Minimum weight design of CNT/fiber reinforced laminates subject to a frequency constraint by optimal distribution of reinforcements across the thickness

Shivaan Anashpaul[1], Georgios A. Drosopoulos[2,1], Sarp Adali[3*]
[1]Discipline of Civil Engineering, University of KwaZulu-Natal, Durban, South Africa
[2]Discipline of Civil Engineering, University of Central Lancashire, Preston, UK
[3]Discipline of Mechanical Engineering, University of KwaZulu-Natal, Durban, South Africa

* Corresponding author, ADALI@ukzn.ac.za

ABSTRACT

Present work involves minimization of the weight of a three-phase, glass and carbon nanotubes (CNT) reinforced angle-ply laminate, subject to a fundamental frequency constraint. Design variables include the optimal (nonuniform) distributions of the fibers and the CNTs across the laminate thickness, the stacking sequence, ply angles and ply thicknesses. In order to assess the effect of different design variables on weight minimization, a sequence of five different optimization problems is solved with an increasing number of design variables. The present approach has the advantage of comparing and assessing the effectiveness of different sets of design variables on the design efficiency and identifying the ones which are more effective in minimizing the weight. Design efficiency is defined as the ratio of the weight of the optimal laminate to the weight of a plate which is not reinforced, but subject to the same frequency constraint. It is observed that fiber and CNT reinforcements are used most effectively if placed close to or in the outer layers. It is also observed that CNT reinforcement affects the optimal values of the ply angles due to the fact that the directional properties of the fibers become less pronounced as the matrix is reinforced by CNTs.

Keywords: Minimum weight design, Nanocomposites, CNT reinforcement, Frequency constraint, Multiscale composite

1. Introduction

Lightweight structures are of major importance in several fields of engineering such as aerospace, marine and automotive engineering. This led to an increased use of composites in many engineering applications due to their high stiffness and high strength to weight ratios. Development of nanocomposites improved the weight advantage of composite materials further by employing nano-scale reinforcements in the design of composite components. This was mainly due to the discovery of carbon nanotubes (CNTs) with exceptional properties in 1991 leading to two-phase nanocomposites in the form of CNT reinforced polymer matrices as noted in [1, 2] and in review articles [3, 4]. The effect of CNTs for improving the properties of polymer composites has been studied extensively and reported in the review articles [5, 6].

Recent studies on components involving two-phase CNT reinforced composites include papers on bending [7, 8], buckling [9, 10] and vibration [9-12]. One drawback of two-phase components involves the mostly random distribution of CNTs in a polymer matrix as noted in [13, 14]. As a result of this, i.e., the random distribution of CNTs in the matrix, two-phase nanocomposites lack directional properties. However, in many applications, directional stiffness and strength properties are important design issues. Further developments in material science and manufacturing processes led to three-phase multiscale nanocomposites by also introducing fibers as reinforcing components in addition to CNTs as pointed out in [15].

Presently, nanocomposites comprising fiber/CNT/polymer combination are being used in the design of several components as outlined in the review article [16]. As such, these composites, having macro, micro and nanoscale elements, can be considered ideal materials in weight sensitive applications as they combine the light weight advantage of CNTs with the directional properties of fibers. Due to the reinforcing effect of CNTs, three-phase nanocomposites are lighter than fiber reinforced composites for the same level of stiffness. In addition to their natural light weight advantage, three-phase composites can be further optimized for minimum weight subject to a certain set of design constraints. The weight content of CNTs in each layer provides another design variable when this parameter is included in the optimization process. Inclusion of the fiber orientations as design variables in addition to the volume contents of CNTs and the fibers in each layer increases the complexity of the design and manufacturing processes of the composite laminates. However, the benefits of using fiber reinforced three-phase nanocomposites are likely to compensate for the negatives (which are mostly due to a more complicated manufacturing process) by increasing the strength and stiffness and reducing the weight of the final design.

Light weight structures tend to be slender and, as a result, susceptible to resonance in the presence of dynamic loads. Vibrations of three-phase nanocomposites have been studied in a number of cases which include viscoelastic composite beams [17], 3D frequency response of laminated annular plates [18], and nonlinear vibrations of cylindrical shells [19, 20].

