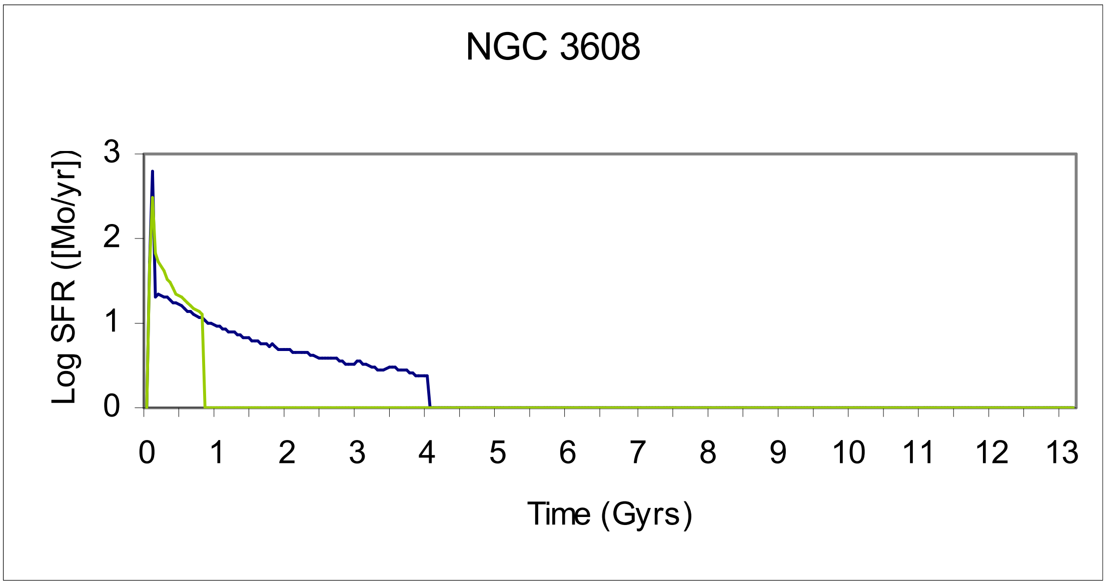
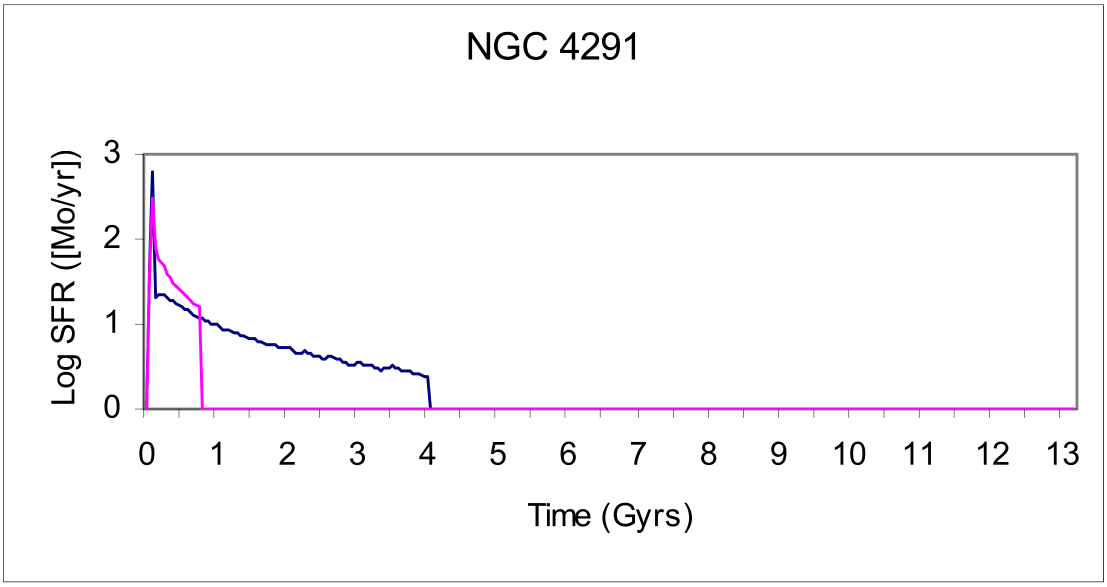


Figure 21/ continued





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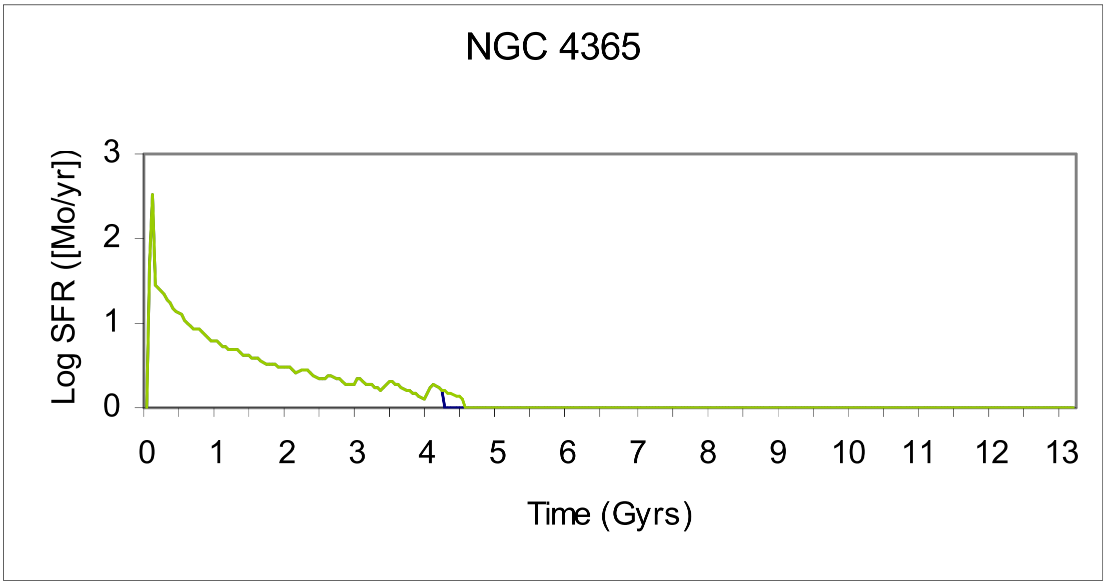

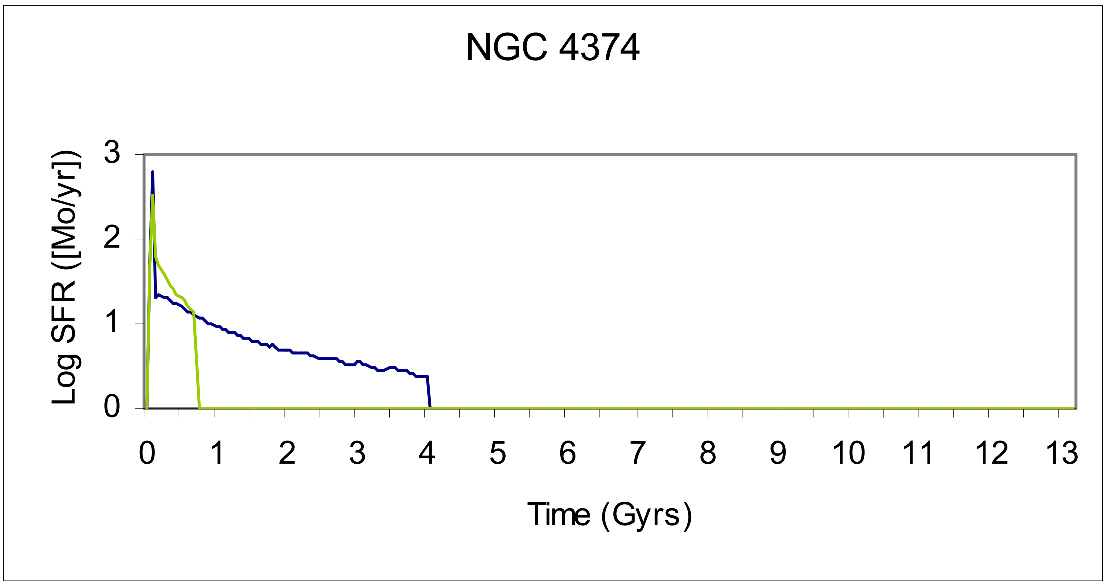

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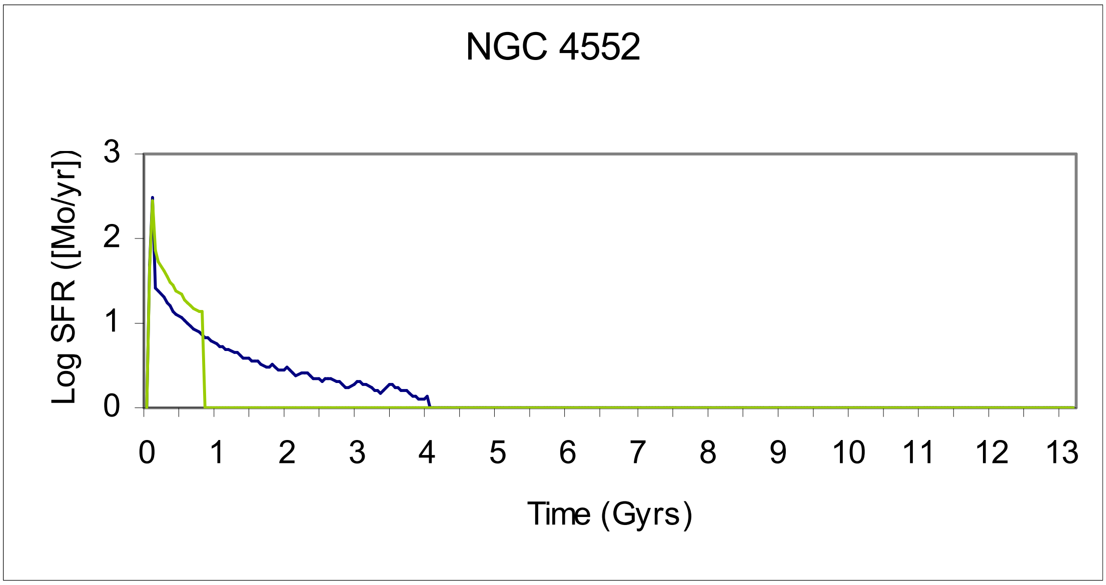


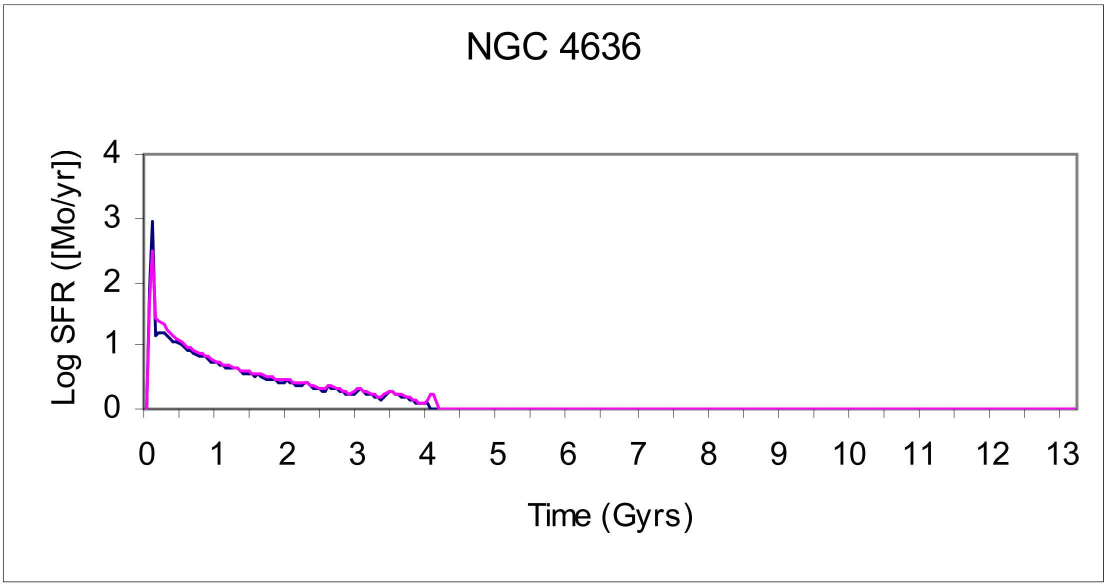
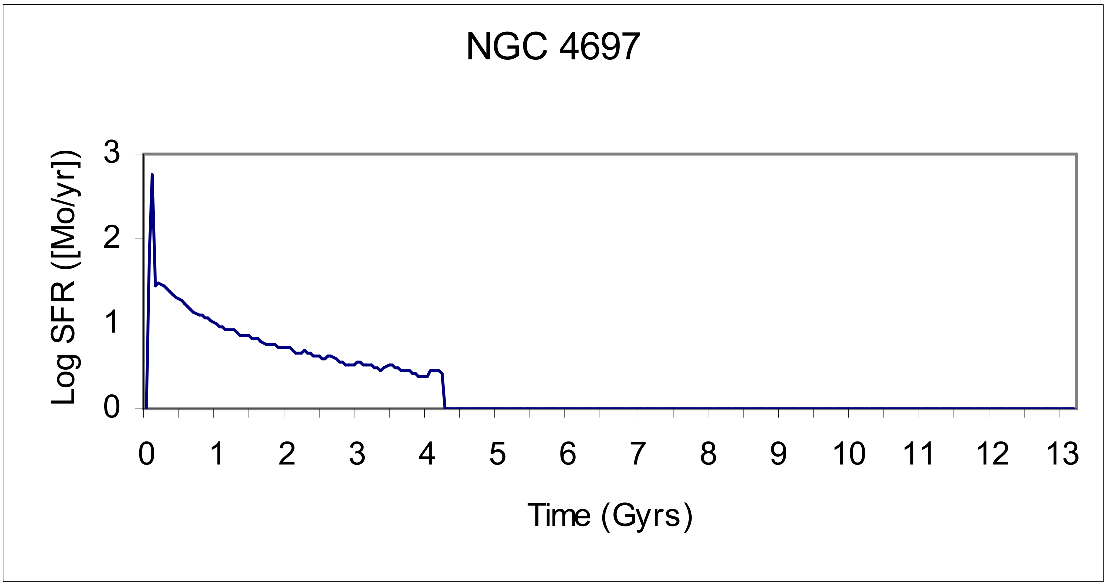


Figure 21/continued



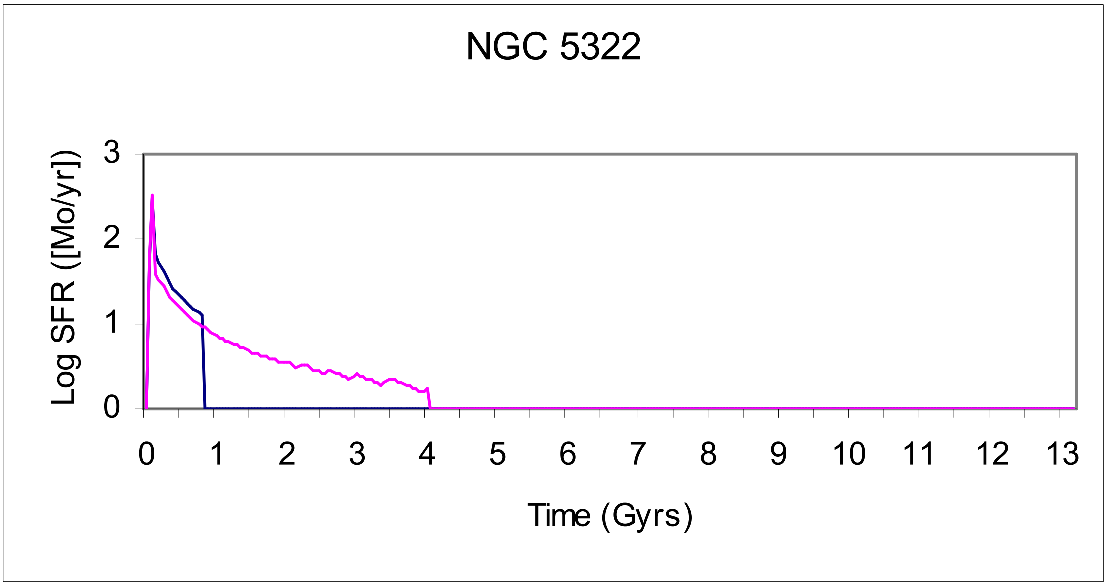


Figure 21/continued

Comparison of PS02 and D05 results

Galaxies NGC 3226, NGC 3608, NGC 4365, NGC 4374 and NGC 5322 had observations taken by both PS02 and D05, enabling results to be compared. D05 data have smaller average uncertainties than the PS02 data. It would therefore be expected to be less likely that a model could be found to fit well across all indices, and this is borne out by β_{ave} across all indices being higher. In fact, no models were found to be able to fit the D05 data with $\beta_{ave} < 3$.

Star formation histories of NGC 4365 and model (a) for NGC 5322, derived using the Phoenix model and the PS02 observations, were confirmed by the code run with D05 observations. This strongly suggests that the SFH of model (b) for NGC 5322 should be rejected. Differences in the observations between PS02 and D05 for galaxies NGC 3226, NGC 3608 and NGC 4374 lead to different star formation histories being deduced using the Phoenix model.

6.3.3 Star formation histories: SB07 data

The above process was repeated for the SB07 data sample. Very small uncertainties on these data made it difficult to find "good" models ($\beta_{ave} < 3$). Best-fit models are given in table 35, together with the best fit-models for the D05 data (where applicable). SNIa rates deduced from the models for these galaxies were checked against "expected" values (table 36) as before. This suggests that the model (a) for NGC 3384, where the galactic wind occurs after 4.4 Gyrs is not a valid model, as the final SNIa rate is outside the expected range (marked in grey). Resulting star formation histories of the SB07 sample are plotted in figure 22.

Galaxy	Model	β_{ave} of best-fit model	Initial galaxy mass ($M_\odot$)	Galaxy age (Gyr)	SFR constant	Timing of galactic wind (Gyr after start)	Percentage of initial gas forming Population III stars in first timestep
NGC 1600 (SB07)	a	19.07	3.7 x 10^{10}	13.26	0.45	4.0	53%
	b	19.19	6.7 x 10^{10}	13.26	0.45	0.765	43%
NGC 1600 (D05)		7.19	4.5 x 10^{10}	13.27	0.57	4.0	53%
NGC 1700 (SB07)	a	18.42	5.5 x 10^{10}	13.26	0.57	0.765	47%
NGC 1700 (D05)		5.46	5.5 x 10^{10}	13.26	0.67	0.74	53%
NGC 3377 (SB07)	a	25.36	6.0 x 10^{10}	13.20	0.65	0.65	46%
NGC 3377 (D05)		10.87	5.5 x 10^{10}	13.26	0.67	0.74	53%
NGC 3379 (SB07)	a	29.44	3.7 x 10^{10}	13.25	0.45	4.0	53%
NGC 3379 (D05)		6.57	5.7 x 10^{10}	13.26	0.45	4.0	37%
NGC 3384 (SB07)	a	37.82	5.0 x 10^{10}	9.00	0.50	4.4	54%
	b	32.67	6.7 x 10^{10}	13.26	0.45	0.765	43%
NGC 3384 (D05)		9.71	6.0 x 10^{10}	12.80	0.65	0.75	52%
NGC 4387 (SB07)	a	9.52	5.5 x 10^{10}	13.26	0.45	0.765	41%
NGC 4458 (SB07)	a	10.23	6.0 x 10^{10}	13.20	0.65	0.65	46%
NGC 4464 (SB07)	a	15.39	6.5 x 10^{10}	13.26	0.67	0.765	43%
NGC 4472 (SB07)	a	27.08	3.7 x 10^{10}	13.26	0.45	4.0	53%
NGC 4551 (SB07)	a	11.31	5.5 x 10^{10}	13.26	0.57	0.765	47%

Table 35 Best fit results for parameter-space searches for the elliptical galaxies in the SB07 sample, together with those for D05 data where available for the SB07 galaxies. One model for NGC 3384 (shaded in grey) is rejected as the final SNIa rates were outside the range given by Turatto et al. 1994.

Galaxy	Final SNu (events/ century/ 10^{10} $L_{\odot}$) (model a)	Final SNu (events/ century/ 10^{10} $L_{\odot}$) (model b where applicable)
NGC 1600	0.0408	0.0388
NGC 1700	0.0409	N/A
NGC 3377	0.0374	N/A
NGC 3379	0.0388	N/A
NGC 3384	0.0011	0.0408
NGC 4387	0.0426	N/A
NGC 4458	0.0374	N/A
NGC 4464	0.0417	N/A
NGC 4472	0.0408	N/A
NGC 4551	0.0409	N/A

Table 36: Final SNIa rates for galaxies in SB07 sample as modelled by Phoenix; expected result is 0.03-0.08 (Turatto et al. 1994). As two best fit models were found for NGC 1600 and NGC 3384, two final SNu results are given.

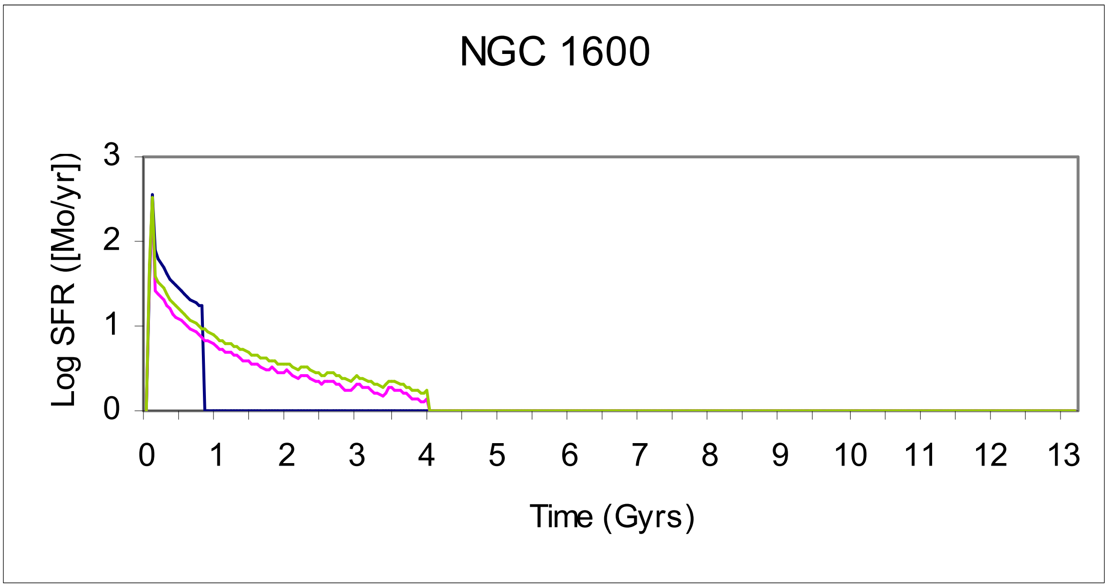
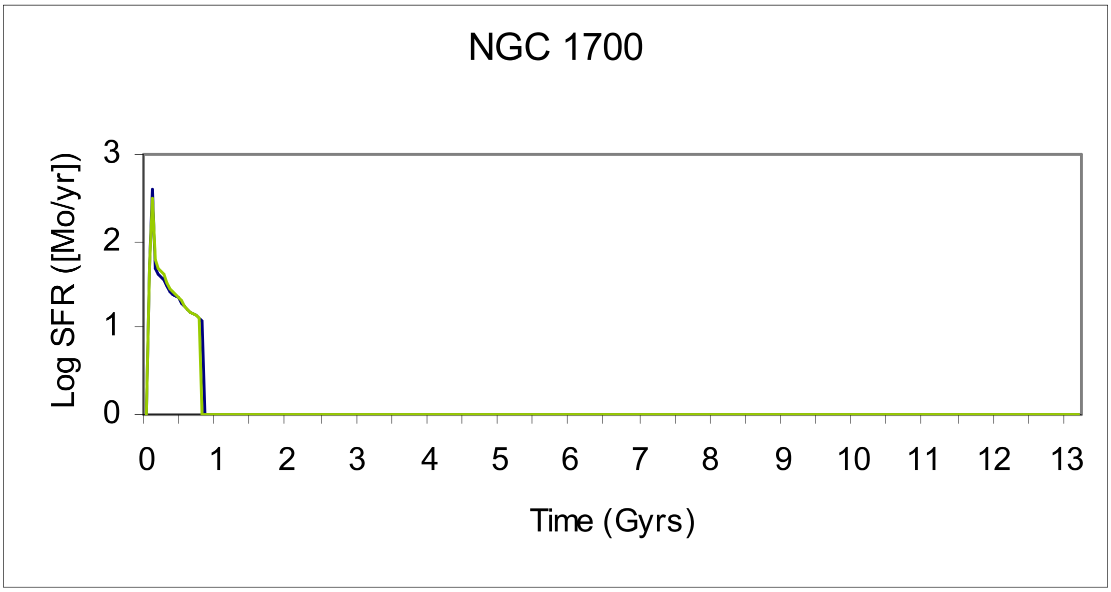
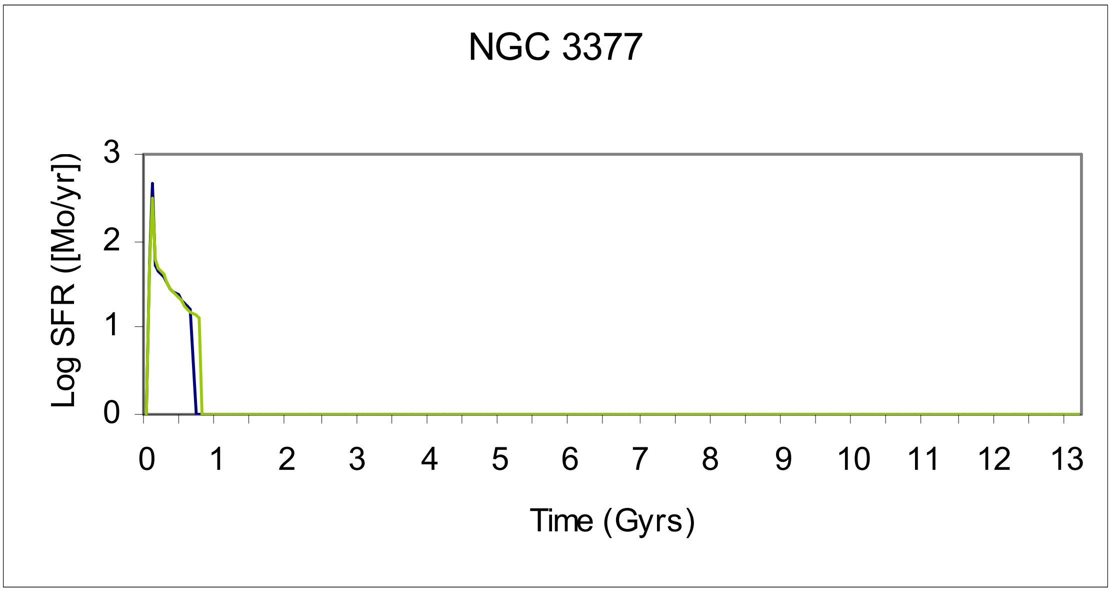


Figure 22: Star formation histories of the SB07 sample, derived using Phoenix model. Model (a) is shown in blue, and where applicable, model (b) is shown in pink. D05 results, where applicable, shown in green.





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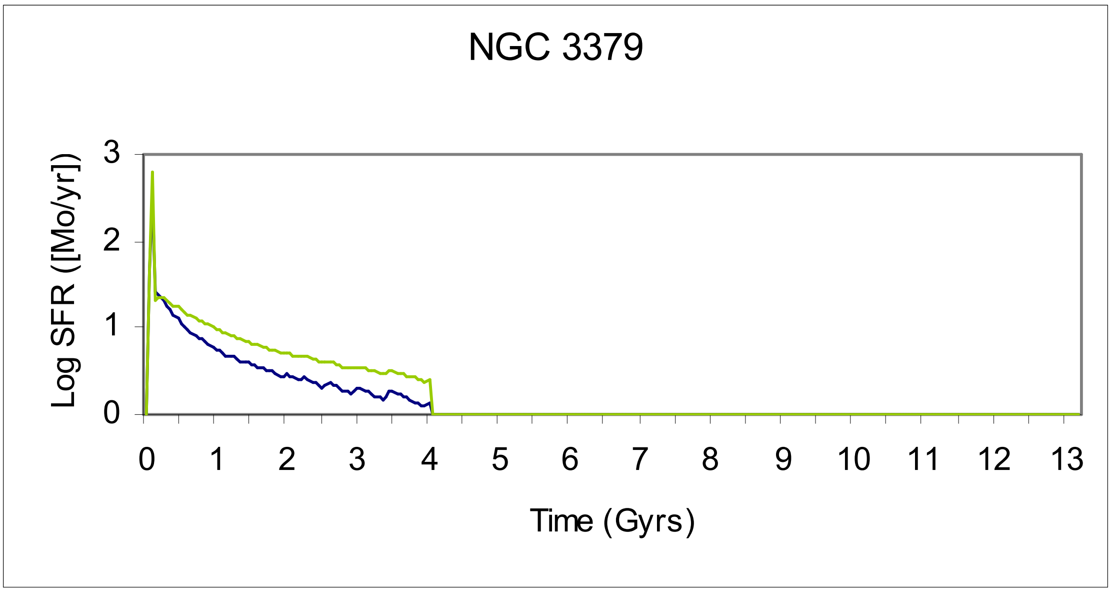

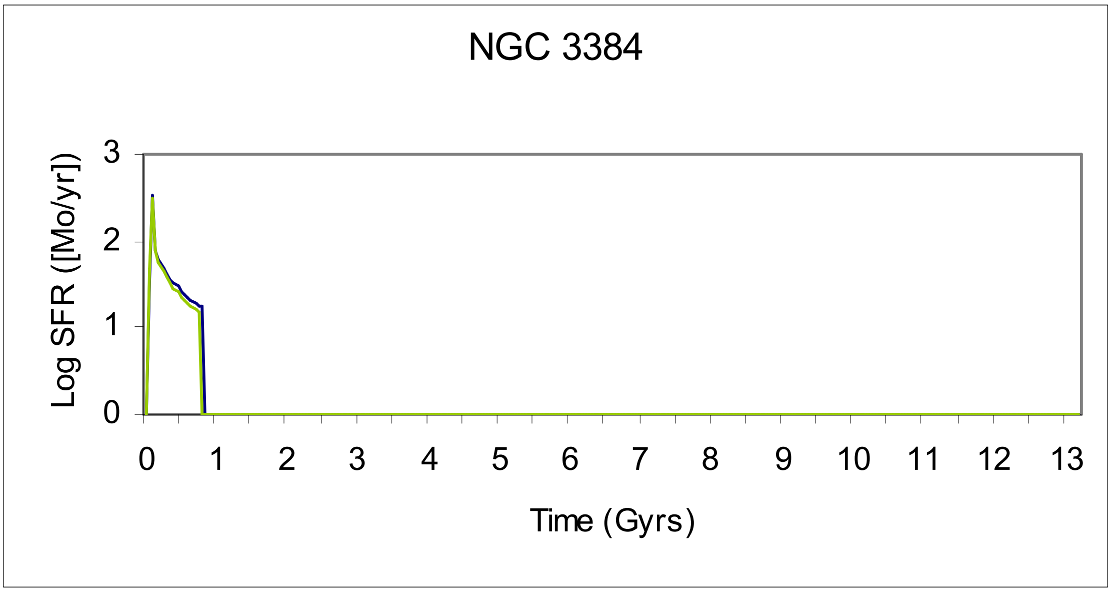

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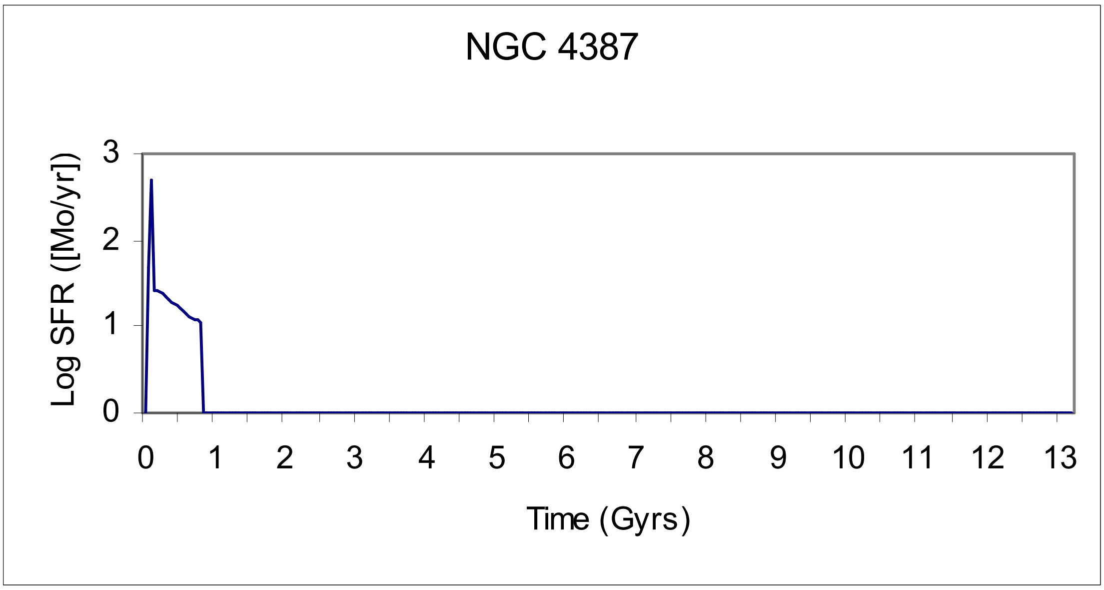


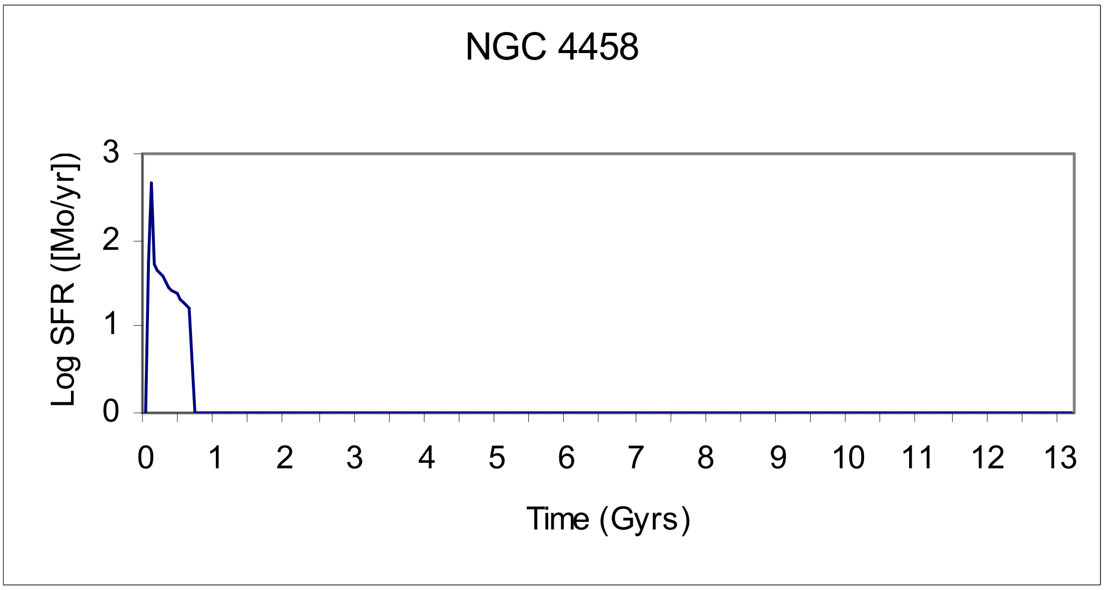
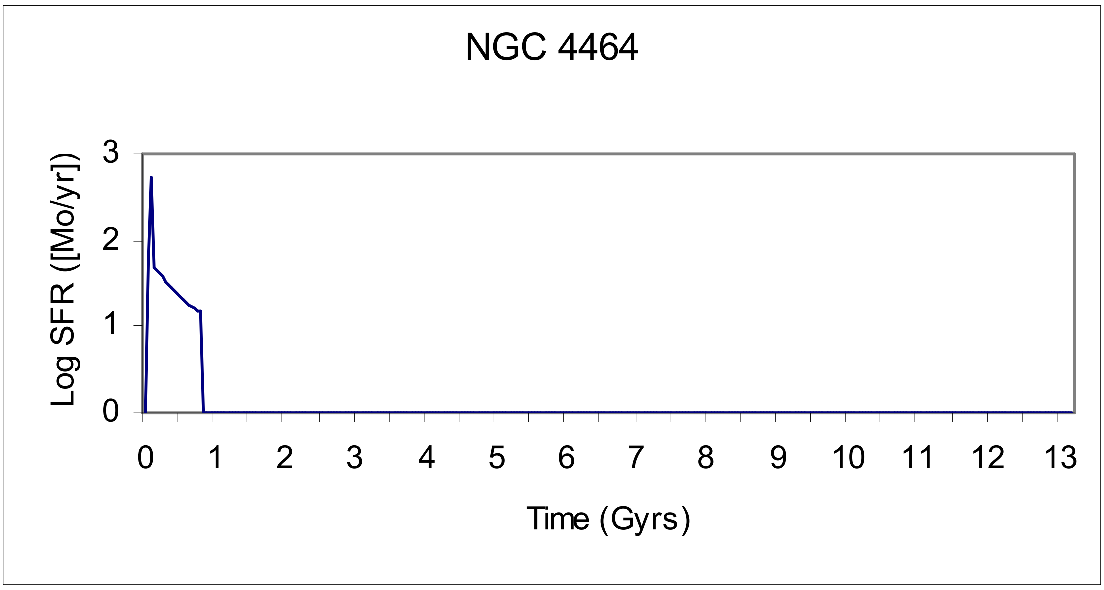


Figure 22/continued



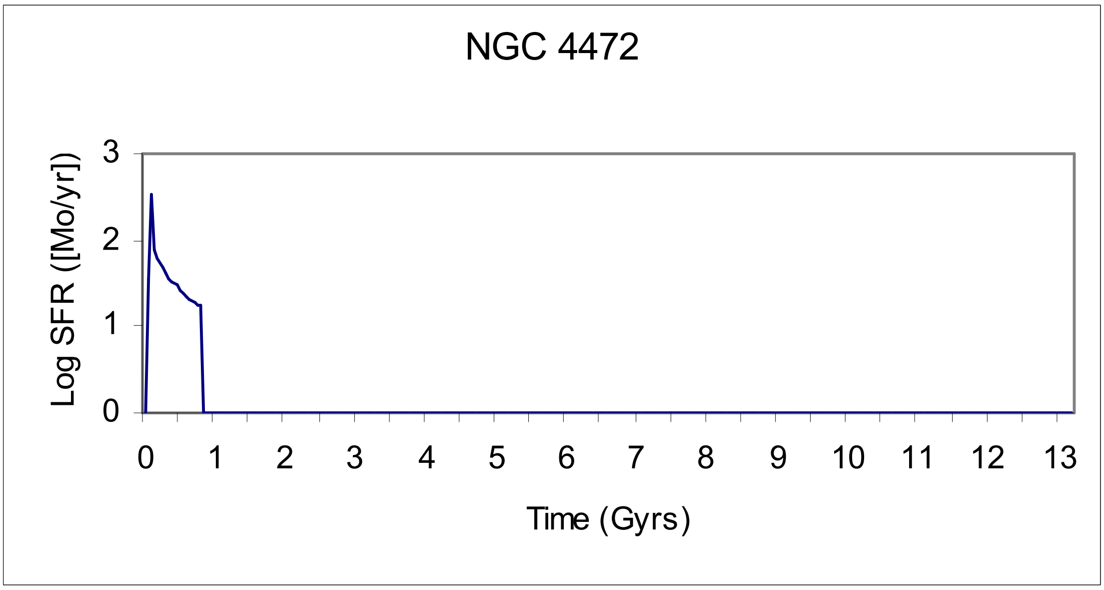


Figure 22/ continued

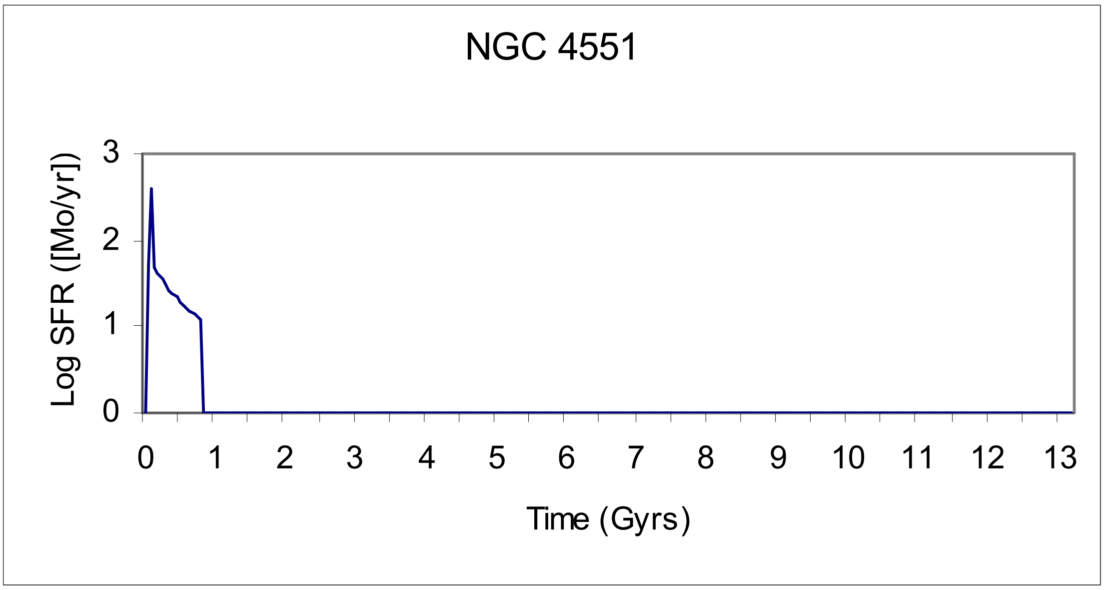


Figure 22/ continued

Comparison of SB07 and D05 results

Galaxies NGC 1600, NGC 1700, NGC 3377, NGC 3379 and NGC 3384 had observations taken by both SB07 and D05, enabling results to be compared. As the D05 data have larger uncertainties than those of SB07, it is easier to fit a model and this can be seen by the lower values of β_{ave} for the D05 galaxies when compared to the corresponding galaxies in the SB07 dataset.

Best-fit models to D05 data were similar to, but not identical with, the best-fit models to SB07 data, with the exception of NGC 3384.

6.4 CHECKING MODEL RESULTS

6.4.1 Comparing results to a separate set of data: a recap and discussion

From figure 20 above, which compares the Lick indices for those galaxies that are in both D05 and either PS02 or SB07, some of the measurements by D05 can be seen to differ from those of PS02. These are minor differences and are generally within the uncertainties of the PS02 observations. However, the derived star formation histories for three of the galaxies (NGC 3226, NGC 3608 and NGC 4374) are very different. On the other hand, the calculated star formation histories for NGC 4365 and NGC 5322 (model a) are supported by the modelling of D05.

Lick indices measured by D05 are similar to those of SB07, with the exception of NGC 1600. It would therefore be expected that similar star formation histories would be found for NGC 1700, NGC 3377, NGC 3379 and NGC 3384 when modelled with either T04 SSPs or Phoenix, and this is indeed true for all except NGC 3384. Whilst the majority of the index observations for this galaxy are similar, HgA and Fe5270 have small differences, and these small differences appear to be enough to produce quite different star formation histories. The two sets of observations of NGC 1600 lead to significantly different star formation histories when modelled by either T04 or Phoenix.

This shows how very sensitive these models are to slight variations in the observational data, and emphasises the importance of using more than one set of observations of the same object (taken from different telescopes and using different data reduction techniques) for establishing star formation histories, a method not used in the literature as standard.

6.4.2 Indices selected for modelling

For the work so far described in this Chapter, the entire set of indices given in the observational data set was used for modelling. However, three further options could have been used:

1. Any individual index that is not modelled within 3 β of the observation could have been ignored.

2. The Mg indices, noted as having uncertainties that do not reflect the actual uncertainty on the data for the PS02 observations taken with the WHT, could be ignored (or treated as having much larger uncertainties).
3. Where the D05 data are used for comparison, only those model indices which were measured in both D05 and the data set being compared could be included, thus ignoring any indices which are in one data set but not the other.

<u>Option 1: disregard individual indices which are poorly modelled</u>

Option 1 is rejected as it would produce results that are not scientifically robust.

<u>Option 2: disregard Mg indices in the PS02 data as the uncertainties are understated</u>

Not surprisingly, lower β_{ave} values are found for the PS02 data when the Mg indices are removed from the sample. This is because they have small uncertainties and as such any model which does not produce accurate Mg indices will have a large value for β for those indices, giving a larger β_{ave}.

