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# Two close large quasar groups of size $\sim 350$ Mpc at $z \sim 1.2$

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## ABSTRACT

The Clowes & Campusano large quasar group (LQG) at  $\bar{z} = 1.28$  has been re-examined using the quasar data from the DR7QSO catalogue of the Sloan Digital Sky Survey. In the 1991 discovery, the LQG impinged on the northern, southern and eastern limits of the survey. In the DR7QSO data, the western, northern and southern boundaries of the LQG remain essentially the same, but an extension eastwards of  $\sim 2^\circ$  is indicated. In the DR7QSO data, the LQG has 34 members, with  $\bar{z} = 1.28$ . A new group of 38 members is indicated at  $\bar{z} = 1.11$  and within  $\sim 2^\circ$  of the Clowes & Campusano LQG. The characteristic sizes of these two LQGs,  $\sim 350$ – $400$  Mpc, appear to be only marginally consistent with the scale of homogeneity in the concordance cosmology. In addition to their intrinsic interest, these two LQGs provide locations in which to investigate early large-scale structure in galaxies and to identify high- $z$  clusters. A method is presented for assessing the statistical significance and overdensity of groups found by linkage of points.

**Key words:** galaxies: clusters: general – quasars: general – large-scale structure of Universe.

## 1 INTRODUCTION

Since 1982 there have been many reports of large-scale structures (LSSs) in the distribution of quasars and related objects. These structures have now become generally known as large quasar groups (LQGs). They are the largest structures so far seen in the early universe ( $z \sim 0.4$ – $2.0$ ), with sizes in the range of  $70$ – $250$  Mpc and memberships  $\gtrsim 5$ . The Clowes & Campusano (1991) LQG, at redshift  $z \sim 1.3$  and with a longest dimension of  $\sim 250$  Mpc, is a particularly large example of LSS in the early universe. For some of the historical development of work on LQGs see, for example Webster (1982), Crampton, Cowley & Hartwick (1987, 1989), Clowes & Campusano (1991), Komberg & Lukash (1994), Graham, Clowes & Campusano (1995), Komberg, Kravtsov & Lukash (1996), Newman et al. (1998), Newman (1999), Clowes, Campusano & Graham (1999), Tesch & Engels (2000), Williger et al. (2002), Brand et al. (2003) (for radio galaxies), Haines, Campusano & Clowes (2004), Miller et al. (2004) and Pilipenko (2007).

LQGs are of interest not only as examples of large-scale features in the early universe but also because of their potential for investigations of the LSS in galaxies, for indicating places in which to

look for high- $z$  clusters and for identifying the environments that favour the formation of quasars.

Komberg et al. (1996) (see also Komberg & Lukash 1994) considered that LQGs denote the precursors at high redshifts of the superclusters seen today. Pilipenko (2007) similarly considered that LQGs may be ‘incipient superclusters’ and also concluded that a substantial fraction of quasars lie two-dimensionally, in sheets. Among some of the earliest papers in this area, de Ruiter & Zuiderwijk (1982) considered the possibility that quasars reside in superclusters to be a natural explanation for the occurrence of wide-angle doublets and triplets of quasars of very similar redshifts. Longo (1991) showed that the ‘Great Wall’ of galaxies (Geller & Huchra 1989) at  $z \sim 0.03$  is traced by its active galactic nuclei. More recently, Mountrichas et al. (2009) found that quasars and luminous red galaxies cross-correlate on scales  $\lesssim 40$  Mpc for  $0.35 \leq z \leq 0.75$ . Söchting, Clowes & Campusano (2002, 2004) showed that, at the low redshifts  $z \sim 0.3$ , quasars tended to follow the LSS in clusters of galaxies, but were preferentially associated with the peripheries of clusters. (See also Sánchez & González-Serrano 1999 and references therein.) For the Crampton et al. (1989) LQG at  $z \sim 1.1$ , a similar result was found by Tanaka et al. (2001) for five of its quasars: they followed a chain of clusters (or groups) but were associated with the peripheries. Ohta et al. (2003) found three quasars in the vicinity of a supercluster at  $z = 1.27$  that are presumed to be

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associated, while being separated from the two-component clusters by  $\sim 7\text{--}17$  Mpc.

For the Clowes & Campusano LQG, Haines et al. (2001) found that one quasar is simultaneously on the peripheries of two clusters (or groups) of red galaxies, together with a band of blue, presumably star-forming, galaxies. The association of two further quasars from the Clowes & Campusano LQG with clusters or their peripheries is less apparent (Haines et al. 2004), and at least one of these may be quite isolated. There is evidence for general sheet-like enhancements of red galaxies with embedded clusters associated with this LQG (Haines et al. 2004). Habertzettl et al. (2009) similarly find evidence for enhancements of candidates for Lyman-break galaxies associated with it. A direct association (i.e. using spectroscopic redshifts rather than photometric estimates) of the Clowes & Campusano LQG with an excess of galaxies has been achieved via Mg II absorbers in background quasars, which showed a  $\sim 200$  per cent excess in the redshift interval  $1.2 < z < 1.4$  (Williger et al. 2002).

The investigation of LQGs was for many years made difficult by the limitations of quasar surveys. Surveys tended to be small, or larger but not very deep, or affected by selection effects, and so on. Compilations of different surveys increased the numbers of quasars but generally worsened the selection effects. In recent years, however, the difficulties with quasar data have been substantially lessened by the Sloan Digital Sky Survey (SDSS; e.g. Vanden Berk et al. 2005; Richards et al. 2006; Schneider et al. 2007, 2010) and the Two-Degree Field (2dF) QSO (quasi-stellar object) Redshift Survey (2QZ; e.g. Croom et al. 2004; Smith et al. 2005).

Miller et al. (2004) and Pilipenko (2007) have investigated LSS in the 2QZ quasars, with Miller et al. finding  $\sim 200$  Mpc structures in a statistical sense, and Pilipenko more specifically giving coordinates and other properties of the LQGs discovered. In particular, Pilipenko (2007) suggests that there might be two categories of LQGs: (i) size  $\sim 85$  Mpc, membership  $\sim 6\text{--}8$  and overdensity  $\sim 10$ ; and (ii) size  $\sim 200$  Mpc, membership  $\gtrsim 15$  and overdensity  $\sim 4$ . Note that these overdensities are substantially larger than those found by Miller et al. (2004) using spherical filtering.

The Clowes & Campusano (1991) LQG is, as mentioned above, a particularly large structure in the early universe. It was discovered effectively by random spatial sampling of quasar candidates from an objective-prism survey. This sampling was necessary for homogeneous coverage of a large area ( $\sim 25$  deg<sup>2</sup>) by single-object spectroscopy at a time when it was not feasible with multi-object spectroscopy.

The field of the Clowes & Campusano LQG is now contained within the SDSS. This paper considers what additional properties of the LQG and its cosmological neighbourhood can be deduced using the SDSS quasars, given their wide-angle coverage and almost complete multi-object spectroscopy.

The results presented here indicate that both the Clowes & Campusano LQG and a newly discovered neighbouring LQG are among the largest features so far seen in the early universe. In this context, we note that there have been some reports of still larger correlations. Nabokov & Baryshev (2008) presented preliminary evidence for Gpc-scale correlations of galaxies, and Padmanabhan et al. (2007) and Thomas, Abdalla & Lahav (2011) both found power on Gpc scales in the power spectrum of SDSS galaxies. A particularly striking result is that of Hutsemékers et al. (2005), who found that the polarization vectors of quasars are correlated on Gpc scales.