For structures subject to dynamic loads, structural resonance needs to be avoided for their safe operation. This can be achieved by imposing frequency constraints at the design stage of the component. In this case, the objective is to keep the natural frequencies away from the resonance frequencies. In most cases of the minimum weight designs, the constraint is imposed on the fundamental frequency to be higher than a certain value. This is also the same approach for the minimum weight design case studied in the present work. Minimum weight designs of composite structures have been studied extensively due to their importance in many fields of engineering with aerospace being a prime area of importance for light weight designs. Weight and cost minimization of carbon and glass fiber hybrid laminates were studied in a number of papers [21, 22]. In [22], design variables were specified as the number of layers and the fiber angles and using an unconventional stacking sequence, minimum weight designs of composite laminates were obtained.

Multiscale composites are studied in [23] involving composite beams and blades. CNTs are assumed to be uniformly distributed and randomly oriented. It was noted that a small amount of CNT improved the stiffness of the beams and blades substantially with the matrix specified as E-glass/epoxy. Vibration, bending and buckling of CNT reinforced hybrid beams are studied in [24]. In [25], weight and cost minimization were investigated for a laminated beam made of carbon and flax fibers subject to a frequency constraint with the layer thicknesses, type, angle and volume contents of fibers specified as design parameters.

Minimization of the weight of a functionally graded beam subject to a buckling constraint was the subject of the work in [26] with the material specified as aluminum and ceramic and taking the material distribution as the design variable. Multiobjective lightweight design of a functionally graded beam was given in [27] specifying the maximum frequency and the buckling load as the design objectives. Weight of a laminated carbon fiber composite wing torsion box was minimized in [28] subject to stiffness and strength constraints using a multi-objective optimization approach. In [28], fiber orientations and the layer thicknesses were taken as the design variables. Optimization of three-phase plates with respect to frequencies was studied in [29] using Mori-Tanaka approach for micromechanical analysis and firefly algorithm for optimization. The effect of the nonuniform distribution of CNTs across laminate thickness was noted in several studies involving functionally graded distributions of the reinforcements [30-33].

In the present study, the weight of a three-phase laminated plate is minimized subject to a fundamental frequency constraint taking the distribution of the fibers and the CNTs nonuniform across the laminate thickness. As such, they serve as design parameters in addition to the stacking sequence, ply angles and the layer thicknesses. Sequential quadratic programming algorithm is employed to solve the formulated optimization problems. In order to assess the effectiveness of each design variable on minimizing the weight, a sequence of five optimization problems is solved with the number of design variables increasing from one design problem to the next. This approach indicates the effectiveness of each design variable as compared to the other design variables in minimizing the weight.

In a composite laminate design, fiber orientations and the stacking sequence are the design variables mostly used in the design process. However, distributions of the reinforcing materials, which are the fibers and the CNTs in the present case, across the thickness have gained importance recently to improve the efficiency of the optimal designs and to reduce the weight by using the reinforcements where they are most effective as noted in [34]. The effect of the distribution of CNTs on the weight has been noted in a number of studies involving functionally graded laminates with the CNTs distributed nonuniformly across the thickness. In this context, it was observed that composite material designs consisting of a combination of fibers and a nano material had substantially improved the design efficiencies in terms of weight as noted in [24, 26, 35]. Their effectiveness in minimizing the weight is studied in the present paper by means of solving five design problems of increasing complexity.

The paper is organized as follows: Section 2 presents the formulation for the minimum weight problem subject to a frequency constraint. Micromechanical equations to compute the material properties of three-phase materials are presented in Section 3. In Section 4, the formulations for the different optimization problems are presented. This includes the design constraints and the design variables used in each optimization problem. Section 5 shows the verification of the current method of solution. This is followed by the numerical results in Section 6 which provides detailed descriptions for each optimization problem and highlighting the effect of carbon nanotubes to weight minimization. Section 7 concludes the paper and summarizes the main findings of the study.

2. Theoretical formulation

Under consideration is a simply supported rectangular laminate of dimensions a in the x direction and b in the y direction as shown in Figure 1. The laminate has a thickness H and the number of layers is denoted by N (Figure 1).