As shown in table 37, best fit models for NGC 2831, NGC 2832 and NGC 4636 (a) when the Mg indices were excluded were the similar to those found when the Mg indices were included. Model (a) of NGC 4291 was similar to model (1), and model (b) of NGC 3226 was similar to that found when the Mg indices were excluded.

However, for the other six galaxies in this sample (NGC 3608, NGC 4365 NGC 4374, NGC 4552, NGC 4697 and NGC 5322, different results were found (highlighted below). This demonstrates that the completeness of the data set can influence the SFH found.

Galaxy (PS02 data)	**Model**	**β_{ave} of best-fit model**	**Initial galaxy mass ($M_{\odot}$)**	**Galaxy age (Gyr)**	**SFR constant**	**Timing of galactic wind (Gyr after start)**	**Percentage of initial gas forming Population III stars in first timestep**
NGC 2831	a	1.57	6.0 x 10^{10}	13.20	0.65	0.75	44%
Excluding Mg indices		1.16	6.0 x 10^{10}	13.20	0.65	0.75	44%
NGC 2832	a	2.86	3.7 x 10^{10}	13.26	0.45	4.1	53%
Excluding Mg indices		1.55	3.7 x 10^{10}	13.26	0.45	4.0	53%
NGC 3226	a	3.39	5.7 x 10^{10}	13.26	0.53	4.0	39%
	b	3.91	6.0 x 10^{10}	13.20	0.65	0.75	42%
Excluding Mg indices		1.70	5.7 x 10^{10}	13.26	0.63	0.76	43%
NGC 3608	a	2.88	5.7 x 10^{10}	13.26	0.53	4.0	39%
Excluding Mg indices		1.75	3.7 x 10^{10}	13.26	0.45	4.1	53%
NGC 4291	a	3.38	5.7 x 10^{10}	13.29	0.45	4.0	37%
	b	3.81	6.2 x 10^{10}	13.25	0.55	0.75	49%
Excluding Mg indices	1	1.88	5.5 x 10^{10}	13.26	0.45	4.0	37%
	2	1.89	6.7 x 10^{10}	13.26	0.45	0.75	43%
NGC 4365	a	3.24	4.0 x 10^{10}	13.20	0.55	4.2	54%
Excluding Mg indices		1.75	5.0 x 10^{10}	13.00	0.10	4.4	60%
NGC 4374	a	2.95	5.7 x 10^{10}	13.26	0.53	4.0	39%
Excluding Mg indices		1.46	6.6 x 10^{10}	13.26	0.55	0.765	49%
NGC 4552	a	3.35	3.7 x 10^{10}	13.27	0.45	4.0	53%
Excluding Mg indices		2.42	4.5 x 10^{10}	13.26	0.57	4.0	53%
NGC 4636	a	2.60	3.7 x 10^{10}	13.26	0.45	4.1	53%
	b	2.60	3.7 x 10^{10}	13.26	0.53	4.0	39%
Excluding Mg indices		1.40	3.7 x 10^{10}	13.26	0.45	4.1	53%
NGC 4697	a	2.74	6.0 x 10^{10}	13.20	0.65	4.2	42%
Excluding Mg indices		1.70	1.0 x 10^{11}	9.00	0.50	4.4	33%
NGC 5322	a	2.58	5.6 x 10^{10}	13.26	0.65	0.765	53%
	b	3.25	4.5 x 10^{10}	13.26	0.57	4.0	53%
Excluding Mg indices		1.51	5.0 x 10^{10}	9.0	0.50	4.4	50%

Table 37: Comparison of best fit models of PS02 data when Mg indices are or are not included. Model (a) and (b) data are as given in table 33.

Option 3: only model indices which are in both data sets

The modelling described in 6.3 above included in each instance the full set of observational indices provided. However, the three data sets (PS02, SB07 and D05) did not all observe the same set of indices:

- neither PS02 nor D05 include the D4000 index, whereas this is in the SB07 data set;
- Fe5406 is in PS02 and D05 but not in SB07; and
- neither PS02 nor SB07 include Fe5709, Fe5782, NaD, Ti01 or TiO2, but these are all included in the D05 data.

If D05 models just use the indices that are in the PS02/SB07 data to which they are being compared, the best fit model is the same as found with the full set of indices for D05 for NGC 5365, NGC 4374 and NGC 5322 and NGC 3384, but different results are found for the other six galaxies, which are given in table 38. Matching the set of indices observed means the SFH found for NGC 1700 is now identical to that found with the SB07 data. The SFH for NGC 3226, NGC 3377 and NGC 3379 are more similar to those found from PS02/SB07 (as applicable) when the data set of D05 is restricted. On the other hand, the SFH for NGC 3608 and NGC 1600 are less similar when compared to the restricted D05 set. The important point to note, however, is that the SFH are different when the data sets are selectively chosen.

Therefore, if other indices had been measured at the time the observational data were taken, or if fewer indices had been observed, it would be expected that different star formation histories could have been found by the modelling.

Galaxy (dataset)	Model	β_{ave} of best-fit model	Initial galaxy mass ($M_{\odot}$)	Galaxy age (Gyr)	SFR constant	Timing of galactic wind (Gyr after start)	Percentage of initial gas forming Population III stars in first timestep
NGC 3226 (PS02)	a	3.39	5.7 x 10^{10}	13.26	0.53	4.0	39%
	b	3.91	6.0 x 10^{10}	13.20	0.65	0.75	42%
NGC 3226 (D05 complete)		6.21	5.6 x 10^{10}	13.26	0.65	0.765	53%
NGC 3226 (D05restricted)		6.01	5.7 x 10^{10}	13.26	0.63	0.765	43%
NGC 3608 (PS02)	a	2.88	5.7 x 10^{10}	13.26	0.53	4.0	39%
NGC 3608 (D05 complete)		11.67	5.5 x 10^{10}	13.26	0.63	0.765	53%
NGC 3608 (D05 restricted)		11.73	4.0 x 10^{10}	13.20	0.55	0.75	54%
NGC 1600 (SB07)	a	19.07	3.7 x 10^{10}	13.26	0.45	4.0	53%
	b	19.19	6.7 x 10^{10}	13.26	0.45	0.765	43%
NGC 1600 (D05 complete)		7.19	4.5 x 10^{10}	13.27	0.57	4.0	53%
NGC 1600 (D05 restricted)		6.07	6.0 x 10^{10}	13.20	0.65	4.2	42%
NGC 1700 (SB07)	a	18.42	5.5 x 10^{10}	13.26	0.57	0.765	47%
NGC 1700 (D05 complete)		5.46	5.5 x 10^{10}	13.26	0.67	0.74	53%
NGC 1700 (D05 restricted)		5.51	5.5 x 10^{10}	13.26	0.57	0.74	47%
NGC 3377 (SB07)	a	25.36	6.0 x 10^{10}	13.20	0.65	0.65	46%
NGC 3377 (D05 complete)		10.87	5.5 x 10^{10}	13.26	0.67	0.74	53%
NGC 3377 (D05 restricted)		10.73	5.7 x 10^{10}	13.26	0.63	0.765	43%
NGC 3379 (SB07)	a	29.44	3.7 x 10^{10}	13.25	0.45	4.0	53%
NGC 3379 (D05 complete)		6.57	5.7 x 10^{10}	13.26	0.45	4.0	37%
NGC 3379 (D05 restricted)		6.39	5.7 x 10^{10}	13.26	0.45	4.0	53%

Table 38: PS02/SB07 model best fits (from tables 33 and 35) compared to D05 (complete set of Lick indices) and D05 (restricted set), where the restricted set models only those indices also observed by PS02/SB07 and where the D05 model changes when the restriction is imposed.

6.4.3 Star formation histories: comparison using different models

NGC 3226 is a dwarf elliptical galaxy currently merging with spiral galaxy NGC 3227 (Rubin and Ford 1968). This merger is triggering star formation outside the boundary of the observed galaxies (Mundell et al. 2004); no molecular gas is observed within the galaxies, indicating that the merger, as far as NGC 3226 is concerned, is dry (Cullen et al. (2006). Gondin et al. (2004) found the galaxy to contain a central black hole with a mass of 1.7 x 10^7 $M_\odot$, and observed X-ray emission away from the galactic nucleus which supports an historical wind. This is therefore a simple galaxy which might be expected to be successfully modelled with an SSP.

A star formation history of this galaxy, using data from PS02, was deduced using the GCE model (in Chapter 2), and subsequently modelled with T04 SSPs and the Phoenix model (this Chapter). Findings from these three models are collated below in table 39. The SSP model requires a pre-enriched gas cloud and proposes a younger-aged galaxy than those proposed by the GCE and Phoenix models. The GCE model requires gas infall at the same chemical composition as the model galaxy's ISM, and star formation continuing for 8.5 Gyrs, whereas the Phoenix model does not require any gas infall and star formation continues for 4.0 Gyrs.

Parameter	GCE model	TO4 SSP model	Phoenix model
Initial galaxy mass	1 x 10^6 $M_\odot$ (hard-coded)	N/A	5.7 x 10^{10} $M_\odot$ (search parameter)
Final galaxy mass	8.5 x 10^6 $M_\odot$ (initial + infall)	N/A	4.7 x 10^{10} $M_\odot$ (calculated by code)
Constant in Schmidt star formation rate equation	5.0 reducing to 4.5 after 0.5 Gyrs and then to zero after a further 7.5 Gyrs (search parameters)	N/A	0.53 constant (search parameter)
Percentage of initial gas forming Population III stars	N/A	N/A	39% (search parameter)
Overall age of the galaxy	12.0 Gyrs (parameter set by user not the "stepping software")	9.0-10.0 Gyrs	13.26 Gyrs (search parameter)
Gas infall (to represent a merger event)	Pre-enriched gas, infalling at a rate of 10^6 $M_\odot$/Gyr, starting when the galaxy was 0.5 Gyrs old and lasting for 7.5 Gyrs (search parameter)	N/A	None (search parameter)
Time of galactic wind	Not included in code but modelled as a cessation of star formation 8.5 Gyrs after start of galaxy (search parameter).	N/A	4.0 Gyrs after start of galaxy (search parameter)
Model fit (β_{ave})	2.79	1.85-1.86	3.39

Table 39: Comparison of the star formation history of NGC3226 found by three models. 'Search parameter' indicates a variable on which the model searches for the best fit.

6.5 DISCUSSION AND CONCLUSIONS

6.5.1 Results from the Phoenix model

The Phoenix model, when applied to the data sets of elliptical galaxies from PS02 and SB07, suggests the following parameter constraints:

- Galaxy age is tightly constrained in the range $13.26^{+0.09}_{-0.06}$ Gyrs;

- The constant C in the Schmidt (1959) equation, modified with Kennicutt (1989) index (SFR=C $\rho^{1.3}$) ranges between 0.45 and 0.67;

- None of the models require inflow of gas at any time during the galaxy evolution. This implies that all mergers are dry; the percentage of initial primordial gas forming the Population III stars ranges between 37% and 54%, with no good-fit models for percentages higher than this;

- Confidence in almost all of the models is given by the final SNIa rates being within the expected range from Turatto et al. (1994);

- Stars in the final modelled galaxy are all < 1 $M_\odot$ as would be expected: more massive stars having reached the end of their lives and no new high mass stars being formed following the galactic wind. On average 38% (by mass) of these are original Population III stars, although note that these will not be major contributors to the overall luminosity of the galaxy and hence not to the luminosity-weighted indices;

- Galactic winds occurs either early, after 0.65-0.765 Gyrs into the galaxy's life, or later, after 4.0-4.2 Gyrs. Four of the models (table 40 below) had good results around both these regions of parameter space; the rest were only well modelled at one or other region. Models for NGC 3384 (SB07) and NGC 2831 (PS02) with the galactic wind at 4.4 Gyrs are rejected because the final SNIa rate is outside the expected range. There is no correlation between the timing of the galactic winds and the galaxy location; indeed, the results suggest that an undetected systematic error in one or other of the two data sets, as the timing of the wind appears to be correlated with the data source rather than any other factor. This is not obvious from the plots of the indices (figure 20).

The D05 results reject one or other of the models where there were two models found with different timing of the galactic wind, and find different timing of the wind for two of the PS02 models (highlighted).

The D05 data set (sub-sampled to select galaxies that were also in either PS02 or SB07) only included one field galaxy (NGC 1600), which was found to have a later timing of the galactic wind; this is insufficient data to draw any conclusions regarding timing of wind to galaxy location or environment.

Data set	**Galaxies with model results either between 0.65-0.75 Gyrs OR between 4.0-4.2 Gyrs for time of galactic wind**		**Galaxies with reasonable models between EITHER 0.65-0.75 Gyrs OR 4.0-4.2 Gyrs for time of galactic wind**
	Galactic wind after 0.65-0.75 Gyrs	**Galactic wind after 4.0-4.2 Gyrs**	
PS02	NGC 2831 field	NGC 2832 field NGC 3608 Leo NGC 4365 Virgo NGC 4374 Virgo NGC 4552 Virgo NGC 4636 Virgo NGC 4697 Virgo	NGC 3226 Leo NGC 4291 Ursa Major NGC 5322 Draco
SB07	NGC 1700 Eridanus NGC 3377 Leo NGC 3384 Leo NGC 4387 Virgo NGC 4458 Virgo NGC 4464 Virgo NGC 4551 Virgo	NGC 3379 Leo NGC 4472 Virgo	NGC 1600 field
D05	NGC 3226 (PS02) NGC 3608 (PS02) NGC 4374 (PS02) NGC 5322 (PS02) NGC 1700 (SB07) NGC 3377 (SB07) NGC 3384 (SB07)	NGC 4365 (PS02) NGC 1600 (SB07) NGC 3379 (SB07)	

Table 40: Galaxies categorised by the timing of the galactic wind. The location of each galaxy is indicated; there is no correlation between these results and the galaxy location, but there does appear to be correlation to data source for PS02 and SB07.

- Correlation between the final (U-V) colour to velocity dispersion modelled is within the expected range from Bower et al. (1992) (figure 23); NGC 4636 (PS02) is the outlier, and it is noted that the SB07 data is less well modelled than the PS02 results.

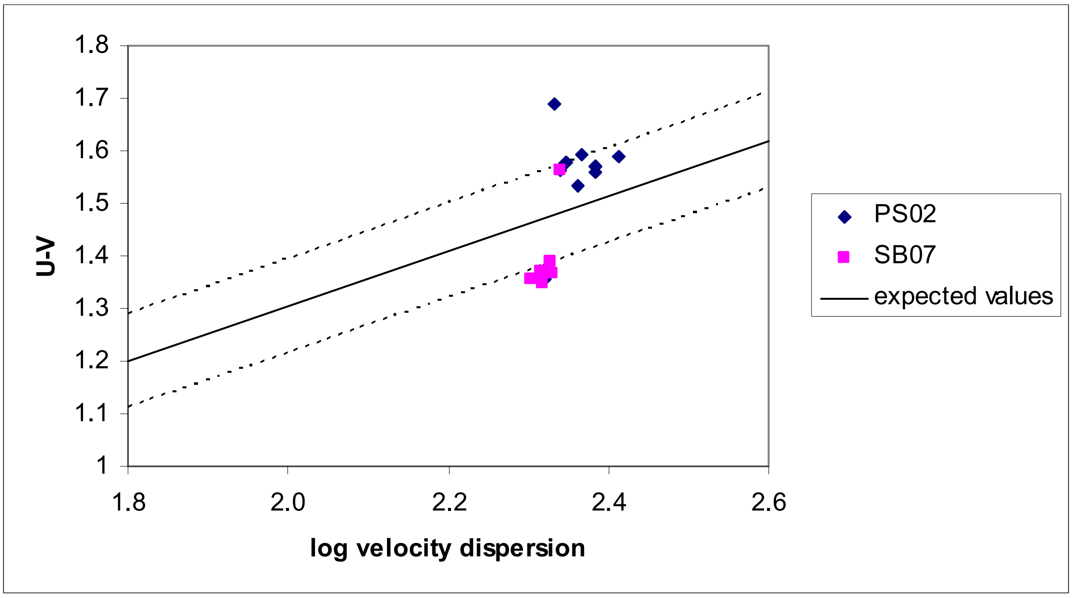


Figure 23: Relationship between the (U-V) colour and velocity dispersion, compared to expected results from Bower, Lucey and Ellis (1992).

6.5.2 Correlations within the results from the Phoenix model

Results from the Phoenix models also demonstrate the following correlations (figure 24 below), although it is noted there is wide scatter in all plots:

- A lower star formation constant and lower galaxy mass correlates to a later timing for the galactic wind, and a higher star formation constant and higher mass correlates to an earlier timing for the galactic wind. This corresponds to the theory of "downsizing" where stars in more massive galaxies tend to have formed earlier and over a shorter timeframe (i.e. have older average ages) than those in smaller galaxies (figure 24 a and b);

- There is almost no correlation between the percentage of Population III stars formed from the initial gas, and the timing of the galactic wind (figure 24 c). A correlation might be expected if stellar winds, which are lower in lower metallicity stars, were a causative agent for galactic winds: a higher percentage of Population III stars would indicate a galaxy with a larger number of low metallicity stars. This suggests that stellar winds are unlikely to be responsible for galactic winds;

- There is also almost no correlation between the percentage of Population III stars formed from the initial gas, and the subsequent star formation efficiency (figure 24 d), suggesting these factors are not linked; and

- Compared to more massive galaxies, lower mass galaxies have a higher proportion of the initial gas cloud forming the Population III stars (figure 24 e) and subsequent stars are formed less efficiently (a lower SFR constant) (figure 24 f). Figure 24 (d) has indicated that these are not correlated to one another i.e. are independently correlated to galaxy mass.

  Ferreras and Silk (2003) find their models of early-type galaxies predict star formation efficiency proportional to galaxy mass, but do not propose any underlying physical reason for this. These findings differ from those of Rownd and Young (1999), who find from their models of spiral galaxies that more massive galaxies are less efficient at star forming than mid-sized galaxies, but again, do not propose a physical mechanism leading to this result.

  Perhaps larger galaxies have more massive central black holes, which selectively remove the hotter gas, leaving the cooler gas to form stars more efficiently, whereas smaller galaxies, with either no black hole or a less efficient one will still contain hot gas which could impede efficient star formation, but, conversely, smaller collapsing gas clouds in the early Universe would lose their energy more quickly and therefore be able to form a higher proportion of Population III stars.

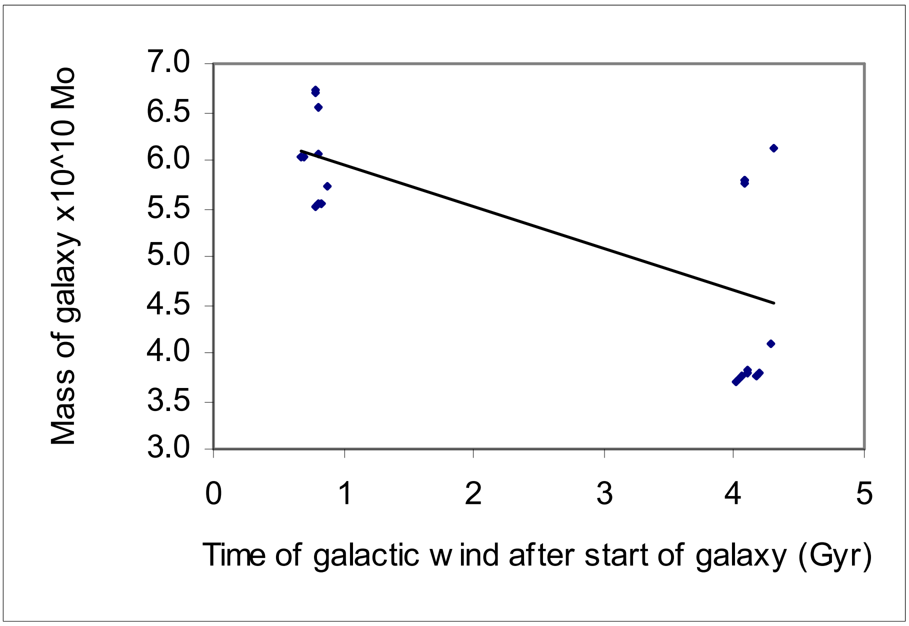


figure (a)

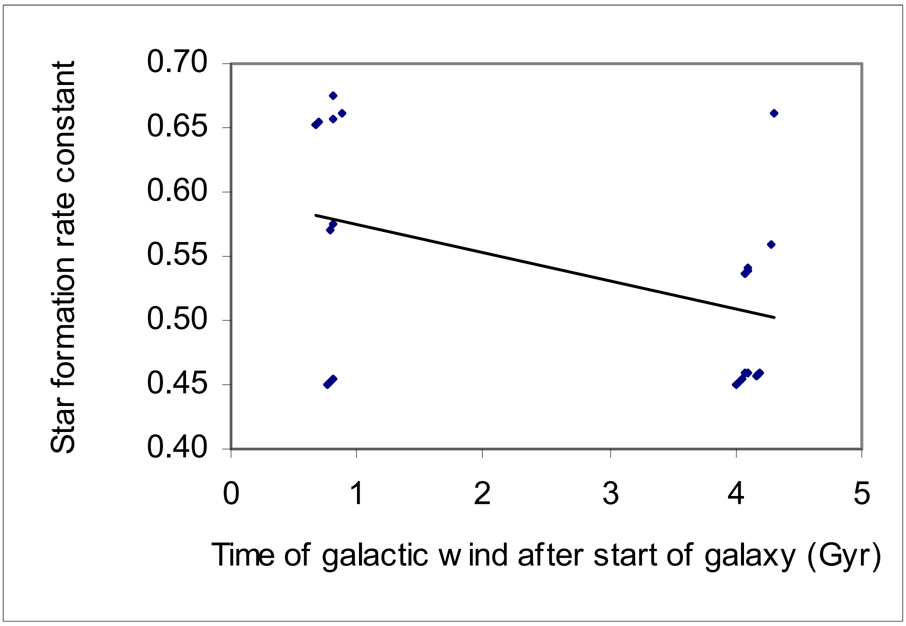


figure (b)

Figure 24: Comparison of parameters from the best-fit models found by Phoenix for the data sets of PS02 and SB07. Small manual adjustments have been made where data points coincided on a graph, in order to make all data points visible.

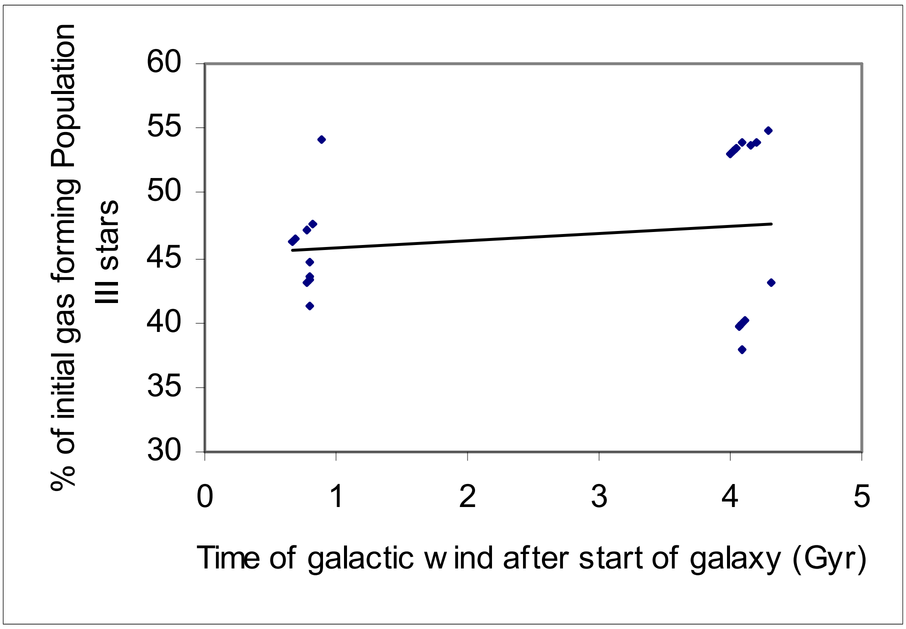


figure (c)

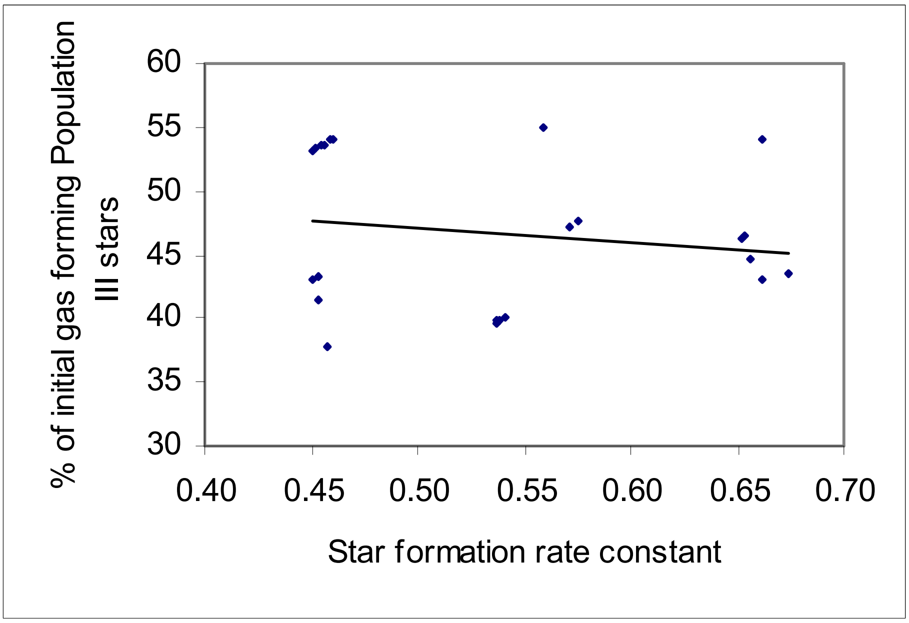


figure (d)

Figure 24/continued

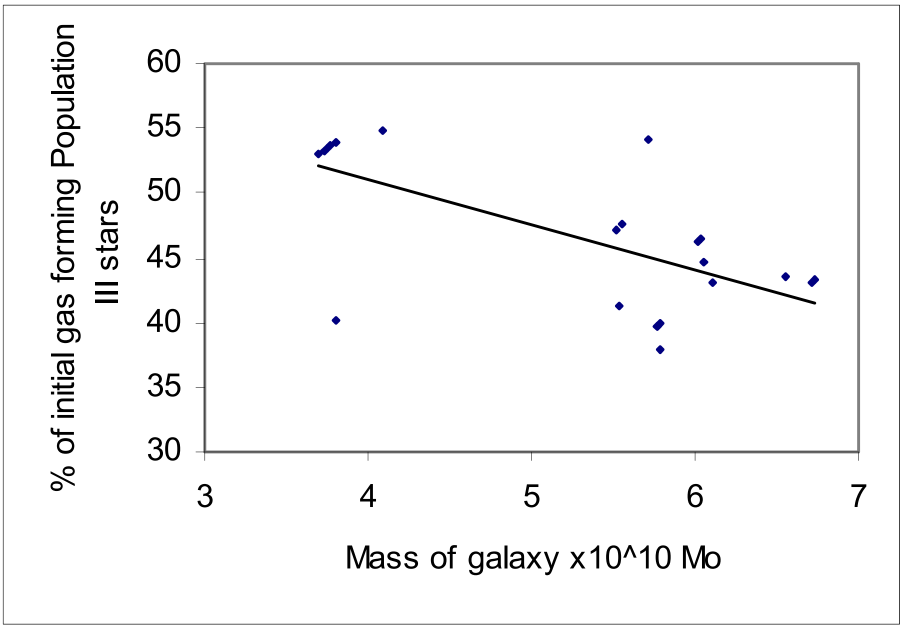


figure (e)

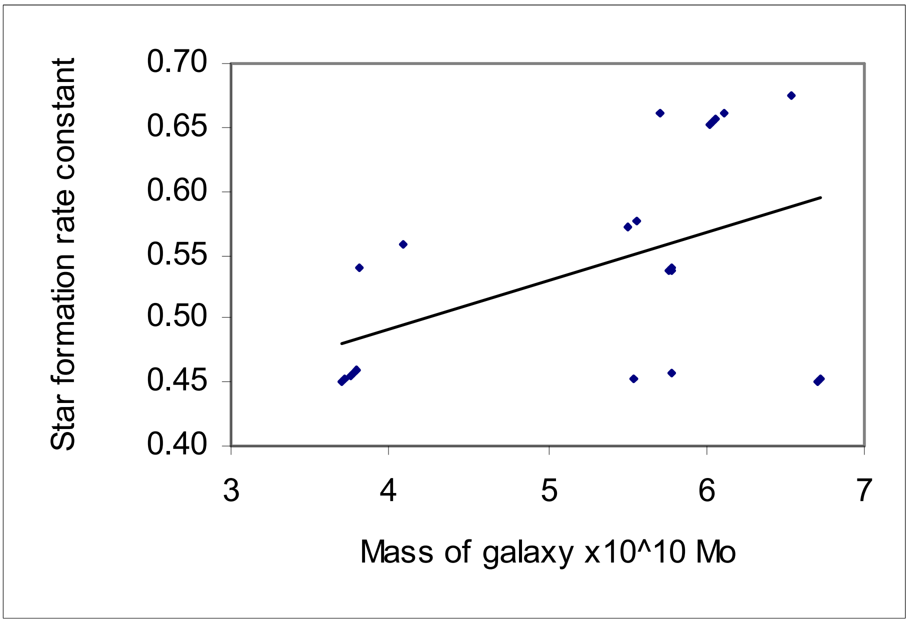


figure (f)

Figure 24/continued

6.5.3 Bimodality of results

It is noted that the results obtained by Phoenix exhibit extreme bimodality, with best-fit models having the galactic wind at either 0.65-0.75 Gyrs or 4.0-4.2 Gyrs. The "searching software" looked for models both between and either side of these values; it is therefore not a consequence of limited searching nor of the model itself, which is allowed to freely scale the timing of the winds.

There is no currently known physical reason for this bimodality; as noted in Section 6.1.2, the similarity in index data for elliptical galaxies suggests a universally similar star formation history. If the results *had* been a single narrow range of times for the galactic wind, this would have supported that observation; on the other hand, the theory of "downsizing" would support a range of timing of the wind, correlated to galaxy size (which is noted within these results, albeit in a bimodal way).

Nothing has been observed to have occurred at either 13.05-12.32 or 9.5-9.7 Gyrs ago (i.e. 0.65-0.75 or 4.0-4.2 Gyrs after galaxy formation) which could have given rise to these peaks. It is of course possible that future instrumentation and telescopes will provide observations over a wider range of wavelengths and redshifts and will find evidence for an astrophysical Event at that point which could have triggered the galactic winds.

Note that the model finds this surprising result for galaxies even within a single cluster; if it was an astrophysical event, it would be expected to perhaps apply to all galaxies within that cluster.

As the bimodality appears to be correlated with the observations, it is suggested that this bimodality is simply a consequence of the ill-conditioning arising within the Lick indices. The results from D05 exhibit the same bimodality, finding the best fit models at the same ranges 0.65-0.75 Gyrs or 4.0-4.2 Gyrs as the results from PS02 and SB07, although not necessarily finding the same results.

6.5.4 Alpha enhancement

It was noted in Section 2.2.4 that other authors had found α-enhanced element abundances when modelling some individual galaxies, but when modelling overall parameters of large data sets of galaxies, elements were not α-enhanced.

The Phoenix model tracks 14 elements insofar as it is able to (limited by the lack of data in the literature for elements other than carbon and oxygen for massive star yields), and uses this to give a value for [α/Fe]. This is then used to select the appropriate SSP data, interpolating in [α/Fe] if necessary. Due to the limited data, the values calculated by the model for [α/Fe] are actually very slightly Fe-enhanced (as would be expected, given the limited α-element data compared to the complete Fe-data from SNIa). This means the model will generally use solar-scaled element abundances, being the data nearest in value to the calculated [α/Fe] value. However, the model does allow for α-enhanced SSP data to be incorporated within the final results, should the model generate α-enhanced abundances.

The isochrones are not α- enhanced, but are only used to calculate the luminosity, and the under/overstatement of the luminosity due to using solar-abundance isochrones is therefore not considered to have a significant net effect on the final galaxy parameters.

The best-fit T04 models, on the other hand, are all α-enhanced; which indicates a previous population and thus they cannot be defined as SSPs, as discussed in Section 6.2.5.

Should more complete data for massive star element yields become available in the literature in the future, the Phoenix model can be easily updated and then fully tested for the effects of α- enhancement.

6.5.5 Conclusions

The two data sets (from PS02 and SB07) can be more precisely modelled using the SSPs from T04 than with the Phoenix model. However:

- Successful SSP models could not exist without at least one previous stellar generation, in order to appropriately enrich the gas and consequently the stars formed from it, and as such are not valid as single populations; and

- The majority of the galaxies in the two samples have observational characteristics that indicate they are not single populations.

The Phoenix model is able to produce star formation histories which are consistent with the literature: intensive star formation for a short period of time followed by passive evolution. The modelled galaxies have final SNIa rates and colours within expected ranges. Gas inflow is not required by the models but the proportion of stars initially forming as Population III appears to be a significant parameter.

It is important to note that the smaller uncertainties on the SB07 data make it harder to simultaneously model the entire set of Lick indices within reasonable multiples of these uncertainties – "good" models of this data set are not statistically good models, as they are >> 3 β_{ave} from the observed data points.

The period of time before the galactic wind required by the model is markedly different for PS02 and SB07, suggesting a systematic error in one or other (or both) sets of observational data, although this is not apparent from the plots of individual indices, and both data sets cover the same galaxy clusters so this is not a function of galaxy location. It may therefore be that there is/are additional parameter(s) which are not included in the Phoenix model but which are important within galaxy evolution. Results from D05 support the theory of a systematic error, as these find earlier times of the galactic wind for four of the PS02 galaxies, and later winds for two of the SB07 galaxies.

This emphasises the importance of using more than one data set, taken from different observational facilities (to remove any instrument bias), before forming any conclusions from models of star formation histories of nearby elliptical galaxies.

CHAPTER 7: CONCLUSIONS AND FURTHER WORK

7.1 MAIN CONCLUSIONS

7.1.1 Main contribution to knowledge from this thesis

The main contribution to knowledge from this thesis is that Lick indices, which are subject to apparently minor variations when observed by different groups using different telescope facilities with different spectrographs and different data reduction techniques, may result in mathematically ill-conditioned results when used in population synthesis modelling.

7.1.2 Implications for the "Population Synthesis" community

Many papers have been published since 1994 using Lick indices as the observational data source against which models are compared. This work shows that, unless the work is verified using observational data of the same object(s) from a separate source, the results may not be reliable.

Examples of works that could be reinvestigated include:

- Confirming whether the Lick indices identified as age/metallicity sensitive actually are, and that it is not a consequence of instrument bias (Worthey 1994);
- Confirming the correlations between thin and thick discs with age, with thinner discs consisting of younger populations (calculated from Lick index observations) actually a function of the instrumentation and not the galaxy? (Yoachim and Dalcanton 2008);
- Checking whether the differences between the SSPs of W94, V99 and T04 might be a consequence of the different observational data in the stellar libraries used;
- Confirming the conclusions of Johnston et al (2012), who found from Lick index analysis of nine galaxies in the Fornax Cluster that bulges in lenticular galaxies appear to have higher metallicities and younger stellar populations than the corresponding discs, thus suggesting that star formation in the disc ceases at the same time as a final burst of star formation takes place in the bulge.

7.2 MODELLING STAR FORMATION HISTORIES OF NEARBY ELLIPTICAL GALAXIES

7.2.1 Summary of this thesis

This thesis represents a contribution to the ongoing work of establishing the formation mechanisms and evolutionary history of elliptical galaxies. It demonstrated why an accepted model from the literature was unable to recreate the indices of individual galaxies, and presented a new model, Phoenix, to propose star formation histories for 21 nearby elliptical galaxies from two data sets. Star formation histories for these 21 galaxies do not currently exist in the literature.

New work contained in this thesis can be briefly summarised as follows, with more detail provided in the remainder of this section:

- enhanced an existing model from the literature (the GCE model);
- audited the GCE model to find out why it didn't work;
- built a new model (Phoenix);
- tested the new model, including comparison to other models in the literature;
- used the new model, and an SSP model from the literature, to find possible SFH of galaxies from 2 data sets;
- found that the results suggested observational bias;
- used a third set of data to verify some of the SFHs found;
- found that minor changes in observational data could result in very different SFHs; and
- found that the results from the Phoenix model supported downsizing and constrained the epoch of initial galaxy formation.

7.2.2 Contribution to knowledge from work on the GCE model

The main reason that the GCE model was unable to suggest appropriate star formation histories of nearby galaxies was found to be due to a coding error whereby one variable, ROO, was used for more than one physical value (mass of stars, mass of gas and density of gas). It was therefore incorrectly updated as the model was run, resulting in excessive star formation rates and hence metallicity within the model galaxy becoming unrealistically high.

This went unnoticed mainly because the model did not give the user a warning when it was obliged to use the nearest yield/ejecta or SSP value from tables taken from the literature: the model reported solar values even when the metallicity was extremely super-solar, thus producing a reasonable output from an unreasonable model.

Limitations due to the method used to luminosity weight the indices, the range of data provided in the literature for yields and ejecta, and errors in the statistical method used to evaluate the galaxy meant that the star formation history given as output was not that actually developed by the model.

The GCE model includes several adjustments to the synthetic magnesium indices which were being modelled against observational data taken from the WHT. Instead of modifying the synthetic indices, the uncertainties on the *observational* data should have been reviewed, as they did not include instrumentation uncertainties on these specific indices which had been observed at the WHT.

The GCE model allows searching through 4 of the 12 model parameters. It was found that

1. TIME – the life of the galaxy in Gyrs - was a more important parameter to search on than the four used in the searching software; and
2. the upper limit for gas inflow duration in the "stepping software" was set to a value lower than required to successfully model an observed galaxy.

Updating the literature sources for planetary nebulae and SSPs did not significantly change the code outputs.

7.2.3 Contribution to knowledge from the Phoenix model

The Phoenix model is new, independent evolutionary population synthesis model based on the 'bottom up' approach, i.e. it evolves a galaxy based on stellar lifetimes and mass and calculates synthetic luminosity-weighted Lick indices, which can be compared to observational data. The model was tested against other models from the literature to give reassurance that that the outputs were reasonable.

The model demonstrated:

- galactic winds modelled as occurring at a specific time (to model AGN) provide better results than galactic winds modelled with gas loading (to model SN), suggesting galactic winds are a result of AGN rather than SN;
- there is little difference between the planetary nebula yield models of RV81, G05 and vdH&G97 when incorporated into the code;
- the galactic radius is an important model parameter;
- the percentage of Population III stars from the initial gas cloud is an important parameter;
- gas inflow is not required to successfully model the galaxies, indicating mergers are dry;
- the theory of "downsizing" is supported by the results, with more massive galaxies having an earlier galactic wind;
- final models are supported by expected colours, SNIa rates and stellar composition;
- elliptical galaxies were formed $13.26^{+0.09}_{-0.06}$ Gyrs ago.

7.2.4 Contribution to knowledge: proposed star formation histories for some nearby elliptical galaxies

The star formation histories of nearby elliptical galaxies as found by the Phoenix model form two distinct groups, distinguished by the timing of the galactic wind:

Parameter	**Model 1**	**Model 2**
Galaxy age (Gyrs)	$13.24^{+0.02}_{-0.04}$	$13.25^{+0.04}_{-0.05}$
Timing of galactic wind (Gyrs after galaxy formation)	$0.74^{+0.02}_{-0.09}$	$4.04^{+0.15}_{-0.05}$
Galaxy mass (x 10^{10} $M_{\odot}$)	$6.03^{+0.67}_{-0.53}$	$4.51^{+1.49}_{-0.81}$
Constant in the Schmidt (1959) star formation equation	$0.58^{+0.09}_{-0.13}$	$0.50^{+0.15}_{-0.05}$
Percentage of primordial gas that forms Population III stars	45^{+8}_{-4}	47^{+7}_{-10}
Gas inflow parameters	Not required	Not required

Table 41: comparison of the two groups of models found with the Phoenix model and the data sets of PS02 and SB07

There is no correlation between these two model groups and the galaxy location; indeed, the results suggest that there could be an undetected systematic error in one or other of the two data sets, as there is noted correlation between model 1 and the SB07 data and model 2 and the PS02 data. This is not obvious from the plots of the indices (figure 20).

Model accuracy is tested by comparing the difference between the synthetic model index and the corresponding observed index, measured in units of the standard deviation on the observational data β. Data from the SB07 data set has considerably smaller uncertainties and consequently is shown as less successfully modelled using this measure.

A lower star formation constant and lower galaxy mass was found to correlate to a later timing for the galactic wind, and a higher star formation constant and higher mass correlates to an earlier timing for the galactic wind. This corresponds to the theory of "downsizing" where stars in more massive galaxies tend to have formed earlier and over a shorter timeframe (i.e. have older average ages) than those in smaller galaxies.

Star formation histories for nearby elliptical galaxies, which have not previously been proposed within the literature, are given in section 6.2 (when modelled with an SSP) and section 6.3 when modelled with Phoenix.

7.2.5 Contributions to knowledge: the importance of a second data set

Some of the star formation histories proposed by the Phoenix model were able to be tested, because the 10 of the galaxies from the PS02 and SP05 data sets were also in a third data set, which had been taken from a separate telescope and spectrograph.

Of the ten galaxies, four produced different star formation histories when the D05 data was used as an alternative. Different star formation histories were also found when the PS02 data was run without the Mg indices, and when the D05 data was restricted to only model the indices that were also in the PS02/SB07 data sets. It is therefore considered essential that at least two sets of observations be taken before drawing any conclusions regarding the star formation history of nearby elliptical galaxies using observed Lick index data.

7.3 FURTHER WORK

7.3.1 Introduction

There are a number of directions for future work. These include updates to the source data used by Phoenix, enhancements to the code to expand its capabilities, and additional observational data against which to compare the model. In an ideal world, there would be additions within the literature in a number of areas, the results from which could be incorporated into this model.

Interesting further work could be undertaken to assess the extent of the ill-conditioning found when using this methodology.

7.3.2 Model development and enhancement

When writing a computer model, there is always a balance between what the code must be able to do as a minimum to achieve the objectives set, and enhancements it would be interesting to add. For the Phoenix model, future code enhancements using data sources currently available in the literature could include:

- Adding the bi-modal equation for SNIa rates given by Matteucci et al. (2006) as an alternative to Timmes et al. (1995) and Scannapieco and Bildsten (2005);

- Increase the number of starbursts modelled (by way of gas inflow) – this would open the door to modelling spiral galaxies, which would also need to include consideration of the effect of dust on the synthetic indices, how best to model inflow of other stellar populations, and would require alternative IMFs such as those of Scalo (1986) and Kroupa (2001) to be tested as alternatives to Salpeter (1955);

- Enhance the model so that it was able to simultaneously model AGN and SN feedback, rather than one or the other, so that relative contributions of these two processes could be compared.

- The isochrone data, which gives the luminosity of the stars at any given point, is currently taken from one source (Padova isochrones of Bertelli et al. 1994). It would be interesting to add an alternative, such as the Yonsei-Yale isochrones (Yi et al. 2001) to compare the results;

- The new MILES library of SSPs (Vazdekis et al. 2010) could be added as an alternative to W94, V99 and T04, although this would require transforming the current observational data sets of Lick indices given in PS02 and SB07 to ensure they are aligned, as the Vazdekis et al. (2010) SSPs are presented using an updated line index system to that given in W94.

7.3.3 Updates to source data from the literature

Computer models such as Phoenix can continuously evolve: as new results become available in the literature, additional tests can be added to the code and its outputs, and new models of processes such as supernova and planetary nebula can provide code updates and user-selectable options. As discussed above, limitations to the code due to limitations in yield/ejecta data is one of the main sources of frustration for current galactic chemical evolution modelling.

SNIa data currently used by Phoenix is from Nomoto et al. (1984) is very out of date. As noted with the tests of planetary nebula yields, this doesn't mean it is wrong – good results were obtained using RV81 results compared to the more recent results from G05 and vdH&G97 – but it would be nice to have a second set of results to compare the Nomoto et al. (1984) results to.

The current source of massive star yield data from the Geneva group unfortunately does not provide detailed chemistry – carbon and oxygen only – although it is understood that whilst their models do include a wider set of elements this data is not yet planned for publication (private communication Hirschi August 2010). Addition of the other elements would improve the data held for initial chemical composition of the next generation of stars, and enable the results from T95 and K05 to adjust for non-solar abundances be investigated as code enhancements.

The equation for main sequence lifetime is taken from Wood (1992) (equation 8) does not take into account metallicity of the stars; because lower metallicity stars have lower stellar winds, it would be expected that they spend longer on the main sequence than stars of comparable mass but higher metallicity. Stellar winds are not a significant in this mass range, and as such the lifetimes may not be significantly different, but with the current equation this physical difference cannot be included in the code.

7.3.4 Additional observational data

A more extensive review of the literature may reveal more data sets where the individual galaxies have been observed using different observational facilities and processed using different techniques. It would be useful to have several sets of observations on a reasonable set of, say, 50 galaxies which includes galaxies in different environments. Having four or five observations on each individual galaxy may be sufficient evidence to establish the cause that leads to this problem being ill-conditioned. This may also provide sufficient evidence to establish with more certainty the star formation histories of these galaxies.