Note that the concordance model is adopted for cosmological calculations, with  $\Omega_T = 1$ ,  $\Omega_M = 0.27$ ,  $\Omega_\Lambda = 0.73$  and  $H_0 =$

$70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . All sizes given are proper sizes at the present epoch.

## 2 THE SDSS QUASARS

The SDSS DR7 quasar ('DR7QSO') catalogue (Schneider et al. 2010) of 105 783 quasars has been used for this work. Schneider et al. note that the catalogue does not constitute a statistical sample (i.e. a sample with homogeneous selection) and refer to Vanden Berk et al. (2005) and Richards et al. (2006) for detailed discussion of the important properties for completeness and efficiency of the parent survey. In particular, Richards et al. (2006) describe how to construct a statistical sample from the DR3QSO catalogue, in a discussion which should also be applicable to the DR7QSO catalogue. However, because the requirement here is for assessing the spatial connectivity of quasars across intervals on the sky of  $\sim 5^\circ$  and intervals of redshift  $\Delta z \sim 0.2$ , rather than for the luminosity function, the criteria of Richards et al. (2006) can be relaxed. First, the redshifts  $z \lesssim 2$  of main interest are well within the limit  $z \lesssim 3$  of the low-redshift strand of selection, so the changes in the SDSS selection algorithms from the initial to the final (Richards et al. 2002) should not be important. Secondly, satisfactory spatial uniformity of selection on the sky for these redshifts should be achievable by selecting  $i \leq 19.1$  (Vanden Berk et al. 2005; Richards et al. 2006), since those quasars are predominantly from the low-redshift selection strand, which is limited to  $i \leq 19.1$ .

For this re-examination of the Clowes & Campusano LQG, we use the entire DR7QSO catalogue of  $\sim 9380$  deg<sup>2</sup>, limited to  $i \leq 19.1$ . The coverage of the catalogue comprises a large main area for the north galactic cap (NGC;  $\sim 7600$  deg<sup>2</sup>) with some jagged boundaries, three equatorial stripes (totalling  $\sim 800$  deg<sup>2</sup>) and several distinct, smaller areas ('special plates'). The LQG is in the main NGC area, and is  $\sim 5$  deg from the nearest boundary. Some nearby holes in the coverage that could be seen in the preceding DR5QSO catalogue appear to be no longer present in DR7QSO. Of course, any candidate LQG elsewhere in the catalogue that encounters a boundary might not be completely identified.

We shall denote the area of  $\sim 9380$  deg<sup>2</sup> as A9380. We also define a control area, designated A3725, of  $\sim 3725$  deg<sup>2</sup> (actually  $3724.5$  deg<sup>2</sup>) by RA  $123^\circ 0\text{--}237^\circ 0$  and Dec.  $15^\circ 0\text{--}56^\circ 0$ . A3725 is chosen to be a large area well separated from the LQG.

Note that the relatively bright limiting magnitude,  $i \leq 19.1$ , of the SDSS low-redshift selection strand is not ideal for the tasks of finding and further investigating LQGs. The difficulty is possibly illustrated by Pilipenko (2007), who finds LQGs in the fainter 2QZ data (see also Miller et al. 2004) but finds no significant groups beyond doublets and triplets in the DR5QSO (Schneider et al. 2007), although the adoption of a small linkage scale (57 Mpc compared with mean separation of 83 Mpc) for the DR5QSO seems likely to have also been a very substantial factor in their apparent absence. The identification of LSSs by algorithms can be quite subtle, especially concerning the effective and objective specification of factors such as the linkage scale and overdensity.

The discovery of the Clowes & Campusano LQG was reported in Clowes & Campusano (1991), with a later revision, following some additional observations, given by Clowes et al. (1999). From these two papers, the LQG was detected as 18 quasars in the redshift range  $1.2 \leq z < 1.4$ , of which 15 were new discoveries and three were previously known. At the time of the observations, multi-object spectroscopy across the whole survey area of  $25.3$  deg<sup>2</sup> was not practicable and, instead, wide-area coverage was achieved effectively by random spatial sampling of the quasar candidates for

single-object spectroscopy. Although the magnitude limit of  $i = 19.1$  for the SDSS low-redshift strand is brighter than the Clowes & Campusano limit of  $B_J \sim 20.4$ , there should be many further SDSS quasars in the range  $1.2 \leq z \leq 1.4$  because of the complete wide-area coverage. Note that of the 18 original LQG members, 10 are present in the DR7QSO catalogue with  $i \leq 19.1$ , and a further two are present with  $i > 19.1$ .

Ten further quasars with  $1.2 \leq z < 1.4$  in the region of the LQG are known from a later UV-excess survey (the Chile–UK Quasar Survey; Newman et al. 1998; Newman 1999) with  $b_J \leq 20$  and of incomplete spatial coverage. Of these 10, eight are present in the DR7QSO catalogue, all with  $i \leq 19.1$ . (One of these eight has a DR7QSO redshift slightly greater than 1.4.)

For the assessment of significance of the LQG, Clowes & Campusano (1991) made use of the two-dimensional minimal spanning tree (MST) of the RA and Dec. coordinates. This paper uses three-dimensional single-linkage hierarchical clustering, which is equivalent to the three-dimensional MST. These MST-type algorithms have the great advantage that they do not require assumptions about the morphology of the structure. With their use there are at least three parameters needed to specify the LQGs – the linkage scale, the number of members and the overdensity. Usually, the analyses concentrate on a chosen linkage scale. Pilipenko (2007) discusses two possibilities for making the choice: (i) the physical, in which one chooses the scale that maximizes the fraction of groups that match closely some specified physical parameters (e.g. size, membership and density); and (ii) the formal, in which, for example, one might choose the scale that maximizes the number of groups found (e.g. Graham et al. 1995). Note that some LQGs found at one linkage scale may be fragments of LQGs that would be found completely at a larger linkage scale.

The mean nearest-neighbour separation is an objective measure that can be used to guide the choice of linkage scale. For the redshifts of interest here,  $z \sim 1.3$ , the mean nearest-neighbour separation calculated from area A3725 (as  $0.55\rho^{-1/3}$ ) is  $\sim 74$  Mpc. Note that this nearest-neighbour separation is a true, physical, separation, determined globally by the number of points in a particular volume. However, the linkage in practice between any two quasars is an apparent separation incorporating the true separation plus distortions that arise from the contributions to the redshifts of observational uncertainties and peculiar velocities. Several papers quote estimates for the observational redshift uncertainties of SDSS quasars: 0.004 (Schneider et al. 2010, DR7), 0.006 (Schneider et al. 2007, DR5),  $< 0.01$  (Trammell et al. 2007, DR3,  $z \lesssim 3.4$ ),  $\sim 0.01$  (Shen et al. 2007, DR5,  $z \geq 2.9$ ) and 0.003–0.01 (Ross et al. 2009, compilation). The value of 0.006 from Schneider et al. (2007) refers to a difference of redshifts, so we should divide it by  $\sqrt{2}$  to obtain the uncertainty for a single measurement, 0.004, which is the same as the value from Schneider et al. (2010). We adopt the Schneider et al. (2010) uncertainty of  $\Delta z_{\text{obs}} \sim 0.004$  as a representative DR7 value. The mean redshift of the DR7QSO quasars with  $i \leq 19.1$  is  $z = 1.38$ , so this value of the uncertainty should be appropriate to the redshifts of interest here. Note that the SDSS data processing (Stoughton et al. 2002; Schneider et al. 2007) attempts to correct for the redshift offsets between high- and low-ionization emission lines ( $\sim 600 \text{ km s}^{-1}$ ; Gaskell 1982) and so this possible source of distortion is not considered further here. Hewett & Wild (2010) have provided a catalogue of revised redshifts for SDSS DR6 quasars that corrects for small systematic offsets in the SDSS redshifts, reduces the redshift uncertainties and provides an estimate of the redshift uncertainty for the individual quasars (typically  $\sim 0.0006$ ). How-

ever, DR6 contains large gaps in the sky coverage compared with DR7.