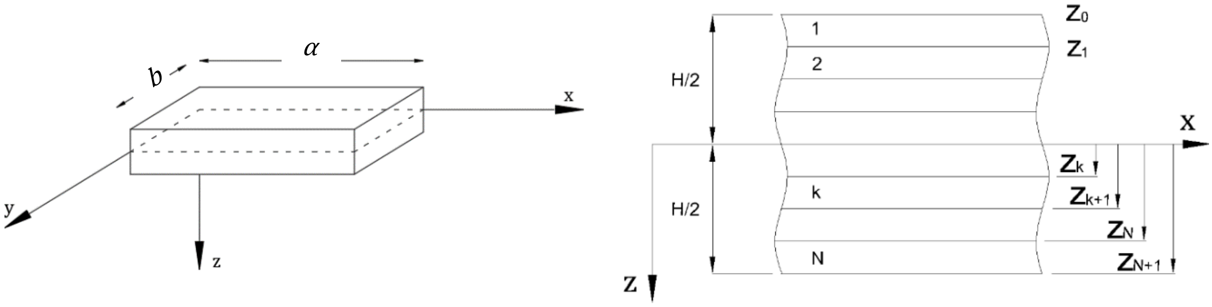


Figure 1: Geometry and notations for the laminated plate.

The equation governing the free vibrations of the laminate is given in [36] as

$$D_{11}\frac{\partial^4 w_0}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w_0}{\partial y^4} + I_0\frac{\partial^2 w_0}{\partial t^2} = 0 \tag{1}$$

where

$$I_0 = \sum_{k=1}^{N} \rho_0^{(k)}(z_{k+1} - z_k) \tag{2}$$

is the mass per unit area with the thickness direction z shown in Figure 1. The stiffness terms D_{ij} are given by:

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} \bar{Q}_{ij}^{(k)}(z_{k+1}^3 - z_k^3) \tag{3}$$

The expressions for the transformed stiffness terms $\bar{Q}_{ij}(\theta)$ are given by [36]:

$$\begin{aligned}
\overline{Q}_{11} &= Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta \\
\overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta) \\
\overline{Q}_{22} &= Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta \\
\overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta)
\end{aligned} \tag{4}$$

The stiffness Q_{ij} for each layer is expressed as

$$Q_{11}=\frac{E_{11}}{1-\nu_{12}\nu_{21}} \quad Q_{12}=\frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} \quad Q_{22}=\frac{E_{22}}{1-\nu_{12}\nu_{21}} \quad Q_{66}=G_{12} \tag{5}$$

In the present study, the influence of bending-twisting coupling terms D_{16} and D_{26} is neglected. The error of neglecting these terms becomes negligible when $\gamma \leq 0.2$ and $\delta \leq 0.2$ where $\gamma = D_{16}(D_{11}^3 D_{22})^{-1/4}$, $\delta = D_{26}(D_{11}D_{22}^3)^{-1/4}$ as noted in a number of studies [37-39].

For the simply supported boundary conditions, the solution of the governing equation (1) can be expressed as

$$w_0(x,y)=C_{mn}\,sin\frac{m\pi x}{a}\;sin\frac{n\pi y}{b} \tag{6}$$

corresponding to the mode numbers m and n. The fundamental frequency ω_{11} is given by [36]:

$$\omega_{11}^2=\frac{\pi^4}{I_0 b^4}\left[D_{11}r^4+2(D_{12}+2D_{66})r^2+D_{22}\right] \tag{7}$$

where r is the aspect ratio defined as $r=b/a$.

3. Effective material properties using micromechanics equations

3.1. Two-phase CNT reinforced matrix composite

For two-phase CNT reinforced matrix, the elastic constants can be determined using the micromechanical equations. The index $CNTM$ is used to denote the effective elastic properties of the two-phase CNT reinforced matrix. To indicate the material properties of the CNT and the matrix, the indices CNT and M are used. Young's modulus of the CNT reinforced matrix is given by [23]:

$$E_{CNTM}=\frac{E_M}{8}\left[5\left(\frac{1+2\beta_{dd}V_{CNT}}{1-\beta_{dd}V_{CNT}}\right)+3\left(\frac{1+2(l_{CNT}/d_{CNT})\beta_{dl}V_{CNT}}{1-\beta_{dl}V_{CNT}}\right)\right] \tag{8}$$

where subscripts *CNT* and *M* correspond to the CNT reinforced matrix and the unreinforced matrix, respectively. The coefficients which appear in equation (8) are given by

$$\beta_{dd}=\frac{(E_{CNT}/E_M)-(d_{CNT}/4t_{CNT})}{(E_{CNT}/E_M)+(d_{CNT}/2t_{CNT})} \tag{9}$$

$$\beta_{dl} = \frac{(E_{CNT}/E_M) - (d_{CNT}/4t_{CNT})}{(E_{CNT}/E_M) + (l_{CNT}/2t_{CNT})} \tag{10}$$