As one of the conclusions from this work is that there may be a systematic difference between the two data sets arising from observational bias, then sourcing Lick indices for all 21 galaxies from the PS02 and SB07 data sets from another telescope may resolve whether this is the case (the D05 results, which were already in the literature, only overlapped with 10 of these galaxies). Alternatively, the PS02 galaxies could be observed using the Keck telescope and the SB07 galaxies observed using the WHT and the results compared.

When instrumentation improves to the point of being able to obtain detailed element abundances rather than relying on Lick indices, this would give an alterative measure of the reliability of the model, provided that more complete predictions of massive star element yields were available in the literature, as the model would also need to be updated.

Further support for the results found in this thesis would come from data, were they to be available, on the actual masses, luminosities, radii, stellar composition and other physical properties of the galaxies in the sample, which could be compared to the final galaxies modelled.

7.3.5 Assessment of ill-conditioning

Further work could be undertaken to systematically establish whether it is specific indices within the Lick data set which give rise to the ill-conditioning. It was noted that when the Mg indices were removed from the PS02 data set, some (but not all) SFH were altered. It would be interesting to run these tests for selectively removing indices that are within, and outside, the uncertainties of the comparison data set (figure 20).

The tests that established the ill-conditioning were found using the Phoenix model run with T04 SSPs. Whilst earlier work showed that there was minimal difference between the SSP data sets when used in the Phoenix model, it would be a useful test to find out whether the same areas of ill-conditioning apply when the SSPs of W94 or V99 are substituted into the Phoenix code.

Vazdekis et al. (2010) have re-observed the Lick index stars and provided a new calibration for these to their set of galaxies, as well as a mechanism for converting existing data to this new paradigm. It would be interesting to find out if this removes the ill-conditioning, although the process for converting the data from the old to the new may itself affect the ill-conditioning.

If further data were to be available, with the PS02 data set obtained and processed in the same way as the SB07 data was, and vice-versa, it may be possible to establish whether the main source of the ill-conditioning lies with the observer, the telescope, the spectrograph and/or the data reduction techniques.

Whilst there are many other potential avenues for future research, the directions outlined above would answer many of the questions raised by the work in this thesis.

LIST OF REFERENCES

Key to journal abbreviations:

A&A	Astronomy and Astrophysics
A&AS	Astronomy and Astrophysics Supplement
AAS	American Astronomical Society
AIPC	American Institute of Physics conference proceedings
AJ	Astronomical Journal
ApJ	Astrophysical Journal
ApJS	Astrophysical Journal Supplement
ASPC	Astronomical Society of the Pacific Conference Series
CompStat	Computational Statistics
CurrSci	Current Science
GeCoA	Geochimica et Cosmochimica Acta
FChPh	Fundamentals of Cosmic Physics
IAUC	International Astronomical Union Circular
IAUS	International Astronomical Union Symposium
JCAP	Journal of Cosmology and Astroparticle Physics
JCoPh	Journal of Computational Physics
MNRAS	Monthly Notices of the Royal Astronomical Society
Natur	Nature
NewA	New Astronomy
NuPhA	Nuclear Physics A
PASP	Publications of the Astronomical Society of the Pacific
PThPh	Progress of Theoretical Physics
RAA	Research in Astronomy and Astrophysics
RPPh	Reports on Progress in Physics
SSRv	Space Science Reviews

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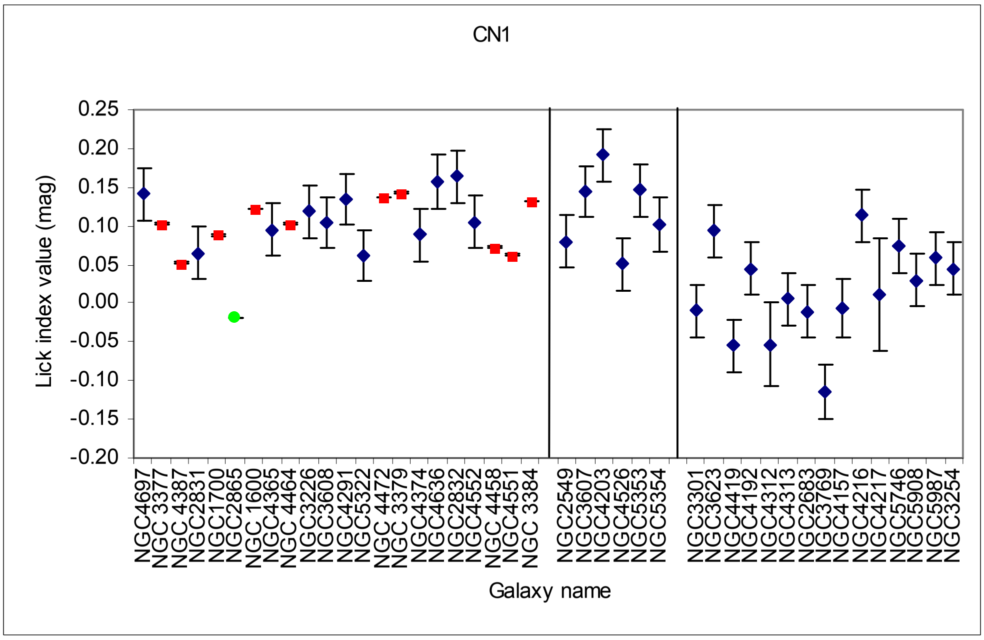
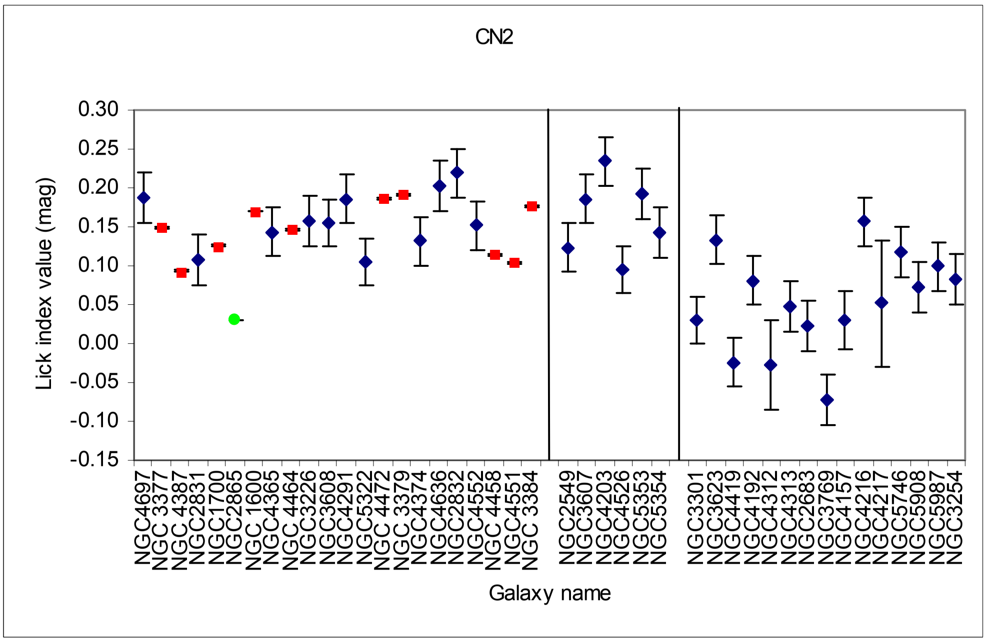
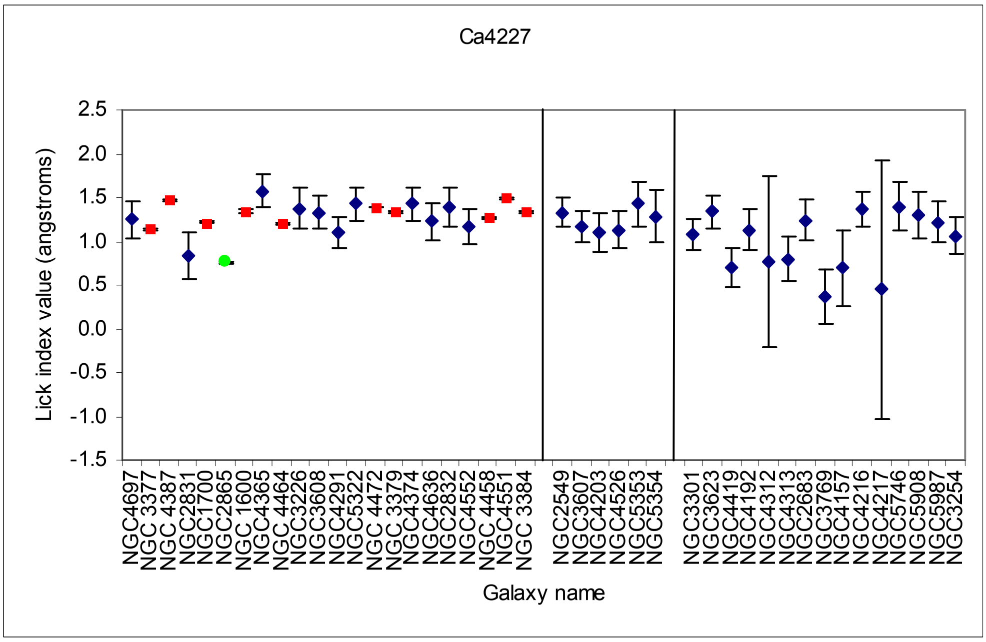
APPENDIX A: Lick index by morphology

Lick index data from the PS02 (blue diamonds) and SB07 (red squares) data sets plotted (from left to right) in order of increasing T-type (de Vaucouleurs et al. 1991), for a sample of three Lick indices. The outlier galaxy NGC 2865 from SB07 is marked with a green star. The three sections delineated with vertical lines are (from left to right) ellipticals, lenticulars, spirals. This is discussed in more detail in Chapter 6.






Ca4227
Lick index value (angstroms)
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
NGC4697
NGC 3377
NGC 4387
NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
NGC3608
NGC4291
NGC5322
NGC 4472
NGC 3379
NGC4374
NGC4636
NGC2832
NGC4552
NGC 4458
NGC4551
NGC 3384
NGC2549
NGC3607
NGC4203
NGC4526
NGC5353
NGC5354
NGC3301
NGC3623
NGC4419
NGC4192
NGC4312
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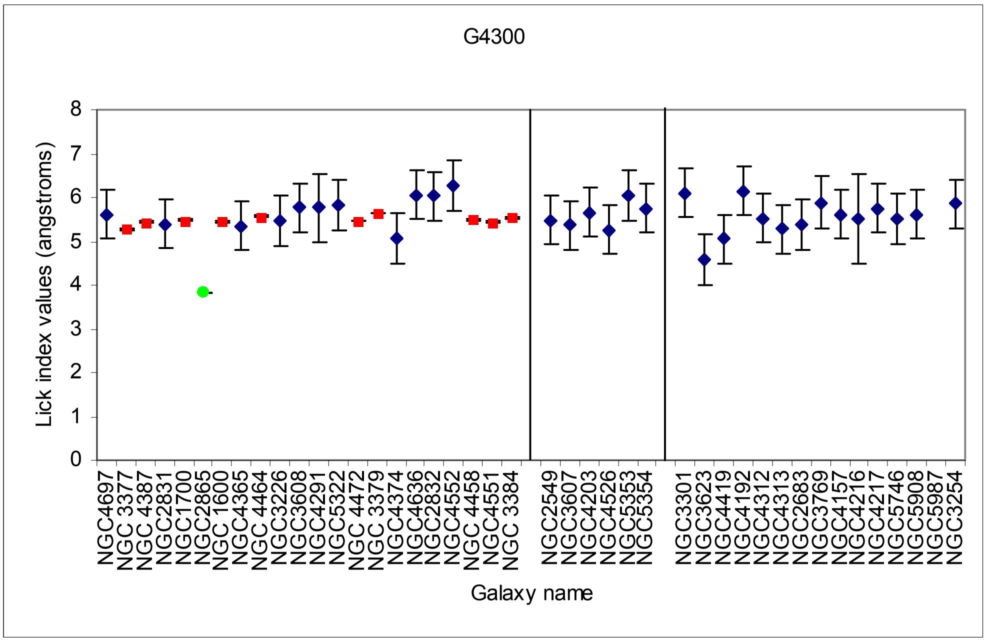
NGC3769
NGC4157
NGC4216
NGC4217
NGC5746
NGC5908
NGC5987
NGC3254
Galaxy name


G4300
Lick index values (angstroms)
8
7
6
5
4
3
2
1
0
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NGC 3377
NGC 4387
NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
NGC3608
NGC4291
NGC5322
NGC 4472
NGC 3379
NGC4374
NGC4636
NGC2832
NGC4552
NGC 4458
NGC4551
NGC 3384
NGC2549
NGC3607
NGC4203
NGC4526
NGC5353
NGC5354
NGC3301
NGC3623
NGC4419
NGC4192
NGC4312
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NGC2683
NGC3769
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NGC4216
NGC4217
NGC5746
NGC5908
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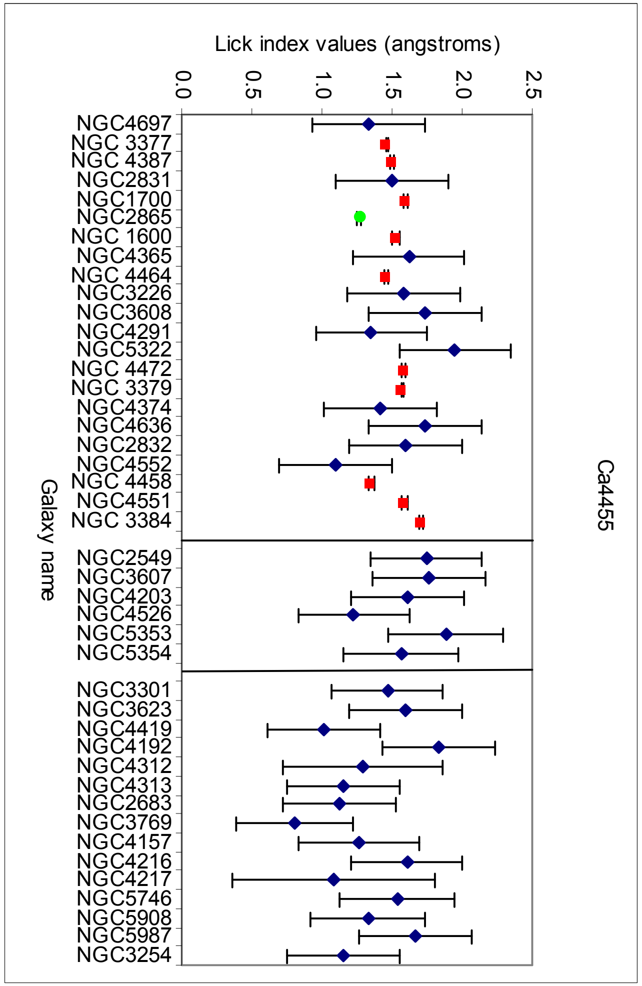
Galaxy name


Ca4455
Lick index values (angstroms)
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0.5
1.0
1.5
2.0
2.5
Galaxy name
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NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
NGC3608
NGC4291
NGC5322
NGC 4472
NGC 3379
NGC4374
NGC4636
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NGC 4458
NGC4551
NGC 3384
NGC2549
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NGC4203
NGC4526
NGC5353
NGC5354
NGC3301
NGC3623
NGC4419
NGC4192
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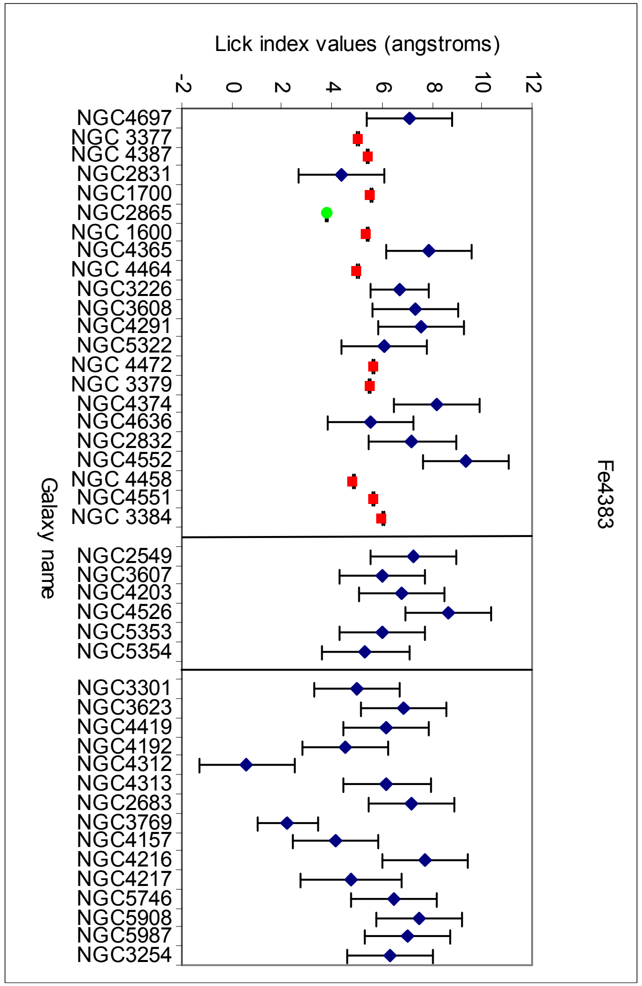
NGC4313
NGC2683
NGC3769
NGC4157
NGC4216
NGC4217
NGC5746
NGC5908
NGC5987
NGC3254


Fe4383
Lick index values (angstroms)
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0
2
4
6
8
10
12
Galaxy name
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NGC 3377
NGC 4387
NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
NGC3608
NGC4291
NGC5322
NGC 4472
NGC 3379
NGC4374
NGC4636
NGC2832
NGC4552
NGC 4458
NGC4551
NGC 3384
NGC2549
NGC3607
NGC4203
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NGC5353
NGC5354
NGC3301
NGC3623
NGC4419
NGC4192
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NGC4217
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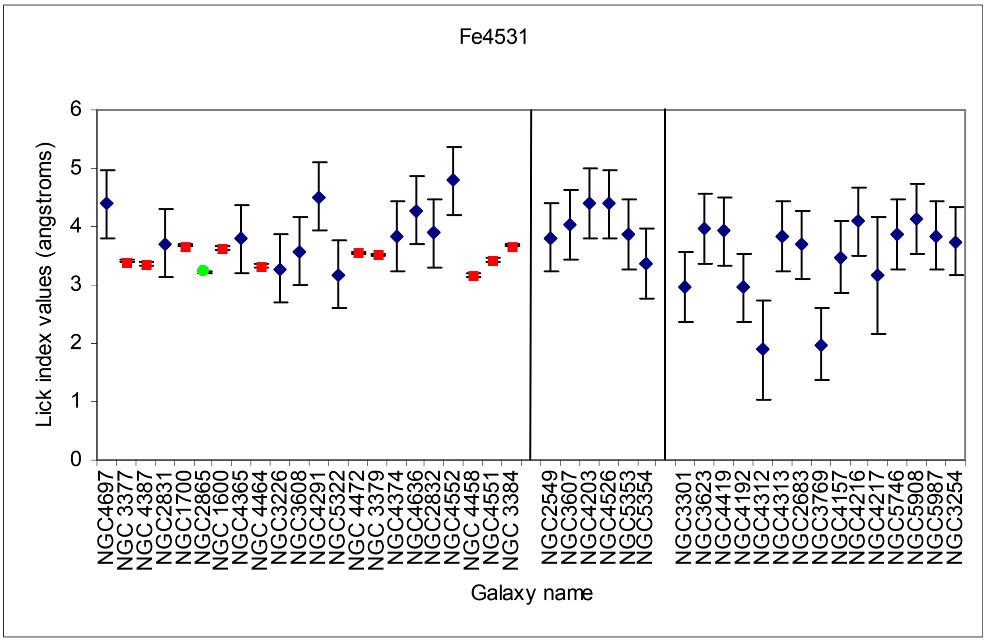
Fe4531
Lick index values (angstroms)
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2
3
4
5
6
NGC4697
NGC 3377
NGC 4387
NGC2831
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NGC 1600
NGC4365
NGC 4464
NGC3226
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NGC4419
NGC4192
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NGC5746
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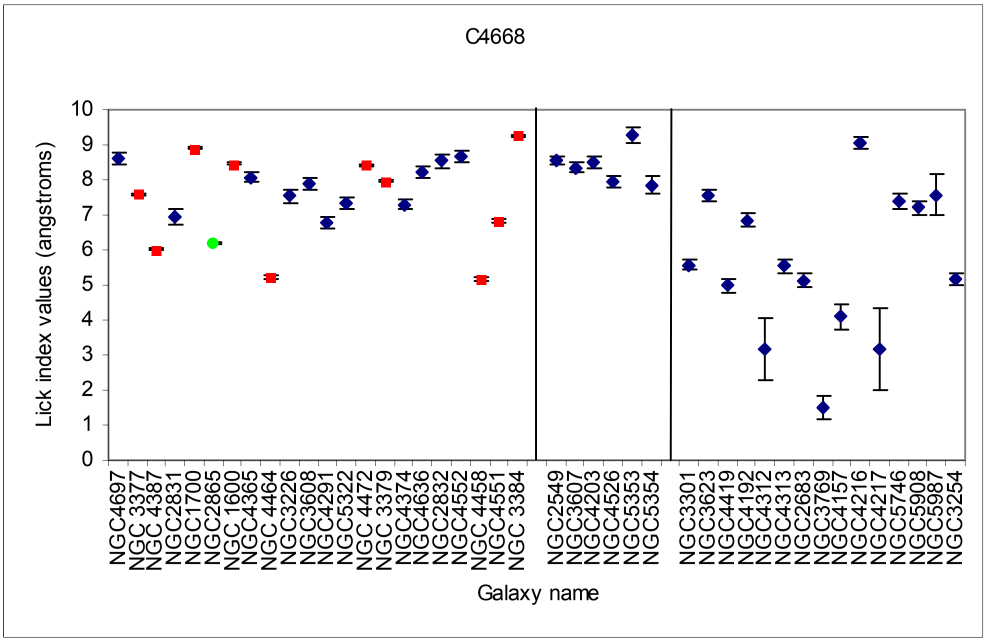
NGC5987
NGC3254
Galaxy name


C4668
Lick index values (angstroms)
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1
2
3
4
5
6
7
8
9
10
NGC4697
NGC 3377
NGC 4387
NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
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NGC5322
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NGC4192
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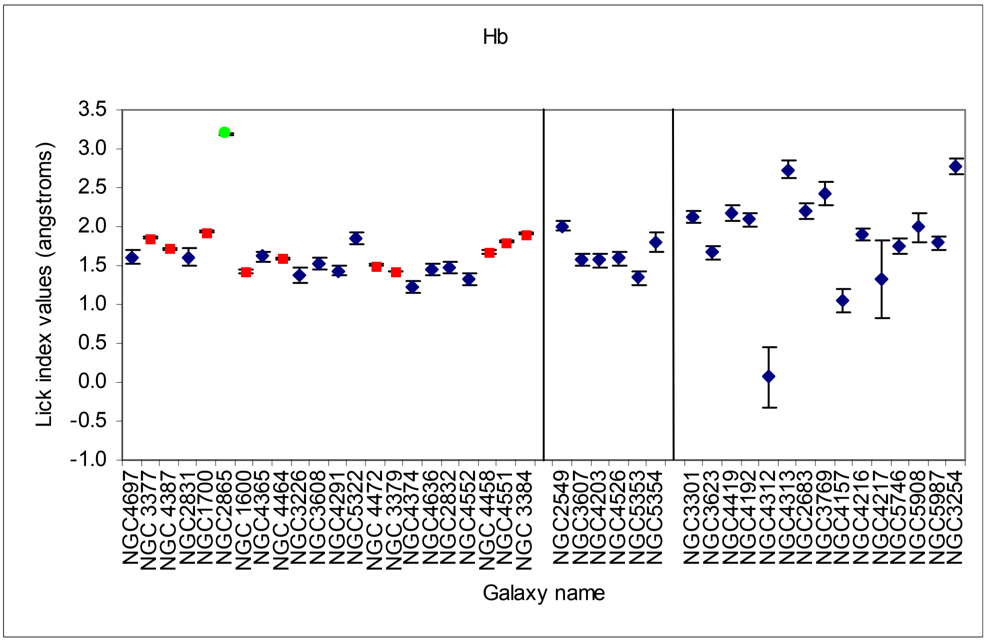
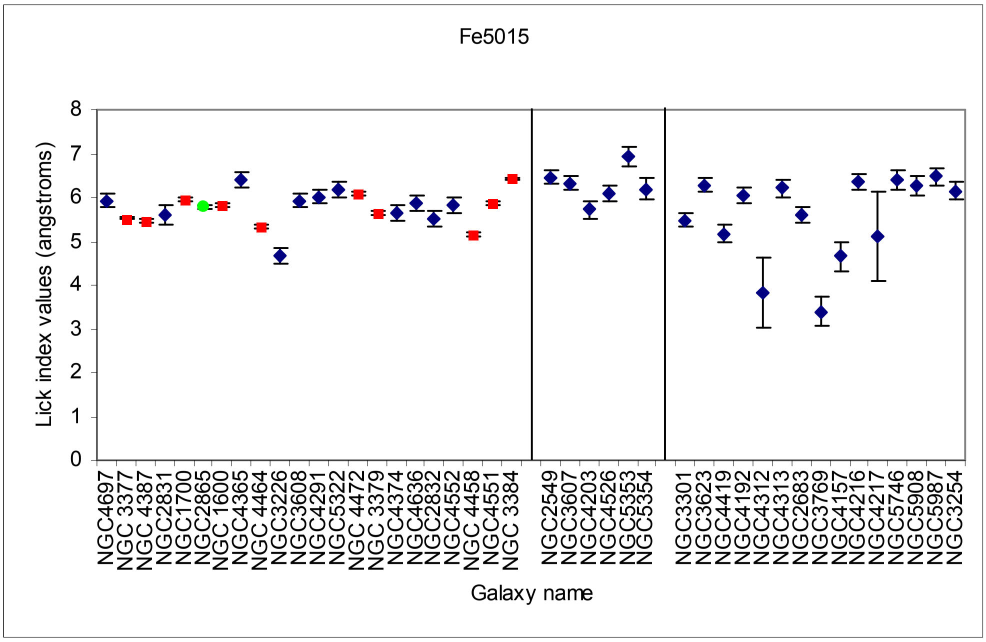
NGC5987
NGC3254
Galaxy name


Hb
Lick index values (angstroms)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
NGC4697
NGC 3377
NGC 4387
NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
NGC3608
NGC4291
NGC5322
NGC 4472
NGC 3379
NGC4374
NGC4636
NGC2832
NGC4552
NGC 4458
NGC4551
NGC 3384
NGC2549
NGC3607
NGC4203
NGC4526
NGC5353
NGC5354
NGC3301
NGC3623
NGC4419
NGC4192
NGC4312
NGC4313
NGC2683
NGC3769
NGC4157
NGC4216
NGC4217
NGC5746
NGC5908
NGC5987
NGC3254
Galaxy name


Fe5015
Lick index values (angstroms)
8
7
6
5
4
3
2
1
0
NGC4697
NGC 3377
NGC 4387
NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
NGC3608
NGC4291
NGC5322
NGC 4472
NGC 3379
NGC4374
NGC4636
NGC2832
NGC4552
NGC 4458
NGC4551
NGC 3384
NGC2549
NGC3607
NGC4203
NGC4526
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NGC3301
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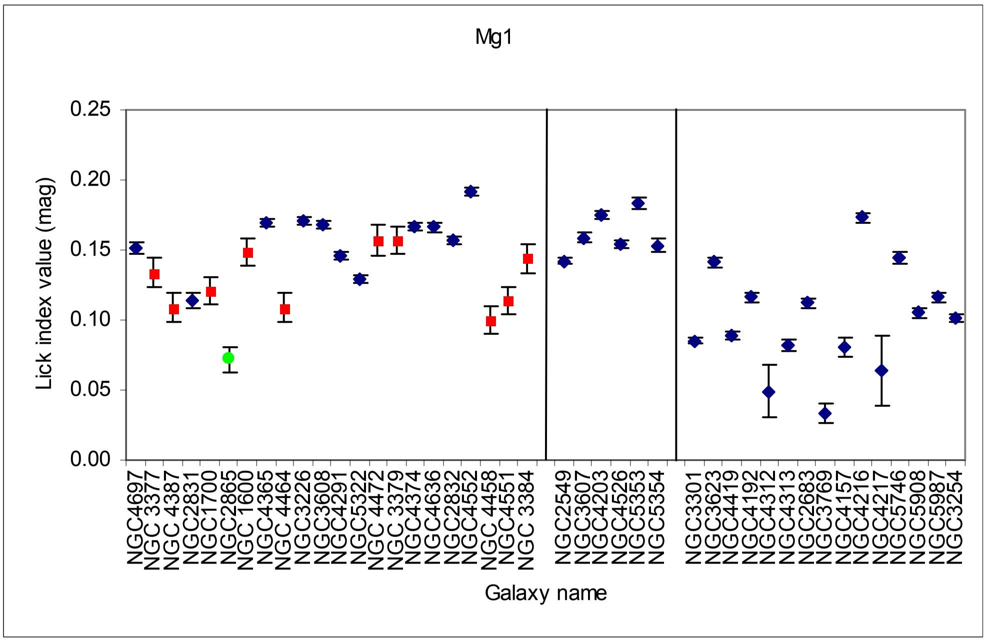
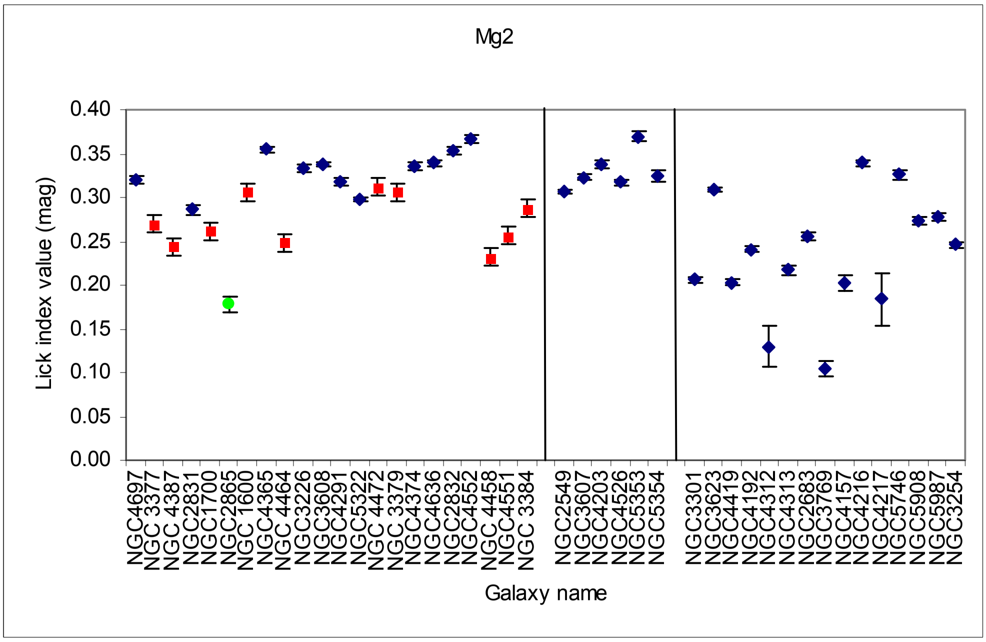
NGC4419
NGC4192
NGC4312
NGC4313
NGC2683
NGC3769
NGC4157
NGC4216
NGC4217
NGC5746
NGC5908
NGC5987
NGC3254
Galaxy name


Mg1
Lick index value (mag)
0.25
0.20
0.15
0.10
0.05
0.00
NGC4697
NGC 3377
NGC 4387
NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
NGC3608
NGC4291
NGC5322
NGC 4472
NGC 3379
NGC4374
NGC4636
NGC2832
NGC4552
NGC 4458
NGC4551
NGC 3384
NGC2549
NGC3607
NGC4203
NGC4526
NGC5353
NGC5354
NGC3301
NGC3623
NGC4419
NGC4192
NGC4312
NGC4313
NGC2683
NGC3769
NGC4157
NGC4216
NGC4217
NGC5746
NGC5908
NGC5987
NGC3254
Galaxy name


Mg2
Lick index value (mag)
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
NGC4697
NGC 3377
NGC 4387
NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
NGC3608
NGC4291
NGC5322
NGC 4472
NGC 3379
NGC4374
NGC4636
NGC2832
NGC4552
NGC 4458
NGC4551
NGC 3384
NGC2549
NGC3607
NGC4203
NGC4526
NGC5353
NGC5354
NGC3301
NGC3623
NGC4419
NGC4192
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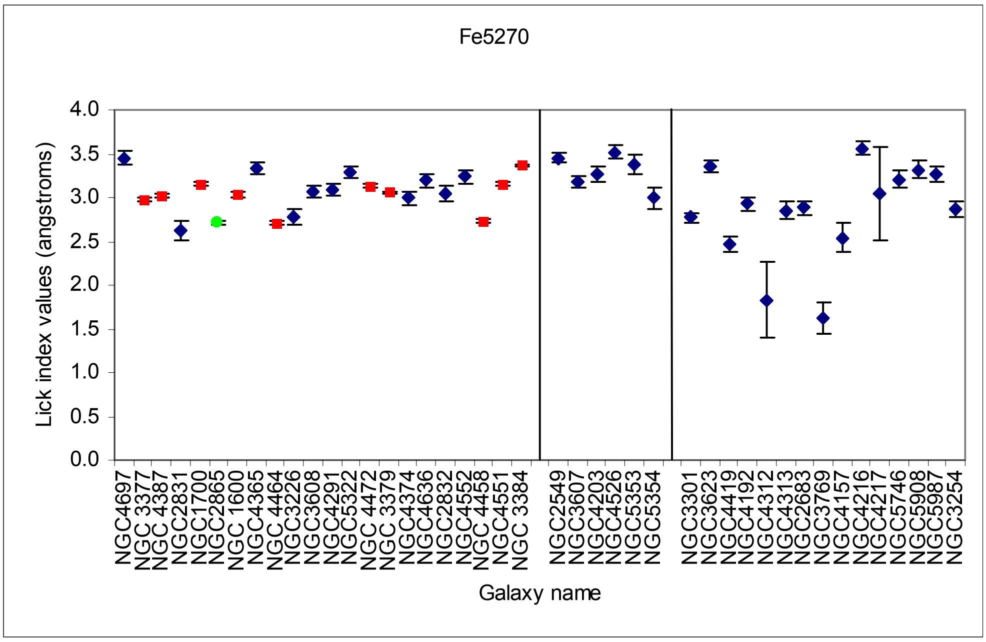
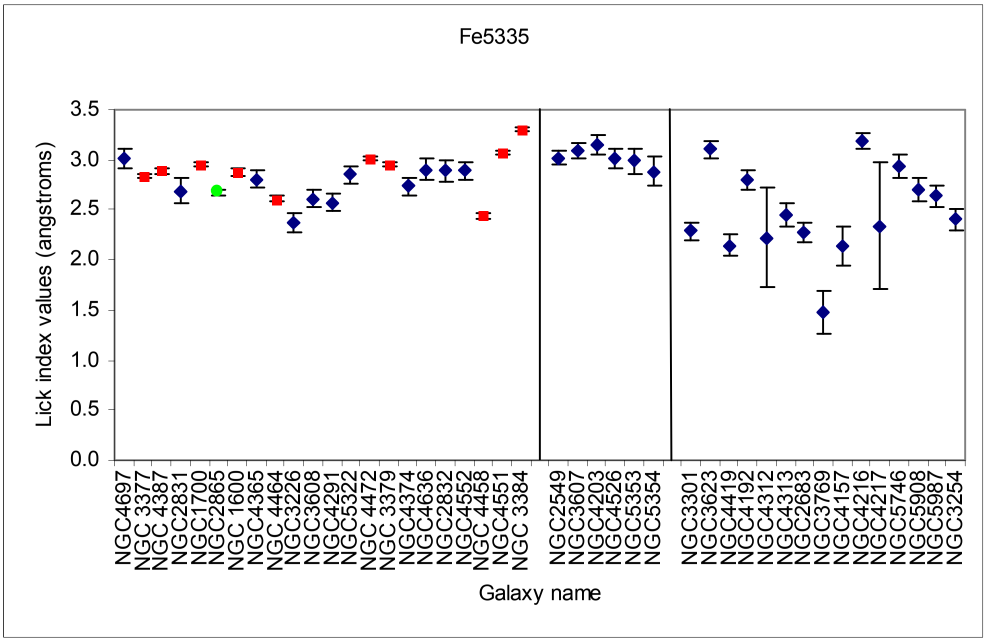
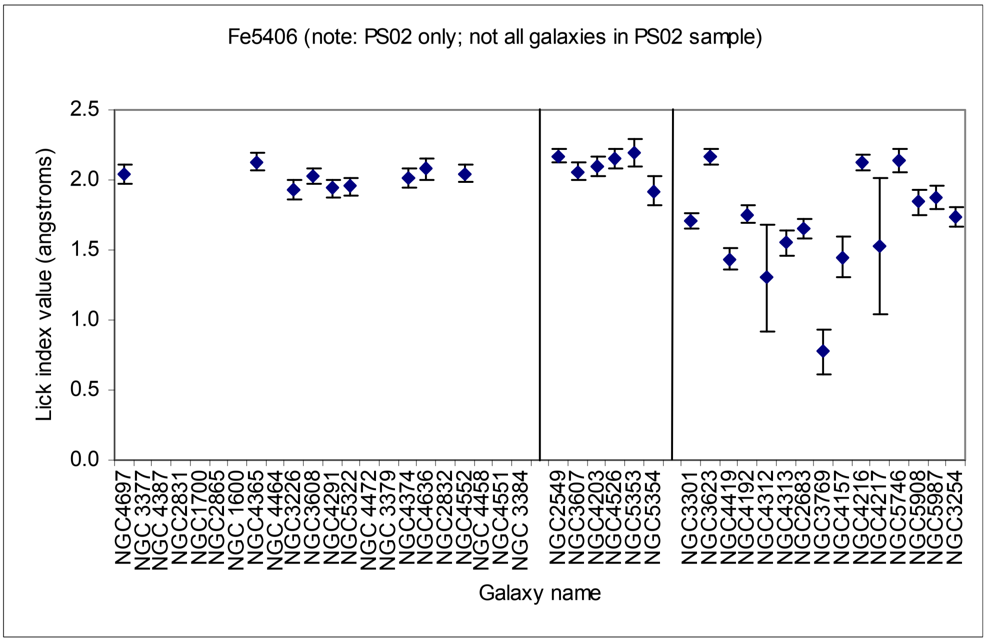
NGC4216
NGC4217
NGC5746
NGC5908
NGC5987
NGC3254
Galaxy name


Fe5270
Lick index values (angstroms)
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
NGC4697 NGC 3377 NGC 4387 NGC2831 NGC1700 NGC2865 NGC 1600 NGC4365 NGC 4464 NGC3226 NGC3608 NGC4291 NGC5322 NGC 4472 NGC 3379 NGC4374 NGC4636 NGC2832 NGC4552 NGC 4458 NGC4551 NGC 3384
NGC2549 NGC3607 NGC4203 NGC4526 NGC5353 NGC5354
NGC3301 NGC3623 NGC4419 NGC4192 NGC4312 NGC4313 NGC2683 NGC3769 NGC4157 NGC4216 NGC4217 NGC5746 NGC5908 NGC5987 NGC3254
Galaxy name


Fe5335
Lick index values (angstroms)
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5
NGC4697 NGC 3377 NGC 4387 NGC2831 NGC1700 NGC2865 NGC 1600 NGC4365 NGC 4464 NGC3226 NGC3608 NGC4291 NGC5322 NGC 4472 NGC 3379 NGC4374 NGC4636 NGC2832 NGC4552 NGC 4458 NGC4551 NGC 3384
NGC2549 NGC3607 NGC4203 NGC4526 NGC5353 NGC5354
NGC3301 NGC3623 NGC4419 NGC4192 NGC4312 NGC4313 NGC2683 NGC3769 NGC4157 NGC4216 NGC4217 NGC5746 NGC5908 NGC5987 NGC3254
Galaxy name


Fe5406 (note: PS02 only; not all galaxies in PS02 sample)
Lick index value (angstroms)
0.0
0.5
1.0
1.5
2.0
2.5
NGC4697
NGC 3377
NGC 4387
NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
NGC3608
NGC4291
NGC5322
NGC 4472
NGC 3379
NGC4374
NGC4636
NGC2832
NGC4552
NGC 4458
NGC4551
NGC 3384
NGC2549
NGC3607
NGC4203
NGC4526
NGC5353
NGC5354
NGC3301
NGC3623
NGC4419
NGC4192
NGC4312
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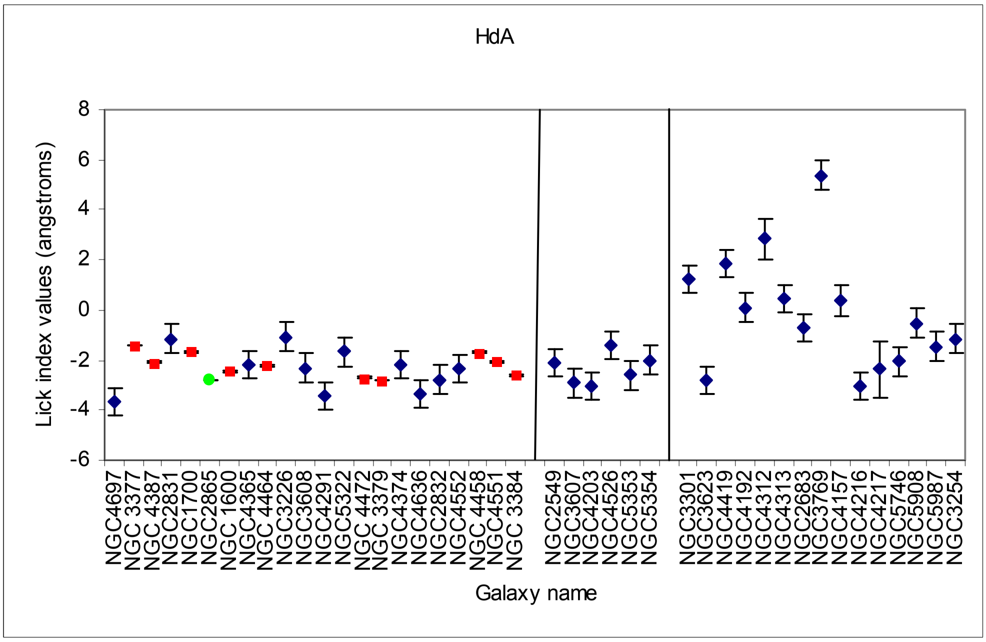
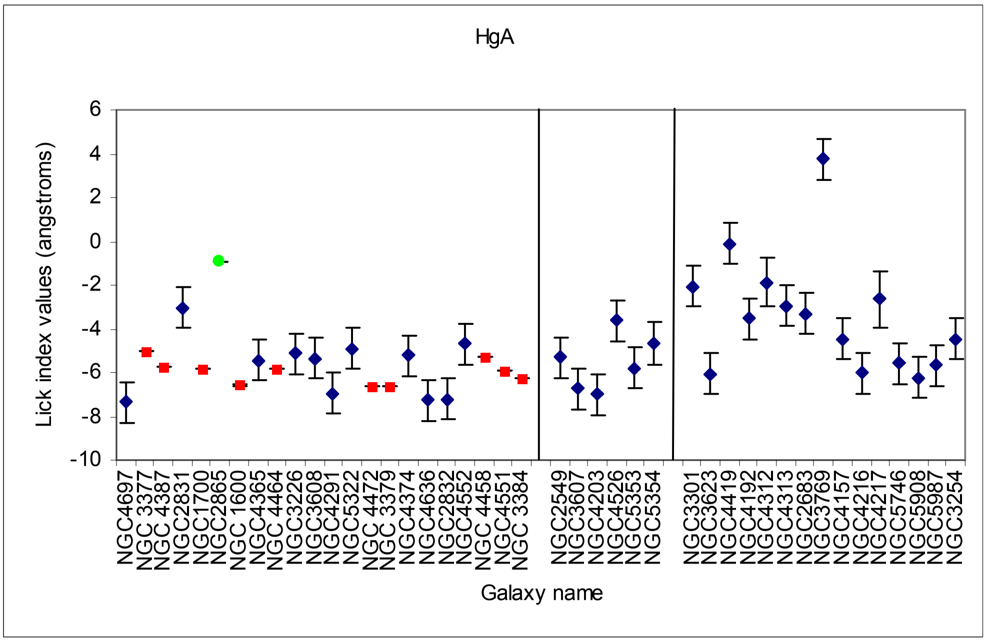
NGC2683
NGC3769
NGC4157
NGC4216
NGC4217
NGC5746
NGC5908
NGC5987
NGC3254
Galaxy name