If we assume that quasars have peculiar velocities in the radial direction of  $\sim 400 \text{ km s}^{-1}$ , then there is a further random component to the redshift  $\Delta z_{\text{pec}} \sim 0.003$  at  $z \sim 1.3$ .<sup>1</sup> Combining  $\Delta z_{\text{obs}} \sim 0.004$  and  $\Delta z_{\text{pec}} \sim 0.003$  in quadrature gives the typical distortion in position of a quasar of  $\sim 11$  Mpc at  $z \sim 1.3$ . The corresponding distortion in pairwise separations is then  $\sim 16$  Mpc. Thus, a set of quasars that forms a unit at a true linkage scale of 73 Mpc would be very likely to appear fragmented for apparent linkage scales  $\sim 90$  Mpc.

The detection of the LQG in the DR7QSO catalogue is considered below for linkage scales in the range of 75–105 Mpc. The upper limit of this range is set to be smaller than the expected percolation radius for a Poisson distribution (Pike & Seager 1974; Martínez & Saar 2002), which, for  $z \sim 1.3$  and A3725, is  $\sim 115$  Mpc. This range of 30 Mpc was explored by a binary chop rather than by a series of equal increments. The redshift distribution for the DR7QSO catalogue with  $i \leq 19.1$  is fairly flat across the interval 1.0–1.8 and so this is the range that has been considered in practice, rather than simply 1.2–1.4.

Using the *agnes* algorithm in the *R package*<sup>2</sup> for single-linkage hierarchical clustering, a unit of 34 quasars emerges at a linkage scale of 100 Mpc that, given evidence presented below, appears to be the LQG. At the other linkage scale considered in the binary chop, 90 Mpc, the LQG does not appear at the specified minimum cluster size of 10 members. The apparent linkage scale of 100 Mpc thus means that the LQG is unlikely to be fragmenting at the true mean nearest-neighbour separation  $\sim 73$  Mpc. (Note that this scale of 100 Mpc is between the two scales  $\{100 h^{-1}, 200 h^{-1}$  Mpc diameters, equivalent to 71 and 143 Mpc radii} for spherical filtering used by Miller et al. 2004 to find LQGs.)

The 34 quasars connected at this 100 Mpc linkage scale are listed in Table 1. Their mean redshift is 1.28, identical to that of the original 18 members of the LQG. They cover the redshift range 1.1865–1.4232, whereas the original 18 cover the range 1.207–1.386.

The centroids of this unit of 34 and of the original 18 are separated by only  $\sim 0^\circ.92$ . The identification as the LQG of this unit of 34, occurring at the correct redshift, within  $0^\circ.92$ , and also (see the next section) with essentially the same western, northern and southern boundaries, therefore seems certain.

In what follows, for convenience, this LQG unit of 34 quasars will be designated U1.28 from its mean redshift, and similarly for two other units of interest that are discussed below.

### 3 PROPERTIES OF THE CLOWES & CAMPUSANO LQG FROM THE SDSS DR7QSO QUASARS

The Clowes & Campusano LQG has been detected by three independent methods (Clowes & Campusano 1991; Newman et al. 1998; Newman 1999; Williger et al. 2002). The detection in the DR7QSO data base as U1.28 adds a fourth method.

The location on the sky of the 34 members of U1.28 corresponds well to that of the original LQG: the western, northern and southern

<sup>1</sup> This estimate, provided by O. Snaith (private communication) from simulations, refers to galaxies on the outskirts of clusters (as indicated for quasar environments) at  $z \sim 1.3$ .

<sup>2</sup> See <http://www.r-project.org>

**Table 1.** U1.28: the set of 34 100-Mpc-linked quasars from the SDSS DR7QSO catalogue that are associated with the Clowes & Campusano (1991) LQG. The columns are: SDSS name; RA, Dec. (2000); redshift;  $i$  magnitude and comments.

SDSS name	RA, Dec. (2000)	$z$	$i$	Comments
103744.89+051834.2	10:37:44.89, +05:18:34.2	1.2280	18.958	b
104114.06+034312.0	10:41:14.06, +03:43:12.0	1.2633	18.588	b
104115.58+051345.0	10:41:15.58, +05:13:45.0	1.2553	18.697	a
104116.79+035511.4	10:41:16.79, +03:55:11.4	1.2444	18.531	b
104149.92+064336.5	10:41:49.92, +06:43:36.5	1.3238	18.923	a
104225.63+035539.1	10:42:25.63, +03:55:39.1	1.2293	17.879	b
104256.38+054937.4	10:42:56.38, +05:49:37.4	1.3555	18.661	
104304.95+052515.6	10:43:04.95, +05:25:15.6	1.1865	18.908	
104321.88+045920.6	10:43:21.88, +04:59:20.6	1.3646	19.001	
104345.39+040300.3	10:43:45.39, +04:03:00.3	1.1884	18.937	
104425.80+060925.6	10:44:25.80, +06:09:25.6	1.2523	18.652	b
104426.79+072754.9	10:44:26.79, +07:27:54.9	1.4232	18.846	
104445.32+054348.8	10:44:45.32, +05:43:48.8	1.1879	18.793	
104556.93+072714.7	10:45:56.93, +07:27:14.7	1.3966	18.907	
104637.30+075318.7	10:46:37.30, +07:53:18.7	1.3635	17.612	
104656.71+054150.3	10:46:56.71, +05:41:50.3	1.2284	17.594	a
104733.16+052454.9	10:47:33.16, +05:24:54.9	1.3341	17.705	a
104752.69+061828.9	10:47:52.69, +06:18:28.9	1.3125	18.954	a
104843.05+064456.8	10:48:43.05, +06:44:56.8	1.3523	18.721	
105010.05+043249.1	10:50:10.05, +04:32:49.1	1.2158	18.151	a
105018.10+052826.4	10:50:18.10, +05:28:26.4	1.3067	19.074	b
105022.81+064621.8	10:50:22.81, +06:46:21.8	1.2900	18.362	b
105149.58+033430.2	10:51:49.58, +03:34:30.2	1.2697	19.044	
105422.47+033719.3	10:54:22.47, +03:37:19.3	1.2278	17.972	
105423.26+051909.8	10:54:23.26, +05:19:09.8	1.2785	18.283	
105512.23+061243.9	10:55:12.23, +06:12:43.9	1.3018	18.413	
105534.66+033028.8	10:55:34.66, +03:30:28.8	1.2495	18.195	
105537.63+040520.0	10:55:37.63, +04:05:20.0	1.2619	18.651	
105719.23+045548.2	10:57:19.23, +04:55:48.2	1.3355	18.429	
105810.30+025145.7	10:58:10.30, +02:51:45.7	1.2761	18.842	
105821.28+053448.9	10:58:21.28, +05:34:48.9	1.2540	18.134	
105833.86+055440.2	10:58:33.86, +05:54:40.2	1.3222	18.758	
110108.00+043849.6	11:01:08.00, +04:38:49.6	1.2516	18.254	
110412.00+044058.2	11:04:12.00, +04:40:58.2	1.2554	18.851	

a: Members of the original LQG. See Clowes & Campusano (1991, 1994) and Clowes et al. (1999).