In Eqs. (9) and (10), l_{CNT} , d_{CNT} and t_{CNT} denote the length, diameter and the thickness of the carbon nanotubes, respectively. The Poisson's ratio and the shear modulus of the CNT reinforced matrix are given by

$$\nu_{CNTM} = \nu_{CNT}\, V_{CNT} + \nu_M\,(1 - V_{CNT}) \tag{11}$$

$$G_{CNTM} = \frac{E_{CNTM}}{2(1 + \nu_{CNTM})} \tag{12}$$

The volume content of carbon nanotubes V_{CNT} can be expressed in terms of the weight fraction, denoted by W_{CNT}, as follows:

$$V_{CNT} = \frac{W_{CNT}}{W_{CNT} + (\rho_{CNT}/\rho_M)(1 - W_{CNT})} \tag{13}$$

The density of CNT reinforced matrix can be calculated from:

$$\rho_{CNTM} = \rho_{CNT}\, V_{CNT} + \rho_M\,(1 - V_{CNT}) \tag{14}$$

3.2. Three-phase fiber/CNT reinforced matrix composite

The elastic moduli and density of each ply for the three-phase laminate (Carbon fiber/CNT/matrix) are computed from the equations given in [23, 40] as

$$E_{11} = E_{F11}V_F + E_{CNTM}\,(1 - V_F) \tag{15}$$

$$E_{22} = \left(\frac{V_F}{E_{F22}} + \frac{1 - V_F}{E_{CNTM}} - V_F(1 - V_F)\frac{\nu_F^2\,(E_{CNTM}/E_{F22}) + \nu_{CNTM}^2(E_{F22}/E_{CNTM}) - 2\nu_F\,\nu_{CNTM}}{V_F\,E_{F22} + (1 - V_F)E_{CNTM}}\right)^{-1} \tag{16}$$

$$G_{12} = \left(\frac{V_F}{G_F} + \frac{1 - V_F}{G_{CNTM}}\right)^{-1} \tag{17}$$

$$\nu_{12} = \nu_{F12}V_F + \nu_{CNTM}\,(1 - V_F) \tag{18}$$

Density ρ of the three-phase ply is given by

$$\rho = \rho_F\, V_F + \rho_{CNT}\, V_{CNT} + \rho_M\,(1 - V_F - V_{CNT}) \tag{19}$$

4. Formulations of minimum weight problems

In the present work, the design objective is to minimize the weight of a simply supported (SS) laminated plate subject to a fundamental frequency constraint. For optimization, the sequential quadratic programming algorithm (SQP) available in MATLAB is implemented. The SQP algorithm is used for nonlinear constrained optimization problems and has been used in a number of studies involving the optimal design of composites [41-43]. For the problems that the fiber angles are considered as design variables (result Sections 6.5 and 6.6), discrete fiber angles are tested using parametric iterations. In each iteration, the SQP algorithm is called to determine the optimal response, considering the discrete-parametric fiber angles as well as the remaining design variables (CTN/fiber content and thickness).

In particular, five optimization problems are formulated with the number of design variables increasing with each design problem. The design variables are fiber volume fractions (V_{Fk}) and CNT weight fractions (W_{CNTk}) of layers, fiber orientation angles (θ_k), and ply thickness ratios h_k/H with $k = 1, 2, \dots, N$. The formulations of the optimization problems are given next.

4.1 Problem 1: Two-phase laminate with non-uniform (optimal) distribution of fibers (Design variables: V_{Fk} across the thickness, no CNT reinforcement)