HdA
Lick index values (angstroms)
-6
-4
-2
0
2
4
6
8
NGC4697
NGC 3377
NGC 4387
NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
NGC3608
NGC4291
NGC5322
NGC 4472
NGC 3379
NGC4374
NGC4636
NGC2832
NGC4552
NGC 4458
NGC4551
NGC 3384
NGC2549
NGC3607
NGC4203
NGC4526
NGC5353
NGC5354
NGC3301
NGC3623
NGC4419
NGC4192
NGC4312
NGC4313
NGC2683
NGC3769
NGC4157
NGC4216
NGC4217
NGC5746
NGC5908
NGC5987
NGC3254
Galaxy name


HgA
Lick index values (angstroms)
Galaxy name
NGC4697
NGC 3377
NGC 4387
NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
NGC3608
NGC4291
NGC5322
NGC 4472
NGC 3379
NGC4374
NGC4636
NGC2832
NGC4552
NGC 4458
NGC4551
NGC 3384
NGC2549
NGC3607
NGC4203
NGC4526
NGC5353
NGC5354
NGC3301
NGC3623
NGC4419
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NGC2683
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NGC4157
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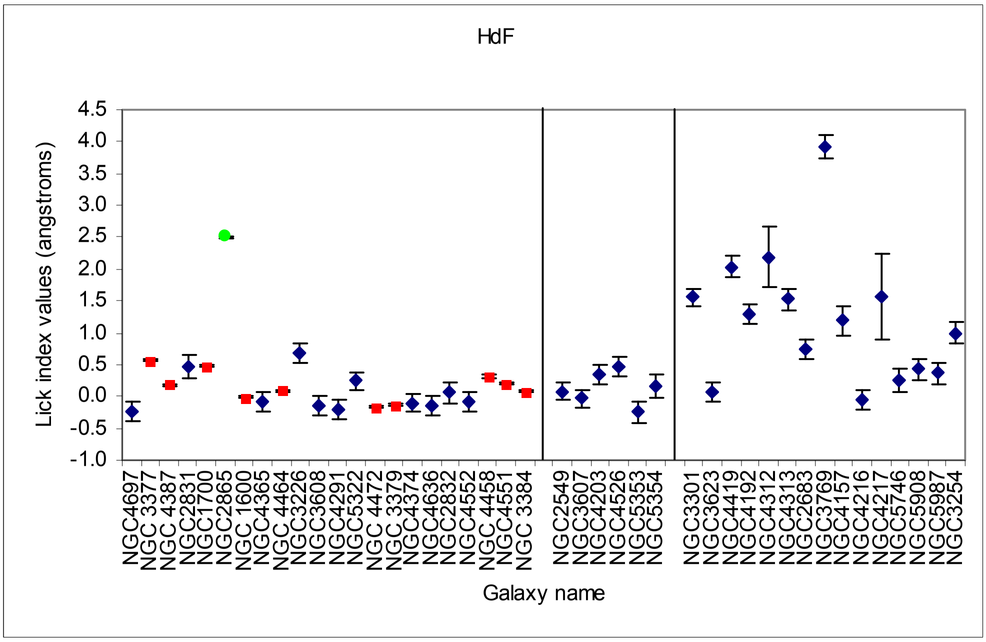
NGC5908
NGC5987
NGC3254


HdF
Lick index values (angstroms)
Galaxy name
NGC4697
NGC 3377
NGC 4387
NGC2831
NGC1700
NGC2865
NGC 1600
NGC4365
NGC 4464
NGC3226
NGC3608
NGC4291
NGC5322
NGC 4472
NGC 3379
NGC4374
NGC4636
NGC2832
NGC4552
NGC 4458
NGC4551
NGC 3384
NGC2549
NGC3607
NGC4203
NGC4526
NGC5353
NGC5354
NGC3301
NGC3623
NGC4419
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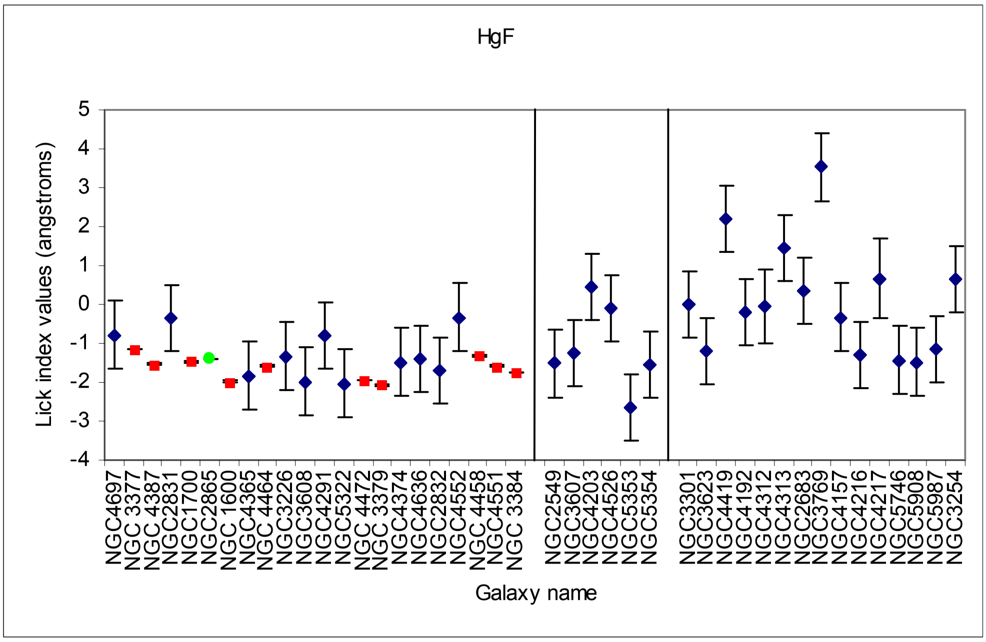
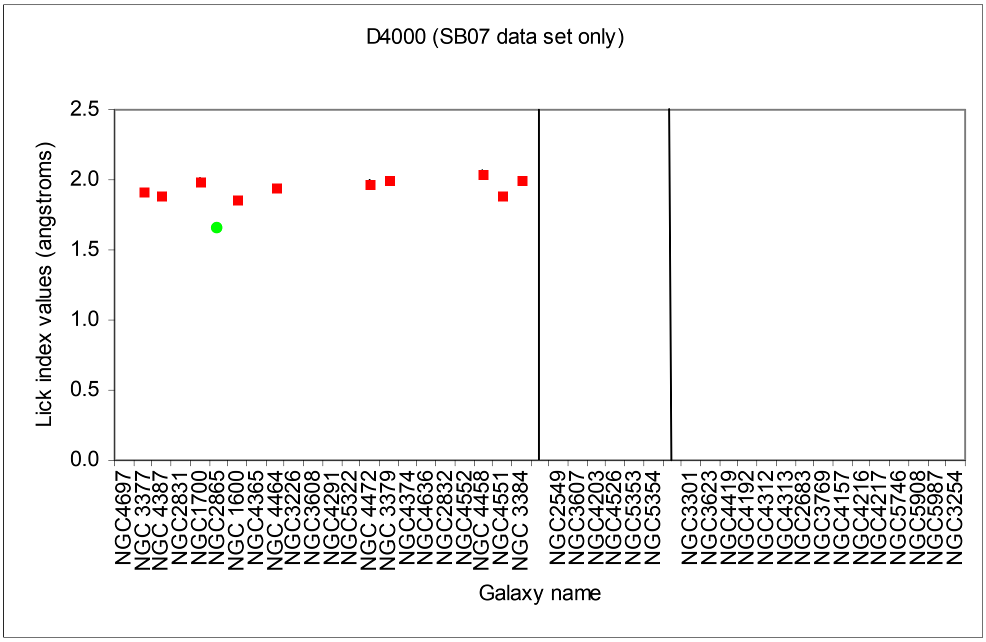
NGC5908
NGC5987
NGC3254


HgF
Lick index values (angstroms)
Galaxy name


D4000 (SB07 data set only)
Lick index values (angstroms)
Galaxy name

APPENDIX B: The Phoenix code

```
!Module of shared data for use by the programme Phoenix

    MODULE SHARED
    IMPLICIT NONE
    SAVE

! Set universal constants
    REAL, PARAMETER :: PI=3.141592654

! Set parameters that define the values of some counters
    INTEGER,PARAMETER :: NINDEX=55                          !Number of indices and colours modelled
    INTEGER,PARAMETER :: NET=14                             !Number of elements tracked; array references listed below
    INTEGER,PARAMETER :: NTMAX=300                          !Maximum number of timesteps (> 13.7 Gyr/ TIMESTEP) 300 OK if TIMESTEP = 0.05
    INTEGER,PARAMETER :: NVALUESIN=18                       !Number of items sent via file values.in

! Set parameters that define the sizes of arrays
    INTEGER,PARAMETER :: NCVZ=48,NZVZ=7,NAGEVZ=18                                      !Dimensions for Vazdekis SSP data
    INTEGER,PARAMETER :: NAGEGV=47,NZGV=4                                              !Dimensions for Garcia-Vargas SSP data
    INTEGER,PARAMETER :: NAGEW94=7,NZW94=8                                             !Number of ages and metallicities in Worthey 94 SSP data
    INTEGER,PARAMETER :: NAGES=20,NZSSPS=10                                            !Number of ages and metallicities in Vazdekis SSP data
    INTEGER,PARAMETER :: NRWT=17,NMWT=11,NZWT=5                                        !Dimensions for WW95 data
    INTEGER,PARAMETER :: NCGENEVA=11,NMGENEVA=11,NZGENEVA=2                            !Dimensions for Geneva Group data
    INTEGER,PARAMETER :: NBITOT=25,NAGET04=20,NZT04=6,NRATIOT04=4                      !Dimensions for Thomas 04 SSP data
    INTEGER,PARAMETER :: NKORNZ=6,NKORNI=25,NKORNC=13                                  !Dimensions for Korn 05 response functions data
    INTEGER,PARAMETER :: NITB95=21,NCTB95=13                                           !Dimensions for Tripicco and Bell 95 response functions data
    INTEGER,PARAMETER :: NISOZ=6,NISOA=50,NISOM=200,NISOC=13,NISOCHRONES=5159          !Dimensions for Bertelli data (NISOCHRONES is variable)
    INTEGER,PARAMETER :: NMASSBINS=209,NMASSCOLS=144                                   !Dimensions for mass bins array.  Remnants stored separately

! Set parameters that are generally constant, but may want to vary with literature updates etc
    REAL, PARAMETER :: POP3=0.50                  !Fraction of initial galaxy forming population III stars
    REAL, PARAMETER :: SFRINDEX=1.3               !Index in Schmidt SFR equation SFR=SFRCONST*GASD**SFRINDEX  1.4=Schmidt, 1.3=Kennicutt, 1.51 Bothwell
    REAL, PARAMETER :: IMFINDEX=1.35              !Power in initial mass fraction (IMF) equation (1.35=Salpeter)
```

```
    REAL,PARAMETER :: CRITICALD=0.3625                 !Critical density for star formation in Msolar/kpc^3, calculated from Dunham et al 2010
    REAL,PARAMETER :: MAXMASS=120.0                    !Upper mass limit for stars made NB if change, amend NMASSBINS
    REAL,PARAMETER :: MINMASS=0.1                      !Lower limit for stars made NB if change, amend NMASSBINS
    REAL,PARAMETER :: MINBLACKHOLE=130.0               !Stars above this size go straight to black holes without evolving (set to >120 to "not work")
    REAL,PARAMETER :: MAXBDWARF=0.08                   !Upper limit for mass of brown dwarf stars
    REAL,PARAMETER :: MINSNII=10.0                     !Lower mass for stars undergoing SNII (=max mass for undergoing PN) NB if change, need to amend MASSBINS
    REAL,PARAMETER :: TIMESTEP=0.1                     !Minimum timesteps in Gyrs (if amend, may need to amend NTMAX~13.7Gyrs/TIMESTEP)
    REAL,PARAMETER :: TWEAK=1.0E-5                     !Small adjuster to clear Fortran rounding errors
    REAL,PARAMETER :: XPRIMORDIALMF=0.7523             !Initial mass fraction of hydrogen !100% - Peimbert 2008
    REAL,PARAMETER :: YPRIMORDIALMF=0.2477             !Initial mass fraction of helium   !From Peimbert 2008
    REAL,PARAMETER :: ZPRIMORDIALMF=0.0000             !Initial mass fraction of metals (only metal is Li, which is at values too low for this model)
    REAL,PARAMETER :: XSUN=0.7155                      !Solar H mass fraction, from Grevesse et al 2010 + 0.0001 so totals 100%.
    REAL,PARAMETER :: YSUN=0.2703                      !Solar He mass fraction, from Grevesse et al 2010
    REAL,PARAMETER :: ZSUN=0.0142                      !Solar metallicity, from Grevesse et al 2010.  If amend, also update details in GETVALS

! Set arrays used within the programme
    REAL :: AGET04(NAGET04)                                  !Age in Gyr as per Thomas 04 data
    REAL :: BERTELLI(NISOZ,NISOA,NISOM,NISOC)                !Array of Bertelli isochrone colours and luminosities averaged over temperatures
    REAL :: BERTELLIAGE(NISOA)                               !Array of ages in Bertelli isochrone data
    INTEGER :: BERTELLIMN(NISOZ,NISOA)                       !Array giving NUMBER of different mass isochrones (NOT MASSES) for each z/age combination
    REAL :: BERTELLIZ(NISOZ)                                 !Array of metallicities in Bertelli isochrone data
    REAL :: BLACKHOLES(NTMAX)                                !Total mass Mo of material that has gone directly to form blackholes
    REAL :: BROWNDWARF(NTMAX)                                !Mass (Mo) held in non-shining brown dwarf stars
    REAL :: EPRIMORDIALMF(NET)                               !'Primordial' mass fractions of elements in initial gas (=zero). Numbered as per list below
    REAL :: ELEMENTSGAS(NET,NTMAX)                           !Mass (Mo) of elements in the ISM - selected elements tracked over time (elements list is below)
    REAL :: EJECTED(NET)                                     !Ejecta - new and recycled material - in Mo for individual elements as a result of PN/SNIA/SNII
    REAL :: FLOW(NTMAX)                                      !Cumulative net flow of gas to end of this timestep (Mo)
    REAL :: FLOWIN(NTMAX)                                    !Mass of gas flowing into model in this timestep (Mo)
    REAL :: FLOWOUT(NTMAX)                                   !Mass of gas flowing out of the model in this timestep (Mo)
    REAL :: GALMASS(NTMAX)                                   !Mass of galaxy at end of timestep NT (Mo)
    REAL :: GASD(NTMAX)                                      !Density of gas in Mo/pc^3 at the start of the timestep
    REAL :: GASMASS(NTMAX)                                   !Total mass in ISM in this timestep
    REAL :: GENEVA(NCGENEVA,NMGENEVA,NZGENEVA)               !3-d array of yield data from Geneva Group
```

```
REAL :: GM(NMGENEVA)                               !1-d array of initial star masses from Geneva Group(ie prior to SNII event)
REAL :: GZ(NZGENEVA)                               !1-d array of metallicities from Geneva Group data
REAL :: GVAGE(NAGEGV)                              !Array of ages from Garcia-Vargas SSP data on calcium triplets
REAL :: GVSSP(NINDEX,NZGV,NAGEGV)                  !Array of SSP data from Garcia-Vargas
REAL :: GVZ(NZGV)                                  !Array of metallicities from Garcia-Vargas
REAL :: INDICES(NINDEX,NTMAX)                      !Composite indices and colours produced over time
REAL :: ISOCHRONE(NISOC)                           !Interpolated luminosity and colours for given stellar age, mass and metallicity
REAL :: KORN(NKORNZ,NKORNI,NKORNC)                 !Table of response functions from Korn 05
REAL :: KORNZ(NKORNZ)                              !Table of metallicities from Korn 05
REAL :: LOGRATIO(NTMAX)                            !Log(alpha/Fe) for stars forming in this timestep
REAL :: MASSBIN(NMASSBINS,NMASSCOLS,NTMAX)         !Array of data about stars of different masses in each timestep.  See table below.
REAL :: MASSCHECK(NTMAX)                           !Conservation of mass check
REAL :: NEWSTARS(NTMAX)                            !Mass of material in Mo converted from gas to stars made in this timestep
REAL :: OBSERVED(NINDEX)                           !Array of observed features
REAL :: OBSERVEDERROR(NINDEX)                      !Array of 1-sigma errors on observed features
REAL :: RADIUS(NTMAX)                              !Galaxy radius in kpc
REAL :: RATIOT04(NRATIOT04)                        !Log alpha/Fe ratio as per Thomas 04 data
REAL :: REMNANTS(NTMAX)                            !Mass in Mo held in white dwarfs, neutrino stars etc, by timestep (may undergo SN1A)
REAL :: SFR(NTMAX)                                 !Star formation rate at timestep NT, calculated using Schmidt formula SFR=SFRCONST*GASD(NT)*SFRINDEX
REAL :: SNIAEVENTS(NTMAX)                          !Number of SNIA events in this timestep (note: may not be integer)
REAL :: SNIARATE(NTMAX)                            !SNIA rate in events per century per 10^10 Mo
REAL :: SNIILEVENTS(NTMAX)                         !Number of SNII events in this timestep as a result of large star explosions
REAL :: SNIIMEVENTS(NTMAX)                         !Number of SNII events in this timestep as a result of massive star explosions
REAL :: SNIIRATE(NTMAX)                            !SNII reate in events per century per 10^10 Mo
REAL :: SOLARMF(NET)                               !Array of solar element mass fractions
REAL :: SSP(NINDEX)                                !Array of Lick indices, colours and M/L from SSP option selected by user
REAL :: STANDARDDEV(NINDEX)                        !Table of standard deviations of model compared to observed data chosen by use
REAL :: STARCHECK(NTMAX)                           !Difference (if any) between mass held in STARMASS and mass held in MASSBINS+REMNANTS
REAL :: STARMASS(NTMAX)                            !Total mass in stars inc those made in this timestep and INC stars held in REMNANTS and BROWNDWARF
REAL :: SY(NTMAX)                                  !Initial helium gas mass fraction at start of timestep stored for next step
REAL :: SZ(NTMAX)                                  !Initial metal mass fraction at start of time step stored for next step
REAL :: TB95(NITB95,NCTB95)                        !Response functions for different elements, for each Lick index from Tripicco & Bell
REAL :: THSSP(NINDEX,NZT04,NAGET04,NRATIOT04)      !4-D array of SSP data from Thomas 04
```

```
    REAL :: T04Z(NZT04)                         !Array of metallicities for Thomas 04 SSPs
    REAL :: TIMENOW(NTMAX)                      !Array of times at end of each step (Gyrs)
    REAL :: TOTLUM(NTMAX)                       !Total luminosity of the galaxy in Lsolar at each timestep
    REAL :: TZV(NZSSPS)                         !Metallicity array from SSP data Vazdekis
    REAL :: VZAGE(NAGEVZ)                       !Array of ages within Vazdekis data
    REAL :: VZSSP(NINDEX,NZVZ,NAGEVZ)           !Array of Vazdekis SSP data
    REAL :: VZZ(NCVZ)                           !Array of Vazdekis metallicities
    REAL :: W94AGE(NAGEW94)                     !1-D output array of ages in Worthey 94 SSP data
    REAL :: W94Z(NZW94)                         !1-D output array of metallicity values in Worthey 94 SSP data (converted in code from [Fe/H])
    REAL :: W94SSP(NINDEX,NZW94,NAGEW94)        !3-D array of SSP indices from Worthey 94 & 97 (H indices)
    REAL :: WWM(NMWT)                           !Array of the typical masses in WW95 large star yield data
    REAL :: WW(NRWT,NMWT,NZWT)                  !Array of WW95 large star yield data
    REAL :: WWZ(NZWT)                           !Array of the typical metallicities in WW95 large star yield data
    REAL :: XMF(NTMAX)                          !Mass fraction of H in gas, over time
    REAL :: XISM(NTMAX)                         !Mass of H in Mo in gas, over time
    REAL :: XSTARS(NTMAX)                       !Mass of H in Mo in stars, over time
    REAL :: YIELDS(4,NTMAX)                     !Track yield of metals in Mo due to SNIa(1),PN(2),SNIIWW(3)and SNIIGeneva(4), per timestep
    REAL :: YMF(NTMAX)                          !Mass fraction of He in gas, over time
    REAL :: YISM(NTMAX)                         !Mass of helium in Mo in ISM, over time
    REAL :: YSTARS(NTMAX)                       !Mass of helium in Mo in stars, over time
    REAL :: ZMF(NTMAX)                          !Mass fraction of metals in gas, over time
    REAL :: ZISM(NTMAX)                         !Mass of metals in Mo in ISM, over time
    REAL :: ZSTARS(NTMAX)                       !Mass of metals in Mo in stars, over time

! Set variables that are used within the programme
    REAL :: AGE                 !Age of the galaxy in Gyrs
    REAL :: AGESTAR             !Age of star in Gyrs
    REAL :: ALPHAMF             !Total mass fraction of alpha-elements in the ISM in the model at this point (see list below for included elements)
    REAL :: ALPHASUNMF          !Total mass fraction of alpha elements in the sun
    REAL :: DECSTARSX           !Decrease in H held in stars, due to this evolutionary process in this timestep
    REAL :: DECSTARSY           !Decease in He held in stars, due to this evolutionary process in this timestep
    REAL :: DECSTARSZ           !Decrease in metals held in stars, due to the evolutionary process in this timestep
    REAL :: DURATION            !Duration in Gyrs of gas inflow, which starts at FLOWINSTART set by user in values.in
```

```
      REAL :: FEPEAKMF               !Total mass fraction of Fe-peak elements in the ISM in the model at this point (see list below for included elements)
      REAL :: FEPEAKSUNMF            !Total mass fraction of Fe-peal elements in the sun
      REAL :: FLOWINSTART            !Time in Gyrs after start of galaxy when gas inflow starts, and which lasts for DURATION
      REAL :: FLOWINRATE             !Flowrate of gas in M0/Gyr
      REAL :: GALMASSI               !Initial mass of galaxy in Mo (note will all be gas in current set up)
      REAL :: GASMASSI               !Initial mass of gas in Mo
      REAL :: GASOUT                 !Time in Gyrs after start of galaxy when gas flows out OR gas loading factor (depends on GASOUTMETHOD) selected by user
      REAL :: LOGZ                   !(LOGZ=[Z/H]=LOG10(Z/H)-LOG10(Z/H)sun)
      REAL :: INCREM                 !Increase in remnants in this process in this timestep due to PN/SNIA/SNII
      REAL :: INCISM                 !Increase in gas in the galaxy (= decrease in stars) in this timestep due to PN/SNIA/SNII
      REAL :: INCISMX                !Increase in hydrogen in the ISM due to this process in this timestep due to PN/SNIA/SNII
      REAL :: INCISMY                !Increase in helium in the ISM due to this process in this timestep due to PN/SNIA/SNII
      REAL :: INCISMZ                !Increase in metals in the ISM due to this process in this timestep due to PN/SNIA/SNII
      REAL :: MASSFRAC               !Mass fraction of stars in the range given
      REAL :: MASSSTEP               !Incremental increase in masses as count through MASSBINS
      REAL :: NTMREAL                !Max number of timesteps for model, converted to a real number
      REAL :: RECYCLE                !Unaltered material ejected into the ISM
      REAL :: SDTOTAL                !Sum total of standard deviations so can get average
      REAL :: SFRCONST               !Current rate of star formation(arbitrary parameter) set by user in values.in
      REAL :: SNIAMASS               !Total mass (Mo) of stars undergoing SNIA in this timestep
      REAL :: SOLARFEPEAK            !Iron peak elements in the sun
      REAL :: TIME                   !Total lifetime for the galaxy in Gyrs set by user in values.in
      REAL :: TIMELAG                !Delay (in Gyrs) in SNIA production from star formation (applies to results from Timmes 1995)
      REAL :: TOTMASS                !Total overall mass of stars (from which GETFRAC can calculate the mass fraction in a given mass range)
      REAL :: TOTRANGE               !Total mass of stars in the given mass range for use by GETFRAC to calculate the Salpeter mass fraction
      REAL :: VOLUME                 !Volume of the modelled galaxy in kpc^3
      REAL :: YSNIA                  !Total yield of helium from SNIA events, in Mo
      REAL :: ZSNIA                  !Total yield of metals from SNIA events, in Mo

    !names for observed indices and their corresponding errors
      REAL :: HDA,HDA_ERR,HGA,HGA_ERR,HDF,HDF_ERR,HGF,HGF_ERR,CN1,CN1_ERR,CN2,CN2_ERR,CA4227,CA4227_ERR
      REAL :: G4300,G4300_ERR,FE4383,FE4383_ERR,CA4455,CA4455_ERR,FE4531,FE4531_ERR,HBETA,HBETA_ERR
      REAL :: FE4668,FE4668_ERR,C4668,C4668_ERR                                        !note FE4668 now renamed as C4668, code will correct
```

```
    REAL :: FE5015,FE5015_ERR, MG1,MG1_ERR,MG2,MG2_ERR,MGB,MGB_ERR,FE5270,FE5270_ERR,FE5335,FE5335_ERR
    REAL :: FE5406,FE5406_ERR,FE5709,FE5709_ERR,FE5782,FE5782_ERR,NAD,NAD_ERR,TIO1,TIO1_ERR,TIO2,TIO2_ERR
    REAL :: CAII1,CAII1_ERR,CAII2,CAII2_ERR,CAII3,CAII3_ERR,MGI,MGI_ERR,CAT,CAT_ERR

!counters used by various DO loops
    INTEGER :: NTM              !Total number of timesteps to run the model
    INTEGER :: NA               !Counter through ages (as tabled in SSP data)
    INTEGER :: NB               !Counter through indices (as tabled in SSP data)
    INTEGER :: NC               !Counter through columns of data
    INTEGER :: ND               !Counter through blank (dummy) rows when reading in data
    INTEGER :: NE               !Counter through elements being tracked
    INTEGER :: NF               !Counter through mass fractions
    INTEGER :: NG               !Counter through data tables
    INTEGER :: NH               !Counter through header rows
    INTEGER :: NI               !Counter through indices
    INTEGER :: NJ               !Counter through general values in an array
    INTEGER :: NL               !Counter through alpha/Fe ratios (as tabled in SSP data)
    INTEGER :: NM               !Counter through masses
    INTEGER :: NN               !Searching software loop counter through galaxy masses (GALMASSI)
    INTEGER :: NO               !Searching software loop counter through gas flow out start times (FLOWOUTSTART)
    INTEGER :: NP               !Counter through massbins   !care - use only in MAKEINDICES and EVOLVE
    INTEGER :: NQ               !Counter from 1 to NT (current timestep) !care! - use only in EVOLVE and MAKEINDICES
    INTEGER :: NR               !Counter through rows
    INTEGER :: NT               !Counter through timesteps (value of timestep held in TIMENOW(NT) CARE! ONLY USE FOR MAIN EVOLUTION!
    INTEGER :: NU               !Searching software loop counter through galaxy lifetime length (TIME)
    INTEGER :: NV               !Searching software loop counter through options for SFR constant (SFRCONST)
    INTEGER :: NW               !Searching software loop counter through gas flow in rates (FLOWINRATE)
    INTEGER :: NX               !Searching software loop counter through gas flow in start times (FLOWINSTART)
    INTEGER :: NY               !Searching software loop counter through gas flow in duration (DURATION)
    INTEGER :: NZ               !Counter through metallicities
    INTEGER :: I,J,K,L,M,N      !Counters through various actions in searching subroutine
    INTEGER :: IOFLAG           !IOSTAT flag for file read-in - error checks read-in process

```

```
!character variables used within code (mainly for reading in data from obs.in and values.in files
    CHARACTER(18) :: ANAME                  !Label for each index read in from obs.in
    CHARACTER(18) :: ANAMES(NINDEX)         !Array of names allocated in standard order
    CHARACTER(10) :: AVALUE                 !Value read in for each index read in from obs.in
    CHARACTER(10) :: AERR                   !Error on each index read in from obs.in
    CHARACTER(20) :: BNAME                  !Descriptor used in values.in to indicate individual information required
    CHARACTER(20) :: BVALUE                 !Descriptor used in values.in being the individual information entered
    CHARACTER(132) :: DUMMY                 !Holding point for unnecessary data during reading-in
    CHARACTER(10) :: GASOUTMETHOD           !Process for gas loss: at a specific TIME, or LOADed to track new stars formed
    CHARACTER(20) :: INFLOWTYPE             !Chemical composition of gas inflow set by user in values.in
    CHARACTER(10) :: LARGE                  !Source file for large stars (SNL to 40.0Mo) set by user in values.in
    CHARACTER(10) :: MASSIVE                !Source file for massive stars (above 40Mo)
    CHARACTER(10) :: MODELTYPE              !Either SINGLE or SEARCH, depending on which model the user wishes to run
    CHARACTER(10) :: NONSOLAR               !Either TB95 or KORN05 for choice of non-solar abundance adjustments
    CHARACTER(10) :: PNDATAIN               !Selection by user of data source to use for planetary nebula yields
    CHARACTER(10) :: SNIATYPE               !User defined source for SNIA rates: Timmes 1995 (Timmes) or Scannapieco and Bildsten 1995 (SB05)
    CHARACTER(1)  :: SSPDATA                !User defined source for SSP data: W (Worthey 1994), V (Vazdekis 1999) or T (Thomas et al 2004)

! Allocation within arrays tracking elements is as follows (as given by Thomas et al 2004):
!   Mg(24,25,26) is for                NE=1  alpha
!   Fe(54,56,57,58) is for             NE=2  Fe-peak
!   Si(28,29,30) is for                NE=3  alpha
!   S(32-36) is for                    NE=4  alpha
!   O(16,17,18) is for                 NE=5  alpha
!   C(12,13) is for                    NE=6
!   Ca(40/2/3/4/6/8) is for            NE=7  alpha
!   N(14,15) is for                    NE=8  alpha
!   Ne(20,21,22) is for                NE=9  alpha
!   Na(23) is for                      NE=10 alpha
!   Al(27) is for                      NE=11
!   Ar(36,38,40) is for                NE=12 alpha
!   Cr(50,52,53,54) is for             NE=13 Fe-peak
!   Ni(56,58,60,61,62,64) is for       NE=14 Fe-peak (Ni 56 decays to Fe) (Nomoto 1995 include as Fe-peak element)
```

```
!The components of the MASSBIN array are:
! MASSBIN(massbins number, X, timestep), with X allocated as follows:
!    1              !Lower mass limit for this mass bin
!    2              !Upper mass limit for this mass bin
!    3              !Average mass of a star in this massbin (not strictly true as not weighted but ok as bins small)
!    4              !Total mass in this bin, calculated using Salpeter(1955)
!    5              !Metal content in Mo of stars formed
!    6              !H content in Mo of stars formed
!    7              !He content in Mo of stars formed
!    8              !Timestep when these stars formed (held here as a real number)
!    9              !No timesteps on MS from Wood 1992 sec 4.6 (formula only for small stars, but gives <1 for larger stars, so ok)
!    10             !Timestep when stars leave MS - assume rest of life happens in next timestep unless dwarf
!    11             !Luminosity of the stars in this mass bin
!    12             !Relative luminosity for these stars in the galaxy at this time
!    13             !Alpha/Fe of stars formed
!    14-20          !Unallocated
!    21-75          !Absolute indices and colours as listed below unweighted
!    76-130         !Weighted indices and colours for the stars in the galaxy at this time
!    131-144        !Original element mass in star when formed, elements 1-14 as before
! Below 10.0 Mo, the massbins increase in size by 0.01Mo from the previous bin, above 10.0 Mo, increments are 0.1 M0

! Allocation within arrays tracking indicies is as follows (SSP is just in NINDEX, MASSBIN is in components):
!   NINDEX=1, component 21(unweighted),76(weighted)    CN1 (mag)                 NINDEX=23, component 43(unweighted),98 (weighted)    U
!   NINDEX=2, component 22(unweighted),77(weighted)    CN2 (mag)                 NINDEX=24, component 44(unweighted),99 (weighted)    B
!   NINDEX=3, component 23(unweighted),78(weighted)    Ca4227 (A)                NINDEX=25, component 45(unweighted),100(weighted)    V
!   NINDEX=4, compoennt 24(unweighted),79(weighted)    G4300 (A)                 NINDEX=26, component 46(unweighted),101(weighted)    Rc
!   NINDEX=5, component 25(unweighted),80(weighted)    Fe4383 (A)                NINDEX=27, component 47(unweighted),102(weighted)    Ic
!   NINDEX=6, component 26(unweighted),81(weighted)    Ca4455 (A)                NINDEX=28, component 48(unweighted),103(weighted)    J
!   NINDEX=7, component 27(unweighted),82(weighted)    Fe4531 (A)                NINDEX=29, component 49(unweighted),104(weighted)    H
!   NINDEX=8, component 28(unweighted),83(weighted)    C4668 (was Fe4668)(A)     NINDEX=30, component 50(unweighted),105(weighted)    K
!   NINDEX=9, component 29(unweighted),84(weighted)    Hb (A)                    NINDEX=31, component 51(unweighted),106(weighted)    L
!   NINDEX=10,component 30(unweighted),85(weighted)    Fe5015 (A)                NINDEX=32, component 52(unweighted),107(weighted)    Ldash
```

```
! NINDEX=11,component 31(unweighted),86(weighted)  Mg1 (mag)      NINDEX=33, component 53(unweighted),108(weighted)  M
! NINDEX=12,component 32(unweighted),87(weighted)  Mg2 (mag)      NINDEX=34, component 54(unweighted),109(weighted)  U-V
! NINDEX=13,component 33(unweighted),88(weighted)  Mgb (A)        NINDEX=35, component 55(unweighted),110(weighted)  B-V
! NINDEX=14,component 34(unweighted),89(weighted)  Fe5270 (A)     NINDEX=36, component 56(unweighted),111(weighted)  V-R
! NINDEX=15,component 35(unweighted),90(weighted)  Fe5335 (A)     NINDEX=37, component 57(unweighted),112(weighted)  V-I
! NINDEX=16,component 36(unweighted),91(weighted)  Fe5406 (A)     NINDEX=38, component 58(unweighted),113(weighted)  V-J
! NINDEX=17,component 37(unweighted),92(weighted)  Fe5709 (A)     NINDEX=39, component 59(unweighted),114(weighted)  V-K
! NINDEX=18,component 38(unweighted),93(weighted)  Fe5782 (A)     NINDEX=40, component 60(unweighted),115(weighted)  J-H
! NINDEX=19,component 39(unweighted),94(weighted)  NaD (A)        NINDEX=41, component 61(unweighted),116(weighted)  J-K
! NINDEX=20,compoennt 40(unweighted),95(weighted)  TiO1 (mag)     NINDEX=42, component 62(unweighted),117(weighted)  J-L
! NINDEX=21,component 41(unweighted),96(weighted)  TiO2 (mag)     NINDEX=43, component 63(unweighted),118(weighted)  J-Ldash
! NINDEX=22,component 42(unweighted),97(weighted)  D4000          NINDEX=44, component 64(unweighted),119(weighted)  J-M

! NINDEX=45,component 65(unweighted),120(weighted)  HdA (A)
! NINDEX=46,component 66(unweighted),121(weighted)  HgA (A)
! NINDEX=47,component 67(unweighted),122(weighted)  HdF (A)
! NINDEX=48,component 68(unweighted),123(weighted)  HgF (A)
! NINDEX=49,component 69(unweighted),124(weighted)  CaT (A)
! NINDEX=50,component 70(unweighted),125(weighted)  CaII1 (A)
! NINDEX=51,compoennt 71(unweighted),126(weighted)  CaII2 (A)
! NINDEX=52,component 72(unweighted),127(weighted)  CaII3 (A)
! NINDEX=53,compoennt 73(unweighted),128(weighted)  MgI (A)
! NINDEX=54,component 74(unweighted),129(weighted)  U-B
! NINDEX=55,compoennt 75(unweighted),130(weighted)  V-H

! The unit number used (which are opened and closed, so could be re-used without error) are:
! 20  file of SSP data from Worthey 94
! 21  file of large star yields from Woosley and Weaver (user selects whether to use this or Geneva group results)
! 22  file of large and massive star yields from Geneva group (user selects which)
! 23  file of Tripicco and bell adjustments for non-solar abundances
! 24  file of Vazdekis SSP data
! 26  file of SSP data from Worthey 97
! 27  file of Korn 05 response functions
```

```
! 28  file of data input to Phoenix by the user giving user-set variables (values.in)
! 29  file of data input to Phoenix by the user giving chosen galaxy observed data (obs.in)
! 30  file of planetary nebula data from selected source
! 31  file of Garcia Vargas SSP data for calcium triplet
! 32  file of Thomas 2004 SSP data
! 33  file of Bruzual and Charlot 2003 colour data
! 34  file of Bertelli 1994 isochrone data
! 50  file of warnings from code eg where looking up in a data table and values are outside range
! 60  file of data output by code for plotting
! 61  file of data output by code of stars remaining in galaxy at end of galaxy life
! 70  file of data results from the searching software


    END MODULE SHARED
```