b: Possible additional members of the original LQG from a survey with incomplete spatial coverage: Newman et al. (1998) and Newman (1999).

boundaries seem to be essentially the same, apart from two compact clumps to the north and south-west (Fig. 1), while the eastern boundary is extended by  $\sim 2^\circ$ . Note that for the original 18 members, only the western boundary did not encounter the limits of the survey, but the extension seen with U1.28 is predominantly eastwards, with no major extensions either northwards or southwards. The coincidence of these sets of 18 and 34 seems particularly striking given that only six quasars are in common (of 10 possible for  $i \leq 19.1$ ).

The intensity map of Fig. 1 is many times ( $\sim 6$ ) larger than the area covered by U1.28 itself. In the upper histogram of Fig. 2 we show the redshift distribution of quasars in a smaller rectangular area of  $\sim 47 \text{ deg}^{-2}$  (A47, actually  $46.3 \text{ deg}^2$ ) that contains both U1.28 and the original members of the LQG. The limits of this area are shown by the grey rectangle in Fig. 1. In the lower histogram, we show the redshift distribution of the control area A3725 for comparison. Both the histograms are derived from the DR7QSO catalogue restricted to  $i \leq 19.1$ , and for clarity they have been further restricted to  $z \leq 2.4$ .

The histogram for A47 (upper) shows a prominent peak for  $1.20 < z \leq 1.35$ , which corresponds to U1.28. Note also the peaks for  $1.10 <$

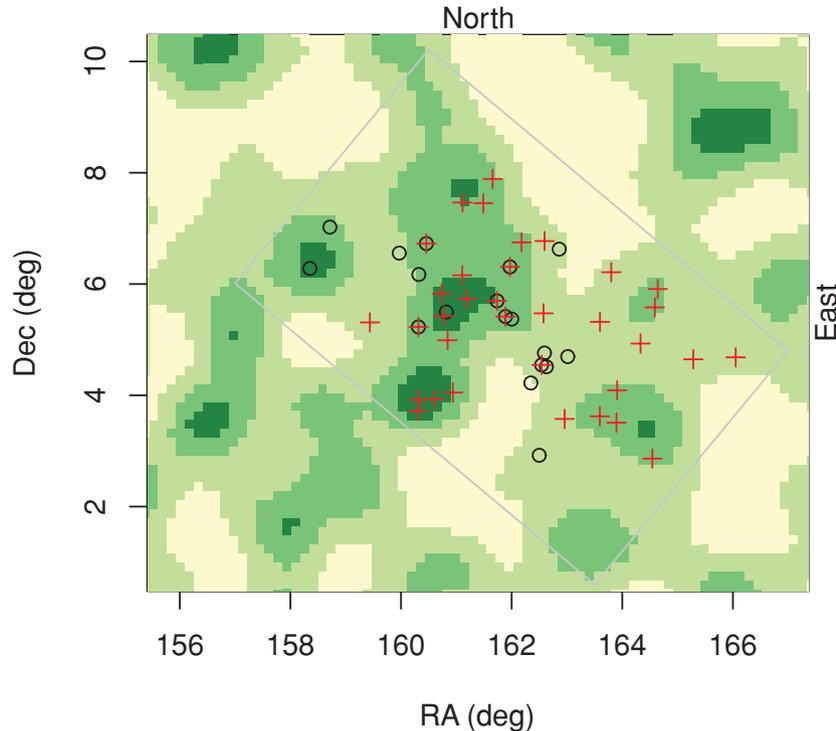
$z \leq 1.15$  and  $1.50 < z \leq 1.60$ , which will be discussed further below.

Fig. 3 shows the redshift distribution within the unit U1.28 of 34 100-Mpc-linked quasars that corresponds to the Clowes & Campusano LQG. Although there are no particularly compelling features in the distribution, there is possibly some concentration of redshifts to the lower half of the range.

#### 4 A NEW LQG AT $z = 1.11$ IN THE SAME DIRECTION

While investigating the appearance of the Clowes & Campusano LQG in the SDSS data, two further (candidate) LQGs became apparent in the same general direction on the sky. We were considering only units with a minimum membership of 20 at the 100-Mpc linkage scale.

These additional candidate LQGs are designated as U1.11 and U1.54, from their mean redshifts. U1.11 has 38 members,  $\bar{z} = 1.11$ , and angular separation (of RA, Dec. centroids) of  $1^\circ 97'$  from U1.28. U1.54 has 21 members,  $\bar{z} = 1.54$ , and angular separation of  $1.62^\circ$



**Figure 1.** The sky distribution of the 34 quasars of U1.28 ( $\bar{z} = 1.28$ ) (crosses) that are connected at the linkage scale of 100 Mpc together with the original 18 LQG members (circles). The area shown is approximately  $12^\circ \times 10^\circ$ , which is  $\sim 6$  times larger than the area covered by U1.28 itself. The DR7QSO quasars are limited to  $i \leq 19.1$ . Superimposed on these distributions is a kernel-smoothed intensity map (isotropic Gaussian kernel,  $\sigma = 0.5$ ), plotted with four linear palette levels ( $\leq 0.8, 0.8-1.6, 1.6-2.4, \geq 2.4 \text{ deg}^{-2}$ ), for all of the quasars in the redshift range of U1.28 ( $z: 1.1865-1.4232$ ). The co-location on the sky of U1.28 and the original 18 LQG members is clear. Although only the western boundary of the original LQG members did not encounter the limits of its survey, the western, northern and southern boundaries of U1.28 and the original 18 are essentially the same, apart from two compact clumps to the north and south-west. The eastern boundary of U1.28 extends further than the original 18 by  $\sim 2^\circ$ . (The grey rectangle shows an area {A47} defined for a following figure.)

from U1.28. We present below a method for assessing the statistical significance and overdensity of groups found by linkage of points. We find that U1.28 and U1.11 are significant, but U1.54 is not. However, we note that U1.54 corresponds to a known LQG (of marginal significance) at  $\bar{z} = 1.53$  (or, as published, median  $z = 1.51$ ) that was discovered by Newman et al. (1998) and Newman (1999) in an independent UV-excess survey (the Chile–UK Quasar Survey). Thirteen members were found in the original discovery, of which 10 are present in this re-discovery.