FILE *'subroutines.f90'*

```
!PHOENIX – a galactic evolution model
!The code produces synthetic Lick indices against which observational indices can be compared.
!The user can select some parameters in a file values.in, and the observational comparison, in a file obs.in
!Final values stored against array parameter NT are the values at the end of the timestep NT

!This code is laid out as follows:
!   PHOENIX        initialise, and establish which format of the model (single or search) is being run
!   SINGLE         run the code once
!   SEARCH         run the code several times, searching parameter space
!   EVALUATE       runs the code
!   EVOLVE         move galaxy on by one timestep, flow gas in and out, make and evolve stars
!   MAKEINDICES    produce synthetic indices at the end of this timestep

!Calculations:
!   GETFRAC        calculate mass fractions using Salpeter IMF
!   INTERPOLATE    linearly interpolate between data read in

!Evolutionary yields
!   SNIAYELDS      SNIA yields from Nomoto 1984 (currently data is hard-coded here rather than read in)
!   PNYIELDS       PN yields from Renzini & Voli 81 OR Gavilan 05 OR van den Hoek and Groenewegen 97
!   SNIIWWYIELDS   SNII yields from Woosley and Weaver 1995
!   SNIIGYIELDS    SNII yields from the Geneva Group (several files to select from)

!Update the galaxy's parameters
!   UPDATE         following evolutionary event, update stars, gas, elements etc

!Make synthetic indices and colours from SSP data
!   W94INDICES     make indices from Worthey 1994 SSPs
!   GV98INDICES    make indices from Garcia-Vargas 1998 SSPs
!   V99INDICES     make indices from Vazdekis 1999 SSPs
!   T04INDICES     make indices from Thomas 2004 SSPs and Bruzual and Charlot 2003 colours
!   B94ISOCHRONES  get luminosity and colour data from Bertelli 94 isochrones

!Calculate statistics, create output files for plotting and on-screen checks
!   SINGLEOUTPUTS  produce results to screen and file from single run of code
!   SEARCHOUTPUTS  produce results to file from parameter space searching

!Remaining subroutines get in data, initialise variables etc
!   READIN         read in the various data files
!   ZERO           initialise arrays and variables (generally to zero)
!   RESET          resets some arrays and variables (to zero)
!   GETVALS        read in user-selected model parameters
!   GETOBS         read in user-selected observational data
!   READPN         planetary nebula data from choice of 3 sources
!   READWW95SNII   data for large stars from Woosley and Weaver 1995
!   READGENEVASNII data for large and massive stars from Geneva Group
!   READWORTHEY94  SSP data from Worthey 1994
!   READGARCIA     SSP data from Garcia-Vargas (extends Worthey by adding Ca indices)
!   READVAZDEKIS   SSP data from Vazdekis 1999
!   READT04        SSP data from Thomas et al 2004
```

```
!    READBERTELLI   Isochrone data from Bertelli et al 1994
!    READTB95       non-solar abundance adjustments from Tripicco and Bell 1995
!    READKORN       non-solar abundance adjustments from Korn et al 2005

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! 2004 - 2012






      SUBROUTINE PHOENIX

      USE SHARED
      IMPLICIT NONE

      REAL :: DBIN,DOF1
      INTEGER :: ICHECK,IL
      INTEGER :: NPNC,NPNM,NPNZ,NPNCT

!Set the allocatable arrays
      REAL,ALLOCATABLE :: PNDATA(:,:,:)
      REAL,ALLOCATABLE :: PNM(:)
      REAL,ALLOCATABLE :: PNZ(:)
      REAL,ALLOCATABLE :: INTERPZ(:,:)
      REAL,ALLOCATABLE :: INTERPZM(:)

! Zero arrays and set inital values
      CALL ZERO

! Get initial data as set by user
      CALL GETVALS(NPNC,NPNM,NPNZ,NPNCT)
      CALL GETOBS

! Set array sizes for PN data
      ALLOCATE (PNDATA(NPNC,NPNM,NPNZ))
      ALLOCATE (PNM(NPNM))
      ALLOCATE (PNZ(NPNZ))
      ALLOCATE (INTERPZ(NPNC,NPNM))
      ALLOCATE (INTERPZM(NPNC))

! Zero these new arrays
      PNDATA=0.0
      PNM=0.0
      PNZ=0.0
      INTERPZ=0.0
      INTERPZM=0.0

!Read in static data
      CALL READIN(NPNC,NPNM,NPNZ,NPNCT,PNDATA,PNM,PNZ,INTERPZ,INTERPZM)

!Establish which form of the model is to be run
      WRITE(*,*)'Select which model type required: single run (SINGLE) or search parameter space (SEARCH)'
      READ (*,*) MODELTYPE
      IF(MODELTYPE=='single'.OR.MODELTYPE=='Single'.OR.MODELTYPE=='SINGLE')THEN
```

```
        CALL SINGLE(NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ,INTERPZ,INTERPZM)
        MODELTYPE='SINGLE'  !eliminate use of different cases
      ELSE IF(MODELTYPE=='search'.OR.MODELTYPE=='Search'.OR.MODELTYPE=='SEARCH')THEN
        CALL SEARCH(NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ,INTERPZ,INTERPZM)
      END IF

! Free up memory
      DEALLOCATE (PNDATA)
      DEALLOCATE (PNM)
      DEALLOCATE (PNZ)
      DEALLOCATE (INTERPZ)
      DEALLOCATE (INTERPZM)

      END SUBROUTINE PHOENIX






!SINGLE runs the code once, for values set by the user in values.in and compared to galaxy in obs.in
!Outputs are to screen and to file plotdata.out, and warnings are sent to warnings.out
      SUBROUTINE SINGLE(NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ,INTERPZ,INTERPZM)

      USE SHARED
      IMPLICIT NONE

      INTEGER :: NPNC,NPNM,NPNZ,NPNCT
      REAL :: PNDATA(NPNC,NPNM,NPNZ)
      REAL :: PNM(NPNM)
      REAL :: PNZ(NPNZ)
      REAL :: INTERPZ(NPNC,NPNM)
      REAL :: INTERPZM(NPNC)


! Inform the user what initial values have been set or determined for this model
      PRINT *
      PRINT *,'INITIAL PARAMETERS:'
      PRINT *
      PRINT *,'Galaxy initial mass  = ',GALMASSI,'Galaxy lifetime     = ',TIME,'Gyrs'
      PRINT *,'Initial SFR constant = ',SFRCONST,' Pop III proportion = ',POP3*100,'%'
      IF (GASOUTMETHOD=='TIME')THEN
        PRINT*,'Flow out occurs after ',GASOUT,'Gyrs'
      ELSE IF (GASOUTMETHOD=='LOAD')THEN
        PRINT*,'Gas flows out at ',GASOUT,'times the mass of stars formed this timestep'
      END IF
      IF (FLOWINRATE/=0.0)THEN
        PRINT *,'Flow in starts at      ',FLOWINSTART,'Gyrs, at a rate of   ',FLOWINRATE,&
          ' Mo/Gyrs and stops at',FLOWINSTART+DURATION,'Gyrs and is ',INFLOWTYPE
      ELSE
        PRINT*,'No gas inflow'
      END IF
      PRINT*
      IF (PNDATAIN=='RV') THEN
        PRINT *, 'Planetary nebula yields from Renzini and Voli 81'
      ELSE IF (PNDATAIN=='VG') THEN
        PRINT *, 'Planetary nebula yields from van den Hoek & Groenewegen 97'
```

```
      ELSE IF (PNDATAIN=='GA') THEN
        PRINT *, 'Planetary nebula yields from Gavilan et al 05'
      END IF
      IF (SNIATYPE=='Timmes') THEN
        PRINT *,'SNIA yields from Nomoto et al 1984 at rates defined by Timmes et al 1995'
      ELSE IF (SNIATYPE=='SB05') THEN
        PRINT*,'SNIA yields from Nomoto et al 1984 at rates defined by Scannapieco & Bildsten 2005'
      END IF
      IF (LARGE==MASSIVE)THEN
        IF (LARGE=='M92wind')THEN
          PRINT *,'SNII yields from Maeder 92 models, including stellar winds'
        ELSE IF (LARGE=='M92nowind') THEN
          PRINT *,'SNII yields from Maeder 92 models, excluding stellar winds'
        ELSE IF (LARGE=='MM02wind') THEN
          PRINT *,'SNII yields from Meynet and Maeder 02 models, including stellar winds but excluding rotation'
        ELSE IF (LARGE=='MM02RW') THEN
          PRINT *,'SNII yields from Meynet and Maeder 02 models, including wind and rotation'
        END IF
      END IF
      IF (LARGE=='WW95') THEN
        PRINT *,'Large star yields from Woosley and Weaver 95 with massive star extension from ',MASSIVE
      END IF
      IF (SSPDATA=='W') THEN
        PRINT *,'SSP data from Worthey 94'
      ELSE IF (SSPDATA=='V') THEN
        PRINT *,'SSP data from Vazdekis 99'
      ELSE IF (SSPDATA=='T') THEN
        PRINT*,'SSP data from Thomas et al'
      END IF
      PRINT *

      !Output any code warnings (eg data out of range) to a file
      OPEN(UNIT=50,FILE='warnings.out',STATUS='REPLACE')

      !run model
      CALL EVALUATE(NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ,INTERPZ,INTERPZM)

      ! Output results to screen and file
      CALL SINGLEOUTPUTS

      CLOSE(UNIT=50)  !warnings file
      CLOSE(UNIT=60)  !outputs file

      END SUBROUTINE SINGLE





!SEARCH runs the code several times, working through parameter space, using the user's settings in 'values.in'
!and comparing to galaxy in obs.in.  Variables entered in values.in are ignored, but model selection is used.
!Outputs are to the file search.out.

      SUBROUTINE SEARCH(NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ,INTERPZ,INTERPZM)

      USE SHARED
      IMPLICIT NONE
```

```
      INTEGER :: NPNC,NPNM,NPNZ,NPNCT
      REAL :: PNDATA(NPNC,NPNM,NPNZ)
      REAL :: PNM(NPNM)
      REAL :: PNZ(NPNZ)
      REAL :: INTERPZ(NPNC,NPNM)
      REAL :: INTERPZM(NPNC)

      INTEGER,PARAMETER :: NGALMASS=2
      INTEGER,PARAMETER :: NTIME=3
      INTEGER,PARAMETER :: NSFR=2
      INTEGER,PARAMETER :: NGASOUT=3
      INTEGER,PARAMETER :: NFLOWINRATE=3
      INTEGER,PARAMETER :: NFLOWINSTART=3
      INTEGER,PARAMETER :: NDURATION=3

      REAL :: SEARCHGALMASS(NGALMASS)
      REAL :: SEARCHTIME(NTIME)
      REAL :: SEARCHSFR(NSFR)
      REAL :: SEARCHGASOUT(NGASOUT)
      REAL :: SEARCHFLOWINRATE(NFLOWINRATE)
      REAL :: SEARCHFLOWINSTART(NFLOWINSTART)
      REAL :: SEARCHDURATION(NDURATION)

!Open file to store output data
      OPEN(UNIT=70,FILE='searchdata.out',STATUS='REPLACE')
      WRITE(70,*)'Output from searching model'
      WRITE(70,*)'Yields: Plan neb from ',PNDATAIN,' Large from ',LARGE,' Massive from ',MASSIVE
      WRITE(70,*)'SNIA from ',SNIATYPE,' SSP from ',SSPDATA,' Population III % of original mass',POP3
      WRITE(70,*)'Gas inflow chemical composition ',INFLOWTYPE,'Non-solar abundance adj from'
      WRITE(70,*)' '
      WRITE(70,*)' Galmass     Time      SFRconst     gasoutmtd gasout  flowinrate    flowinstart    &
         duration        sdaverage     sdmax      Z%      &
         CN1          &
         CN2          Ca4227        G4300         Fe4383        Ca4455        &
         Fe4531       C4668         Hb            Fe5015        Mg1           &
         Mg2          Mgb           Fe5270        Fe5335        Fe5406        &
         Fe5709       Fe5782        NaD           Ti01          Ti02          &
         D4000        U             B             V             Rc            &
         Ic           J             H             K             L             &
         Ldash        M             U-V           B-V           V-R           &
         V-I          V-J           V-K           J-H           J-K           &
         J-L          J-Ldash       J-M           HdA           HgA           &
         HdF          HgF           CaT           CaII1         CaII2         &
         CaII3        MgI           U-B           V-H'

      WRITE(70,*)' '
!Set parameters to be searched
      SEARCHGALMASS=(/0.5E12,0.5E11/)        !Galaxy mass in Mo
      SEARCHTIME=(/9.0,12.0,13.0/)           !Galaxy age in Gyrs
      SEARCHSFR=(/0.1,0.5/)                  !Constant in Schmidt star formation rate formula
      SEARCHGASOUT=(/0.44,0.7,4.4/)          !note timestep is 0.05 so don't test with earlier than this
      SEARCHFLOWINRATE=(/0.0,1E11,1E12/)     !Gas inflow in Mo per Gyr
      SEARCHFLOWINSTART=(/2.0,4.0,8.0/)      !Time in Gyrs after start of galaxy when gas flows in
      SEARCHDURATION=(/0.5,2.0,4.0/)         !Duration of gas inflow in Gyrs
```

```
!Run model with different combinations of parameters and store output of each run to file
    DO NN=1,NGALMASS
      GALMASSI=SEARCHGALMASS(NN)
      DO NU=1,NTIME
        TIME=SEARCHTIME(NU)
        DO NV=1,NSFR
          SFRCONST=SEARCHSFR(NV)
          DO NO=1,NGASOUT
            GASOUT=SEARCHGASOUT(NO)
              DO NW=1,NFLOWINRATE
              FLOWINRATE=SEARCHFLOWINRATE(NW)
              IF (FLOWINRATE==0.0) THEN
                FLOWINSTART=0.0
                DURATION=0.0
                PRINT*,'Searching software running',NN,NU,NV,NO,NW,'      N/A      N/A'
                !run model and output results to file
                CALL EVALUATE(NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ,INTERPZ,INTERPZM)
                CALL SEARCHOUTPUTS
              ELSE
                DO NX=1,NFLOWINSTART
                FLOWINSTART=SEARCHFLOWINSTART(NX)
                  DO NY=1,NDURATION
                    DURATION=SEARCHDURATION(NY)
                    PRINT*,'Searching software running',NN,NU,NV,NO,NW,NX,NY
                    !run model and output results to file
                    CALL EVALUATE(NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ,INTERPZ,INTERPZM)
                    CALL SEARCHOUTPUTS
                  END DO
                END DO
              END IF
            END DO
          END DO
        END DO
      END DO
    END DO
    PRINT*,'Searching routine has finished; refer to file searchdata.out for results'

    CLOSE (UNIT=70)   !search outputs data file

    END SUBROUTINE SEARCH






!EVALUATE runs the code once

    SUBROUTINE EVALUATE(NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ,INTERPZ,INTERPZM)

    USE SHARED
    IMPLICIT NONE

    INTEGER :: NPNC,NPNM,NPNZ,NPNCT
    REAL :: PNDATA(NPNC,NPNM,NPNZ)
    REAL :: PNM(NPNM)
    REAL :: PNZ(NPNZ)
```

```
      REAL :: INTERPZ(NPNC,NPNM)
      REAL :: INTERPZM(NPNC)

      !Reset arrays and variables
      CALL RESET
      INTERPZ=0.0
      INTERPZM=0.0

      ! Evaluate model
      NTMREAL=TIME/TIMESTEP  !for some reason code doesn't like going straight to INT
      NTM=INT(NTMREAL)     !calculate maximum timesteps for this model
      DO NT=1,NTM
        !evolve the galaxy for one timestep
        CALL EVOLVE(NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ,INTERPZ,INTERPZM)
        !make the indicies that are observed at the end of this timestep
        CALL MAKEINDICES
      END DO

      END SUBROUTINE EVALUATE



!EVOLVE Subroutine to flow gas in/ and out of galaxy, and then make new stars from gas.  Evolve any stars at end of
!their main sequence life in this timestep.

      SUBROUTINE EVOLVE(NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ,INTERPZ,INTERPZM)

      USE SHARED
      IMPLICIT NONE

      REAL :: ADJUST,TOTAL,TEST
      REAL :: INCE(NET)

! Allocatable arrays used in this subroutine need to be re-declared:
      REAL :: INTERPZ(NPNC,NPNM)
      REAL :: PNDATA(NPNC,NPNM,NPNZ)
      REAL :: PNM(NPNM),PNZ(NPNZ)
      REAL :: INTERPZM(NPNC)

      INTEGER :: NPNC,NPNM,NPNZ

! Set time in Gyrs at end of current bin
      TIMENOW(NT)=NT*TIMESTEP

! Set up galaxy masses and mass fractions at this point.  The final value stored is the value at the END of the timestep
      IF (NT==1)THEN
        GALMASS(NT)=GALMASSI
        GASMASS(NT)=GALMASSI*(1-POP3)  !as initially all gas
        STARMASS(NT)=GALMASSI*POP3
        REMNANTS(NT)=0.0        !remnants are a subset of STARMASS
        FLOWIN(NT)=0.0
        XMF(NT)=XPRIMORDIALMF              !set to initial mass fractions
        YMF(NT)=YPRIMORDIALMF
        ZMF(NT)=ZPRIMORDIALMF
```

```
        XISM(NT)=XMF(NT)*GASMASS(NT)  !Mass H (Mo) in the gas
        YISM(NT)=YMF(NT)*GASMASS(NT)  !Mass He (Mo) in the gas
        ZISM(NT)=ZMF(NT)*GASMASS(NT)  !Mass metals (Mo) in the gas
        DO NE=1,NET          !set masses of each element by primordial mass fractions (note: zero)
          ELEMENTSGAS(NE,NT)=EPRIMORDIALMF(NE)*GASMASS(NT)
        END DO

      ELSE                   !set starting point as end point of previous loop - these values update as work through EVOLVE
        GALMASS(NT)=GASMASS(NT-1)+STARMASS(NT-1)
        GASMASS(NT)=GASMASS(NT-1)
        STARMASS(NT)=STARMASS(NT-1)
        REMNANTS(NT)=REMNANTS(NT-1)
        FLOW(NT)=FLOW(NT-1)
        XMF(NT)=XMF(NT-1)         !mass fractions in the galaxy
        YMF(NT)=YMF(NT-1)
        ZMF(NT)=ZMF(NT-1)
        XISM(NT)=XISM(NT-1)      !Mass H (Mo) in the gas
        YISM(NT)=YISM(NT-1)      !Mass He (Mo)in the gas
        ZISM(NT)=ZISM(NT-1)      !Mass metals (Mo) in the gas
        DO NE=1,NET
          ELEMENTSGAS(NE,NT)=ELEMENTSGAS(NE,NT-1)
        END DO
      END IF


! Gas flowing in from outside model galaxy with chemical composition as defined by the user
! All inflow is gas so no update to XSTARS/YSTARS/ZSTARS.
      IF(TIMENOW(NT)>FLOWINSTART.AND.TIMENOW(NT-1)<FLOWINSTART)THEN   !partial inflow in this timestep
        FLOWIN(NT)=FLOWINRATE*TIMESTEP*(TIMENOW(NT)-(FLOWINSTART)/TIMESTEP)
      ELSE IF(TIMENOW(NT)>FLOWINSTART+DURATION.AND.TIMENOW(NT-1)<FLOWINSTART+DURATION)THEN  !partial inflow in this timestep
        FLOWIN(NT)=FLOWINRATE*TIMESTEP*((FLOWINSTART+DURATION-TIMENOW(NT-1))/TIMESTEP)
      ELSE IF (TIMENOW(NT)>FLOWINSTART.AND.TIMENOW(NT)<=FLOWINSTART+DURATION)THEN
        FLOWIN(NT)=FLOWINRATE*TIMESTEP    !flow in for the whole of this timestep
      ELSE
        FLOWIN(NT)=0.0
      END IF

      IF(FLOWIN(NT)/=0.0)THEN
        IF (INFLOWTYPE=='PRIMORDIAL') THEN !merging with a primordial gas cloud
          XISM(NT)=XISM(NT)+(FLOWIN(NT)*XPRIMORDIALMF)
          YISM(NT)=YISM(NT)+(FLOWIN(NT)*YPRIMORDIALMF)
          ZISM(NT)=ZISM(NT)+(FLOWIN(NT)*ZPRIMORDIALMF)
          DO NE=1,NET
            ELEMENTSGAS(NE,NT)=ELEMENTSGAS(NE,NT)+(FLOWIN(NT)*EPRIMORDIALMF(NE))
          END DO
        ELSE IF (INFLOWTYPE=='SAME')THEN !merging with gas of the same chemical composition as current galaxy
          IF(TIMENOW(NT)>FLOWINSTART+DURATION)THEN   !No current gas so no current chemical composition
            PRINT*,'WARNING! gas inflow cannot be tretaed as "SAME" as no current gas -default to solar'
            INFLOWTYPE='SOLAR'
          END IF
          XISM(NT)=XISM(NT)+(FLOWIN(NT)*XMF(NT))
          YISM(NT)=YISM(NT)+(FLOWIN(NT)*YMF(NT))
          ZISM(NT)=ZISM(NT)+(FLOWIN(NT)*ZMF(NT))
          DO NE=1,NET
ELEMENTSGAS(NE,NT)=ELEMENTSGAS(NE,NT)+(FLOWIN(NT)*(ELEMENTSGAS(NE,NT)/GASMASS(NT))) !Gal mass not updated yet with inflow
```

```
      END DO
    ELSE IF (INFLOWTYPE=='SOLAR') THEN !merging with gas of solar composition
      XISM(NT)=XISM(NT)+(FLOWIN(NT)*XSUN)
      YISM(NT)=YISM(NT)+(FLOWIN(NT)*YSUN)
      ZISM(NT)=ZISM(NT)+(FLOWIN(NT)*ZSUN)
      DO NE=1,NET
        ELEMENTSGAS(NE,NT)=ELEMENTSGAS(NE,NT)+(FLOWIN(NT)*SOLARMF(NE))
      END DO
    ELSE IF (INFLOWTYPE=='ENHANCED')THEN !merging with gas of metallicity twice solar
      XISM(NT)=XISM(NT)+(FLOWIN(NT)*(XSUN-((XSUN/(XSUN+YSUN))*ZSUN)))  !weight reduction in H mass fraction by solar
      YISM(NT)=YISM(NT)+(FLOWIN(NT)*(YSUN-((YSUN/(XSUN+YSUN))*ZSUN)))  !weight reduction in He mass fraction by solar
      ZISM(NT)=ZISM(NT)+(FLOWIN(NT)*(ZSUN*2))
      DO NE=1,NET
        ELEMENTSGAS(NE,NT)=ELEMENTSGAS(NE,NT)+(FLOWIN(NT)*SOLARMF(NE)*2)
      END DO
    END IF
    !Update galaxy for gas flowing in
    FLOW(NT)=FLOW(NT)+FLOWIN(NT)                   !Cumulative net flow to this timestep
    GASMASS(NT)=GASMASS(NT)+FLOWIN(NT)
    GALMASS(NT)=GALMASS(NT)+FLOWIN(NT)
    XMF(NT)=XISM(NT)/GASMASS(NT)
    YMF(NT)=YISM(NT)/GASMASS(NT)
    ZMF(NT)=ZISM(NT)/GALMASS(NT)
  END IF


! Calculate dimensions of galaxy at this point from the total mass, assuming galaxy to be spherical.
  IF(NT==1)THEN  !use the revised Shen 2007 formula
    RADIUS(NT)=(2.88E-6)*(GALMASS(NT)**0.56)*(2**(1/3))   !as just forming stars, use total mass.  Cube root of 2 to convert from half light radius to full radius
  ELSE
    RADIUS(NT)=(2.88E-6)*(STARMASS(NT)**0.56)*(2**(1/3))
  END IF
  IF (GALMASS(NT)<4.0E8.OR.GALMASS(NT)>1E12)THEN
    PRINT*,'Warning!  Mass outside range for which Shen 2007 radius formula is valid'
  END IF
  VOLUME=(4.0/3.0)*PI*(RADIUS(NT)**3)          !volume in kpc^3
  GASD(NT)=GASMASS(NT)/VOLUME              !density in Msolar kpc^-3
! Star formation rate (Schmidt) in this time step (>0 only if above critical density)
  IF(GASD(NT)>=CRITICALD) THEN
    SFR(NT)=SFRCONST*(GASD(NT)**SFRINDEX)
  ELSE
    SFR(NT)=0.0
  END IF

! Mass going into stars this step
  NEWSTARS(NT)=SFR(NT)*TIMESTEP
  !Check: reduce NEWSTARS(NT) if there is not enough gas left for this
  IF(NEWSTARS(NT)>GASMASS(NT)) THEN
    NEWSTARS(NT)=GASMASS(NT)*0.95   !arbitrary value: assume will not be 100% converted to stars in one timestep
    IF(MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'Not enough gas in timestep',NT,'to form stars at desired SFR; all remaining gas converted to stars'
    END IF
  END IF
  IF(NT==1)THEN
    NEWSTARS(NT)=STARMASS(NT)   !overwrite for first timestep with amount setup for popIII stars
```

```
    END IF

! Calculate current alpha/Fe ratio in the ISM (nb: ratio, not log ratio)
    IF(GASMASS(NT)/=0.0)THEN
ALPHAMF=(ELEMENTSGAS(1,NT)+ELEMENTSGAS(3,NT)+ELEMENTSGAS(4,NT)+ELEMENTSGAS(5,NT)+&
        ELEMENTSGAS(7,NT)+ELEMENTSGAS(8,NT)+ELEMENTSGAS(9,NT)+ELEMENTSGAS(10,NT)+&
        ELEMENTSGAS(12,NT))/(GASMASS(NT))                    !Mg + Si + S + O + Ca + N + Ne + Na + Ar
      FEPEAKMF=(ELEMENTSGAS(2,NT)+ELEMENTSGAS(13,NT)+ELEMENTSGAS(14,NT))/(GASMASS(NT)) !Fe + Cr + Ni
    END IF
    IF(FEPEAKMF==0.0.OR.FEPEAKSUNMF==0.0) THEN     !Trap any zero denominators
      LOGRATIO(NT)=0.0
    ELSE
      LOGRATIO(NT)=LOG10(ALPHAMF/FEPEAKMF)-LOG10(ALPHASUNMF/FEPEAKSUNMF)
    END IF

!Set the amount of stars in each mass bin and hold the metallicity, alpha/fe ratio (etc) at the time of formation.
!Calculate the timestep when these stars will leave the MS, using Wood 1992 formula (not valid for massive stars, but formula
!gives life < 1 timestep, so this is ok astrophysically, if not logically extrapolated from this paper)
    IF (NEWSTARS(NT)/=0.0)THEN
      DO NP=1,NMASSBINS
        IF (NP<100) THEN
          MASSSTEP=0.1                          !for range 0.1 - 10.0 Mo, count in increments of 0.1 Mo
          MASSBIN(NP,1,NT)=MINMASS+((NP-1)*MASSSTEP)          !Lower mass limit for this mass bin
          MASSBIN(NP,2,NT)=MASSBIN(NP,1,NT)+MASSSTEP          !Upper mass limit for this mass bin
        ELSE
          MASSSTEP=1                            !above 10.0 Mo, count in increments of 1 Mo
          MASSBIN(NP,1,NT)=10.0+((NP-100)*MASSSTEP)           !Lower mass limit for this mass bin
          MASSBIN(NP,2,NT)=MASSBIN(NP,1,NT)+MASSSTEP          !Upper mass limit for this mass bin
        END IF
        MASSBIN(NP,3,NT)=(MASSBIN(NP,1,NT)+MASSBIN(NP,2,NT))/2     !Average mass in this massbin (not strictly true as not weighted)
        CALL GETFRAC(MASSBIN(NP,1,NT),MASSBIN(NP,2,NT))          !Calculate mass fraction in this bin using Salpeter(1955)
        MASSBIN(NP,4,NT)=MASSFRAC*NEWSTARS(NT)              !Total mass in this bin, in Mo
        MASSBIN(NP,5,NT)=ZMF(NT)*MASSBIN(NP,4,NT)       !Metallicity content in Mo of stars formed (=Z of gas at time formed)
        MASSBIN(NP,6,NT)=XMF(NT)*MASSBIN(NP,4,NT)       !H content in Mo of stars formed
        MASSBIN(NP,7,NT)=YMF(NT)*MASSBIN(NP,4,NT)            !He content in Mo of stars formed
        MASSBIN(NP,8,NT)=REAL(NT)                  !Timestep when these stars formed (convert to real number)
        MASSBIN(NP,9,NT)=(10*(MASSBIN(NP,3,NT)**(-2.5)))/TIMESTEP   !No timesteps on MS from Wood 1992
        MASSBIN(NP,10,NT)=MASSBIN(NP,8,NT)+MASSBIN(NP,9,NT)          !Timestep when stars leave MS
        MASSBIN(NP,13,NT)=LOGRATIO(NT)             ![Alpha/Fe] of stars formed.
        DO NE=1,NET                   !Original element content total mass Mo in this bin for the elements being tracked
          MASSBIN(NP,130+NE,NT)=(ELEMENTSGAS(NE,NT)/GASMASS(NT))*MASSBIN(NP,4,NT)
        END DO
      END DO

    !Tweak top mass bin(s) if necessary to ensure amount allocated to bins =  mass created in this timestep
      TOTAL=0.0        !Total mass allocated into mass bins
      DO NP=1,NMASSBINS
        TOTAL=TOTAL+MASSBIN(NP,4,NT)
      END DO
      ADJUST=NEWSTARS(NT)-TOTAL
      MASSBIN(NMASSBINS,4,NT)=MASSBIN(NMASSBINS,4,NT)+ADJUST

    !If the top mass bin then goes negative, this needs to be cleared by 'smoothing' into the top bins until cleared
      IF (MASSBIN(NMASSBINS,4,NT)<0.0) THEN
```

```
          IF(MODELTYPE=='SINGLE')THEN
            WRITE(50,*)'Massbin smoothing resulted in negative top mass bin at NT=',NT
          END IF
          DO NP=NMASSBINS,1,-1
            IF(MASSBIN(NP,4,NT)>=0.0) EXIT     !stop check if not negative, otherwise process next two lines of code
              MASSBIN(NP-1,4,NT)=MASSBIN(NP-1,4,NT)+MASSBIN(NP,4,NT)
              MASSBIN(NP,4,NT)=0.0          !no END IF statement required
          END DO
        END IF
      END IF

! Reduce gas/increase stars by the amount converted from gas into stars in this timestep
      IF(NEWSTARS(NT)>0.0.AND.NT/=1) THEN  !if NT=1 these are as initially set - although note this ignores any stars made by gas inflow in NT=1
        XISM(NT)=XISM(NT)-(NEWSTARS(NT)*(XISM(NT)/GASMASS(NT)))
        YISM(NT)=YISM(NT)-(NEWSTARS(NT)*(YISM(NT)/GASMASS(NT)))
        ZISM(NT)=ZISM(NT)-(NEWSTARS(NT)*(ZISM(NT)/GASMASS(NT)))
        GASMASS(NT)=GASMASS(NT)-NEWSTARS(NT)
        STARMASS(NT)=STARMASS(NT)+NEWSTARS(NT)
        DO NE=1,NET
          ELEMENTSGAS(NE,NT)=ELEMENTSGAS(NE,NT)-((ELEMENTSGAS(NE,NT)/GASMASS(NT))*NEWSTARS(NT))
        END DO
      END IF

! Stars formed that are below MAXBDWARF should go straight to BROWNDWARF as they do not shine
      DO NP=1,NMASSBINS
        IF (MASSBIN(NP,2,NT)<MAXBDWARF) THEN
          BROWNDWARF(NT)=BROWNDWARF(NT)+MASSBIN(NP,4,NT)
          DO NC=1,NMASSCOLS
            MASSBIN(NP,NC,NT)=0.0                 !empty this bin as contents have gone to remnants
          END DO
        END IF
      END DO

! Stars formed that are above MINBLACKHOLE should go straight to BLACKHOLES as they do not shine/emit/etc but collapse straight away.
      DO NP=1,NMASSBINS
        IF (MASSBIN(NP,1,NT)>=MINBLACKHOLE)THEN
          BLACKHOLES(NT)=BLACKHOLES(NT)+MASSBIN(NP,4,NT)
          DO NC=1,NMASSCOLS
            MASSBIN(NP,NC,NT)=0.0   !empty this mass bin
          END DO
        END IF
      END DO

! Some material currently held in remnants (white dwarfs, neutron stars etc) will explode as SNIA
! Calculate number of SNIA events in this timestep using methodology selected by user in values.in
      IF (SNIATYPE=='Timmes') THEN            !Use results from Timmes 1995 ApJSS 98,617
        TIMELAG=3.0                       !Delay in SNIA production, in Gyrs interpreted from graph in Timmes
          IF (REAL(NT)<(TIMELAG/TIMESTEP)) THEN
            SNIAEVENTS(NT)=0.0
            SNIARATE(NT)=0.0
          ELSE
            SNIARATE(NT)=0.53/6.0          !Events per Gyr per 10^10Mo: 0.53 events per century for Milky Way of mass 6x10^10Mo
            !calculate number of events this timestep based on star mass TIMELAG ago.
            !Ignores the fact some of these stars will have already evolved.
            SNIAEVENTS(NT)=SNIARATE(NT)*TIMESTEP*STARMASS(NT-INT(TIMELAG/TIMESTEP))/(10**3)
```

```
        END IF
      ELSE IF (SNIATYPE=='SB05')THEN              !Use results from Scannapieco and Bildsten 2005 ApJ 629,L85 converted to per Gyr
        !S&B formula: rate per century= delay component (tracks total mass 0.7Gya) + prompt component (tracks SFR)
        IF(NT-INT(0.7/TIMESTEP)<=0) THEN                    !time delay on delay component hasn't kicked in yet
          SNIARATE(NT)=(2.6*(SFR(NT)))/10**5/10**5                    !SNIA rate per century for this gal per 10^10 Mo
        ELSE
          SNIARATE(NT)=((0.044*STARMASS(NT-INT(0.7/TIMESTEP))/10**5/10**5)+(2.6*(SFR(NT))/10**5/10**5))
        END IF
        SNIAEVENTS(NT)=SNIARATE(NT)*STARMASS(NT)/1000/TIMESTEP   !Convert to events per timestep for this galaxy
        IF(REMNANTS(NT)==0.0) SNIARATE(NT)=0.0  !Reset rate/century if events are zero due to zero mass in remnants
      END IF

! Calculate total yields in Mo from the SNIA events in this timestep
      CALL SNIAYIELDS

! Update masses held in stars and gas, and individual elements, as a result of the SNIA events in this timestep
      CALL UPDATE(1)

! Note that material recycled within REMNANTS here, so no update to massbins.
! Stars formed that are above MAXBDWARF and below SNIILOW will form Planetary Nebula if at the end of their life in this timestep
      DO NP=1,NMASSBINS
        DO NQ=1,NT
          IF (MASSBIN(NP,3,NQ)<=MINSNII.AND.MASSBIN(NP,4,NQ)/=0.0) THEN !Select massbins where the stars are the right size to undergo PN
            IF (REAL(NT+1)>MASSBIN(NP,10,NQ)) THEN  !Select from these the massbins where the stars end their life in this timestep
              CALL PNYIELDS(MASSBIN(NP,3,NQ),MASSBIN(NP,4,NQ),MASSBIN(NP,5,NQ),MASSBIN(NP,6,NQ),&
                MASSBIN(NP,7,NQ),MASSBIN(NP,136,NQ),MASSBIN(NP,135,NQ),MASSBIN(NP,138,NQ),&
                NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ)
              DO NC=1,NMASSCOLS
                MASSBIN(NP,NC,NQ)=0.0   !empty this mass bin
              END DO
            END IF
          END IF
        END DO
      END DO

      CALL UPDATE(2)


! Stars formed that are above SNIILOW will form SNII if at the end of their life in this timestep
! If have selected WW data for massive stars, process these, then process all massive stars by Geneva yields (which will give either a
! Geneva extension to WW data, if WW selected, or will process all large and massive stars with Geneva yields).

! If MINSNII<12Mo, then stars of mass between MINSNII and 12Mo (the lowest mass in the WW data) are scaled down from 12).
      IF (LARGE=='WW95')THEN
        DO NP=1,NMASSBINS
          DO NQ=1,NT
            IF (MASSBIN(NP,3,NQ)>MINSNII.AND.MASSBIN(NP,3,NQ)<=40.0) THEN    !Cutoff for Woosley and Weaver 95 data
              IF (REAL(NT+1)>MASSBIN(NP,10,NQ).AND.MASSBIN(NP,4,NQ)/=0.0) THEN    !these stars end their MS lifecycle in this timestep
                CALL SNIIWWYIELDS(MASSBIN(NP,3,NQ),MASSBIN(NP,4,NQ),MASSBIN(NP,5,NQ),&
                  MASSBIN(NP,6,NQ),MASSBIN(NP,7,NQ))
                DO NC=1,NMASSCOLS
```

```
            MASSBIN(NP,NC,NQ)=0.0   !empty this mass bin
          END DO
        END IF
      END IF
    END DO
  END DO

  CALL UPDATE(3)
 END IF

!now process very massive stars - if selected WW, then only massive stars use Geneva yields (large star bins now empty; this code will
! still try to process them but working with empty bins, so will just process the massive stars with Geneva yields)
!Else user has selected all large + massive stars on Geneva yields.
 DO NP=1,NMASSBINS
  DO NQ=1,NT
    IF (MASSBIN(NP,3,NQ)>=MINSNII.AND.MASSBIN(NP,4,NQ)/=0.0) THEN
      IF (REAL(NT+1)>MASSBIN(NP,10,NQ)) THEN  !Star explodes in current timestep: process using results from the Maeder Group
        CALL SNIIGYIELDS(MASSBIN(NP,3,NQ),MASSBIN(NP,4,NQ),MASSBIN(NP,5,NQ),&
          MASSBIN(NP,6,NQ),MASSBIN(NP,7,NQ),MASSBIN(NP,136,NQ),MASSBIN(NP,135,NQ))
        DO NC=1,NMASSCOLS
          MASSBIN(NP,NC,NQ)=0.0  !empty the massbin
        END DO
      END IF
    END IF
  END DO
 END DO

 CALL UPDATE(4)

! Calculate number of SNII events per century per 10^10 Mo stars
SNIIRATE(NT)=(SNIILEVENTS(NT)+SNIIMEVENTS(NT))*(1.0/(TIMESTEP*10**7))*(STARMASS(NT)/10**5/10**5)

! Gas flowing out of the model galaxy
!(cannot easily use energy calcs with this model so use fixed time, or proportional to stars formed, as set by user)
!GASOUT parameter used differently depending on option: either TIME in Gyrs of outflow or LOADing factor
 IF (GASOUTMETHOD=='TIME')THEN            !gas loss at a specific time
  IF (TIMENOW(NT)>=GASOUT)THEN
    FLOWOUT(NT)=FLOWOUT(NT)+GASMASS(NT)    !all the gas flows out in this timestep, including any subsequent gas as a consequence of post-wind evolution
    FLOW(NT)=FLOW(NT)-FLOWOUT(NT)
    GASMASS(NT)=0.0
    GALMASS(NT)=GALMASS(NT)-FLOWOUT(NT)
    XISM(NT)=0.0
    YISM(NT)=0.0
    ZISM(NT)=0.0
    XMF(NT)=0.0
    YMF(NT)=0.0
    ZMF(NT)=0.0
    DO NE=1,NET
      ELEMENTSGAS(NE,NT)=0.0
    END DO
  ELSE
    FLOWOUT(NT)=0.0                        !Currently do not flow out gas
  END IF
 ELSE IF (GASOUTMETHOD=='LOAD')THEN   !gas loss proportional to new stars made
```

```
      IF (GASMASS(NT)>=GASOUT*NEWSTARS(NT))THEN
        FLOWOUT(NT)=GASOUT*NEWSTARS(NT)
      ELSE
        FLOWOUT(NT)=GASMASS(NT)     !Limit outflow if not enough gas to use the gas loading
      END IF
      IF(FLOWOUT(NT)/=0.0)THEN ! Some gas outflow in this timestep so update galaxy parameters
        !Flow out elements, keeping proportions (i.e. do not assume outflow is differentiated)
        XISM(NT)=XISM(NT)*(1-(FLOWOUT(NT)/GASMASS(NT)))
        YISM(NT)=YISM(NT)*(1-(FLOWOUT(NT)/GASMASS(NT)))
        ZISM(NT)=ZISM(NT)*(1-(FLOWOUT(NT)/GASMASS(NT)))
        DO NE=1,NET
          ELEMENTSGAS(NE,NT)=ELEMENTSGAS(NE,NT)*(FLOWOUT(NT)/GASMASS(NT))
        END DO
        FLOW(NT)=FLOW(NT)-FLOWOUT(NT)
        GASMASS(NT)=GASMASS(NT)-FLOWOUT(NT)  !update after updating the elements
        GALMASS(NT)=GALMASS(NT)-FLOWOUT(NT)  !mass fractions remain unchanged as outflow not differentiated
      END IF
    END IF

! Update galaxy parameters at end of this timestep
    IF (GASMASS(NT)/=0.0)THEN
      XMF(NT)=XISM(NT)/GASMASS(NT)
      YMF(NT)=YISM(NT)/GASMASS(NT)
      ZMF(NT)=ZISM(NT)/GASMASS(NT)
    ELSE
      XMF(NT)=0.0
      YMF(NT)=0.0
      ZMF(NT)=0.0
    END IF

! Deal with computer rounding errors
! Adjust the greater of gas or stars to clear rounding errors, and send warning to file if non-trivial
    MASSCHECK(NT)=GALMASS(NT)-STARMASS(NT)-GASMASS(NT)
    IF(MASSCHECK(NT)>GALMASS(NT)/10**6.AND.MODELTYPE=='SINGLE') THEN   !roundings below this may occur due to way numbers stored in code
      WRITE(50,*)'Mass conservation error not due to roundings in NT=',NT,'Mass check=',MASSCHECK(NT)
    END IF
    IF(GASMASS(NT)>=STARMASS(NT))THEN
      GASMASS(NT)=GASMASS(NT)+MASSCHECK(NT)
    ELSE
      STARMASS(NT)=STARMASS(NT)+MASSCHECK(NT)
    END IF

! Check total held in mass bins and remnants = total held in starmass
    DO NP=1,NMASSBINS
      DO NQ=1,NT
        STARCHECK(NT)=STARCHECK(NT)+MASSBIN(NP,4,NQ)
      END DO
    END DO
    STARCHECK(NT)=STARCHECK(NT)+REMNANTS(NT)  !total held in MASSBINS + REMNANTS
    STARCHECK(NT)=STARCHECK(NT)-STARMASS(NT)  !difference between STARMASS and (MASSBINS+REMNANTS)
    STARMASS(NT)=STARMASS(NT)+STARCHECK(NT)   !adjust starmass if necessary
    IF(STARCHECK(NT)>STARMASS(NT)/10**6.AND.MODELTYPE=='SINGLE')THEN  !roundings below this may occur due to way numbers stored in code
      WRITE(50,*)'Stellar mass conservation error not due to roundings in NT=',NT,'Star check=',STARCHECK(NT)
    END IF