The peaks in the upper redshift histogram of Fig. 2 for the intervals  $1.10 < z \leq 1.15$  and  $1.50 < z \leq 1.60$ , which were mentioned briefly in the earlier discussion of the figure, correspond to U1.11 and U1.54.

U1.11, however, is a new discovery, notable for both its appearance in the same cosmological neighbourhood as U1.28 and its similarly large number of members. The 38 quasars of U1.11 are listed in Table 2. The distribution of redshifts for U1.11 is shown in Fig. 4. Again, there are no particularly compelling features in the distribution, but in this case there is possibly some concentration of redshifts to the upper half of the range.

Fig. 5 shows three projections (Dec.–RA, RA– $z$  and Dec.– $z$ ) of the spatial distributions of U1.11 and U1.28.

In the entire DR7QSO catalogue (i.e. area A9380), we find a total of 15 LQG candidates of such high membership ( $N \geq 34$ ). Of these, U1.28 and U1.11 are the closest, with a separation of centroids of  $\sim 410$  Mpc. From the density of such candidates, the probability of a pair within this separation occurring by chance somewhere within the coverage of the whole catalogue (and  $1.0 \leq z \leq 1.8$ ) is  $\sim 0.5$ , and so this pair is consistent. Note, however, that U1.28 and U1.11

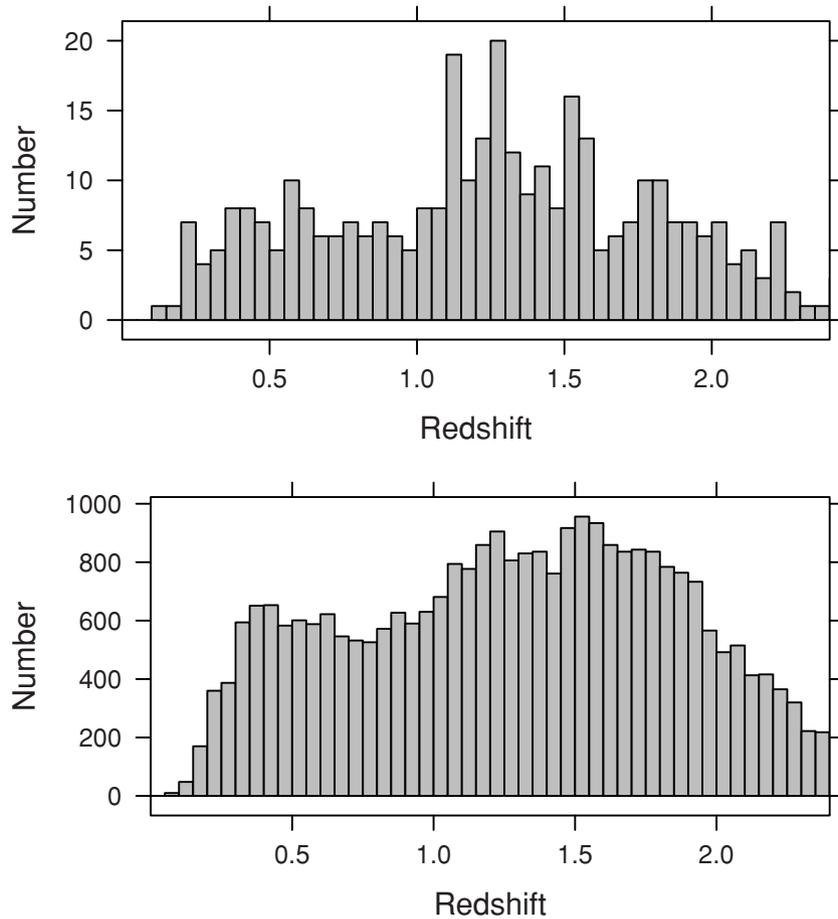
do appear to be quite distinct: a small increase in the linkage scale does not lead to their merger as a single unit. The volume occupied by U1.28 and U1.11 together thus appears rather distinctive for quasars on a large scale and would presumably correspond to some notable features in the cosmic web if the distribution of galaxies was accessible to observation. As mentioned above, and as is apparent from Fig. 5, U1.28 and U1.11 are also quite closely aligned with the line of sight.

## 5 ASSESSMENT OF STATISTICAL SIGNIFICANCE AND OVERDENSITIES

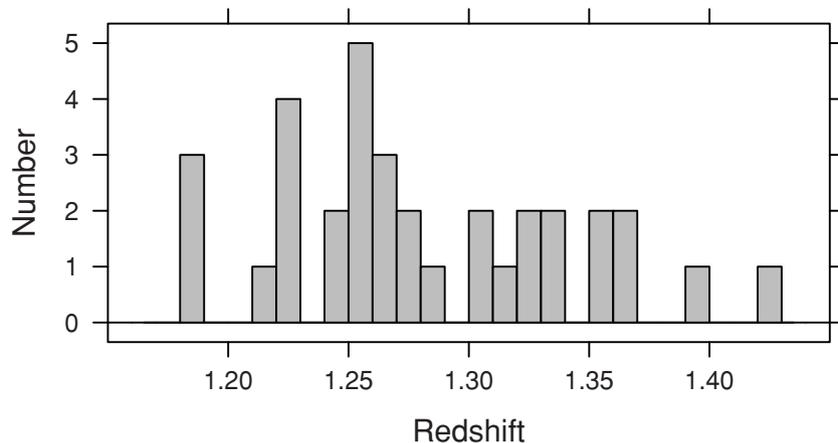
Groups found by the linkage of points generally require a separate procedure to assess statistical significance. Graham et al. (1995) considered this problem previously and created the  $m, \sigma$  method, which requires that groups show significant sub-clustering; Pilipenko (2007) addressed it differently by counting the frequency of comparable groups in random sets. A procedure is also needed to estimate overdensities. Pilipenko (2007) estimated overdensity from properties of the MST, which method is convenient but was not justified in that paper. In this section, we present a new procedure based on the convex hull that we can use for both statistical significance and overdensity.

We use a measure of the volume occupied by an LQG to assess the statistical significance and estimate the overdensity: an LQG must occupy a smaller volume than that expected for the same number of random points.

A simple way to estimate the volume is to define a ‘RA–Dec.– $z$  box’ with corners determined by the RA, Dec. and  $z$  limits of the



**Figure 2.** The upper histogram shows the redshift histogram for all DR7QSO quasars ( $i \leq 19.1$ ) in the area A47 that contains both U1.28 and the original members of the LQG. The area A47 is shown by the grey rectangle in Fig. 1. The lower histogram shows the redshift distribution ( $i \leq 19.1$ ) of the control area A3725 for comparison. Both histograms have been restricted to  $z \leq 2.4$  for clarity. The histogram for A47 shows a prominent peak for  $1.20 < z \leq 1.35$ , which corresponds to U1.28. Note also the peaks for  $1.10 < z \leq 1.15$  and  $1.50 < z \leq 1.60$ .



**Figure 3.** The redshift distribution within the unit U1.28 of 34 100-Mpc-linked quasars that corresponds to the Clowes & Campusano LQG. The histogram is for a bin size of  $\Delta z = 0.01$ .

LQG. However, since LQGs are typically not cuboids aligned with the coordinate axes, the box typically overestimates the volume quite severely. The RA–Dec.– $z$  boxes will generally also include non-members and so their volumes are not then solely those of the LQGs.