```

```
! Tweak hydrogen mass fraction to clear any rounding errors; give warning if not immaterial
    IF(GASMASS(NT)/=0.0)THEN
      IF(ABS(XMF(NT)+YMF(NT)+ZMF(NT)-1.0)>TWEAK.AND.MODELTYPE=='SINGLE') THEN
        WRITE(50,*)'Hydrogen rounding not minor. XMF=',XMF(NT),'YMF=',YMF(NT),'ZMF=',ZMF(NT),&
            'Total',XMF(NT)+YMF(NT)+ZMF(NT),'NT=',NT
        PRINT*,'Hydrogen rounding not minor XMF=',XMF(NT),'YMF=',YMF(NT),'ZMF=',ZMF(NT),&
            'Total',XMF(NT)+YMF(NT)+ZMF(NT),'NT=',NT
      END IF
      XMF(NT)=XMF(NT)-(XMF(NT)+YMF(NT)+ZMF(NT)-1.0)
    END IF

    IF (NT==NTM.AND.MODELTYPE=='SEARCH')THEN  ! Test print so can track searching software/follow progress
      PRINT*,'Model just run: GALMASS',GALMASSI,'TIME',TIME,'SFR',SFRCONST,'RATE',&
        FLOWINRATE,'START',FLOWINSTART,'DUR',DURATION
    END IF

    END SUBROUTINE EVOLVE






!MAKEINDICES Subroutine to create luminosity-weighted Lick indices from the stars available at the end of each timestep (ie after EVOLVE)

    SUBROUTINE MAKEINDICES

    USE SHARED
    IMPLICIT NONE

    REAL :: STARAGE    !Current age of stars from historic massbins, needed to obtain correct SSP

    DO NQ=1,NT
      DO NP=1,NMASSBINS
        IF(MASSBIN(NP,4,NQ)/=0.0)THEN !There is mass in this bin so get corresponding indices and colours
          CALL B94ISOCHRONES

          !Store the luminosity in Lsolar of the stars in this mass bin
          IF(MASSBIN(NP,3,NQ)==0.0)THEN   !Trap any zero denominators
            MASSBIN(NP,11,NQ)=0.0
          ELSE
            MASSBIN(NP,11,NQ)=ISOCHRONE(4)*(MASSBIN(NP,4,NQ)/MASSBIN(NP,3,NQ))
          END IF
          !Store the colours from the isochrones as unweighted in the MASSBINS array
          MASSBIN(NP,45,NQ)=ISOCHRONE(5)
          DO NI=7,10
            MASSBIN(NP,48+NI,NQ)=ISOCHRONE(NI)
          END DO
          MASSBIN(NP,74,NQ)=ISOCHRONE(6)
          MASSBIN(NP,75,NQ)=ISOCHRONE(11)
          MASSBIN(NP,59,NQ)=ISOCHRONE(12)

          ! Set current age of the stars in this mass bin by comparing timestep when stars formed to current timestep
          STARAGE=(REAL(NT)-MASSBIN(NP,8,NQ))*TIMESTEP

          !Get the SSP indices as selected by user
```

```
          IF (SSPDATA=='W') THEN
            CALL W94INDICES(STARAGE)
            CALL GV98INDICES(STARAGE)      !check GV98 only overwrites index 49 in output SSP array from W94
          ELSE IF (SSPDATA=='V') THEN
            CALL V99INDICES(STARAGE)
            CALL GV98INDICES(STARAGE)
          ELSE IF (SSPDATA=='T') THEN
            CALL T04INDICES(STARAGE)            !Thomas 04 SSPs and colours from Bruzual and Charlot 03
          END IF

          !Get the unweighted indices/other colours for the stars in this mass bin from the selected SSP.
          !This may overwrite above colours.
          DO NI=1,NINDEX
            IF(SSP(NI)/=0.0)THEN
              MASSBIN(NP,NI+20,NQ)=SSP(NI)
            END IF
          END DO
        END IF  !The check that there is mass in this bin
      END DO  !The NP loop
    END DO  !THE NQ=1,NT LOOP - have now updated all mass bins with appropriate indices and luminosity for the current time

    !Calculate the total luminosity of the galaxy in Lsolar at this time= luminosity of all current stars
    DO NP=1,NMASSBINS
      DO NQ=1,NT
        IF(MASSBIN(NP,4,NQ)/=0.0)THEN      !only total up for bins which contain stars
          TOTLUM(NT)=TOTLUM(NT)+MASSBIN(NP,11,NQ)
        END IF
      END DO
    END DO

    !convert all colours, and indices held as magnitudes, for linear combination
    DO NP=1,NMASSBINS
      DO NQ=1,NT
        DO NC=1,NMASSCOLS
          IF (NC==21.OR.NC==22.OR.NC==31.OR.NC==32.OR.NC==40.OR.NC==41.OR.(NC>=54.AND.NC<=64).OR.&
            NC==74.OR.NC==75)THEN
            IF(MASSBIN(NP,NC,NQ)/=0.0)THEN
              MASSBIN(NP,NC,NQ)=10**((MASSBIN(NP,NC,NQ))/(-2.5))
            END IF
          END IF
       END DO
      END DO
    END DO

    !Calculate weighting factor of each mass bin and weight all the colours and indices
    DO NP=1,NMASSBINS
      DO NQ=1,NT
        IF(MASSBIN(NP,4,NQ)/=0.0)THEN      !only do this weighting and totalling for bins which contain stars
          MASSBIN(NP,12,NQ)=MASSBIN(NP,11,NQ)/TOTLUM(NT)
          DO NI=1,NINDEX
            MASSBIN(NP,NI+75,NQ)=MASSBIN(NP,NI+20,NQ)*MASSBIN(NP,12,NQ)
          END DO
        END IF
      END DO
    END DO
```

```
    !Calculate the overall integrated luminosity weighted indices at this time
    DO NP=1,NMASSBINS
      DO NQ=1,NT
        IF(MASSBIN(NP,4,NQ)/=0.0)THEN   !only do this weighting for bins which contain stars
          DO NI=1,NINDEX
            INDICES(NI,NT)=INDICES(NI,NT)+MASSBIN(NP,NI+75,NQ)
          END DO
         END IF
      END DO
    END DO

    !Convert magnitudes back
    DO NP=1,NMASSBINS
      DO NQ=1,NT
        DO NC=1,NMASSCOLS
          IF (NC==21.OR.NC==22.OR.NC==31.OR.NC==32.OR.NC==40.OR.NC==41.OR.(NC>=54.AND.NC<=64).OR.&
            NC==74.OR.NC==75.OR.NC==76.OR.NC==77.OR.NC==86.OR.NC==87.OR.NC==95.OR.NC==96.OR.&
            (NC>=109.AND.NC<=119).OR.NC==129.OR.NC==130)THEN
            IF (MASSBIN(NP,NC,NQ)/=0.0)THEN
              MASSBIN(NP,NC,NQ)=(-2.5)*(LOG10(MASSBIN(NP,NC,NQ)))
            END IF
          END IF
         END DO
      END DO
    END DO
    DO NI=1,NINDEX
      IF (NI==1.OR.NI==2.OR.NI==11.OR.NI==12.OR.NI==20.OR.NI==21.OR.(NI>=34.AND.NI<=44).OR.NI==54.OR.NI==55)THEN
        IF (INDICES(NI,NT)/=0.0)THEN
          INDICES(NI,NT)=(-2.5)*(LOG10(INDICES(NI,NT)))
        END IF
      END IF
    END DO

    END SUBROUTINE MAKEINDICES






!GETFRAC Subroutine to get mass fraction of stars in a specified mass range using Salpeter IMF.  MINMASS and MAXMASS are the lower and
!upper limits for any star, MASSBIN(N,1,NT) and MASSBIN(N,2,NT) are the lower and upper limits for this selected range.

    SUBROUTINE GETFRAC(LOWER,UPPER)

    USE SHARED
    IMPLICIT NONE

    REAL :: LOWER,UPPER     !Input  Lower and upper mass values for the range being evaluated

    IF(SFRCONST==0.0)THEN
      MASSFRAC=0.0         !stops the calculation trying to divide by zero
    ELSE
      TOTRANGE=(LOWER**(1.0-IMFINDEX))-(UPPER**(1.0-IMFINDEX))    !Total mass in this range
```

```
    TOTMASS=(MINMASS**(1.0-IMFINDEX))-(MAXMASS**(1.0-IMFINDEX)) !Total overall mass in stars
    MASSFRAC=TOTRANGE/TOTMASS                         !Mass fraction formed
  END IF

  END SUBROUTINE GETFRAC






!INTERPOLATE Subroutine to find which values to interpolate between, and derive weightings, for interpolating data from monotonically
! increasing one-dimensional ARRAY.
! Output flags are 0: failed, 1:success, 2:TESTVAL too low, used nearest values, 3:TESTVAL too high, used nearest values


  SUBROUTINE INTERPOLATE(TESTVAL,NGRID,ARRAY,LOWVAL,WEIGHTLOW,WEIGHTHIGH,FLAG)

  USE SHARED
  IMPLICIT NONE

  REAL :: DIFFERENCE          !In code Difference between two consecutive values in ARRAY
  REAL :: CHECKHIGH           !In code Test the value in ARRAY to see if it's above the TESTVAL
  REAL :: CHECKLOW            !In code Test the value in ARRAY to see if it's below the TESTVAL
  INTEGER :: FLAG           !Output  Flag = 1 if in tabulated range, 2 if too low, 3 if too high, 0 otherwise
  INTEGER :: LOWVAL           !Output  Position in ARRAY which is the last item lower than the one being tested
  INTEGER :: NGRID            !Input   Number of grid values in ARRAY
  REAL :: TESTVAL             !Input   Value of parameter being tested against grid values
  REAL :: WEIGHTHIGH          !Output  Weighting for data value at high grid value (WEIGHTLOW + WEIGHTHIGH = 1)
  REAL :: WEIGHTLOW           !Output  Weighting for data value at low grid value

  REAL ::ARRAY(NGRID)         !Input   1-d Array of grid values

! Zero the variables used in this subroutine
  LOWVAL=0
  WEIGHTLOW=0.0
  WEIGHTHIGH=0.0
  FLAG=0

! For test values below tabulated lower limit of ARRAY, set to use lowest value in ARRAY and set warning flag to 2
  IF (TESTVAL<ARRAY(1)) THEN
    WEIGHTLOW=1.0
    WEIGHTHIGH=0.0
    LOWVAL=1
    FLAG=2

! For test values above tabulated upper limit of ARRAY, set to use highest value in ARRAY and set warning flag to 3
  ELSE IF (TESTVAL>ARRAY(NGRID)) THEN
    WEIGHTLOW=0.0
    WEIGHTHIGH=1.0
    LOWVAL=NGRID-1
    FLAG=3
  END IF

! Find value to go from if neither of the above is true ie TESTVAL is within the range of ARRAY, or set to min/max value in ARRAY
```

```
      DO NJ=1,(NGRID-1)
        IF (TESTVAL>=ARRAY(NJ).AND.TESTVAL<=ARRAY(NJ+1)) THEN  !TESTVAL is between these two values in ARRAY
          CHECKLOW=ARRAY(NJ)
          CHECKHIGH=ARRAY(NJ+1)
          DIFFERENCE=CHECKHIGH-CHECKLOW
          IF (DIFFERENCE>0.0) THEN
            WEIGHTLOW=(CHECKHIGH-TESTVAL)/DIFFERENCE
          ELSE IF (DIFFERENCE==0.0) THEN !trap if two rows are the same
            WEIGHTLOW=0.0
          ELSE IF (DIFFERENCE<0.0.AND.MODELTYPE=='SINGLE') THEN  !send error message
            WRITE (50,*) 'WARNING! Input data not monotonic within INTERPOLATE'
          END IF

! Set outputs from this subroutine if TESTVAL is correctly interpolated within ARRAY
          WEIGHTHIGH=1.0-WEIGHTLOW
          LOWVAL=NJ     !Lower grid value to interpolate between
          FLAG=1        !Flag set to indicate success
        END IF
      END DO

      END SUBROUTINE INTERPOLATE






! SNIA YIELDS Subroutine to give yields for SNIA events in the current timestep, using data from Nomoto et al. 1984 ApJ 286,644 table 4
! model W7 (stable isotopes).  This assumes all SNIA result from accreting white dwarf stars, each of the same mass 1.385Mo
! Note this means assuming mass of REMNANTS is magically all stars of this mass.
! This subroutines updates values for EJECTED(NET) (new and recycled material ejected in Mo).  XSNIA is zero, by definition.

      SUBROUTINE SNIAYIELDS
      USE SHARED
      IMPLICIT NONE

      SNIAMASS=1.378*SNIAEVENTS(NT)        !1.378 is total of elements taken from tables 1&4 from Nomoto = total mass disrupted into ISM = total star (no remnant)

 !Adjust SNIAMASS downwards if necessary
      IF (SNIAMASS>REMNANTS(NT)) THEN
        SNIAMASS=REMNANTS(NT)
        IF(MODELTYPE=='SINGLE')THEN
          WRITE(50,*)'Too many SNIA events calculated for timestep',NT,'not enough material &
            held in REMNANTS so corrected to max REMNANTS'
        END IF
      END IF

!Calculate total yield in Mo for metals, helium and tracked elements, for the SNIA events in this timestep
      INCREM=INCREM-SNIAMASS       !Star totally destroyed
      INCISM=INCISM+SNIAMASS
!INCISMX, INCISMY, DECSTARSX and DECSTARSY are all zero, as initial star consists only of metals
      INCISMX=0.0
      INCISMY=0.0
      DECSTARSX=0.0
```

```
      DECSTARSY=0.0
      INCISMZ=INCISMZ+SNIAMASS          !Mass of metals from SNIA ie whole star forms metals in ISM
      DECSTARSZ=DECSTARSZ-SNIAMASS

      EJECTED(1)=(.023)*SNIAMASS     ! Mg
      EJECTED(2)=(.771)*SNIAMASS     ! Fe
      EJECTED(3)=(.165)*SNIAMASS     ! Si
      EJECTED(4)=(.084)*SNIAMASS     ! S
      EJECTED(5)=(.140)*SNIAMASS     ! O
      EJECTED(6)=(.032)*SNIAMASS     ! C
      EJECTED(7)=(.041)*SNIAMASS     ! Ca
      EJECTED(8)=(2.58E-8)*SNIAMASS  ! N
      EJECTED(9)=(0.0125)*SNIAMASS   ! Ne
      EJECTED(10)=(1.8E-5)*SNIAMASS  ! Na
      EJECTED(11)=(6.6E-4)*SNIAMASS  ! Al
      EJECTED(12)=(0.0230)*SNIAMASS  ! Ar
      EJECTED(13)=(0.01086)*SNIAMASS ! Cr
      EJECTED(14)=(0.07228)*SNIAMASS ! Ni

      END SUBROUTINE SNIAYIELDS






! PN YIELDS Subroutine to obtain table of yield data at input Z from linear interpolation between tabulated values for Planetary Nebula.
! Data source as specified at start of GCE_main - from Renzini & Voli 1981, Gavilan et al 2005 or van den Hoek and Groenewegen 1997.
! If mass or metallicity out of range of the data being used, INTERPOLATE will "pull back" to the max/min as appropriate.

      SUBROUTINE PNYIELDS(AVSTAR,MASSINBIN,ZSTAR,HSTAR,HESTAR,CSTAR,OSTAR,NSTAR,NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ)

      USE SHARED
      IMPLICIT NONE

      REAL :: AVSTAR               !Input   Average mass of a star in this mass range (MASSBIN 3)
      REAL :: CSTAR                !Input   Mass in Mo of carbon in this massbin
      INTEGER :: FLAGM,FLAGZ
      REAL :: HSTAR                !Mass in Mo of hydrogen in this massbin
      REAL :: HESTAR               !Mass in Mo of helium in this massbin
      INTEGER :: LM                !In code Lower mass to interpolate from (output from INTERPOLATE)
      INTEGER :: LZ                !In code Lower metallicity to interpolate from (output from INTERPOLATE)
      REAL :: MASSINBIN            !Input   Total mass of stars in the range being worked with  (MASSBIN 4)
      INTEGER :: NPNC              !Input   No. of components for planetary nebula (columns in selected PN data)
      INTEGER :: NPNM              !Input   No. of masses for planetary nebula (rows in selected PN data)
      INTEGER :: NPNZ              !Input   Max number of metallicities (tables in selected PN data)
      REAL :: NSTAR                !Input   Mass in Mo of nitrogen in this massbin
      REAL :: NSTARS               !In code Number of stars in this mass bin
      REAL :: OSTAR                !Input   Mass in Mo of oxygen in this mass bin
      REAL :: WMHI                 !In code Upper value of mass to interpolate between (output from INTERPOLATE)
      REAL :: WMLOW                !In code Lower value of mass to interpolate between (output from INTERPOLATE)
      REAL :: WZHI                 !In code Upper value of metallicity to interpolate between (output from INTERPOLATE)
      REAL :: WZLOW                !In code Lower value of metallicity to interpolate between (output from INTERPOLATE)
      REAL :: ZSTAR                !Input   Mass in Mo of metals in this massbin (MASSBIN 5)
```

```
      REAL :: INTERPZ(NPNC,NPNM)     !In code 2-d Array of component masses interpolated in Z
      REAL :: INTERPZM(NPNC)         !In code 1-d Array of interpolated (in Z and mass) element yields from selected INTERPZ
      REAL :: PNDATA(NPNC,NPNM,NPNZ) !Input 3-d Array of ejected masses for PNs, from selected paper (RV, GA, VG)
      REAL :: PNM(NPNM)              !Input   1-d arrays of characteristic masses for PN data (RV,GA,VG)
      REAL :: PNZ(NPNZ)              !Input  1-d Array of metallicities at which selected data are tabulated (RV,GA,VG)

! Work out which metallicities to interpolate between and derive weightings for linear interpolation.
      CALL INTERPOLATE(ZSTAR/MASSINBIN,NPNZ,PNZ,LZ,WZLOW,WZHI,FLAGZ)   ! (LZ=Lower metallicity to interpolate from)
      IF(MODELTYPE=='SINGLE')THEN
        IF(FLAGZ==2)THEN
          WRITE(50,*)'Metallicity too low in PNYIELDS, used nearest values. NT=',NT,'historic timestep=',NQ,&
            'Massbin number=',NP,'AVSTAR=',AVSTAR,'MASSINBIN=',MASSINBIN,'ZSTAR=',ZSTAR
        ELSE IF (FLAGZ==3)THEN
          WRITE(50,*)'Metallicity too high in PNYIELDS, used nearest values. NT=',NT,'historic timestep=',NQ,&
            'Massbin number=',NP,'AVSTAR=',AVSTAR,'MASSINBIN=',MASSINBIN,'ZSTAR=',ZSTAR
        END IF
       END IF

! Interpolate in metallicity to get 2-d array of ejected masses at the correct value of Z (INTERPZ)
      DO NM=1,NPNM
        DO NC=1,NPNC
          INTERPZ(NC,NM)=WZLOW*PNDATA(NC,NM,LZ)+WZHI*PNDATA(NC,NM,(LZ+1))
        END DO
      END DO

! Work out which masses to interpolate between and derive weightings for linear interpolation
      CALL INTERPOLATE(AVSTAR,NPNM,PNM,LM,WMLOW,WMHI,FLAGM)

! Interpolate in mass, to get 1-d array of data for this typical mass size at the correct value of Z (INTERPZM)
      DO NC=1,NPNC
        INTERPZM(NC)=WMLOW*INTERPZ(NC,LM)+WMHI*INTERPZ(NC,(LM+1))
      END DO

! If AVSTAR is not in the data range, flag will be 2 or 3, and yields from the nearest available mass in the table.
! Scale yields up or down as appropriate
      IF(FLAGM==2)THEN
        DO NC=1,NPNC
          IF(PNM(1)==0.0) THEN   !Trap any zero denominators
            INTERPZM(NC)=0.0
          ELSE
            INTERPZM(NC)=INTERPZM(NC)*(AVSTAR/PNM(1)) !scale the data down (absolute yields)
          END IF
        END DO
        IF(MODELTYPE=='SINGLE')THEN
          WRITE(50,*)'Mass too low in PNYIELDS, used nearest values NT=',NT,'historic timestep=',NQ,&
            'Massbin number=',NP,'AVSTAR=',AVSTAR,'MASSINBIN=',MASSINBIN,'ZSTAR=',ZSTAR
        END IF
      ELSE IF (FLAGM==3)THEN
        DO NC=1,NPNC
          IF(PNM(NPNM)==0.0)THEN  !Trap any zero denominators
            INTERPZM(NC)=0.0
          ELSE
            INTERPZM(NC)=INTERPZM(NC)*(AVSTAR/PNM(NPNM))  !scale the data up (absolute yields)
          END IF
```

```
      END DO
      IF(MODELTYPE=='SINGLE')THEN
        WRITE(50,*)'Mass too high in PNYIELDS (expected as SNIILOW > data in file), used nearest values NT=',&
          NT,'historic timestep=',NQ,'Massbin number=',NP,'AVSTAR=',AVSTAR,'MASSINBIN=',&
          MASSINBIN,'ZSTAR=',ZSTAR
      END IF
    END IF

! Check no errors
    IF(FLAGM==0.OR.FLAGZ==0.AND.MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'WARNING! Subroutine INTERPOLATE has FAILED within PNYIELDS'
    END IF

!Coding note: INTERPZM(1) is remnant, (2) total metal yield, (3) He yield, (4) Oxygen yield, (5) Carbon yield and (6) Nitrogen yield.
!This is the new material made AND ejected.  Some new material will be made but not ejected (left in remnant), and some original material will be ejected unchanged.

! Work out yields and movements of mass between stars and ISM, in Mo, from the stars in this mass bin all becoming planetary nebula
    IF(AVSTAR==0.0)THEN !Trap any zero denominators
      NSTARS=0.0
    ELSE
      NSTARS=MASSINBIN/AVSTAR                    !number of stars in this mass bin
    END IF

    !calculate changes to ISM and remnants, in Mo.  Increase in ISM=decrease in stars.
    INCREM=INCREM+(INTERPZM(1)*NSTARS)            !Total mass of remnants after PN - assumed to be CO white dwarf no H or He
    INCISM=INCISM+(MASSINBIN-(INTERPZM(1)*NSTARS)) !Total mass of material that was stars and is now returned to ISM as gas

 ! PRINT*,'MASSINBIN',MASSINBIN,'CSTAR',CSTAR,(CSTAR/MASSINBIN)*100,'OSTAR',OSTAR,(OSTAR/MASSINBIN)*100   !TEST
    !calculate ejecta in Mo of newly synthesised material plus recycled existing material
    EJECTED(5)=EJECTED(5)+(INTERPZM(4)*NSTARS)+OSTAR      !newly synthesised plus recycled and ejected oxygen
    EJECTED(6)=EJECTED(6)+(INTERPZM(5)*NSTARS)+CSTAR      !newly synthesised plus recycled and ejected carbon
    EJECTED(8)=EJECTED(8)+(INTERPZM(6)*NSTARS)+NSTAR      !newly synthesised plus recycled and ejected nitrogen

    !Empty the variables
    OSTAR=0.0
    CSTAR=0.0
    NSTAR=0.0

    !calculate movements in mass of X,Y,Z in ISM and stars - consists of both new material and recycled material
    !Total mass of H added to ISM = H in star less H converted to He and metals, some of which will remain in remnant
    INCISMX=INCISMX+HSTAR-((+INTERPZM(3)+INTERPZM(2)+(INTERPZM(1)*(HSTAR/(HSTAR+HESTAR))))*NSTARS)
    !Total mass of He added to ISM = He in star plus He converted from H less He converted to metals, some of which will remain in remnant
    INCISMY=INCISMY+HESTAR+((INTERPZM(3)-(INTERPZM(1)*(HESTAR/(HSTAR+HESTAR))))*NSTARS)
    !Total mass of metals added to ISM = original metals + new metals
    INCISMZ=INCISMZ+ZSTAR+(INTERPZM(2)*NSTARS)

    DECSTARSX=DECSTARSX-HSTAR  !Total mass of H lost from stars - ejected or converted = total H as all lost from remnant
    DECSTARSY=DECSTARSY-HESTAR  !Total mass of He lost from stars - ejected or converted = total He as all lost from remnant
    DECSTARSZ=DECSTARSZ-ZSTAR+(INTERPZM(1)*NSTARS)  !Total mass of metals lost from stars (actually a net increase - smaller star but 100% metal)