Another way to estimate the volume is to use the volume of the convex hull.<sup>3</sup> However, the convex hull typically underestimates

<sup>3</sup> The 3D convex hull of a set of points is the polyhedron of minimum volume that contains the lines connecting all pairs of points.

**Table 2.** U1.11: the set of 38 100-Mpc-linked quasars from the SDSS DR7QSO catalogue. The columns are: SDSS name; RA, Dec. (2000); redshift and  $i$  magnitude.

SDSS name	RA, Dec. (2000)	$z$	$i$
102907.22+021552.4	10:29:07.22, +02:15:52.4	1.0173	19.050
103200.20+022056.4	10:32:00.20, +02:20:56.4	1.0038	18.901
103300.11+042116.9	10:33:00.11, +04:21:16.9	1.0144	17.875
103552.43+032537.2	10:35:52.43, +03:25:37.2	1.0553	18.980
103626.33+045436.4	10:36:26.33, +04:54:36.4	1.0477	18.404
103639.63+022553.5	10:36:39.63, +02:25:53.5	1.0525	18.817
103641.96+050941.1	10:36:41.96, +05:09:41.1	1.0631	18.553
103709.33+022055.7	10:37:09.33, +02:20:55.7	1.0143	18.449
103739.49+034946.9	10:37:39.49, +03:49:46.9	1.0321	18.924
103743.97+040233.8	10:37:43.97, +04:02:33.8	1.0932	18.671
103748.36+040242.1	10:37:48.36, +04:02:42.1	1.0869	17.857
103806.57+020234.3	10:38:06.57, +02:02:34.3	1.0526	19.068
104012.14+043904.6	10:40:12.14, +04:39:04.6	1.1195	18.578
104309.70+075317.8	10:43:09.70, +07:53:17.8	1.1823	18.872
104410.13+072305.6	10:44:10.13, +07:23:05.6	1.1514	18.189
104446.16+070651.4	10:44:46.16, +07:06:51.4	1.1292	17.973
104506.44+051627.4	10:45:06.44, +05:16:27.4	1.1116	18.753
104509.93+063559.0	10:45:09.93, +06:35:59.0	1.1184	19.001
104636.93+082437.4	10:46:36.93, +08:24:37.4	1.1398	18.747
104752.93+022408.1	10:47:52.93, +02:24:08.1	1.1390	19.012
104835.72+000002.3	10:48:35.72, +00:00:02.3	1.1429	18.641
104901.71+005534.0	10:49:01.71, +00:55:34.0	1.1630	18.215
104932.22+050531.7	10:49:32.22, +05:05:31.7	1.1136	18.699
105017.31+012450.9	10:50:17.31, +01:24:50.9	1.2007	18.800
105048.25+032328.6	10:50:48.25, +03:23:28.6	1.1509	18.914
105118.61+015755.9	10:51:18.61, +01:57:55.9	1.1812	18.305
105229.50+031131.5	10:52:29.50, +03:11:31.5	1.1665	18.832
105234.24,-001501.0	10:52:34.24 -00:15:01.0	1.1607	17.694
105352.72+050043.9	10:53:52.72, +05:00:43.9	1.1320	18.865
105414.09,-001803.5	10:54:14.09 -00:18:03.5	1.1618	18.883
105459.34+031151.3	10:54:59.34, +03:11:51.3	1.1819	18.582
105527.67+002001.5	10:55:27.67, +00:20:01.5	1.1448	18.782
105543.01+001001.7	10:55:43.01, +00:10:01.7	1.1093	19.052
105549.72+031324.1	10:55:49.72, +03:13:24.1	1.1348	18.618
105703.23+040526.8	10:57:03.23, +04:05:26.8	1.1330	18.078
105837.95+033124.9	10:58:37.95, +03:31:24.9	1.1215	18.857
105943.44+024418.2	10:59:43.44, +02:44:18.2	1.1024	18.618
110121.27+023333.0	11:01:21.27, +02:33:33.0	1.0874	18.306

the volume (although overestimates are also possible, depending on morphology) because it can wrap tightly around the surface points and does not then consider any surrounding region as belonging to them.

We can construct a measure of volume that should better reflect the volume occupied by the LQG than either the RA–Dec. box or the convex hull of the points. We do this by expanding each member point of a unit to be a sphere, with radius set to be half of the mean linkage (MST edge length) of the unit. In this way, each point is associated with a spherical volume. We then take the volume of the LQG to be the volume of the convex hull of these spheres. We shall refer to this method as the CHMS method – convex hull of member spheres. Note that this measure refers to the LQG members only, unlike the RA–Dec.– $z$  box, which typically incorporates non-members too.

The distribution of CHMS volumes resulting from random points that have been distributed with known density allows the statistical significance of an LQG to be assessed. The CHMS volumes for sets of random points can also be used to estimate residual biases and,

if required, make corrections to the CHMS volumes, densities and overdensities for LQGs.

The residual bias is expressed as a volume correction, which is the ratio of the known volume of the sets of random points to their mean CHMS volume. It corrects for imperfections, of consequence only at the lower memberships, in reproducing the true volume with the above (natural) choice of sphere radius. The observed CHMS volume for an LQG can then be corrected by multiplying by the appropriate volume correction.

For assessing the statistical significance of an LQG of membership  $N$ , we compare the departure of its CHMS volume (uncorrected, since there is no need for the corrections here) from the mean of the distribution of CHMS volume for 1000 random sets of  $N$ . Each random set is defined by distributing  $N$  points in a cube, of volume such that the density in the cube corresponds to the density in A3725 for the redshift limits of the LQG. In this way, we find that the departures from random expectations for U1.28, U1.11 and U1.54 are respectively 3.57, 2.95 and 1.75 $\sigma$ .

U1.54 thus appears as not significant, although, as mentioned above, it is a re-discovery, with an independent sample, of an LQG that was previously known (Newman et al. 1998; Newman 1999), albeit also at marginal significance then.

After correcting the CHMS volumes for residual bias, the estimated overdensities of the significant LQGs are  $\delta_q = \delta\rho_q/\rho_q = 0.83, 0.55$  for U1.28 and U1.11, respectively. (The volume corrections are  $\sim 10$  and 8 per cent, respectively.)

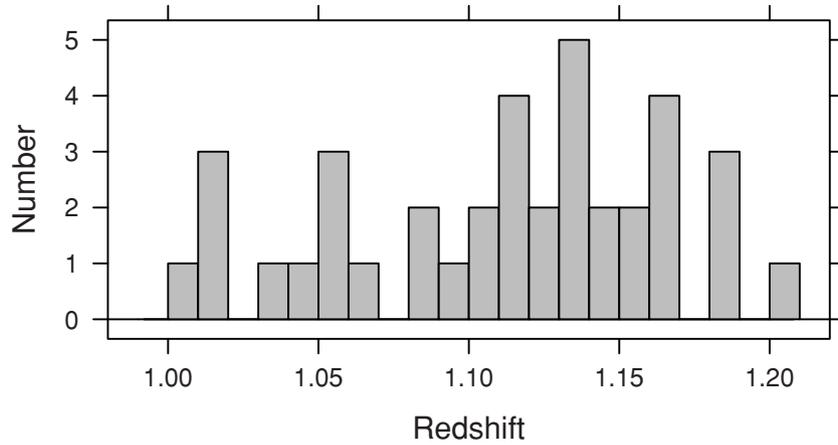
Pilipenko (2007) gives a method for determining the overdensity of an LQG that avoids defining the containing volume, but gave no justification for it and did not discuss possible biases. In this method, the overdensity is defined to be  $\delta_q = \langle l_0^3 \rangle / \langle l^3 \rangle$ , where  $l$  is MST edge length for the LQG and  $l_0$  is MST edge length for a control area elsewhere. Here we modify this definition by including the ‘-1’ that is more usual for overdensities, giving  $\delta_q = (\langle l_0^3 \rangle / \langle l^3 \rangle) - 1$ . Recall that the MST is equivalent to the single-linkage hierarchical clustering used in this paper. For this method, we obtain  $\delta_q = 0.78, 1.31$  for U1.28 and U1.11, respectively, with  $l_0$  having been obtained from the control area A3725 separately for the redshift range of each unit. The overdensities by the Pilipenko method are thus higher for U1.11 than for our CHMS method.

The CHMS overdensities here,  $\delta_q \sim 0.5$ – $0.9$ , are substantially lower than those found by Pilipenko (2007) and a little higher than those found by Miller et al. (2004). The two Pilipenko categories, as mentioned above are: (i) size  $\sim 85$  Mpc, membership  $\sim 6$ – $8$  and overdensity  $\sim 10$  (or 9 with the ‘-1’) and (ii) size  $\sim 200$  Mpc, membership  $\gtrsim 15$  and overdensity  $\sim 4$  (or 3 with the ‘-1’). The overdensities from Miller et al. (2004) are calculated for spherical tophat filtering, giving  $\delta_q < 0.44$  for 100  $h^{-1}$  Mpc diameters and  $\delta_q < 0.17$  for 200  $h^{-1}$  Mpc diameters.

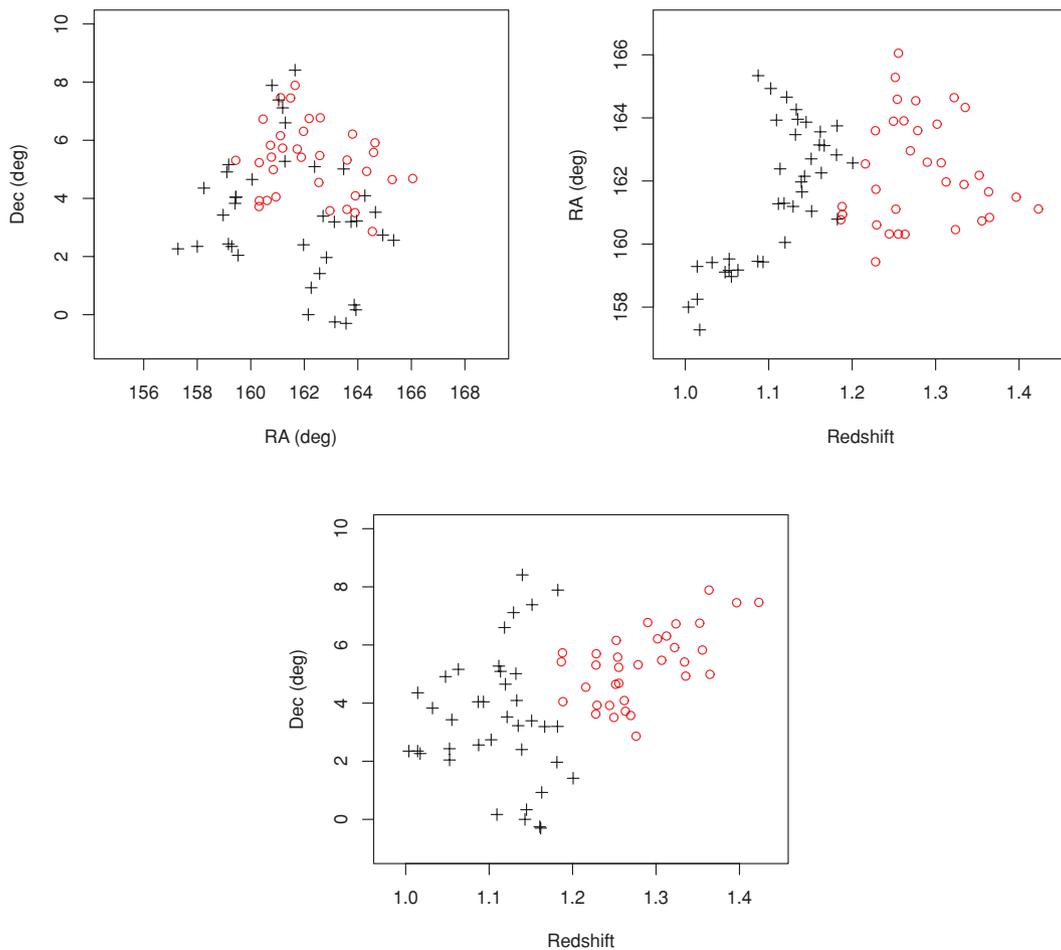
## 6 DISCUSSION AND CONCLUSIONS

This paper has re-examined the Clowes & Campusano (1991) LQG, originally known with 18 members, using data from the SDSS DR7QSO catalogue. It is found as the unit U1.28 of 34 100-Mpc-linked quasars, with mean redshift  $\bar{z} = 1.28$ . While the western, northern and southern boundaries remain essentially unchanged, apart from two compact clumps to the north and south-west, there is an extension eastwards, beyond the original survey, of  $\sim 2^\circ$ .

A new LQG, U1.11, was discovered in the same direction – 1:97 from U1.28. It has 38 members and mean redshift  $\bar{z} = 1.11$ . A third candidate, U1.54, in the same direction at 1:62 from U1.28,



**Figure 4.** The redshift distribution within the unit U1.11 of 38 100-Mpc-linked quasars. The histogram is for a bin size of  $\Delta z = 0.01$ .



**Figure 5.** Projections of the spatial distributions of the 100-Mpc-linked units of U1.11 (crosses) and U1.28 (circles), shown as plots of Dec. against RA, RA against redshift and Dec. against redshift.

appeared statistically insignificant, although it is a re-discovery of a known (marginal) LQG (Newman et al. 1998; Newman 1999).

We have also presented the ‘CHMS method’ for assessing the statistical significance and overdensity of groups such as LQGs that have been found by linkage of points.

Attention was first drawn to peculiarities in this area of sky by Cannon & Oke (private communication) who, in the early days of quasar surveys, noted the unusual ease with which they could

find the then hard-to-find quasars with  $z \sim 1.0$ – $1.6$ . It may be that, now we know there are not one but two LQGs of unusually high membership in this direction, we finally have the complete explanation of their result.

A simple measure of the characteristic size of the LQGs, which takes no account of morphology, is the cube root of the corrected CHMS volumes, giving  $\sim 350$  and  $380$  Mpc for U1.28 and U1.11, respectively. Clearly, these are very large sizes, placing these two

LQGs among the largest features so far seen in the early universe. For comparison, Yadav, Bagla & Khandai (2010) give an idealized limit to the scale of homogeneity in the concordance cosmology as  $\sim 370$  Mpc: it should not be possible to find departures from homogeneity above this scale. The LQGs appear to be only marginally consistent with this scale of homogeneity.