```

```
    !Adjust any rounding errors and send warning to file if non-trivial
IF(INCISMX+INCISMY+INCISMZ+DECSTARSX+DECSTARSY+DECSTARSZ>100.0.AND.MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'Rounding errors in PNYIELDS non-trivial'
    END IF
    INCISMX=INCISMX-(INCISMX+INCISMY+INCISMZ+DECSTARSX+DECSTARSY+DECSTARSZ)

    END SUBROUTINE PNYIELDS

!SNII WW YIELDS Subroutine to evaluate yields of ejected mass of components from linear interpolation  between tabulated values for SNIIs
!from Woosley and Weaver 1995.  This only covers stars up to 40Mo, so more massive stars will use yields from the Geneva group.  If the
!range or "large" stars starts below 12Mo, the starting point of this data, the yields are scaled downwards.

    SUBROUTINE SNIIWWYIELDS(AVSTAR,MASSINBIN,ZSTAR,HSTAR,HESTAR)

    USE SHARED
    IMPLICIT NONE

    REAL :: AVSTAR              !Input   Average mass of a star in this mass range (MASSBIN 3)
    INTEGER :: FLAGM,FLAGZ
    REAL :: HSTAR               !Input   Mass in Mo of H in the stars being evolved
    REAL :: HESTAR              !Input   Mass in Mo of He in the stars being evolved
    INTEGER :: LM               !In code Lower mass to interpolate from (output from INTERPOLATE)
    INTEGER :: LZ               !In code Lower metallicity to interpolate from (output from INTERPOLATE)
    REAL :: MASSINBIN           !Input   Total mass of stars in the range being worked with  (MASSBIN 4)
    REAL :: NSTARS              !In code Number of stars in this mass bin
    REAL :: WMHI                !In code Upper weighting for mass to interpolate between (output from INTERPOLATE)
    REAL :: WMLOW               !In code Lower weighting for mass to interpolate between (output from INTERPOLATE)
    REAL :: WZHI                !In code Upper weighting for metallicity to interpolate between (output from INTERPOLATE)
    REAL :: WZLOW               !In code Lower weighting for metallicity to interpolate between (output from INTERPOLATE)
    REAL :: ZSTAR               !Input   Metals in this mass bin  (MASSBIN 5)

    REAL :: INTERPWWZ(NRWT,NMWT)  !In code 2-d array of components and masses, interpolated in Z
    REAL :: INTERPWWZM(NRWT)      !In code 1-d array of components, interpolated in Z and M

! Work out which metallicities to interpolate between and derive weightings for linear interpolation.
    CALL INTERPOLATE(ZSTAR/MASSINBIN,NZWT,WWZ,LZ,WZLOW,WZHI,FLAGZ)
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGZ==2)THEN
        WRITE(50,*)'Metallicity too low in SNIIWWYIELDS, used nearest value. NT=',NT,'historic timestep=',NQ,&
          'Massbin number=',NP,'AVSTAR=',AVSTAR,'MASSINBIN=',MASSINBIN,'ZSTAR=',ZSTAR
      ELSE IF (FLAGZ==3)THEN
        WRITE(50,*)'Metallicity too high in SNIIWWYIELDS, used nearest value. NT=',NT,'historic timestep=',NQ,&
          'Massbin number=',NP,'AVSTAR=',AVSTAR,'MASSINBIN=',MASSINBIN,'ZSTAR=',ZSTAR
      END IF
    END IF
```

```
! Interpolate in metallicity to get 2-d array of data for this Z
    DO NM=1,NMWT
      DO NR=1,NRWT
        INTERPWWZ(NR,NM)=WZLOW*WW(NR,NM,LZ)+WZHI*WW(NR,NM,(LZ+1))
      END DO
    END DO

! Work out which masses to interpolate between and derive weightings for linear interpolation
    CALL INTERPOLATE(AVSTAR,NMWT,WWM,LM,WMLOW,WMHI,FLAGM)
    IF(FLAGM==2)THEN
      DO NR=1,NRWT
        IF (WWM(1)==0.0)THEN !Trap any zero denominators
          INTERPWWZM(NR)=0.0
        ELSE
          INTERPWWZM(NR)=INTERPWWZM(NR)*(AVSTAR/WWM(1)) !scale the data down (absolute yields)
        END IF
      END DO
      IF(AVSTAR<MINSNII.AND.MODELTYPE=='SINGLE')THEN     !some stars will fall into this but are allowed for below with scaling
        WRITE(50,*)'Mass too low in SNIIWWYIELDS, used nearest value. NT=',NT,'historic timestep=',NQ,&
          'Massbin number=',NP,'AVSTAR=',AVSTAR,'MASSINBIN=',MASSINBIN,'ZSTAR=',ZSTAR
      END IF
    ELSE IF (FLAGM==3)THEN
      IF(MODELTYPE=='SINGLE')THEN
        WRITE(50,*)'Mass too high in SNIIWWYIELDS, used nearest value. (note: should not happen) NT=',NT,&
          'historic timestep=',NQ,'Massbin number=',NP,'AVSTAR=',AVSTAR,'MASSINBIN=',MASSINBIN,'ZSTAR=',ZSTAR
      END IF
      DO NR=1,NRWT
        IF(WWM(NMWT)==0.0) THEN    !Trap any zero denominators
          INTERPWWZM(NR)=0.0
        ELSE
          INTERPWWZM(NR)=INTERPWWZM(NR)*(AVSTAR/WWM(NMWT))  !scale the data up (absolute yields)
        END IF
      END DO
    END IF

! Interpolate in mass, to get 1-d array of data for this typical mass size at the correct value of Z (INTERPWWZM)
    DO NR=1,NRWT
      INTERPWWZM(NR)=WMLOW*INTERPWWZ(NR,LM)+WMHI*INTERPWWZ(NR,(LM+1))
    END DO

! Check no errors
    IF(FLAGM==0.OR.FLAGZ==0.AND.MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'WARNING! Subroutine INTERPOLATE has FAILED within SNIIWWYIELDS'
    END IF

!Code note: INTERPWWZM(1) = total mass ejected, (2)= H+He ejected, (3)=He ejected, (4)-(17)=Mg,Ge,Si,S,O,C,Ca,N,Ne,Na,Al,Ar,Cr,Ni ejected

! Work out yields and movements of mass between stars and ISM. in Mo, from the stars in this mass bin all becoming SNII
    IF(AVSTAR==0.0)THEN   !Trap any zero denominators
      NSTARS=0.0
    ELSE
      NSTARS=MASSINBIN/AVSTAR                          !number of stars in this mass bin
    END IF

    !calculate changes to ISM and remnants, in Mo.  Increase in ISM = decrease in stars
```

```
    INCREM=INCREM+((AVSTAR-INTERPWWZM(1))*NSTARS)        !Total mass of remnants after SNII
    INCISM=INCISM+(INTERPWWZM(1)*NSTARS)           !Total mass of material that was stars and is now returned to ISM as gas

    !calculate individual elements ejected ie new plus recycled material
    DO NE=1,NET
      EJECTED(NE)=EJECTED(NE)+(INTERPWWZM(3+NE)*NSTARS)
      IF(EJECTED(NE)<0.0.AND.ZMF(NT-1)/=0.0)THEN    !send a warning
        PRINT*,'negative elements in ww, NT= ',NT,'ELEMENT =',NE,'AV STAR=',AVSTAR
      END IF
    END DO

    !calculate movements in mass of X,Y,Z in ISM and stars - both new material and recycled material
    !WW95 data is for total ejecta, not new yields.
    INCISMX=INCISMX+((INTERPWWZM(2)-INTERPWWZM(3))*NSTARS)                !Total mass of H added to the ISM - H envelope fully ejected
    DECSTARSX=DECSTARSX-HSTAR                                    !Assume all H in star is converted to heavier elements or released to ISM
    INCISMY=INCISMY+(INTERPWWZM(3)*NSTARS)                          !Total mass of helium returned to ISM
    DECSTARSY=DECSTARSY-HESTAR                                   !Assume all He in star is converted to heavier elements or released to ISM
    INCISMZ=INCISMZ+((INTERPWWZM(1)-INTERPWWZM(2))*NSTARS)               !Total mass of  metals returned to ISM
    DECSTARSZ=DECSTARSZ-ZSTAR+((AVSTAR-INTERPWWZM(1))*NSTARS)             !Total mass of metals removed from stars (remnant all metal)


    !Adjust any rounding errors and send warning to file if non-trivial
IF(INCISMX+INCISMY+INCISMZ+DECSTARSX+DECSTARSY+DECSTARSZ>100.0.AND.MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'Rounding errors in WWYIELDS non-trivial'
    END IF
    INCISMX=INCISMX-(INCISMX+INCISMY+INCISMZ+DECSTARSX+DECSTARSY+DECSTARSZ)

    !Number of SNII events from large stars = number of stars exploding in this timestep
    SNIILEVENTS(NT)=SNIILEVENTS(NT)+NSTARS

    END SUBROUTINE SNIIWWYIELDS






!SNII G YIELDS Subroutine to evaluate yields of ejected mass of components from linear interpolation between tabulated values for SNIIs from
!Geneva group (as selected in values.in). If selected WW95 for stars up to 40Mo, this will just give yields for the stars more massive than
!this, else will give yields for all large and massive stars (M>MINSNII)

    SUBROUTINE SNIIGYIELDS(AVSTAR,MASSINBIN,ZSTAR,HSTAR,HESTAR,CSTAR,OSTAR)

    USE SHARED
    IMPLICIT NONE

    REAL :: AVSTAR              !Input   Average mass of a star in this mass range (MASSBIN 3)
    REAL :: CSTAR              !Input   Total mass in Mo of carbon in these stars before the SN event
    INTEGER :: FLAGM,FLAGZ       !Indicator from INTERPOLATE as to whether data in or out of range
    REAL :: HSTAR              !Input   Total mass in Mo of H in these stars
    REAL :: HESTAR              !Input   Total mass in Mo of He in these stars
    INTEGER :: LM              !In code Lower mass to interpolate from (output from INTERPOLATE)
```

```
    INTEGER :: LZ                !In code Lower metallicity to interpolate from (output from INTERPOLATE)
    REAL :: MASSINBIN            !Input   Total mass of stars in the range being worked with  (MASSBIN 4)
    REAL :: NSTARS               !In code Number of stars in this mass bin
    REAL :: OSTAR                !Input   Total mass in Mo of oxygen in these stars before the SN event
    REAL :: WMHI                 !In code Upper value of mass to interpolate between (output from INTERPOLATE)
    REAL :: WMLOW                !In code Lower value of mass to interpolate between (output from INTERPOLATE)
    REAL :: WZHI                 !In code Upper value of metallicity to interpolate between (output from INTERPOLATE)
    REAL :: WZLOW                !In code Lower value of metallicity to interpolate between (output from INTERPOLATE)
    REAL :: ZSTAR                !Input   Metals in this massbin (MASSBIN 5)

    REAL :: INTERPZ(NCGENEVA,NMGENEVA)     !In code 2-d array of components and masses, interpolated in Z
    REAL :: INTERPZM(NMGENEVA)         !In code 1-d array of components, interpolated in Z and M

! Reset the output arrays
    INTERPZ=0.0
    INTERPZM=0.0

! Work out which metallicities to interpolate between and derive weightings for linear interpolation.
    CALL INTERPOLATE(ZSTAR/MASSINBIN,NZGENEVA,GZ,LZ,WZLOW,WZHI,FLAGZ)

! Interpolate in metallicity to get 2-d array of data for this Z
    DO NM=1,NMGENEVA
      DO NC=1,NCGENEVA
        INTERPZ(NC,NM)=WZLOW*GENEVA(NC,NM,LZ)+WZHI*GENEVA(NC,NM,(LZ+1))
      END DO
    END DO

! If code has used nearest value because metallicity is outside range of data held, send warning to file
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGZ==2)THEN
        WRITE(50,*)'Metallicity too low in SNIIGYIELDS, used nearest value. NT=',NT,'historic timestep=',NQ,&
          'Massbin number=',NP,'AVSTAR=',AVSTAR,'MASSINBIN=',MASSINBIN,'ZSTAR=',ZSTAR
      ELSE IF (FLAGZ==3)THEN
        WRITE(50,*)'Metallicity too high in SNIIGYIELDS, used nearest value. NT=',NT,'historic timestep=',NQ,&
          'Massbin number=',NP,'AVSTAR=',AVSTAR,'MASSINBIN=',MASSINBIN,'ZSTAR=',ZSTAR
      END IF
    END IF

! Work out which masses to interpolate between and derive weightings for linear interpolation
    CALL INTERPOLATE(AVSTAR,NMGENEVA,GM,LM,WMLOW,WMHI,FLAGM)

! Interpolate in mass, to get 1-d array of data for this typical mass size at the correct value of Z (INTERPZM)
    DO NC=1,NCGENEVA
      INTERPZM(NC)=WMLOW*INTERPZ(NC,LM)+WMHI*INTERPZ(NC,(LM+1))
    END DO

! If code has used nearest value because mass is outside range of data held, scale up/down nearest result, and send warning to file
    IF(FLAGM==2)THEN
      DO NC=1,NCGENEVA
        IF(GM(1)==0.0)THEN !Trap any zero denominators
          INTERPZM(NC)=0.0
        ELSE
          INTERPZM(NC)=INTERPZM(NC)*(AVSTAR/GM(1)) !scale the data down (absolute yields)
        END IF
      END DO
      IF(MODELTYPE=='SINGLE')THEN
```

```
        WRITE(50,*)'Mass too low in SNIIGYIELDS, used nearest value. NT=',NT,'historic timestep=',NQ,&
          'Massbin number=',NP,'AVSTAR=',AVSTAR,'MASSINBIN=',MASSINBIN,'ZSTAR=',ZSTAR
      END IF
    ELSE IF (FLAGM==3)THEN
      DO NC=1,NCGENEVA
        IF(GM(NMGENEVA)==0.0)THEN !Trap any zero denominators
          INTERPZM(NC)=0.0
        ELSE
          INTERPZM(NC)=INTERPZM(NC)*(AVSTAR/GM(NMGENEVA))  !scale the data up (absolute yields)
        END IF
      END DO
      IF(MODELTYPE=='SINGLE')THEN
        WRITE(50,*)'Mass too high in SNIIGYIELDS, used nearest value. NT=',NT,'historic timestep=',NQ,&
          'Massbin number=',NP,'AVSTAR=',AVSTAR,'MASSINBIN=',MASSINBIN,'ZSTAR=',ZSTAR
      END IF
    END IF

! Check no errors
    IF(FLAGM==0.OR.FLAGZ==0.AND.MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'WARNING! Subroutine INTERPOLATE has FAILED within SNIIGYIELDS'
    END IF

! Work out yields and movements of mass between stars and ISM, in Mo, from the stars in this mass bin all becoming SNII
    IF(AVSTAR==0.0)THEN    !Trap any zero denominators
      NSTARS=0.0
    ELSE
      NSTARS=MASSINBIN/AVSTAR                              !number of stars in this mass bin
    END IF

    !Calculate changes to ISM and remnants, in Mo.  Increase in ISM=decrease in stars
    INCREM=INCREM+(INTERPZM(5)*NSTARS)                  !Total mass of remnants after SNII
    INCISM=INCISM+(MASSINBIN-(INTERPZM(5)*NSTARS)) !Total mass of material that was stars and is now returned to ISM as gas


    !Calculate movements in mass of X,Y,Z in ISM and stars - both new material and recycled material
    !RECYCLE = Initial mass less remnant(assumed to be all new material), metal yield and helium yield= material per star ejected unaltered into ISM
    RECYCLE=AVSTAR-INTERPZM(5)-INTERPZM(4)-INTERPZM(7)
    INCISMX=INCISMX+(RECYCLE*(HSTAR/MASSINBIN)*NSTARS)                    !Total mass of hydrogen returned to ISM
    DECSTARSX=DECSTARSX-HSTAR                                    !No hydrogen in remnant stars
    INCISMY=INCISMY+(INTERPZM(7)+((HESTAR/MASSINBIN)*(RECYCLE)))*NSTARS      !Total mass of helium returned to ISM = yield plus recycled
    DECSTARSY=DECSTARSY-HESTAR                                    !No He in remnant stars
    INCISMZ=INCISMZ+(INTERPZM(4)+((ZSTAR/MASSINBIN)*RECYCLE))*NSTARS                 !Total mass of metals returned to ISM
    DECSTARSZ=DECSTARSZ-ZSTAR+(INTERPZM(5)*NSTARS)                        !remnant all metal

    !Calculate ejecta of tracked elements: new + recycled material NB: other elements would have yields but these are not inc by Geneva Group so are not tracked here
    EJECTED(6)=EJECTED(6)+(INTERPZM(9)*NSTARS)+(RECYCLE*(CSTAR/MASSINBIN))   !carbon new plus recycled
    EJECTED(5)=EJECTED(5)+(INTERPZM(11)*NSTARS)+(RECYCLE*(OSTAR/MASSINBIN))  !oxygen new plus recycled


    !Adjust any rounding errors and send warning to file if non-trivial
IF(INCISMX+INCISMY+INCISMZ+DECSTARSX+DECSTARSY+DECSTARSZ>100.0.AND.MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'Rounding errors in GYIELDS non-trivial'
    END IF
```

```
    INCISMX=INCISMX-(INCISMX+INCISMY+INCISMZ+DECSTARSX+DECSTARSY+DECSTARSZ)


    !Number of SNII events from massive stars = number of stars exploding in this timestep
    SNIIMEVENTS(NT)=SNIIMEVENTS(NT)+NSTARS

    END SUBROUTINE SNIIGYIELDS






!UPDATE is a subroutine to update the values held in the ISM, in the stars, and in the counters monitoring yields of different elements.
!It is called after each processing event (SNIA, planetary nebulae,SNII)

    SUBROUTINE UPDATE(EVOLUTION)

    USE SHARED
    IMPLICIT NONE

    INTEGER :: EVOLUTION   !a code to indicate the process being run: 1=SNIA, 2=planetary nebula 3=large stars and 4=massive stars

    !update galaxy parameters
    REMNANTS(NT)=REMNANTS(NT)+INCREM
    GASMASS(NT)=GASMASS(NT)+INCISM
    STARMASS(NT)=STARMASS(NT)-INCISM        !star mass includes that held in remnants
    XISM(NT)=XISM(NT)+INCISMX
    YISM(NT)=YISM(NT)+INCISMY
    ZISM(NT)=ZISM(NT)+INCISMZ
    DO NE=1,NET
      ELEMENTSGAS(NE,NT)=ELEMENTSGAS(NE,NT)+EJECTED(NE) !EJECTED is new and recycled  material ejected, so all gas.
      IF(ELEMENTSGAS(NE,NT)<0.0.AND.GASMASS(NT-1)==0.0.AND.EVOLUTION==4)THEN
        PRINT*,'WARNING! Negative elements at end of timestep NT=',NT,'Process=',EVOLUTION,'Element=',NE
      END IF
    END DO

    !update yields monitor - yield of metals in this timestep - new AND RECYCKED material
    YIELDS(EVOLUTION,NT)=INCISMZ

    !update gas mass fractions
    IF(GASMASS(NT)/=0.0)THEN
      XMF(NT)=XISM(NT)/GASMASS(NT)
      YMF(NT)=YISM(NT)/GASMASS(NT)
      ZMF(NT)=ZISM(NT)/GASMASS(NT)
    END IF

    !reset variables and arrays
    INCREM=0.0
    INCISM=0.0
    INCISMX=0.0
    INCISMY=0.0
    INCISMZ=0.0
    DECSTARSX=0.0
    DECSTARSY=0.0
```

```
      DECSTARSZ=0.0
      EJECTED=0.0
      RECYCLE=0.0


      END SUBROUTINE UPDATE






!W94INDICES Subroutine to evaluate spectral features from linear interpolation between tabulated values for Worthey 94 & 97 SSPs.
!Returns all features (including luminosities and mass-to-light ratios) for specified AGE and COMPOSITION.(Note Fluctuation mags and colours
!are overwritten by mags and colours).  Where required data is outside tabulated range, returns warning and uses nearest values.

      SUBROUTINE W94INDICES(STARAGE)

      USE SHARED
      IMPLICIT NONE

      REAL :: INTERPW94A(NINDEX,NZW94)   !W94 SSP data interpolated in age
      REAL :: WALOW,WAHI,WZLOW,WZHI   !Interpolation weightings for age and metallicity
      REAL :: STARAGE
      INTEGER :: FLAGA,FLAGZ  !Check within interpolate whether data being looked up is within data range available
      INTEGER :: LA,LZ     !Position in array of lower age/metallicity to interpolate from

! Work out which ages to interpolate between (LA= lower age to interpolate from)
      CALL INTERPOLATE(STARAGE,NAGEW94,W94AGE,LA,WALOW,WAHI,FLAGA)
      IF(MODELTYPE=='SINGLE')THEN
        IF(FLAGA==2)THEN
          WRITE(50,*)'Age too low in W94INDICES, used nearest value.  Age=',AGE,'NT=',NT
        ELSE IF (FLAGA==3)THEN
          WRITE(50,*),'Age too high in W94INDICES, used nearest value. Age=',AGE,'NT=',NT
        ELSE IF (FLAGA==0.AND.STARAGE<8.0.AND.MASSBIN(NP,5,NQ)/MASSBIN(NP,4,NQ)<0.01)THEN
          WRITE(50,*),'Low age and metal poor star in W94INDICES' !data in table backfilled in READWORTHEY94
        END IF
      END IF

! Interpolate in age to get intermediate array, INTERPW94A
      DO NI=1,NINDEX
        DO NZ=1,NZW94
          INTERPW94A(NI,NZ)=WALOW*W94SSP(NI,NZ,LA)+WAHI*W94SSP(NI,NZ,LA+1)
        END DO
      END DO

! Work out which metallicities to interpolate between (LM=lower metallicity to interpolate from) using Z of stars in current
!mass bin being checked.
      CALL INTERPOLATE(MASSBIN(NP,5,NQ)/MASSBIN(NP,4,NQ),NZW94,W94Z,LZ,WZLOW,WZHI,FLAGZ)
      IF(MODELTYPE=='SINGLE')THEN
        IF(FLAGZ==2)THEN
          WRITE(50,*)'Metallicity too low in W94INDICES, used nearest value. Z=',&
            MASSBIN(NP,5,NQ)/MASSBIN(NP,4,NQ),'NT=',NT
        ELSE IF (FLAGZ==3)THEN
          WRITE(50,*),'Metallicity too high in W94INDICES, used nearest value. Z=',&
```

```
          MASSBIN(NP,5,NQ)/MASSBIN(NP,4,NQ),'NT=',NT
      END IF
    END IF

! Interpolate in metallicity to get array of interpolated W94 indices
    DO NI=1,NINDEX
      SSP(NI)=WZLOW*INTERPW94A(NI,LZ)+WZHI*INTERPW94A(NI,LZ+1)
    END DO

! Check no errors
    IF(FLAGA==0.OR.FLAGZ==0.AND.MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'WARNING! Subroutine INTERPOLATE has FAILED within W94INDICES'
    END IF

    END SUBROUTINE W94INDICES






!GV98INDICES Subroutine to evaluate spectral features from linear interpolation between tabulated values for CaT (Garcia-Vargas et al 98)

    SUBROUTINE GV98INDICES(STARAGE)

    USE SHARED
    IMPLICIT NONE

    REAL :: WALOW,WAHI,WZLOW,WZHI !interpolation weightings
    INTEGER :: LA,LZ,FLAGA,FLAGZ  !outputs from interpolation
    REAL :: INTERPGVA(NINDEX,NZGV)  !intermediate array of Garcia-Vargas data interpolated in age
    REAL :: STARAGE

! Reset flags
    FLAGZ=0
    FLAGA=0

! Interpolate in age
    CALL INTERPOLATE(STARAGE,NAGEGV,GVAGE,LA,WALOW,WAHI,FLAGA) !LA=lower age to interpolate from
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGA==2)THEN
        WRITE(50,*),'Age too low in GV98INDICES, used nearest value. Age=',STARAGE,'NT=',NT
      ELSE IF (FLAGA==3)THEN
        WRITE(50,*)'Age too high in GV98INDICES, used nearest value. Age=',STARAGE,'NT=',NT
      END IF
    END IF
    DO NZ=1,NZGV
      INTERPGVA(49,NZ)=WALOW*GVSSP(49,NZ,LA)+WAHI*GVSSP(49,NZ,LA+1)
    END DO

! Interpolate in metallicity using Z for mass bin currently being checked.
    CALL INTERPOLATE(MASSBIN(NP,5,NQ)/MASSBIN(NP,4,NQ),NZGV,GVZ,LZ,WZLOW,WZHI,FLAGZ) !LS=lower metalicity to interpolate from
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGZ==2)THEN
        WRITE(50,*)'Metallicity too low in GV98INDICES, used nearest value. Z=',MASSBIN(NP,5,NQ)/MASSBIN(NP,4,NQ),'NT=',NT
```

```
      ELSE IF (FLAGZ==3)THEN
        WRITE(50,*)'Metallicity too high in GV98INDICES, used nearest value. Z=',MASSBIN(NP,5,NQ)/MASSBIN(NP,4,NQ),'NT=',NT
      END IF
    END IF

!Update the SSP with the calcuim triplet data
    SSP(49)=WZLOW*INTERPGVA(49,LZ)+WZHI*INTERPGVA(49,LZ+1)

! Check no errors
    IF(FLAGA==0.OR.FLAGZ==0.AND.MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'WARNING! Subroutine INTERPOLATE has FAILED within GV98YIELDS'
    END IF

    END SUBROUTINE GV98INDICES






!V99INDICES Subroutine to evaluate spectral features from linear interpolation between tabulated values for Vazdekis 1999 (an update to Vazdekis 1996)

    SUBROUTINE V99INDICES(STARAGE)

    USE SHARED
    IMPLICIT NONE

    REAL    :: INTERPV99A(NINDEX,NZVZ)
    REAL    :: WZLOW,WZHI,WMOL,WALOW,WAHI,FBIL,FBIH
    INTEGER :: LA,LS,FLAGA,FLAGZ
    REAL    :: STARAGE

!Work out which ages to interpolate between (LA=lower age to interpolate from)
    CALL INTERPOLATE(STARAGE,NAGEVZ,VZAGE,LA,WALOW,WAHI,FLAGA)
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGA==2)THEN
        WRITE(50,*)'Age too low in V99INDICES, nearest value used'
      ELSE IF (FLAGA==3)THEN
        WRITE(50,*)'Agevalue too high in V99INDICES, nearest value used.'
      END IF
    END IF

!Interpolate in age to get intermediate array, INTERPV99A
    DO NI=1,NINDEX
      DO NZ=1,NZVZ
        INTERPV99A(NI,NZ)=WALOW*VZSSP(NI,NZ,LA)+WAHI*VZSSP(NI,NZ,LA+1)
      END DO
    END DO

!Work out which metallicities to interpolate between (LS=lower metallicity to interpolate from)
    CALL INTERPOLATE(MASSBIN(NP,5,NQ)/MASSBIN(NP,4,NQ),NZVZ,TZV,LS,WZLOW,WZHI,FLAGZ)
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGZ==2)THEN
        WRITE(50,*)'Metallicity too low in V99INDICES, nearest value used'
      ELSE IF (FLAGZ==3)THEN
        WRITE(50,*)'Metallicity too high in V99INDICES, nearest value used'
```

```
      END IF
    END IF

!Interpolate in metallicity to get array of interpolated V99 indices
    DO NI=1,NINDEX
      SSP(NI)=WZLOW*INTERPV99A(NI,LS)+WZHI*INTERPV99A(NI,LS+1)
    END DO

! Check no errors
    IF(FLAGZ==0.OR.FLAGA==0.AND.MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'WARNING! Subroutine INTERPOLATE has FAILED within V99INDICES'
    END IF

    END SUBROUTINE V99INDICES






!T04INDICES Subroutine to linearly interpolate between data given by Thomas 04 to give an output array (T04SSP) at the values of AGE,
!LOGZ and RATIO input by the main programme.  It is an alternative to using SSPs by Worthey (W94INDICES) or Vazdekis (V99INDICES).  Includes
!colours from Bruzual and Charlot 2003.

    SUBROUTINE T04INDICES(STARAGE)

    USE SHARED
    IMPLICIT NONE

    REAL :: arrayA(NINDEX),arrayB(NINDEX),arrayC(NINDEX)
    REAL :: arrayD(NINDEX),arrayE(NINDEX),arrayF(NINDEX)        !holding arrays
    INTEGER:: LR,LZ,LA                                          !grid values for interpolation
    REAL :: WLR,WHR,WLZ,WHZ,WLA,WHA                             !weightings for interpolation
    INTEGER :: FLAGR,FLAGZ,FLAGA
    REAL :: STARAGE

! Zero the holding arrays
    arrayA = 0.0
    arrayB = 0.0
    arrayC = 0.0
    arrayD = 0.0
    arrayE = 0.0
    arrayF = 0.0

! Establish values to look up in Thomas data (imported STARAGE with the call, and RATIO is stored as log within MASSBINS)
    IF(MASSBIN(NP,6,NQ)==0.0)THEN   !Trap any zero denominators
      LOGZ=0.0
    ELSE
      LOGZ=LOG10(MASSBIN(NP,5,NQ)/MASSBIN(NP,6,NQ))-LOG10(ZSUN/XSUN)
    END IF

! Find lower value of alpha/fe ratio to interpolate from
      CALL INTERPOLATE(MASSBIN(NP,13,NQ),NRATIOT04,RATIOT04,LR,WLR,WHR,FLAGR)
      IF(MODELTYPE=='SINGLE')THEN
        IF(FLAGR==2)THEN
```

```
        WRITE(50,*)'Alpha/Fe ratio too low in T04INDICES, nearest value used. NT=',NT,'Historic t/step=',NQ,&
          'Alpha/Fe=',MASSBIN(NP,13,NQ)
      ELSE IF (FLAGR==3)THEN
        WRITE(50,*)'Alpha/Fe ratio too high in T04INDICES, nearest value used. NT=',NT,'Historic t/step=',NQ,&
         'Alpha/Fe=',MASSBIN(NP,13,NQ)
      END IF
    END IF

! Find lower value of metallicity to interpolate from
    CALL INTERPOLATE(LOGZ,NZT04,T04Z,LZ,WLZ,WHZ,FLAGZ)
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGZ==2)THEN
        WRITE(50,*)'Metallicity value too low in T04INDICES, nearest value used. NT=',NT,'Historic t/step=',NQ,&
         'Z=',LOGZ
      ELSE IF (FLAGZ==3)THEN
        WRITE(50,*)'Metallicity value too high in T04INDICES, nearest value used. NT=',NT,'Historic t/step=',NQ,&
         'Z=',LOGZ
      END IF
    END IF

! Find lower value of age to interpolate from
    CALL INTERPOLATE(STARAGE,NAGET04,AGET04,LA,WLA,WHA,FLAGA)
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGA==2)THEN
        WRITE(50,*)'Age value too low in T04INDICES, nearest value used. NT=',NT,'Historic t/step=',NQ,&
         'Age=',STARAGE
      ELSE IF (FLAGA==3)THEN
        WRITE(50,*)'Age value too high in T04INDICES, nearest value used. NT=',NT,'Historic t/step=',NQ,&
         'Age=',STARAGE
      END IF
    END IF

! Check no errors
    IF(FLAGR==0.OR.FLAGZ==0.OR.FLAGA==0.AND.MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'WARNING! Subroutine INTERPOLATE has FAILED within T04INDICES'
    END IF

!Interpolate in age then metallicity then ratio if within tabulated area
!   Select the following 1-D arrays (ie rows) from the Thomas data, and interpolate as shown to collapse to single interpolated array:
!   arrayG (upper age, upper Z, upper R)
!   arrayH (lower age, upper Z, upper R)
!                 interpolate these by age to give arrayC (interp age, upperZ, upper R)
!   arrayI (upper age, lower Z, upper R)
!   arrayJ (lower age, lower Z, upper R)
!                 interpolate these by age to give arrayD (interp age, lowerZ, upper R)
!   arrayK (upper age, upper Z, lower R)
!   arrayL (lower age, upper Z, lower R)
!                 interpolate these by age to give arrayE (interp age, upper Z, lowerR)
!   arrayM (upper age, lower Z, lower R)
!   arrayN (lower age, lower Z, lower R)
!                 interpolate these by age to give arrayF (interp age, upper Z, lower R)
!
!   then interpolate arrayC and arrayD by metallicity to give arrayA (interp age, interp Z,upper R)
!   and interpolate arrayE and arrayF by mettallicity to give arrayB (interp age, interp Z, lower R)
!
!   finally interpolate arrayA and arrayB by ratio to give final output interpolated array TH04SSP (interp age, interp Z, interp R)
```

```
    DO NI=1,NINDEX
      arrayF(NI) =WLA*THSSP(NI,LZ,LA,LR)+WHA*THSSP(NI,LZ,(LA+1),LR)                  !interpolate arrayN and arrayM by age
      arrayE(NI) =WLA*THSSP(NI,(LZ+1),LA,LR)+WHA*THSSP(NI,(LZ+1),(LA+1),LR)          !interpolate arrayL and arrayK by age
      arrayB(NI) =WLZ*arrayF(NI)+WHZ*arrayE(NI)                          !interpolate these by metallicity
      arrayC(NI) =WLA*THSSP(NI,(LZ+1),LA,(LR+1))+WHA*THSSP(NI,(LZ+1),(LA+1),(LR+1))   !interpolate arrayH and arrayG by age
      arrayD(NI) =WLA*THSSP(NI,LZ,LA,(LR+1))+WHA*THSSP(NI,LZ,(LA+1),(LR+1))          !interpolate arrayJ and arrayI by age
      arrayA(NI) =WLZ*arrayD(NI)+WHZ*arrayC(NI)                          !interpolate these by metallicity
      SSP(NI)=WLR*arrayB(NI)+WHR*arrayA(NI)                              !interpolate arrayB and arrayA by ratio
    END DO

! Check no errors
    IF(FLAGR==0.OR.FLAGZ==0.OR.FLAGA==0.AND.MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'WARNING! Subroutine INTERPOLATE has FAILED within T04INDICES'
    END IF

    END SUBROUTINE T04INDICES






!B94ISOCHRONES subroutine to get the luminosity and colour data from Bertelli et al 1994 isochrones.  During the read-in of the data
!(READBERTELLI), the age is converted from log(age) to actual age in years, and the bolometric magnitude is converted to the luminosity
!in solar units, which will enable the indices to be weighted by the luminosity in the MAKEINDICES subroutine.

!ISOCHRONE data is all averaged over stars of different temperatures but same mass, age and metallicity, as follows:
! ISOCHRONE(1)= interpolated age of isochrone
! ISOCHRONE(2)= interpolated mass of isochrone
! ISOCHRONE(3)= average log effective temperature of stars of this mass, age, metallicity
! ISOCHRONE(4)= luminosity in solar units (converted from Mbol within subroutine READBERTELLI)
! ISOCHRONE(5)= absolute visual magnitude
! ISOCHRONE(6)to(12)= colour indices as follows: (U-B),(B-V),(V-R),(V-I),(V-J),(V-H),(V-K)
! ISOCHRONE(13)= luminosity function for the case of the Salpeter law


    SUBROUTINE B94ISOCHRONES

    USE SHARED
    IMPLICIT NONE

!Set up values for lower limits for interpolation, for use in this subroutine only, and indicator flags
    INTEGER :: ZLOW,ALOW,PLOW,QLOW,RLOW,SLOW,TLOW,ULOW,VLOW,WLOW,XLOW,YLOW
    INTEGER :: FLAGA,FLAGZ,FLAGP,FLAGQ,FLAGR,FLAGS

!Set up weightings for upper and lower limits for interpolation, for use in this subroutine only
    REAL :: WLOWZ,WLOWA,WLOWP,WLOWQ,WLOWR,WLOWS,WLOWT,WLOWU,WLOWV,WLOWW,WLOWX,WLOWY
    REAL :: WHIZ,WHIA,WHIP,WHIQ,WHIR,WHIS,WHIT,WHIU,WHIV,WHIW,WHIY

!Set up temporary arrays for use in this subroutine only
    REAL :: ArrayP(NISOM,NISOC),ArrayQ(NISOM,NISOC),ArrayR(NISOM,NISOC),ArrayS(NISOM,NISOC)
    REAL :: ArrayT(NISOC),ArrayU(NISOC),ArrayV(NISOC),ArrayW(NISOC),ArrayX(NISOC),ArrayY(NISOC)
```

```
    REAL :: HoldMass(NISOM)

!Zero the temporary arrays
    ArrayP=0.0
    ArrayQ=0.0
    ArrayR=0.0
    ArrayS=0.0
    ArrayT=0.0
    ArrayU=0.0
    ArrayV=0.0
    ArrayW=0.0
    ArrayX=0.0
    ArrayY=0.0
    HoldMass=0.0

!Find out which tables of metallicity are needed
    CALL INTERPOLATE(MASSBIN(NP,5,NQ)/MASSBIN(NP,4,NQ),NISOZ,BERTELLIZ,ZLOW,WLOWZ,WHIZ,FLAGZ)
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGZ==2)THEN
        WRITE(50,*)'Metallicity too low in B94ISOCHRONES, nearest value used. NT=',NT,'Historic t/step=',&
          NQ,'Z=',MASSBIN(NP,5,NQ)/MASSBIN(NP,4,NQ)
      ELSE IF(FLAGZ==3) THEN
        WRITE(50,*)'Metallicity too high in B94ISOCHRONES, nearest value used. NT=',NT,'Historic t/step=',&
          NQ,'Z=',MASSBIN(NP,5,NQ)/MASSBIN(NP,4,NQ)
      END IF
    END IF

!Calculate the age of the star being checked
    AGESTAR=TIMENOW(NT)-(MASSBIN(NP,8,NQ)*TIMESTEP) !TESTTIMENOW-(TESTAGE*TIMESTEP)
    AGESTAR=AGESTAR*(10**9)   !Bertelli ages are in years not Gyrs

!Find out which ages are needed
    CALL INTERPOLATE(AGESTAR,NISOA,BERTELLIAGE,ALOW,WLOWA,WHIA,FLAGA)
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGA==2) THEN
        WRITE(50,*)'Age too low in B94ISOCHRONES, nearest value used. NT=',NT,'Historic t/step=',NQ,'Age=',AGESTAR
      ELSE IF(FLAGA==3) THEN
        WRITE(50,*)'Age too high in B94ISOCHRONES, nearest value used. NT=',NT,'Historic t/step=',NQ,'Age=',AGESTAR
      END IF
    END IF

!Create intermediate 2-D arrays giving all masses at the upper and lower ages and metallicities
!Due to nature of Bertelli data, these will have different numbers of rows (ie blank rows at bottom of array)
    DO NM=1,BERTELLIMN(ZLOW,ALOW)
      DO NC=1,NISOC
        ArrayP(NM,NC)=BERTELLI(ZLOW,ALOW,NM,NC)
      END DO
    END DO
    DO NM=1,BERTELLIMN(ZLOW,ALOW+1)
      DO NC=1,NISOC
        ArrayQ(NM,NC)=BERTELLI(ZLOW,ALOW+1,NM,NC)
      END DO
    END DO
    DO NM=1,BERTELLIMN(ZLOW+1,ALOW)
      DO NC=1,NISOC
```

```
        ArrayR(NM,NC)=BERTELLI(ZLOW+1,ALOW,NM,NC)
      END DO
    END DO
    DO NM=1,BERTELLIMN(ZLOW+1,ALOW+1)
      DO NC=1,NISOC
        ArrayS(NM,NC)=BERTELLI(ZLOW+1,ALOW+1,NM,NC)
      END DO
    END DO

!Find the location of the upper and lower masses in these arrays at each combination of upper and lower ages and metallicities
    DO NM=1,BERTELLIMN(ZLOW,ALOW)
      HoldMass(NM)=ArrayP(NM,2)
    END DO
    CALL INTERPOLATE(MASSBIN(NP,3,NQ),BERTELLIMN(ZLOW,ALOW),HoldMass,PLOW,WLOWP,WHIP,FLAGP)
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGP==2) THEN
        WRITE(50,*)'Mass (P) too low in B94ISOCHRONES, nearest value used.  NT=',NT,'Historic timestep=',&
          NQ,'Mass=',MASSBIN(NP,3,NQ)
      ELSE IF(FLAGP==3) THEN
        WRITE(50,*)'Mass (P) too high in B94ISOCHRONES, nearest value used.  NT=',NT,'Historic timestep=',&
           NQ,'Mass=',MASSBIN(NP,3,NQ)
      END IF
    END IF

    HoldMass=0.0   !reset

    DO NM=1,BERTELLIMN(ZLOW,ALOW+1)
      HoldMass(NM)=ArrayQ(NM,2)
    END DO
    CALL INTERPOLATE(MASSBIN(NP,3,NQ),BERTELLIMN(ZLOW,ALOW+1),HoldMass,QLOW,WLOWQ,WHIQ,FLAGQ)
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGQ==2) THEN
        WRITE(50,*)'Mass (Q) too low in B94ISOCHRONES, nearest value used.  NT=',NT,'Historic timestep=',&
           NQ,'Mass=',MASSBIN(NP,3,NQ)
      ELSE IF(FLAGQ==3) THEN
        WRITE(50,*)'Mass (Q) too high in B94ISOCHRONES, nearest value used.  NT=',NT,'Historic timestep=',&
           NQ,'Mass=',MASSBIN(NP,3,NQ)
      END IF
    END IF

    HoldMass=0.0   !reset

    DO NM=1,BERTELLIMN(ZLOW+1,ALOW)
      HoldMass(NM)=ArrayR(NM,2)
    END DO
    CALL INTERPOLATE(MASSBIN(NP,3,NQ),BERTELLIMN(ZLOW+1,ALOW),HoldMass,RLOW,WLOWR,WHIR,FLAGR)
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGR==2) THEN
        WRITE(50,*)'Mass (R) too low in B94ISOCHRONES, nearest value used.  NT=',NT,'Historic timestep=',&
           NQ,'Mass=',MASSBIN(NP,3,NQ)
      ELSE IF(FLAGR==3)THEN
        WRITE(50,*)'Mass (R) too high in B94ISOCHRONES, nearest value used.  NT=',NT,'Historic timestep=',&
           NQ,'Mass=',MASSBIN(NP,3,NQ)
      END IF
    END IF
```

```
    HoldMass=0.0   !reset

    DO NM=1,BERTELLIMN(ZLOW,ALOW)
      HoldMass(NM)=ArrayS(NM,2)
    END DO

    CALL INTERPOLATE(MASSBIN(NP,3,NQ),BERTELLIMN(ZLOW+1,ALOW+1),HoldMass,SLOW,WLOWS,WHIS,FLAGS)
    IF(MODELTYPE=='SINGLE')THEN
      IF(FLAGS==2)THEN
        WRITE(50,*)'Mass (S) too low in B94ISOCHRONES, nearest value used.  NT=',NT,&
          'Historic timestep=',NQ,'Mass=',MASSBIN(NP,3,NQ)
      ELSE IF(FLAGS==3)THEN
        WRITE(50,*)'Mass (S) too high in B94ISOCHRONES, nearest value used.  NT=',NT,&
          'Historic timestep=',NQ,'Mass=',MASSBIN(NP,3,NQ)
      END IF
    END IF

    HoldMass=0.0   !reset

!Create intermediate 1-D arrays giving interpolated masses at the upper and lower ages and metallicities
    DO NC=1,NISOC
      ArrayT(NC)=WLOWP*ArrayP(PLOW,NC)+WHIP*ArrayP(PLOW+1,NC)
      ArrayU(NC)=WLOWQ*ArrayQ(QLOW,NC)+WHIQ*ArrayQ(QLOW+1,NC)
      ArrayV(NC)=WLOWR*ArrayR(RLOW,NC)+WHIR*ArrayR(RLOW+1,NC)
      ArrayW(NC)=WLOWS*ArrayS(SLOW,NC)+WHIS*ArrayS(SLOW+1,NC)
    END DO

!Create intermediate 1-D arrays giving interpolated masses and ages at the upper and lower metallicities
    DO NC=1,NISOC
      ArrayX(NC)=WLOWA*ArrayT(NC)+WHIA*ArrayU(NC)
      ArrayY(NC)=WLOWA*ArrayV(NC)+WHIA*ArrayW(NC)
    END DO

!Create final 1-D array giving colours and luminosity data at interpolated mass, age and metallicity
    DO NC=1,NISOC
      ISOCHRONE(NC)=WLOWZ*ArrayX(NC)+WHIZ*ArrayY(NC)
    END DO

! Check no errors
IF(FLAGA==0.OR.FLAGZ==0.OR.FLAGP==0.OR.FLAGQ==0.OR.FLAGR==0.OR.FLAGS==0.AND.MODELTYPE=='SINGLE')THEN
      WRITE(50,*)'WARNING! Subroutine INTERPOLATE has FAILED within B94ISOCHRONES'
    END IF

    END SUBROUTINE B94ISOCHRONES





!SINGLEOUTPUTS A subroutine to print some details with time, and to store detailed values over time into an output
!file for separate graphing, from single run of code.
!Also to produce a table of final synthetic indices and compare these with the selected input observable data
! Brad's list was for the following, plotted against time:
```

```
! Z, Mg/Fe, SNIA rates, SNII rates, SFR, Gas mass, gas density
! add: all elements, colours and Lick indices with time?

    SUBROUTINE SINGLEOUTPUTS

    USE SHARED
    IMPLICIT NONE

 754  FORMAT(I5,96F50.10)                                   !plotdata.out
 757  FORMAT(' ',F10.2,8E15.3,F10.2)                        !table 1
 759  FORMAT(' ',F10.2,4E20.4,F20.4,A,E20.6)                !table 2
 756  FORMAT(' ',F10.2,2E20.4,F20.2,5(F20.4,A))             !table 2a
 758  FORMAT(' ',A,F6.3,A)                                  !table 3
 753  FORMAT(' ',A,5F15.3)                                  !table 4

!Output data to a file which can be used separatly for plotting graphs
    OPEN(UNIT=60,FILE='plotdata.out',STATUS='REPLACE')
    WRITE(60,*),'NT      T      X      Y      Z      StarsFormed      SNII      SNIA      &
        PN          MassInStars      MassInGas      MassInRems      MassinBH      &
        MassinBD      TotalMass      FlowIn      FlowOut      Luminosity      &
        Radius      SFR      SNIArate      SNIIrate      (BLANK)      &
        Mg-gas      Fe-gas      Si-gas      S-gas      O-gas      &
        C-gas      Ca-gas      N-gas      Ne-gas      Na-gas      &
        Al-gas      Ar-gas      Cr-gas      Ni-gas      CN1      &
        CN2      Ca4227      G4300      Fe4383      Ca4455      &
        Fe4531      C4668      Hb      Fe5015      Mg1      &
        Mg2      Mgb      Fe5270      Fe5335      Fe5406      &
        Fe5709      Fe5782      NaD      Ti01      Ti02      &
        D4000      U      B      V      Rc      &
        Ic      J      H      K      L      &
        Ldash      M      U-V      B-V      V-R      &
        V-I      V-J      V-K      J-H      J-K      &
        J-L      J-Ldash      J-M      HdA      HgA      &
        HdF      HgF      CaT      CaII1      CaII2      &
        CaII3      MgI      U-B      V-H      alpha/Fe      &
        masscheck      starcheck      ZISM      (BLANK)'
    WRITE(60,*),'count   (Gyrs)  %      %      %      Mo/timestep      Yield/Gyr      Yield/Gyr      &
        Yield/Gyr      Total(Mo)      Total(Mo)      Total(Mo)      Total(Mo)      &
        Total(Mo)      Mo      Mo      Mo      Lo      &
        kpc      Mo/Gyr      events/cent/10^10Mo events/cent/10^10Mo (BLANK)      &
        Mo      Mo      Mo      Mo      Mo      &
        Mo      Mo      Mo      Mo      Mo      &
        Mo      Mo      Mo      Mo      .      &
        .      .      .      .      .      &
        .      (was_Fe)      Mo      (BLANK)'

    !First line of results are for absolute start of galaxy, at T=0
WRITE(60,754),0,0.0,XPRIMORDIALMF*100,YPRIMORDIALMF*100,ZPRIMORDIALMF*100,0.0,0.0,0.0,0.0,0.0,0.0,GALMASSI,0.0,0.0,&
        0.0,GALMASSI,0.0,0.0,TOTLUM(1),RADIUS(1),0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,&
        0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,&
        0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,&
        0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0
    DO NT=1,NTM     !just run for the timesteps in the model   !******XMF etc now replaced with separate for stars and gas??
      WRITE(60,754),NT,TIMENOW(NT),XMF(NT)*100,YMF(NT)*100,ZMF(NT)*100,NEWSTARS(NT),&
        (YIELDS(3,NT)+YIELDS(4,NT))/TIMESTEP,&
```

```
        YIELDS(1,NT)/TIMESTEP,YIELDS(2,NT)/TIMESTEP,STARMASS(NT),GASMASS(NT),REMNANTS(NT),&
        BLACKHOLES(NT),BROWNDWARF(NT),&
        GALMASS(NT),FLOWIN(NT),FLOWOUT(NT),TOTLUM(NT),RADIUS(NT),&
        SFR(NT),SNIARATE(NT),SNIIRATE(NT),0.0,&
(ELEMENTSGAS(NE,NT),NE=1,NET),(INDICES(NI,NT),NI=1,NINDEX),LOGRATIO(NT),MASSCHECK(NT),STARCHECK(NT),ZISM(NT),&
        0.0
    END DO

!Collate data on final stars left in galaxy
!note this excludes stars converted to remnants as currently code does not replace initial star with remnant after evolution
    OPEN(UNIT=61,FILE='finalstars.out',STATUS='REPLACE')
    WRITE(61,*),'lowermass     uppermass    avmasss    totalmass'
    DO NP=1,NMASSBINS
      DO NQ=1,NTM
        IF(MASSBIN(NP,4,NQ)/=0.0)THEN
          WRITE(61,*),(MASSBIN(NP,NC,NQ),NC=1,NMASSCOLS)
        END IF
      END DO
    END DO

!Output some tables to screen, for review of model
    PRINT*,'Table 1: some anticipated outputs'
    !COLOURS, MG/FE, OTHER VALUES AND EXPECTED RESULTS FROM THE LITERATURE, GET COMPUTER TO MARK IF OK OR NOT
    !SNIA rates: 0.86 events per centurey per 10^10Msolar Scannapieco & Bildsten 2005. 0.072 events per century for 3.5x 10^10Mo
    !Valiante et al 2009
    PRINT*
    WRITE(*,758)'Final SN1A events/century/10^10 Mo=',SNIARATE(NTM),'  Expect 0.02-0.86'
    WRITE(*,758)'Final B-V=',INDICES(35,NTM),'  Expect 0.91-0.96 from table in Gibson 1997'
    WRITE(*,758)'Final V-K=',INDICES(39,NTM),'  Expect 3.29-3.48 ditto'
    WRITE(*,758)'Final galaxy metallicity',ZMF(NTM)*100,'%'
    PRINT*,'Final galaxy [alpha/Fe] ratio',LOGRATIO(NTM),'(solar=0)'
    WRITE(*,758)'Final mass/light ratio',GALMASS(NTM)/TOTLUM(NTM),' Expect 4.48 from Gavazzi et al 2007' !ApJ 667 Issue 1 p 166-190
    PRINT*
    PRINT*

    PRINT*,'Table 2: Model compared to observational data'
    PRINT*,'Index             Model         Observed      Error on obs   Standard devs'
    NR=0         !Reset integer counter through number of observational data points supplied
    SDTOTAL=0.0 !Reset total of standard deviations (so can calculate average)
    DO NI=1,NINDEX
      IF (OBSERVEDERROR(NI)/=0.0) THEN !have obs. data; calculate  number of standard devs the model is from observed (=beta)
        NR=NR+1
        STANDARDDEV(NI)=(ABS(OBSERVED(NI)-INDICES(NI,NTM)))/OBSERVEDERROR(NI)
      END IF
WRITE(*,753)ANAMES(NI),INDICES(NI,NTM),OBSERVED(NI),OBSERVEDERROR(NI),STANDARDDEV(NI)
      SDTOTAL=SDTOTAL+STANDARDDEV(NI)
    END DO
    PRINT*
    PRINT*,'Average model variation (s/be less than 2)',SDTOTAL/NR
    PRINT*,'Max model variation (s/be less than 2)',MAXVAL(STANDARDDEV)
    PRINT*
    PRINT*
```

```
      CLOSE (UNIT=60)   !plotdata.out

      END SUBROUTINE SINGLEOUTPUTS






!SEARCHOUTPUTS stores data from the parameter-searching version of the code in file searchdata.out; unit in code=70
      SUBROUTINE SEARCHOUTPUTS

      USE SHARED
      IMPLICIT NONE

!Calculate overall average standard deviations the model is from the observed
      NR=0         !Reset integer counter through number of observational data points supplied
      SDTOTAL=0.0 !Reset total of standard deviations (so can calculate average)
      STANDARDDEV=0.0  !Reset standard deviation on each index
      DO NI=1,NINDEX
        IF (OBSERVEDERROR(NI)/=0.0) THEN !have obs. data; calculate chi-squared and number of standard devs the model is from observed
          STANDARDDEV(NI)=(ABS(OBSERVED(NI)-INDICES(NI,NTM)))/OBSERVEDERROR(NI)  !Observed error is at one sigma
          NR=NR+1
        END IF
        SDTOTAL=SDTOTAL+STANDARDDEV(NI)
      END DO

!Write results to file
WRITE(70,*)GALMASSI,TIME,SFRCONST,GASOUTMETHOD,GASOUT,FLOWINRATE,FLOWINSTART,DURATION,&
         SDTOTAL/NR,MAXVAL(STANDARDDEV),&
         ZMF(NTM)*100,(INDICES(NI,NTM),NI=1,NINDEX)

      END SUBROUTINE SEARCHOUTPUTS






!READIN  Subroutine to read in various static data

      SUBROUTINE READIN(NPNC,NPNM,NPNZ,NPNCT,PNDATA,PNM,PNZ,INTERPZ,INTERPZM)

      USE SHARED
      IMPLICIT NONE

      INTEGER :: NPNC,NPNM,NPNZ,NPNCT
      REAL :: PNDATA(NPNC,NPNM,NPNZ)
      REAL :: PNM(NPNM)
      REAL :: PNZ(NPNZ)
      REAL :: INTERPZ(NPNC,NPNM)
      REAL :: INTERPZM(NPNC)

! Read in yields
! SNIIs from Woosley and Weaver 1995
```

```
      IF (LARGE=='WW95') THEN
        CALL READWW95
      END IF
! SNIIs from Geneva Group - options on values.in for different models
      CALL READGENEVA

! Planetary nebula data for intermediate mass star as selected on values.in
      CALL READPN(NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ,NPNCT)

! Read in SSP indices as selected by user
      IF (SSPDATA=='W') THEN
        CALL READWORTHEY94
        CALL READGARCIA
      ELSE IF (SSPDATA=='V') THEN
        CALL READVAZDEKIS
        CALL READGARCIA
      ELSE IF (SSPDATA=='T') THEN
        CALL READT04
      ELSE
        PRINT*,'Error in values.in file for SSP selection'
      END IF

! Read in isochrone data for stellar luminosities and colours
      CALL READBERTELLI

! Calculate solar alpha and Fe peak mass fractions for future comparisons
      ALPHASUNMF=SOLARMF(1)+SOLARMF(3)+SOLARMF(4)+SOLARMF(5)+&
        SOLARMF(7)+SOLARMF(8)+SOLARMF(9)+SOLARMF(10)+&
        SOLARMF(12)   !Mg + Si + S + O + Ca + N + Ne + Na + Ar
      FEPEAKSUNMF=SOLARMF(2)+SOLARMF(13)+SOLARMF(14)  !Fe + Cr + Ni

      END SUBROUTINE READIN



!ZERO Subroutine to set arrays initially to zero or blank (if character arrays)
      SUBROUTINE ZERO

      USE SHARED
      IMPLICIT NONE

! Zero  some arrays - some are set to zero in RESET
      AGET04=0.0
      BERTELLI=0.0
      BERTELLIAGE=0.0
      BERTELLIMN=0    !is an array of integers
      BERTELLIZ=0.0
      BLACKHOLES=0.0
      BROWNDWARF=0.0
      CN1=0.0
      CN1_ERR=0.0
      CN2=0.0
      CN2_ERR=0.0
      CA4227=0.0
```

```
CA4227_ERR=0.0
CA4455=0.0
CA4455_ERR=0.0
C4668=0.0
C4668_ERR=0.0
CAII1=0.0
CAII1_ERR=0.0
CAII2=0.0
CAII2_ERR=0.0
CAII3=0.0
CAII3_ERR=0.0
CAT=0.0
CAT_ERR=0.0
EPRIMORDIALMF=0.0
ELEMENTSGAS=0.0
EJECTED=0.0
FLOW=0.0
FLOWIN=0.0
FLOWOUT=0.0
FE4383=0.0
FE4383_ERR=0.0
FE4531=0.0
FE4531_ERR=0.0
FE4668=0.0
FE4668_ERR=0.0
FE5015=0.0
FE5015_ERR=0.0
FE5270=0.0
FE5270_ERR=0.0
FE5335=0.0
FE5335_ERR=0.0
FE5406=0.0
FE5406_ERR=0.0
FE5709=0.0
FE5709_ERR=0.0
FE5782=0.0
FE5782_ERR=0.0
G4300=0.0
G4300_ERR=0.0
GALMASS=0.0
GASD=0.0
GASMASS=0.0
GENEVA=0.0
GM=0.0
GZ=0.0
GVAGE=0.0
GVSSP=0.0
GVZ=0.0
HBETA=0.0
HBETA_ERR=0.0
HDA=0.0
HDA_ERR=0.0
HGA=0.0
HGA_ERR=0.0
HDF=0.0
HDF_ERR=0.0
HGF=0.0
```

```
HGF_ERR=0.0
INDICES=0.0
ISOCHRONE=0.0
KORN=0.0
KORNZ=0.0
MASSBIN=0.0
MG1=0.0
MG1_ERR=0.0
MG2=0.0
MG2_ERR=0.0
MGB=0.0
MGB_ERR=0.0
MGI=0.0
MGI_ERR=0.0
NEWSTARS=0.0
NAD=0.0
NAD_ERR=0.0
OBSERVED=0.0
OBSERVEDERROR=0.0
RATIOT04=0.0
REMNANTS=0.0
SFR=0.0
SNIAEVENTS=0.0
SNIILEVENTS=0.0
SNIIMEVENTS=0.0
SSP=0.0
STANDARDDEV=0.0
STARMASS=0.0
TB95=0.0
THSSP=0.0
TIMENOW=0.0
TOTLUM=0.0
TIO1=0.0
TIO1_ERR=0.0
TIO2=0.0
TIO2_ERR=0.0
VZAGE=0.0
VZSSP=0.0
VZZ=0.0
W94AGE=0.0
W94Z=0.0
W94SSP=0.0
WWM=0.0
WW=0.0
WWZ=0.0
XMF=0.0
XISM=0.0
YIELDS=0.0
YMF=0.0
YISM=0.0
ZMF=0.0
ZISM=0.0
ANAMES='                '

END SUBROUTINE ZERO
```

```
!RESET Subroutine to reset some arrays to zero
      SUBROUTINE RESET

      USE SHARED
      IMPLICIT NONE

! Zero all arrays that are updated as code runs when using the searching option
      BLACKHOLES=0.0
      BROWNDWARF=0.0
      DECSTARSX=0.0
      DECSTARSY=0.0
      DECSTARSZ=0.0
      EPRIMORDIALMF=0.0
      ELEMENTSGAS=0.0
      EJECTED=0.0
      FLOW=0.0
      FLOWIN=0.0
      FLOWOUT=0.0
      GASMASS=0.0
      GASD=0.0
      INCISMX=0.0
      INCISMY=0.0
      INCISMZ=0.0
      INCREM=0.0
      INCISM=0.0
      INDICES=0.0
      MASSBIN=0.0
      MASSCHECK=0.0
      NEWSTARS=0.0
      RADIUS=0.0
      REMNANTS=0.0
      SFR=0.0
      SNIAEVENTS=0.0
      SNIILEVENTS=0.0
      SNIIMEVENTS=0.0
      STANDARDDEV=0.0
      STARCHECK=0.0
      STARMASS=0.0
      TIMENOW=0.0
      TOTLUM=0.0
      XMF=0.0
      XISM=0.0
      YIELDS=0.0
      YMF=0.0
      YISM=0.0
      ZMF=0.0
      ZISM=0.0

!Zero some variables used in code (shouldn't need to do this?)
      AGE=0.0
      AGESTAR=0.0
      DECSTARSX=0.0
      DECSTARSY=0.0
      DECSTARSZ=0.0
      INCREM=0.0
```

```
      INCISM=0.0
      INCISMX=0.0
      INCISMY=0.0
      INCISMZ=0.0
      MASSFRAC=0.0
      SDTOTAL=0.0
      SNIARATE=0.0
      TIMELAG=0.0
      TOTMASS=0.0
      TOTRANGE=0.0
      VOLUME=0.0
      YSNIA=0.0
      ZSNIA=0.0

      END SUBROUTINE RESET





      SUBROUTINE GETVALS(NPNC,NPNM,NPNZ,NPNCT)   !output the array sizes for planetary nebula work

      USE SHARED
      IMPLICIT NONE

      INTEGER :: NC1              !Output Counter used with stepping software
      INTEGER :: NT1              !Output Counter used with stepping software
      INTEGER :: ND1              !Output Counter used with stepping software
      INTEGER :: NF1              !Output Counter used with stepping software
      INTEGER :: NPNC             !Output Set value for number of yields in planetary nebula data
      INTEGER :: NPNM             !Output Set value for number of masses in planetary nebula data
      INTEGER :: NPNZ             !Output Set value for number of metallicities in planetary nebula data
      INTEGER :: NPNCT            !Output Set value for max number of yields in planetary nebula data
      CHARACTER(60) :: VALFILE    !In code File name selector for values.in file

   96  FORMAT (A10)
   97  FORMAT (A60)
   98  FORMAT (A1)
   99  FORMAT (A20,A20)

!Obtain file of input values from user
      VALFILE='values.in'
      OPEN (UNIT=28,FILE=VALFILE,STATUS='OLD')

!Read in the data from the input values file selected
      DO K=1,NVALUESIN              !note if delete/add to values.in and amend below, need to amend value of
NVALUESIN to exact number
        READ (28,99) BNAME,BVALUE      !28 is the unit number for the file values.in for processing in code (ie not for
prints/plots)
        IF (BNAME(1:15)=='GALMASSI       ') READ(BVALUE(1:20),*) GALMASSI
        IF (BNAME(1:15)=='SFRCONST       ') READ(BVALUE(1:20),*) SFRCONST
        IF (BNAME(1:15)=='LARGE          ') READ(BVALUE(1:20),96) LARGE
        IF (BNAME(1:15)=='SNIATYPE       ') READ(BVALUE(1:20),96) SNIATYPE
        IF (BNAME(1:15)=='SSPDATA        ') READ(BVALUE(1:20),98) SSPDATA
        IF (BNAME(1:15)=='FLOWINRATE     ') READ(BVALUE(1:20),*) FLOWINRATE
        IF (BNAME(1:15)=='FLOWINSTART    ') READ(BVALUE(1:20),*) FLOWINSTART
        IF (BNAME(1:15)=='DURATION       ') READ(BVALUE(1:20),*) DURATION
        IF (BNAME(1:15)=='GASOUT         ') READ(BVALUE(1:20),*) GASOUT
```

```
      IF (BNAME(1:15)=='TIME           ') READ(BVALUE(1:20),*) TIME
      IF (BNAME(1:15)=='INFLOWTYPE     ') READ(BVALUE(1:20),96) INFLOWTYPE
      IF (BNAME(1:15)=='NON-SOLAR      ') READ(BVALUE(1:20),96) NONSOLAR
      IF (BNAME(1:15)=='GASOUTMETHOD   ') READ(BVALUE(1:20),96) GASOUTMETHOD
      IF (BNAME(1:15)=='NF1            ') READ(BVALUE(1:20),*) NF1
      IF (BNAME(1:15)=='PLAN NEB       ') READ(BVALUE(1:20),96) PNDATAIN
      IF (BNAME(1:15)=='MASSIVE        ') READ(BVALUE(1:20),96) MASSIVE
    END DO

    PRINT*

!Set array sizes for planetary nebula data
    IF (PNDATAIN=='RV')THEN  !Renzini & Voli 1981 PN data
      NPNC=6    !number of columns of data yields of PN required
      NPNM=8    !number of masses
      NPNZ=2    !number of metallicities
      NPNCT=6   !number of columns in original data file exc mass col
    ELSE IF (PNDATAIN=='GA')THEN !Gavilan 2005 IMS data
      NPNC=6
      NPNM=52
      NPNZ=5
      NPNCT=12
    ELSE IF (PNDATAIN=='VG')THEN !van den Hoek & Groenewegen 1997 data
      NPNC=6
      NPNM=13
      NPNZ=5
      NPNCT=10
    ELSE
      IF(MODELTYPE=='SINGLE')THEN
        PRINT*,'No file read in for planetary nebula data: check values.in'
      END IF
    END IF

! Set up Solar mass fractions from Grevesse, Asplund, Sauval and Scott 2010
    SOLARMF(1)=0.00070513   ! magnesium
    SOLARMF(2)=0.00130691   ! iron
    SOLARMF(3)=0.00066867   ! silicon
    SOLARMF(4)=0.00031132   ! sulphur
    SOLARMF(5)=0.00578331   ! oxygen
    SOLARMF(6)=0.00238362   ! carbon
    SOLARMF(7)=0.00006458   ! calcium
    SOLARMF(8)=0.00069853   ! nitrogen
    SOLARMF(9)=0.00125628   ! neon
    SOLARMF(10)=0.00002950   ! sodium
    SOLARMF(11)=0.00006909   ! aluminium
    SOLARMF(12)=0.00007415   ! argon
    SOLARMF(13)=0.00001675   ! chromium
    SOLARMF(14)=0.00007226   ! nickel



!Close units
    CLOSE (UNIT=28)  !values.in for computer processing

    END SUBROUTINE GETVALS
```

```
!GETOBS Get observed values and errors.  If using new observational data, may need to check here to ensure code for all indices observed.

      SUBROUTINE GETOBS

      USE SHARED
      IMPLICIT NONE

      CHARACTER INFIL*60

   77  FORMAT (A60)
   79  FORMAT (A18,A10,A10)

! Open data file
      INFIL='obs.in'
      OPEN (UNIT=29,FILE=INFIL,STATUS='OLD',IOSTAT=IOFLAG)

! Read in data from file for each index
       READ (29,*) DUMMY  !Galaxy name
       DO
         READ (29,79) ANAME,AVALUE,AERR
         IF (ANAME(1:9)=='Hdelta_A ') THEN
           READ(AVALUE(1:10),*) OBSERVED(45)
           READ(AERR(1:10),*) OBSERVEDERROR(45)
         ELSE IF (ANAME(1:9)=='Hdelta_F ') THEN
           READ(AVALUE(1:10),*) OBSERVED(47)
           READ(AERR(1:10),*) OBSERVEDERROR(47)
         ELSE IF (ANAME(1:9)=='CN1      ') THEN
           READ(AVALUE(1:10),*) OBSERVED(1)
           READ(AERR(1:10),*) OBSERVEDERROR(1)
         ELSE IF (ANAME(1:9)=='CN2      ') THEN
           READ(AVALUE(1:10),*) OBSERVED(2)
           READ(AERR(1:10),*) OBSERVEDERROR(2)
         ELSE IF (ANAME(1:9)=='Ca4227   ') THEN
           READ(AVALUE(1:10),*) OBSERVED(3)
           READ(AERR(1:10),*) OBSERVEDERROR(3)
         ELSE IF (ANAME(1:9)=='G4300    ') THEN
           READ(AVALUE(1:10),*) OBSERVED(4)
           READ(AERR(1:10),*) OBSERVEDERROR(4)
         ELSE IF (ANAME(1:9)=='Hgamma_A ') THEN
           READ(AVALUE(1:10),*) OBSERVED(46)
           READ(AERR(1:10),*) OBSERVEDERROR(46)
         ELSE IF (ANAME(1:9)=='Hgamma_F ') THEN
           READ(AVALUE(1:10),*) OBSERVED(48)
           READ(AERR(1:10),*) OBSERVEDERROR(48)
         ELSE IF (ANAME(1:9)=='Fe4383   ') THEN
           READ(AVALUE(1:10),*) OBSERVED(5)
           READ(AERR(1:10),*) OBSERVEDERROR(5)
         ELSE IF (ANAME(1:9)=='Ca4455   ') THEN
           READ(AVALUE(1:10),*) OBSERVED(6)
           READ(AERR(1:10),*) OBSERVEDERROR(6)
         ELSE IF (ANAME(1:9)=='Fe4531   ') THEN
           READ(AVALUE(1:10),*) OBSERVED(7)
```

```
  READ(AERR(1:10),*) OBSERVEDERROR(7)
ELSE IF (ANAME(1:9)=='Fe4668   ') THEN    !note this has been renamed; code here will pick up either Fe4668 or C4668 and file
  READ(AVALUE(1:10),*) OBSERVED(8)          !as C4668, so don't need to correct input files for old names
  READ(AERR(1:10),*) OBSERVEDERROR(8)
ELSE IF (ANAME(1:9)=='C4668    ') THEN
  READ(AVALUE(1:10),*) OBSERVED(8)
  READ(AERR(1:10),*) OBSERVEDERROR(8)
ELSE IF (ANAME(1:9)=='Hbeta    ') THEN
  READ(AVALUE(1:10),*) OBSERVED(9)
  READ(AERR(1:10),*) OBSERVEDERROR(9)
ELSE IF (ANAME(1:9)=='Fe5015   ') THEN
  READ(AVALUE(1:10),*) OBSERVED(10)
  READ(AERR(1:10),*) OBSERVEDERROR(10)
ELSE IF (ANAME(1:9)=='Mg1      ') THEN
  READ(AVALUE(1:10),*) OBSERVED(11)
  READ(AERR(1:10),*) OBSERVEDERROR(11)
ELSE IF (ANAME(1:9)=='Mg2      ') THEN
  READ(AVALUE(1:10),*) OBSERVED(12)
  READ(AERR(1:10),*) OBSERVEDERROR(12)
ELSE IF (ANAME(1:9)=='Mgb      ') THEN
  READ(AVALUE(1:10),*) OBSERVED(13)
  READ(AERR(1:10),*) OBSERVEDERROR(13)
ELSE IF (ANAME(1:9)=='Fe5270   ') THEN
  READ(AVALUE(1:10),*) OBSERVED(14)
  READ(AERR(1:10),*) OBSERVEDERROR(14)
ELSE IF (ANAME(1:9)=='Fe5335   ') THEN
  READ(AVALUE(1:10),*) OBSERVED(15)
  READ(AERR(1:10),*) OBSERVEDERROR(15)
ELSE IF (ANAME(1:9)=='Fe5406   ') THEN
  READ(AVALUE(1:10),*) OBSERVED(16)
  READ(AERR(1:10),*) OBSERVEDERROR(16)
ELSE IF (ANAME(1:9)=='Fe5709   ') THEN
  READ(AVALUE(1:10),*) OBSERVED(17)
  READ(AERR(1:10),*) OBSERVEDERROR(17)
ELSE IF (ANAME(1:9)=='Fe5782   ') THEN
  READ(AVALUE(1:10),*) OBSERVED(18)
  READ(AERR(1:10),*) OBSERVEDERROR(18)
ELSE IF (ANAME(1:9)=='NaD      ') THEN
  READ(AVALUE(1:10),*) OBSERVED(19)
  READ(AERR(1:10),*) OBSERVEDERROR(19)
ELSE IF (ANAME(1:9)=='TiO1     ') THEN
  READ(AVALUE(1:10),*) OBSERVED(20)
  READ(AERR(1:10),*) OBSERVEDERROR(20)
ELSE IF (ANAME(1:9)=='TiO2     ') THEN
  READ(AVALUE(1:10),*) OBSERVED(21)
  READ(AERR(1:10),*) OBSERVEDERROR(21)
ELSE IF (ANAME(1:9)=='D4000    ') THEN
  READ(AVALUE(1:10),*) OBSERVED(22)
  READ(AERR(1:10),*) OBSERVEDERROR(22)
ELSE IF (ANAME(1:9)=='CaII_1   ') THEN
  READ(AVALUE(1:10),*) OBSERVED(50)
  READ(AERR(1:10),*) OBSERVEDERROR(50)
ELSE IF (ANAME(1:9)=='CaII_2   ') THEN
  READ(AVALUE(1:10),*) OBSERVED(51)
  READ(AERR(1:10),*) OBSERVEDERROR(51)
ELSE IF (ANAME(1:9)=='CaII_3   ') THEN
```

```
          READ(AVALUE(1:10),*) OBSERVED(52)
          READ(AERR(1:10),*) OBSERVEDERROR(52)
        ELSE IF (ANAME(1:9)=='CaT      ') THEN
          READ(AVALUE(1:10),*) OBSERVED(49)
          READ(AERR(1:10),*) OBSERVEDERROR(49)
        ELSE IF (ANAME(1:9)=='MgI      ') THEN
          READ(AVALUE(1:10),*) OBSERVED(53)
          READ(AERR(1:10),*) OBSERVEDERROR(53)
        ELSE IF (ANAME(1:9)=='         ') THEN
          EXIT
        END IF
      END DO

! Names of features
      ANAMES(1)= 'CN1 (mag)       '
      ANAMES(2)= 'CN2 (mag)       '
      ANAMES(3)= 'Ca4227 (A)      '
      ANAMES(4)= 'G4300 (A)       '
      ANAMES(5)= 'Fe4383 (A)      '
      ANAMES(6)= 'Ca4455 (A)      '
      ANAMES(7)= 'Fe4531 (A)      '
      ANAMES(8)= 'C4668 (A)       '  !was Fe4668
      ANAMES(9)= 'Hb (A)          '
      ANAMES(10)='Fe5015 (A)      '
      ANAMES(11)='Mg1 (mag)       '
      ANAMES(12)='Mg2 (mag)       '
      ANAMES(13)='Mgb (A)         '
      ANAMES(14)='Fe5270 (A)      '
      ANAMES(15)='Fe5335 (A)      '
      ANAMES(16)='Fe5406 (A)      '
      ANAMES(17)='Fe5709 (A)      '
      ANAMES(18)='Fe5782 (A)      '
      ANAMES(19)='NaD (A)         '
      ANAMES(20)='TiO1 (mag)      '
      ANAMES(21)='TiO2 (mag)      '
      ANAMES(22)='D(4000)         '
      ANAMES(23)='U               '
      ANAMES(24)='B               '
      ANAMES(25)='V               '
      ANAMES(26)='Rc              '
      ANAMES(27)='Ic              '
      ANAMES(28)='J               '
      ANAMES(29)='H               '
      ANAMES(30)='K               '
      ANAMES(31)='L               '
      ANAMES(32)='Ldash           '
      ANAMES(33)='M               '
      ANAMES(34)='U-V             '
      ANAMES(35)='B-V             '
      ANAMES(36)='V-R             '
      ANAMES(37)='V-I             '
      ANAMES(38)='V-J             '
      ANAMES(39)='V-K             '
      ANAMES(40)='J-H             '
      ANAMES(41)='J-K             '
      ANAMES(42)='J-L             '
      ANAMES(43)='J-Ldash         '
```

```
      ANAMES(44)='J-M              '
      ANAMES(45)='Hdelta_A (A)      '
      ANAMES(46)='Hgamma_A (A)      '
      ANAMES(47)='Hdelta_F (A)      '
      ANAMES(48)='Hgamma_F (A)      '
      ANAMES(49)='CaT (A)           '
      ANAMES(50)='CaII_1 (A)        '
      ANAMES(51)='CaII_2 (A)        '
      ANAMES(52)='CaII_3 (A)        '
      ANAMES(53)='MGI (A)           '
      ANAMES(54)='U-B               '
      ANAMES(55)='V-H               '

      CLOSE (UNIT=29)   !obs.in

      END SUBROUTINE GETOBS






!READPN Subroutine to read in intermediate mass star (IMS) data on planetary nebula from Renzini and Voli (1981), A&A, 94, 175 OR
! van den Hoek and Groenewegen (1997) A&ASS, 123, 305-328 OR Gavilan et al (2005) A&A, 432, 861-877 (as selected in values.in)

      SUBROUTINE READPN(NPNC,NPNM,NPNZ,PNDATA,PNM,PNZ,NPNCT)

      USE SHARED
      IMPLICIT NONE

      INTEGER :: NBLANK         !In code Number of blank rows in source data (eg between data tables - not required and skipped over)
      INTEGER :: NHEADER        !In code Number of header rows in source data (not required and so skipped over)
      INTEGER :: NPNC           !Input   Number of components (columns) of data after tidying to uniform format
      INTEGER :: NPNCT          !Input   Number of components in source data (variable depending on author)
      INTEGER :: NPNM           !Input   Number of star masses (rows) of data (variable depending on author)
      INTEGER :: NPNZ           !Input   Number of metallicities (tables) of data (variable depending on author)
      CHARACTER(60) :: PNTABLE  !In code Path to find the selected data tables from DATAFILES directory

      REAL :: HOLD(NPNCT,NPNM,NPNZ)  !In code Temp array prior to sorting columns into consistent order
      REAL :: PNDATA(NPNC,NPNM,NPNZ) !Output  Array of data for planetary nebula from the selected author
      REAL :: PNM(NPNM)              !Output  1-d array of initial masses for planetary nebula from the selected author
      REAL :: PNZ(NPNZ)              !Output  1-d array of initial metallicities for planetary nebula from the selected author

   89  FORMAT (A132)

! zero the arrays
      PNM=0.0
      PNZ=0.0

! Open the data file
      IF (PNDATAIN=='RV') THEN
        PNTABLE='DATAFILES/rv.data'
        NHEADER=4
        NBLANK=1   !number of blank rows between metallicity tables
      ELSE IF (PNDATAIN=='GA')THEN
```

```
        PNTABLE='DATAFILES/gavilan.data'
        NHEADER=17
        NBLANK=1
      ELSE IF (PNDATAIN=='VG')THEN
        PNTABLE='DATAFILES/vandenhoek.data'
        NHEADER=6
        NBLANK=1
      END IF

      IOFLAG=0    !reset before file opened
      OPEN (UNIT=30,FILE=PNTABLE,STATUS='OLD',IOSTAT=IOFLAG)

! Skip over header lines
      DO NH=1,NHEADER
        READ (30,89) DUMMY
      END DO

! Read in data for a given initial metallicity
      DO NZ=1,NPNZ
        ! Read in metallicity
        READ (30,*) PNZ(NZ)
        ! Read in initial masses and yields
        DO NM=1,NPNM
          READ (30,*) PNM(NM),(HOLD(NC,NM,NZ),NC=1,NPNCT)
        END DO
        ! Skip blank lines between metallicity tables
        DO ND=1,NBLANK
          READ (30,89) DUMMY
        END DO
      END DO

! Convert from holding array to actual array required elsewhere in programme
      IF (PNDATAIN=='RV')THEN
        DO NZ=1,NPNZ
          DO NM=1,NPNM
            DO NC=1,NPNCT
              PNDATA(NC,NM,NZ)=HOLD(NC,NM,NZ)
            END DO
          END DO
        END DO
      ELSE IF (PNDATAIN=='GA')THEN
        DO NZ=1,NPNZ
          PNZ(NZ)=LOG10(PNZ(NZ)/0.02)  !convert back to log(Z/Zo) where Zo is 0.02 as per paper
          PNZ(NZ)=ZSUN*(10**(PNZ(NZ)))  !convert based on current value of Zo
          DO NM=1,NPNM
            PNDATA(1,NM,NZ)=HOLD(11,NM,NZ)!Remnant mass
            PNDATA(2,NM,NZ)=HOLD(12,NM,NZ)!Total metal yield =C+N+O
            PNDATA(3,NM,NZ)=HOLD(2,NM,NZ) !Helium
            PNDATA(4,NM,NZ)=HOLD(5,NM,NZ)+HOLD(9,NM,NZ)  !Oxygen
            PNDATA(5,NM,NZ)=HOLD(3,NM,NZ)+HOLD(6,NM,NZ)+HOLD(7,NM,NZ)+HOLD(10,NM,NZ)!Carbon
            PNDATA(6,NM,NZ)=HOLD(4,NM,NZ)+HOLD(8,NM,NZ) !Nitrogen
          END DO
        END DO

      ELSE IF (PNDATAIN=='VG')THEN
        DO NZ=1,NPNZ
          DO NM=1,NPNM
```

```
          PNDATA(1,NM,NZ)=HOLD(10,NM,NZ)!Remnant mass
          PNDATA(2,NM,NZ)=HOLD(8,NM,NZ) !Total metal yield =C+N+O+minor trace
          PNDATA(3,NM,NZ)=HOLD(2,NM,NZ) !Helium
          PNDATA(4,NM,NZ)=HOLD(6,NM,NZ) !Oxygen
          PNDATA(5,NM,NZ)=HOLD(3,NM,NZ)+HOLD(4,NM,NZ)  !Carbon
          PNDATA(6,NM,NZ)=HOLD(5,NM,NZ) !Nitrogen
        END DO
      END DO
    END IF

    CLOSE (UNIT=30)  !planetary nebula data file as selected by user

    END SUBROUTINE READPN





!READWW95 Subroutine to read in SNII data from Woosley and Weaver 1995 ApJSS 101, 181. (WW) for large stars (in range 12-40Mo)
!Following Timmes, Woosley and Weaver 1995, data from models A are used for the mass range 11-25Mo and models B for the mass range 30-40Mo

    SUBROUTINE READWW95

    USE SHARED
    IMPLICIT NONE

    REAL :: CORRECTION
    INTEGER :: NHEADER        !In code Number of header rows
    CHARACTER(60) :: WTABLE
 89  FORMAT (A132)

! Open input text table
    WTABLE='DATAFILES/ww.data'
    OPEN (UNIT=21,FILE=WTABLE,STATUS='OLD')

! Read in for each metalicity - from tables 5,10,12,14,16 (Ejected masses)
    NHEADER=4
! Skip over header
    DO NH=1,NHEADER
      READ (21,89) DUMMY
    END DO

! Read in mass line
    READ (21,89) DUMMY
    READ (DUMMY(7:132),*) (WWM(NM),NM=1,NMWT-1)  !ignore first col (row headers) then read in mass line (hence NMWT-1)
    CORRECTION=2.0/1.9891                !Correction for ww non-integer initial masses
    WWM=WWM*CORRECTION

! Skip a blank line
    READ (21,89) DUMMY

! For each metallicity (ie each table of data)
    DO NZ=1,NZWT
    ! Read metallicity line (values relative to solar (ZSUN))
```

```
      READ (21,*) WWZ(NZ)
      ! Convert metallicities to mass fractions
      WWZ(NZ)=WWZ(NZ)*0.0189    !Zsolar for Anders&Grevesse 1989, as used in this paper

      ! Read in ejecta for different initial masses
      DO NR=1,NRWT
        READ (21,89)DUMMY
        READ (DUMMY(7:132),*) (WW(NR,NM,NZ),NM=1,NMWT-1)
      END DO
      ! Skip over a blank line between metallicities
      READ (21,89) DUMMY
    END DO

    CLOSE (UNIT=21) !ww.data

    END SUBROUTINE READWW95






!READGENEVA Subroutine to read in mass information for massive stars from Geneva Group - specific choice selected in values.in
! by the choice of MASSIVE.  Used as extension only to WW95 if this has been selected as LARGE.  Note that yield information is only
!given for He, C and O, missing the important yields for other elements such as Mg and Fe


    SUBROUTINE READGENEVA

    USE SHARED
    IMPLICIT NONE

    CHARACTER(60) :: GENEVAFILE   !In code Set to the selected file of data
    INTEGER :: NHEADER         !Input   Number of header rows in the data
 89  FORMAT (A132)
! Select the file to be read
    IF (MASSIVE=='M92wind') THEN            !Maeder 1992 A&A 264, 105
      GENEVAFILE='DATAFILES/M92_wind.data'
    ELSE IF (MASSIVE=='M92nowind') THEN     !Maeder 1992 A&A 264, 105
      GENEVAFILE='DATAFILES/M92_no_wind.data'
    ELSE IF (MASSIVE=='MM02RW') THEN        !Meynet and Maeder 2002 fig 19
      GENEVAFILE='DATAFILES/MM02r_w.data'
    ELSE IF (MASSIVE=='MM02wind') THEN      !Meynet and Maeder 2002 fig 19
      GENEVAFILE='DATAFILES/MM02wind.data'
    END IF

! Open file and read in data to arrays
    OPEN (UNIT=22,FILE=GENEVAFILE,STATUS='OLD')

! Skip header lines
    NHEADER=4
    DO NH=1,NHEADER
      READ (22,89) DUMMY
    END DO
    ! Read in a block of masses for each metallicity tabulated
    DO NZ=1,NZGENEVA
```

```
       ! Read in metallicity
       READ (22,*) GZ(NZ)
       ! Read in initial and other masses (at above metallicity)
       DO NM=1,NMGENEVA
         READ (22,*) GM(NM),(GENEVA(NC,NM,NZ),NC=1,NCGENEVA)
       END DO
       ! Skip a line
       READ (22,89) DUMMY
     END DO

     CLOSE (UNIT=22)   !geneva data file as selected by user

     END SUBROUTINE READGENEVA






!READ WORTHEY94 Subroutine to read in Simple Stellar Population (SSPs - single age and metallicity) features
!from tables 5A (including luminosities and colours) and 5B (including line and band strengths) from Worthey 1994 ApJSS, 95, 107.
!Note no data for low ages at low metallicities.
!Output is 3-D array of W94SSPS and two 1-D arrays: W94AGES and W94Z, with the characteristic age/metallicities for this data.
!As Worthey94 does not include H indices, these are added using data from Worthey & Ottaviani 1997 ApJSS, 111, 377 table 6
!As Worthey94 does not include Ca indices, these are added separately using READGARCIA


     SUBROUTINE READWORTHEY94

     USE SHARED
     IMPLICIT NONE

     REAL :: Hold(NZW94)          !Temporary array to hold data whilst moved into standardised order
     INTEGER :: COUNTER             !Counter to facilitate moving data into standard order
     INTEGER :: NBLOCKS             !Number of blocks of data to be read in in each of tables A and B
     INTEGER :: NHEADA,NHEADB        !Number of header rows in tables A and B
     CHARACTER(60) :: W94DATA,HDATA   !File locations for Worthey 94 SSPs and Worthey 97 H indices
  89  FORMAT (A132)

! Open input text table
     W94DATA='DATAFILES/Worthey94.data'
     OPEN (UNIT=20,FILE=W94DATA,STATUS='OLD')

! Read in a block at a time from table 5A (luminosities)
     NBLOCKS=5
     NHEADA=4
     COUNTER=1

     DO NG=1,NBLOCKS
! Skip over header
       DO NH=1,NHEADA
         READ (20,89) DUMMY
       END DO

! Read age line and store ages in array W94AGE using array Hold to facilitate.
```

```
      READ (20,89) DUMMY
      READ (DUMMY(23:132),*) (Hold(NC),NC=1,8)
      IF (Hold(1)==Hold(8)) THEN
        W94AGE(COUNTER)=Hold(1)
        COUNTER=COUNTER+1
      ELSE
        W94AGE(COUNTER)=Hold(1)
        W94AGE(COUNTER+1)=Hold(8)
        COUNTER=COUNTER+2
      END IF

! Read metallicity line. Note: uses [Fe/H], however, tracking back through references Worthey 94 to Worthey et al 94 to Burstein et al 86
! to Faber et al 85 where it states 'mean heavy element abundances here equated to [Fe/H]' so can equate to [Z].
      IF (NG==NBLOCKS) THEN   !For simplicity, just use the last block's data
        READ (20,89)DUMMY
        READ (DUMMY(23:132),*) (W94Z(NC),NC=1,NZW94)
        DO NC=1,NZW94   !convert to actual metallicities
          W94Z(NC)=10**W94Z(NC) !converts to units of Zsolar.  Worthey 94 takes Zsolar as 0.0169
          W94Z(NC)=W94Z(NC)*0.0169   !converts to absolute values
        END DO
      ELSE
        READ (20,89) DUMMY !Skip this row
      END IF
      READ (20,89) DUMMY  !Skip blank row

! Read in colours to SSP array
      IF (NG==1.OR.NG==2)  THEN !no data for metal poor stars at young ages; each table has 2 sets of age data
        DO NR=1,3    !Not using the first three rows RGB Tip Mass, Log L/L0 or BCv
          READ (20,89) DUMMY
        END DO
        DO NI=23,33     !placement within "standard order" for indices in this code
          READ (20,89) DUMMY
          READ (DUMMY(23:132),*)(W94SSP(NI,NC,COUNTER-2),NC=5,8),(W94SSP(NI,NC,COUNTER-1),NC=5,8)
          DO NJ=1,4    !set missing values as lowest available values
            W94SSP(NI,NJ,COUNTER-2)=W94SSP(NI,5,COUNTER-2)
            W94SSP(NI,NJ,COUNTER-1)=W94SSP(NI,5,COUNTER-1)
          END DO
        END DO
        DO NR=1,12
          READ (20,89) DUMMY   !not using the M/L data
        END DO
        DO NI=34,44     !placement within "standard order" for indices in this code
          READ (20,89) DUMMY
          READ (DUMMY(23:132),*)(W94SSP(NI,NC,COUNTER-2),NC=5,8),(W94SSP(NI,NC,COUNTER-1),NC=5,8)
        END DO
      ELSE
        DO NR=1,3    !Not using the first three rows RGB Tip Mass, Log L/L0 or BCv
          READ (20,89) DUMMY
        END DO
        DO NI=23,33    !placement within "standard order" for indices in this code
          READ (20,89) DUMMY
          READ (DUMMY(23:132),*)(W94SSP(NI,NC,COUNTER-1),NC=1,8)
        END DO
        DO NR=1,12
          READ (20,89) DUMMY   !not using the M/L data
```

```
        END DO
        DO NI=34,44    !placement within "standard order" for indices in this code
          READ (20,89) DUMMY
          READ (DUMMY(23:132),*)(W94SSP(NI,NC,COUNTER-1),NC=1,8)
        END DO
      END IF
    END DO

! Now read in table 5B Lick indices to the W94SSP array
    NBLOCKS=5
    NHEADB=7 !includes age and metallicity rows as details read in above

    DO NG=1,NBLOCKS
! Skip over header and age/metallicity info
      DO NH=1,NHEADB
        READ (20,89) DUMMY
      END DO

! Read in indices to W94SSP array
      IF (NG==1.OR.NG==2) THEN
        IF(NG==1)COUNTER=1
        IF(NG==2)COUNTER=3
        DO NB=1,22   !placement within "standard order" for indices in this code
          READ (20,89) DUMMY
          READ (DUMMY(23:132),*)(W94SSP(NB,NC,COUNTER),NC=5,8),(W94SSP(NB,NC,COUNTER+1),NC=5,8)
          DO NJ=1,4 !use lowest available data for young metal poor star
            W94SSP(NB,NJ,COUNTER)=W94SSP(NB,5,COUNTER)
            W94SSP(NB,NJ,COUNTER+1)=W94SSP(NB,5,COUNTER+1)
          END DO
        END DO
      ELSE
        DO NB=1,22    !placement within "standard order" for indices in this code
          READ (20,89)DUMMY
          READ (DUMMY(23:132),*)(W94SSP(NB,NC,NG+2),NC=1,8)
        END DO
      END IF
      ! Read in remainder of table to dummy array
      DO NB=1,22  !further 22 rows
        READ(20,89)DUMMY
      END DO
    END DO

! Add in the H indicies from Worthey & Ottaviani 1997
! Open input text table
    HDATA='DATAFILES/Worthey97.data'
    OPEN (UNIT=26,FILE=HDATA,STATUS='OLD')

! Reset counter and arrays
    COUNTER=0
    Hold=0.0

! Read in data from table
    DO NA=1,NAGEW94
! Read age line
      READ (26,89) DUMMY
      IF (DUMMY(1:4)=='AGE=') THEN
```

```
          READ (DUMMY(5:8),*) Hold(NA)
        END IF
        IF (Hold(NA)<7.9) COUNTER=5
        IF (Hold(NA)>=7.9) COUNTER=1
! Read in H data for that age
        DO NB=COUNTER,NZW94
          READ (26,89) DUMMY
          READ (DUMMY(1:80),*)Hold(NA),(W94SSP(NC,NB,NA),NC=45,48)
          IF(COUNTER==5)THEN   !put lowest values into array spaces
            DO NC=45,48
              DO NJ=1,4
                W94SSP(NC,NJ,NA)=W94SSP(NC,5,NA)
              END DO
            END DO
          END IF
        END DO
      END DO

      CLOSE (UNIT=20)   !SSPsWorthey94.data
      CLOSE (UNIT=26)   !SSPsWorthey97.data

      END SUBROUTINE READWORTHEY94






!READGARCIA Subroutine to read in Simple Stellar Population features for the Calcium triplet in the near-IR.
!Data from Garcia-Vargas, Molla and Bressan 1998 A&AS, 130, 513. (47 ages, 15>1.5Gyrs; 4 metallicities)
!(Ages 10**-3 to 13.18 Gyrs) (Metals 0.2 to 2.5 solar).

      SUBROUTINE READGARCIA

      USE SHARED
      IMPLICIT NONE

      REAL :: NOTNEEDED    !Holding point for data in table that is not required
      CHARACTER :: GVTABLE*60
  89  FORMAT (A132)

! Open input text table
      GVTABLE='DATAFILES/Garcia-Vargas.data'
      OPEN (UNIT=31,FILE=GVTABLE,STATUS='OLD')

! Read in data from table
! Skip over header lines
      DO NH=1,23
        READ (31,89) DUMMY
      END DO
! Read data for each metallicity (store metal poor to rich so reverse order from data file)
      DO NG=NZGV,1,-1
       ! Read data for each age
        DO NA=1,NAGEGV
          READ (31,89) DUMMY
          READ (DUMMY(1:80),*)
NOTNEEDED,NOTNEEDED,GVAGE(NA),GVZ(NG),NOTNEEDED,NOTNEEDED,GVSSP(49,NG,NA)
```

```
            GVAGE(NA)=10**(GVAGE(NA)-9.0)  !Ages to Gyrs from Log(yrs)
            GVZ(NG)=0.02*GVZ(NG)        !Convert to absolute metallicity (units were in Zsolar, which is given as 0.02)
          END DO
        END DO

        CLOSE (UNIT=31)  !Garcia-Vargas.data

        END SUBROUTINE READGARCIA





!READVAZDEKIS  Subroutine to read in data from an SSP based on Vazdekis et al 1996: tables from "MODELS 1999" on
! http://www.iac.es/galeria/vazdekis/vazdekis_models_ssp_linescolors.html

!VZZ was ST and declared here - have moved to shared but think is actually just a holding file and not one of metallicities??

        SUBROUTINE READVAZDEKIS

! Output VZSSP  Arrays of feature values and luminosities
! Output VZAGE,VZZ  Arrays of ages (Gyrs) and metallicities {Z}

        USE SHARED
        IMPLICIT NONE

        REAL :: Hold(NCVZ)     !Holding array whilst reading in data
        CHARACTER(60) :: VFILE !File name for Vazdekis data

   81  FORMAT (F4.2,A4,2F5.2,33F7.3,2F8.3,F7.3,8F9.3)
   89  FORMAT (A132)

! Open data file and read in data to array
        VFILE='DATAFILES/Vazdekis.data'
        OPEN (UNIT=24,FILE=VFILE,STATUS='OLD')

! Skip over header lines
        DO NH=1,4
          READ (24,89)DUMMY
        END DO
!Read in and sort data into standard order
        DO NG=1,5
          DO NZ=1,NZVZ
            DO NA=1,NAGEVZ
              READ (24,81) (Hold(NC),NC=1,NCVZ)
              IF(Hold(1)==1.3)THEN     !Salpeter data
                VZZ(NZ)=Hold(3)
                VZAGE(NA)=Hold(4)
                DO NI=1,21
                  VZSSP(NI,NZ,NA)=Hold(NI+11)
                END DO
                DO NI=23,30
                  VZSSP(NI,NZ,NA)=Hold(NI+18)
                END DO
                DO NI=34,40
```

```
          VZSSP(NI,NZ,NA)=Hold(NI-29)
         END DO
         DO NI=45,48
          VZSSP(NI,NZ,NA)=Hold(NI-8)
         END DO
         DO NI=50,53
          VZSSP(NI,NZ,NA)=Hold(NI-17)
         END DO
        END IF
       END DO
      END DO
     END DO

! Convert from log
     VZZ=0.02*10**(VZZ)

     CLOSE (UNIT=24)  !Vazdekis.data

     END SUBROUTINE READVAZDEKIS






! READT04 Subroutine to read in data from a SSP based on Thomas Maraston Korn 2004 MNRAS 351, L19-23 with datatable from
! http:\\www.dsg.port.ac.uk/~thomas/tms/alpha-models.dat (this is an updated file from the original paper with additional results)
! updated file put into this code 27 August 2009.  Note the file order changed from earlier versions.  Also reads in M/L ratios and colours
! interpolated from Bruzual and Charlot 2003 by Pierre Ocvirk
!
! The data is in a file that has 31 lines of heading, then presents 24 synthetic lick indices from SSPs at 20 different ages
! (0.1 - 15 Gyr).  This is repeated for 6 different values of [Z/H] (-2.250, -1.350, -0.033, 0.000, 0.350 and 0.670), and then this
! cycle is repeated for 4 (was 3) different values of [alpha/Fe] (-0.3, 0.0, 0.3 and 0.5) (-0.3 is new), giving a total of 480 rows of
! data over 28 columns of data as follows:
!        1: age Gyr
!        2: log metallicity [Z/H]  (code converts current vals to this when using this data)
!        3: log alpha:iron ratio [alpha/Fe]
!        4 - 28: synthetic indices for SSPs at the above age/[Z/H]/[alpha/H]

!   The data will be an alternative to the subroutines READVASDEKIS and READWORTHEY.  The alpha ratio will be an additional index.
!   CARE! metallicity and alpha ratios given as log values.  Solar Z taken as 0.02 see Thomas Maraston Bender 2003
!This data has been copied into a file called thomas04update.data

     SUBROUTINE READT04

     USE SHARED
     IMPLICIT NONE

     INTEGER :: FLAG
     INTEGER :: NBC03
     REAL :: readt(NBITOT,NZT04,NAGET04,NRATIOT04)    !holding array
     CHARACTER(60) :: T04TABLE, BC03TABLE                    !filenames and holding point for header rows
  89  FORMAT (A132)
     NBC03=14                                         !Number of entities stored in BC03TABLE
```

```

! Zero the working array
    readt=0.0

! Open the source data files
! Line strengths (from Thomas et al. 2004)
    T04TABLE = 'DATAFILES/thomas04update.data'
    OPEN (UNIT=32,FILE=T04TABLE,STATUS="OLD")
! M/L and colours (interpolated from Bruzual and Charlot 2003)
    BC03TABLE='DATAFILES/BC03.data'
    OPEN (UNIT=33,FILE=BC03TABLE,STATUS="OLD")

! Skip over the header rows
    DO NH = 1,31
      READ (32,89) DUMMY
    END DO

! Read in data from the file to holding file, readt, which has columns in same order as per the source Thomas file
    DO NL = 1,NRATIOT04                    !log alpha/Fe ratios are -0.3, 0, 0.3, 0.5
      DO NZ = 1,NZT04                      !log metallicity values are -2.250, -1.350, -0.033, 0.000,0.350, 0.670
        DO NA = 1,NAGET04                  !ages are 0.1, 0.2, 0.4, 0.6, 0.8, 1-15 Gyrs
          READ (32,*) AGET04(NA),T04Z(NZT04),RATIOT04(NL),(readt(NB,NZ,NA,NL),NB=1,NBITOT)
        END DO
      END DO
    END DO

! Re-order data to match standard indices list order (note: some columns will be zero)
    DO NL = 1, NRATIOT04
      DO NA= 1,NAGET04
        DO NZ = 1,NZT04
          THSSP(45,NZ,NA,NL) = readt(1,NZ,NA,NL)  !HdA
          THSSP(46,NZ,NA,NL) = readt(7,NZ,NA,NL)  !HgA
          THSSP(47,NZ,NA,NL) = readt(2,NZ,NA,NL)  !HdF
          THSSP(48,NZ,NA,NL) = readt(8,NZ,NA,NL)  !HgF
          DO NJ  = 1,4
            THSSP(NJ,NZ,NA,NL) = readt(NJ+2,NZ,NA,NL)  !CN1, CN2, Ca4227, G4300
          END DO
          DO NJ  = 1,21
            THSSP(NJ+4,NZ,NA,NL) = readt(NJ+8,NZ,NA,NL)  !remaining indices
          END DO
        END DO
      END DO
    END DO

! Now read in M/L ratios and colours (from interpolations of BC03 data)
! Skip over the header rows
    DO NH = 1, 15
      READ (33,89) DUMMY
    END DO

! Zero the working array
    readt=0.0

! Read in data from the file to holding file, readt
    DO NZ = 1,NZT04                        !log metallicity values are -2.250, -1.350, -0.033, 0.000,0.350, 0.670
      DO NA = 1, NAGET04                   !ages are 0.1, 0.2, 0.4, 0.6, 0.8, 1-15 Gyrs
        READ (33,*) AGET04(NA),T04Z(NZ),(readt(NB,NZ,NA,1),NB=1,NBC03)
```

```
      END DO
    END DO

! Re-order data, and include zero columns, to match standard NINDEX file format
    DO NL = 1, NRATIOT04
      DO NA= 1,NAGET04
        DO NZ = 1,NZT04
! Mass-to-light ratios
          THSSP(23,NZ,NA,NL) = readt(1,NZ,NA,1) ! (M/L)U
          THSSP(24,NZ,NA,NL) = readt(2,NZ,NA,1) ! (M/L)B
          THSSP(25,NZ,NA,NL) = readt(3,NZ,NA,1) ! (M/L)V
          THSSP(26,NZ,NA,NL) = readt(4,NZ,NA,1) ! (M/L)Rc
          THSSP(27,NZ,NA,NL) = readt(5,NZ,NA,1) ! (M/L)Ic
          THSSP(28,NZ,NA,NL) = readt(6,NZ,NA,1) ! (M/L)J
          THSSP(29,NZ,NA,NL) = readt(7,NZ,NA,1) ! (M/L)H
          THSSP(30,NZ,NA,NL) = readt(8,NZ,NA,1) ! (M/L)K
! Colours
          THSSP(34,NZ,NA,NL) = readt(9,NZ,NA,1)  ! (U-V)
          THSSP(35,NZ,NA,NL) = readt(10,NZ,NA,1) ! (B-V)
          THSSP(36,NZ,NA,NL) = readt(11,NZ,NA,1) ! (V-R)
          THSSP(37,NZ,NA,NL) = readt(12,NZ,NA,1) ! (V-I)
          THSSP(38,NZ,NA,NL) = readt(13,NZ,NA,1) ! (V-J)
          THSSP(40,NZ,NA,NL) = readt(14,NZ,NA,1) ! (V-K)
        END DO
      END DO
    END DO

    CLOSE (UNIT=32)   !thomas04.data
    CLOSE (UNIT=33)   !BC03.data

    END SUBROUTINE READT04






!READBERTELLI Subroutine to read in tables from Bertelli et al 1994.  These isochrones give colour as well as luminosity for stars of
!different masses and temperatures at different ages and metallicities.  As this model (Phoenix) does not model stars of different
!temperatures, where stars of the same age and mass are given, the average luminosity and colours are taken for the range of temperatures
!provided by Bertelli et al.
!This is done by first reading the row into a temporary array, CHECK, then comparing it to the previous row(s) held in HOLD.
!If the current isochrone has the same mass and age, it is added into HOLD, and a denominator counter, J is increased by 1.
!As soon as an isochrone with a different age and mass is read in, the totals in HOLD are averaged over the number of isochrones stored
!there (=J), and put into the (nearly) final BERTELLI array.
!Note that the tables are not all of the same size and whilst ages are stepped through methodically, masses are not, nor are masses
!repeated in subsequent tables.
!In addition, the ages are in descending order, and within age, the masses both decrease and increase, so data needs to be sorted into
!ascending age and, within each age, ascending mass, to enable the subroutine B94ISOCHRONES to find the appropriate information.
!Even when removing the rows which just differ by temperature, the number of rows in the final datatable for each z will be different.
!The first item in the original data file is the row counter, which is not included in the final BERTELLI array.
! The next item is log age, then mass of star, then temp, then bolometric magnitude, then colours x 8 and then the luminosity.
```

```
!The final BERTELLI array stores z, age (years), mass (msolar), temp/colours/luminosity as a 3 dimensional array.


    SUBROUTINE READBERTELLI

    USE SHARED
    IMPLICIT NONE

    INTEGER,PARAMETER :: MAXROWS=6709   !Length of longest table in Bertelli data
    INTEGER :: COUNTER     !Count number of rows that have repeated age and mass but at different temperatures
    INTEGER :: NROWS       !Number of isochrones in each table of Bertelli data (variabe) before tidying for repeated temperatures
    INTEGER :: ROW      !note of the row for swapping whilst sorting
    REAL :: SORT(MAXROWS,NISOC+1)!temp array to hold initial read-in array, sort it into ascending order before duplicates removed
    REAL :: CHECK(NISOC+1) !temp array to hold data whilst checking if it's a duplicate for age and mass to previous rows read in
    REAL :: HOLD(NISOC+1)  !temp array to store cumulative data where repeated ages and masses but at different temperatures
    REAL :: POINTER(NISOC+1)
    REAL :: TEMP(NISOC+1)
    CHARACTER(60) :: BERTTABLE

89   FORMAT (A132)

! Zero the temporary arrays
    SORT=0.0
! Open the source data file
    BERTTABLE = 'DATAFILES/Bertelli.data'
    OPEN (UNIT=34,FILE=BERTTABLE,STATUS="OLD")

! Skip over the header rows
    DO NH = 1,10
      READ (34,89) DUMMY
    END DO

! Read in the data tables, reading the metallicity into an array
    DO NF=1,NISOZ                     !Work through the tables
      READ (34,*) BERTELLIZ(NF)       !Read the metallicity for the table
      READ (34,89) DUMMY              !Skip the next line
      COUNTER=1                       !Reset counter for averaging repeated rows (same age, mass, metallicity but different temp)
      HOLD=0.0                        !Reset temp holding array
      TEMP=0.0                        !ditto
      CHECK=0.0                       !ditto
      POINTER=0.0                     !ditto
      NM=1                            !Reset counter through rows in the final BERTELLI array
      NA=1                            !Reset counter through rows in the final BERTELLIAGES array

      IF (NF==1) NROWS=6351           !The source data tables are of different lengths
      IF (NF==2) NROWS=5936
      IF (NF==3) NROWS=6610
      IF (NF==4) NROWS=6689
      IF (NF==5) NROWS=6593
      IF (NF==6) NROWS=6454

! Read in the first table into a temporary array for sorting into ascending order
      DO NR=1,NROWS
        READ (34,*) (SORT(NR,NC),NC=1,NISOC+1)
```

```
      END DO

! Sort this data into ascending order of ages, and then within each age, into ascending order of masses
! First, sort by age (there are repeated age rows)
      DO NR=1,NROWS-1
        DO NC=1,NISOC+1
          POINTER(NC)=SORT(NR,NC)    !Put the row being checked into POINTER
        END DO
        ROW=NR    !initially set
        DO COUNTER=NR+1,NROWS   !work through data in front of pointer, and see if the age less than the age of the row held in pointer
          IF(SORT(COUNTER,2)<POINTER(2))THEN    !if it finds a value less than the pointer, make that the pointer
            ROW=COUNTER                !make a note of the row number with the value less than the pointer
            DO NC=1,NISOC+1
              POINTER(NC)=SORT(COUNTER,NC)
            END DO
          END IF
        END DO
        !swap the line you were looking at with the lowest line found, via a TEMP array
        TEMP(1:NISOC+1)=SORT(NR,1:NISOC+1)
        SORT(NR,1:NISOC+1)=POINTER(1:NISOC+1)
        SORT(ROW,1:NISOC+1)=TEMP(1:NISOC+1)
      END DO

! Now sort by mass within each age
      DO NR=1,NROWS-1
        DO NC=1,NISOC+1
          POINTER(NC)=SORT(NR,NC)    !Put the row being checked into POINTER
        END DO
        ROW=NR     !initially set
        DO COUNTER=NR+1,NROWS   !work through data in front of pointer, and see if the mass is < the mass of the row held in pointer
          IF(SORT(COUNTER,2)==POINTER(2))THEN !same group of data by age so can go ahead to check for masses
            IF(SORT(COUNTER,3)<POINTER(3))THEN
              ROW=COUNTER                !make a note of the row number
              DO NC=1,NISOC+1
                POINTER(NC)=SORT(COUNTER,NC)  !if it finds a value less than the pointer, make that the pointer
              END DO
            END IF
          END IF
        END DO
        !swap the line you were looking at with the lowest line found, via a TEMP array
        TEMP(1:NISOC+1)=SORT(NR,1:NISOC+1)
        SORT(NR,1:NISOC+1)=POINTER(1:NISOC+1)
        SORT(ROW,1:NISOC+1)=TEMP(1:NISOC+1)
      END DO   !now have temp array SORT in ascending order of ages, and within each age, ascending order by mass

!check output only whilst testing
!     IF (NF==3) THEN   !CHECK OTHER TABLES HERE JUST CHECKING ONE TABLE AT A TIME
!       OPEN(UNIT=50,FILE='bertelliMASS.out',STATUS='REPLACE')
!       WRITE(50,*),'SORTED BY AGE AND MASS BERTELLI DATA FOR VERIFICATION TABLE=',NF
!       DO NR=1,NROWS
!         WRITE(50,*),(SORT(NR,NC),NC=1,NISOC+1)
!       END DO
!     END IF

!Remove repeated rows (Bertelli data has several temperature stars for a give mass, age and metallicity: here just use averages)
```

```
      DO NR=1,NROWS                    !Work through the rows of the sorted data for this metallicity
        DO NC=1,NISOC+1
          CHECK(NC)=SORT(NR,NC)          !Read single row into temp array for checking, inc the extra column (the row counter)
        END DO

        IF (HOLD(1)==0.0) THEN           !Then am dealing with the first line of data in a new table
          HOLD=CHECK                     !Copy the line just read in into the holding array
          CHECK=0.0                      !Clear the temporary array ready for the next row for checking
          COUNTER=1                      !Reset counter for denominator when repeated temperatures for same age and mass
          NM=1                           !Counter throuh masses
          NA=1                           !Counter through ages

        !Check if age and mass read into CHECK are the same as previous row(s) (held in HOLD)
        ELSE IF (CHECK(2)==HOLD(2).AND.CHECK(3)==HOLD(3)) THEN
          DO NI=4,14                     !NI is the counter through the columns before the data tidied
            HOLD(NI)=HOLD(NI)+CHECK(NI)    !Add the data to the previous data for stars with this mass and age
          END DO
          COUNTER=COUNTER+1              !Increase the counter (gives denominator when working out the averages)
          CHECK=0.0                      !Clear the temporary array ready for the next row for checking

        ELSE                             !not on first line of datatable and not on a repeated mass (may be on repeated age)
          BERTELLI(NF,NA,NM,1)=10**HOLD(2)    !transfer the age value to the final array (note: convert from [age] as held here)
          BERTELLI(NF,NA,NM,2)=HOLD(3)        !transfer the mass value to the final array
          !transfer the previous isochrone/the average previous isochrone to the final array
          DO NJ=3,13
            BERTELLI(NF,NA,NM,NJ)=(HOLD(NJ+1)/COUNTER)
          END DO
          BERTELLI(NF,NA,NM,4)=10**((BERTELLI(NF,NA,NM,4)-4.72)/2.5)!Substitute bolometric luminosity with actual luminosity in Lsolar
                                              ! formula from Oxford dictionary of astronomy
          NM=NM+1                             !increment the mass counter (resets below if have incremented age)
          BERTELLIMN(NF,NA)=BERTELLIMN(NF,NA)+1     !increment the array counting the number of masses per age/z combination

          IF (CHECK(2)/=HOLD(2))THEN          !have moved onto a new age
            BERTELLIAGE(NA)=10**HOLD(2)         !put the age just passed into the age array
            NA=NA+1                             !increment the age counter
            NM=1                                !reset the mass counter
          END IF

          HOLD=CHECK                     !transfer the isochrone just read in into the holding array
          CHECK=0.0                      !clear the checking array for the next line to be read in
          COUNTER=1                      !reset the 'repeat rows counter'

        END IF                           !this IF statement is processing depending on the uniqueness of the isochrone read in
      END DO                             !go to next row in that source data table

!at end of each table - transfer last line from the temporary arrays to the final BERTELLI array
      BERTELLI(NF,NA,NM,1)=10**HOLD(2)     !Transfer the age of the final isochrone to the final array (note: convert from [age])
      BERTELLI(NF,NA,NM,2)=HOLD(3)         !Transfer the mass of the star in the final isochrone
      !Transfer the average colours and luminosities for the final isochrone to the final array
      DO L=3,13
        BERTELLI(NF,NA,NM,L)=(HOLD(L+1)/COUNTER)
      END DO
      BERTELLIMN(NF,NA)=BERTELLIMN(NF,NA)+1    !Increment the mass counter array
      BERTELLIAGE(NA)=10**HOLD(2)              !put the age just passed into the age array
```

```
        HOLD=0.0                    !reset the holding array
        CHECK=0.0                    !reset the check array

!If not at last table, skip blank rows between tables before reading in the next table
        IF (NF/=6) THEN
          DO ND=1,3
            READ (34,89) DUMMY
          END DO
        END IF

      END DO  !go to next table of Bertelli data

      CLOSE (UNIT=34)  !Bertelli isochrones data

      END SUBROUTINE READBERTELLI






!READTB95 Subroutine to read in tables from Tripicco and Bell 1995,  AJ, 110, 3035, which model the effects of non-solar
!abundance ratios on 21 Lick indices, using the methodology specified in Trager et al 2000  AJ 119 p 1645-1676 (paper 1).

      SUBROUTINE READTB95

! Output  TB95      Response functions for different elements, for each Lick index
      USE SHARED
      IMPLICIT NONE

      REAL ASSUMEFRACTION(3),TB95read(NITB95,NCTB95,3),TB95sort(NITB95,NCTB95)
      CHARACTER NAME(21)*7,TBTABLE*60
   89  FORMAT (A132)
   20  FORMAT (A7,F8.2,F8.3,11F6.1)

! Specify the mix of stellar types using the mix assumption from Trager et al 2000 see table 5
      ASSUMEFRACTION(1)=0.53   !Cool giants
      ASSUMEFRACTION(2)=0.44   !Turnoff stars
      ASSUMEFRACTION(3)=0.03   !Cool dwarfs

! Zero array for summation
      TB95sort=0.0

! Read and combine TB95 sensitivities.
      TBTABLE='DATAFILES/TB95.data'
      OPEN (UNIT=23,FILE=TBTABLE,STATUS='OLD')
      DO NG=1,3
        ! Skip over header
        DO NH=1,7
          READ (23,89) DUMMY
        END DO
        ! Read in for each Lick index
        DO NR=1,NITB95
          READ (23,20) NAME(NR),(TB95read(NR,NC,NG),NC=1,13)
          DO NC=1,NCTB95
            IF (NC==1) THEN          !Column giving 'standard' Lick indices
```

```
          IF (NR==1.OR.NR==2.OR.NR==11.OR.NR==12.OR.NR==20.OR.NR==21) THEN
            ! Convert band indices CN1,CN2,MG1,MG2,TIO1,TIO2 from magnitudes for linear combination
            TB95sort(NR,NC)=TB95sort(NR,NC)+ASSUMEFRACTION(NG)*10**(TB95read(NR,NC,NG)/(-2.5))
          ELSE
            ! Leave line indices as linear
            TB95sort(NR,NC)=TB95sort(NR,NC)+ASSUMEFRACTION(NG)*TB95read(NR,NC,NG)
          END IF
        ELSE IF (NC==2) THEN     !Column giving 'standard' error on Lick indices
          TB95sort(NR,NC)=TB95read(NR,NC,NG)
        ELSE IF (NC>=3) THEN     !Response functions when element abundance is doubled
          TB95sort(NR,NC)=TB95sort(NR,NC)+ASSUMEFRACTION(NG)*TB95read(NR,NC,NG)
        END IF
      END DO
    END DO
  END DO

! Put band indices back into magnitudes
  TB95sort(1,1)=-2.5*LOG10(TB95sort(1,1))
  TB95sort(2,1)=-2.5*LOG10(TB95sort(2,1))
  TB95sort(11,1)=-2.5*LOG10(TB95sort(11,1))
  TB95sort(12,1)=-2.5*LOG10(TB95sort(12,1))

! Evaluate response functions
  DO NC=1,2
    DO NR=1,NITB95
      TB95(NR,NC)=TB95sort(NR,NC)  !Standard indices and errors
    END DO
  END DO
  DO NC=3,NCTB95
    DO NR=1,NITB95
      TB95(NR,NC)=TB95sort(NR,NC)
    END DO
  END DO

  CLOSE (UNIT=23)  !TB95.data

  END SUBROUTINE READTB95






!READKORN Response functions from Korn, Maraston and Thomas 2005 A&A 438 issue 2, p 685-704 'The sensitivity of Lick indices
! to abundance variations'.  As nearly all the stars in this model will not be turnoff or giant branch for more than one timestep,
! just use the main sequence data.

  SUBROUTINE READKORN

  USE SHARED
  IMPLICIT NONE

  CHARACTER(len=60) :: KTABLE

! Set formats
 780  FORMAT (A132)
```

```
 781  FORMAT (A10,13F8.3)
 782  FORMAT (A11,F6.2)
 783  FORMAT (F6.2)
 784  FORMAT (F6.0,3F6.2)

  !DEL   READ (21,89) DUMMY
 !DEL     READ (DUMMY(7:132),*) (WWM(NM),NM=1,NMWT-1)  !ignore first col (row headers) then read in mass line (hence NMWT-1)

! Read in data from Korn et al 2005 tables 6,9,12,18,21,27,30
      KTABLE = 'DATAFILES/korn.data'
      OPEN (UNIT=27,FILE=KTABLE,STATUS='OLD')

      DO NG=1,NKORNZ
        ! Skip over header
        DO NH=1,4
          READ (27,780) DUMMY
        END DO

        ! Read in [Z/H]
        READ (27,*) DUMMY, DUMMY, KORNZ(NG)

        ! Skip over rest of header
        DO NH=1,2
          READ (27,780) DUMMY
        END DO

        ! Read in the element response for each Lick index
        DO NR=1,NKORNI
          READ (27,*)DUMMY,(KORN(NG,NR,NC),NC=1,NKORNC)
        END DO
      END DO

      CLOSE (UNIT=27)

      END SUBROUTINE READKORN
```