Calculation of the inertia tensor for these two LQGs shows a ratio  $\sim 2.5$  for the longest and shortest principal axes in both cases. They are therefore substantially elongated. By this measure, the longest axes are  $\sim 630$  and  $780$  Mpc for U1.28 and U1.11, respectively. Their morphologies appear to be markedly oblate, like a thick lens, each with two comparably large long axes and a short axis that is smaller by the factor  $\sim 2.5$ . Clearly, the long axes do exceed the expected scale of homogeneity.

The estimated overdensities are  $\delta_q = \delta\rho_q/\rho_q = 0.83$  and  $0.55$  for U1.28 and U1.11, respectively. These overdensities are substantially lower than those found by Pilipenko (2007) and a little higher than those found by Miller et al. (2004).

The occurrence of structure on a particular scale is naturally taken to mean that the universe is not homogeneous on that scale. These two LQGs, U1.28 and U1.11, as overdensities of the amplitudes and scales indicated, thus raise a question of compatibility with the scale of homogeneity in the concordance cosmology, if the Yadav et al. (2010) fractal calculations are adopted as reference. A counter argument could be made that these LQGs are chance associations of groups on sub-homogeneity scales, analogous to the finding of Einasto et al. (2011) that the component superclusters of the Sloan Great Wall have different evolutionary histories. Even if this were true, homogeneity asserts that any global property of sufficiently large volumes should be the same within the expected statistical variations, so the density of quasars in these LQGs would remain distinctive. Of course, unknown observational biases or selection effects could conceivably also affect the overall dimensions of the LQGs.

In finding compatibility of LQGs with concordance cosmology, Miller et al. (2004) noted that they had not considered questions of shape and topology. Given both the large sizes and elongated morphology that we find for U1.28 and U1.11, we have begun a programme to re-investigate compatibility, using the full set of LQGs that we find from the DR7QSO catalogue.

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## REFERENCES

- Brand K., Rawlings S., Hill G. J., Lacy M., Mitchell E., Tufts J., 2003, *MNRAS*, 344, 283
- Clowes R. G., Campusano L. E., 1991, *MNRAS*, 249, 218
- Clowes R. G., Campusano L. E., 1994, *MNRAS*, 266, 317
- Clowes R. G., Campusano L. E., Graham M. J., 1999, *MNRAS*, 309, 48
- Crampton D., Cowley A. P., Hartwick F. D. A., 1987, *ApJ*, 314, 129
- Crampton D., Cowley A. P., Hartwick F. D. A., 1989, *ApJ*, 345, 59
- Croom S. M., Smith R. J., Boyle B. J., Shanks T., Miller L., Outram P. J., Loaring N. S., 2004, *MNRAS*, 349, 1397
- de Ruiter H. R., Zuiderwijk E. J., 1982, *A&A*, 105, 254
- Einasto M. et al., 2011, *ApJ*, 736, 51
- Gaskell C. M., 1982, *ApJ*, 263, 79
- Geller M. J., Huchra J. P., 1989, *Sci*, 246, 897
- Graham M. J., Clowes R. G., Campusano L. E., 1995, *MNRAS*, 275, 790
- Haberzettl L. et al., 2009, *ApJ*, 702, 506
- Haines C. P., Clowes R. G., Campusano L. E., Adamson A. J., 2001, *MNRAS*, 323, 688
- Haines C. P., Campusano L. E., Clowes R. G., 2004, *A&A*, 421, 157
- Hewett P. C., Wild V., 2010, *MNRAS*, 405, 2302
- Hutsemékers D., Cabanac R., Lamy H., Sluse D., 2005, *A&A*, 441, 915
- Komberg B. V., Lukash V. N., 1994, *MNRAS*, 269, 277
- Komberg B. V., Kravtsov A. V., Lukash V. N., 1996, *MNRAS*, 282, 713
- Longo M. J., 1991, *ApJ*, 372, L59
- Martínez V. J., Saar E., 2002, *Statistics of the Galaxy Distribution*. Chapman & Hall/CRC, Boca Raton
- Miller L., Croom S. M., Boyle B. J., Loaring N. S., Smith R. J., Shanks T., Outram P., 2004, *MNRAS*, 355, 385
- Mountrichas G., Sawangwit U., Shanks T., Croom S. M., Schneider D. P., Myers A. D., Pimblett K., 2009, *MNRAS*, 394, 2050
- Nabokov N. V., Baryshev Yu. V., 2008, in Baryshev Yu., Taganov I., Teerikorpi P., eds, *Practical Cosmology*, Proc. International Conference ‘Problems of Practical Cosmology’, TIN, St. Petersburg, p. 69, preprint (arXiv:0809.2390)
- Newman P. R., 1999, PhD thesis, Univ. Central Lancashire
- Newman P. R., Clowes R. G., Campusano L. E., Graham M. J., 1998, in Müller V., Gottlöber S., Mücke J. P., Wambsgans J., eds, *Large Scale Structure: Tracks and Traces*. World Scientific, Singapore, p. 133
- Ohta K. et al., 2003, *ApJ*, 598, 210
- Padmanabhan N. et al., 2007, *MNRAS*, 378, 852
- Pike G. E., Seager C. H., 1974, *Phys. Rev. B*, 10, 1421
- Pilipenko S. V., 2007, *Astron. Rep.*, 51, 820
- Richards G. T. et al., 2002, *AJ*, 123, 2945
- Richards G. T. et al., 2006, *AJ*, 131, 2766
- Ross N. P. et al., 2009, *ApJ*, 697, 1634
- Sánchez S. F., González-Serrano J. I., 1999, *A&A*, 352, 383
- Schneider D. P. et al., 2007, *AJ*, 134, 102
- Schneider D. P. et al., 2010, *AJ*, 139, 2360
- Shen Y. et al., 2007, *AJ*, 133, 2222
- Smith R. J., Croom S. M., Boyle B. J., Shanks T., Miller L., Loaring N. S., 2005, *MNRAS*, 359, 57
- Söchtig I. K., Clowes R. G., Campusano L. E., 2002, *MNRAS*, 331, 569
- Söchtig I. K., Clowes R. G., Campusano L. E., 2004, *MNRAS*, 347, 1241
- Stoughton C. et al., 2002, *AJ*, 123, 485

Tanaka I., Yamada T., Turner E. L., Suto Y., 2001, *ApJ*, 547, 521  
Tesch F., Engels D., 2000, *MNRAS*, 313, 377  
Thomas S. A., Abdalla F. B., Lahav O., 2011, *Phys. Rev. Lett.*, 106, 241301  
Trammell G. B., Vanden Berk D. E., Schneider D. P., Richards G. T., Hall  
P. B., Anderson S. F., Brinkmann J., 2007, *AJ*, 133, 1780  
Vanden Berk D. E. et al., 2005, *AJ*, 129, 2047

Webster A. S., 1982, *MNRAS*, 199, 683  
Williger G. M., Campusano L. E., Clowes R. G., Graham M. J., 2002, *ApJ*,  
578, 708  
Yadav J. K., Bagla J. S., Khandai N., 2010, *MNRAS*, 405, 2009

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