APPENDIX C: Abbreviations used in this thesis

Abbrev.	**Pg.**	**Definition/ paper reference**
β	81	$\beta = \frac{\lvert(\text{observed index} - \text{synthetic index from model})\rvert}{\text{error on observed index } (= 1 \text{ standard deviation})}$
β_{ave}	81	$\beta_{ave} = \frac{\Sigma \beta}{\text{Number of indices observed for galaxy}}$
β_{max}	81	β_{max} = maximum β from all calculated for that galaxy
AGN	19	Active Galactic Nucleus
D05	134	Lick index data on 10 from the set of 52 elliptical galaxies taken on the Observatorio Astrofisico Guillermo Haro Mexico, published by Denicoló et al. (2005)
G05	43	Synthetic yields for planetary nebulae: Gavilán et al. (2005)
GCE	37	Galactic Chemical Evolution Model developed by Sansom and first described in SP98
Geneva Group	48	Massive star research from Maeder, Meynet, Hirschi; here uses as M92 with MM02 correction for stars > 40 $M_\odot$
IMF	16	Initial mass function
K05	47	Lick index response functions: Korn et al. (2005)
M92	38	Synthetic yields for SNII for stars of initial mass 9 to 120 $M_\odot$: Maeder (1992)
MM02	38	Update to M92 data on synthetic yields for very massive stars 40 to 120 $M_\odot$: Meynet and Maeder (2002)
PS02	42	Lick index data on 11 elliptical galaxies taken on WHT: Proctor and Sansom (2002)
RV81	42	Synthetic yields for planetary nebulae: Renzini and Voli (1981)
SAMs	26	Semi-analytic models
SB07	134	Lick index data on 11 elliptical galaxies taken on KeckII: Sanchez-Blazquez et al. (2007)
SDSS	20	Sloan Digital Sky Survey, various data releases
SFH	15	Star formation history
SFR	21	Star formation rate, usually in solar masses produced per unit time.
SSP	22	Single stellar population i.e. stars with same age, metallicity, and, where given, [α/Fe]
SN	19	Supernova(e)
SP98	42	Introduction to the GCE model: Sansom and Proctor (1998)
SPH	25	Smoothed particle hydrodynamics
'toy' galaxy	42	'best guess' generalised model input parameters for a given galaxy morphology
T04	48	SSP models: Thomas et al. (2004)
TB95	47	Lick index response functions: Tripicco and Bell (1995)
V99	39	SSP models: Vazdekis et al. 1999
vdH&G97	43	Synthetic yields for planetary nebulae: van den Hoek and Groenewegen (1997)
W94	39	SSP models: Worthey (1994)
WHT	58	William Herschel telescope, La Palma
WW95	38	Synthetic yields for SNII for stars of initial mass 11 to 40 $M_\odot$: Woolsey and Weaver (1995)