First optimization problem involves the optimization of a fiber reinforced laminate with uniform layer thicknesses and non-uniform (optimal) distribution of fibers across the thickness and with no CNT content. The total volume of fibers (Vol_{Fk}) in the k^{th} layer is given by $Vol_{Fk} = abhV_{Fk}$ where $h = H/N$ and V_{Fk} is the fiber volume content of the k^{th} layer. The total volume of fibers in a laminate with N layers is given by

$$Vol_{FTotal} = \sum_{k=1}^{N} Vol_{Fk} = abh \sum_{k=1}^{N} V_{Fk} \tag{20}$$

The maximum volume of non-uniformly distributed fibers is given by $Vol_{Fmax} = abHV_{Fmax}$ where V_{Fmax} is the maximum volume fraction of fibers in each layer, that is:

$$V_{Fk} \leq V_{Fmax} \text{ for } k = 1, 2, \dots, N \tag{21}$$

As the design objective is the minimization of the weight, the optimal solution will use the minimum amount of fibers. For a laminate with N layers and fiber volume fraction V_{Fk}

of each layer as the design variable, the minimum weight problem subject to a frequency constraint Ω_O can be stated as follows:

$$\min W_L(V_{Fk}) \text{ wrt } V_{Fk} \tag{22}$$

where W_L is the weight of the laminate and is given by

$$W_L(V_{Fk}) = \sum_{k=1}^{N}(W_{Fk} + W_{Mk}) = abh\sum_{k=1}^{N}[\rho_F\, V_{Fk} + \rho_M\,(1 - V_{Fk})] \tag{23}$$

$$\text{subject to} \qquad \frac{\omega_{11}}{\omega_{IP}} \geq \Omega_O \tag{24}$$

$$0 \leq V_{Fk} \leq V_{Fmax} \qquad \text{for } k = 1, 2, \dots, N \tag{25}$$

where W_{Fk} and W_{Mk} are the weights of the fibers and the matrix of the k^{th} layer, and ρ_F and ρ_M are the densities of the fibers and the matrix, respectively. The minimization is carried out with respect to the fiber contents of layers, i.e., V_{Fk} . Equation (23) defines the objective function with the fiber content V_{Fk} being the design parameter for the k^{th} layer. The design constraint in equation (24) refers to the frequency constraint defined as the ratio $\frac{\omega_{11}}{\omega_{IP}}$ of the fundamental frequencies ω_{11} and ω_{IP} where ω_{IP} is the frequency of the isotropic plate of the same thickness as the minimum weight laminate made of the matrix material only, that is, without any reinforcement. The constraint (24) indicates that the ratio $\frac{\omega_{11}}{\omega_{IP}}$ should be equal to or higher than the frequency constraint Ω_O. The constraint (25) limits the fiber content V_{Fk} of each layer.

To provide a quantitative measure of the weight minimization and to evaluate the effectiveness of the optimal solutions, a design efficiency factor η is introduced. This factor is defined as the ratio of the weight W_L of the optimized laminate and the weight W_0 of the reference plate and is given by

$$\eta = \frac{W_L(V_{Fk})}{W_0(H_0)} = \frac{abh\sum_{k=1}^{N}[\rho_F\, V_{Fk} + \rho_M\,(1 - V_{Fk})]}{abH_0\rho_M} \tag{26}$$

The reference plate is defined as an isotropic plate of the same material as the matrix and with its thickness H_0 defined in terms of the fundamental frequency of the plate. The fundamental frequency for a simply supported isotropic plate is given by

$$\omega_{IP} = \left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\right]\left(\frac{D}{\rho_M H_0}\right)^{1/2} \tag{27}$$

where D is the flexural rigidity which is determined from equation (28) as

$$D = \omega_{IP}^2 \rho_M H_0 \left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\right]^{-2} \tag{28}$$

Noting that

$$D = \frac{E_M\, H_0^3}{12(1-\nu_M^2)} \tag{29}$$

the thickness H_0 of the isotropic plate can be determined from Eqs. (28) and (29) as

$$H_0 = \omega_{IP} \left(\frac{12\, \rho_M (1-\nu_M^2)}{E_M}\right)^{1/2} \left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\right]^{-1} \tag{30}$$

where ω_{IP} is equal to the fundamental frequency of the optimal laminate. In equation (29), E_M and ν_M are the Young's modulus and the Poisson's ratio of the matrix. Definition of the design efficiency factor η given by equation (26) indicates that for minimum weight laminates, $\eta < 1$ since the isotropic plate with the same frequency as the optimal laminate will be thicker and consequently, heavier. As such, the design efficiency factor indicates how light the optimal design is, as compared to an unreinforced plate with the same fundamental frequency. A lower value of η indicates a more efficient design. It is noted that the efficiency factor η given by equation (26) is basically the non-dimensional weight of the optimized laminate. It seems that the design efficiency factor as formulated above has not been used in publications involving laminates designed for minimum weight.

4.2 Problem 2: Three-phase laminate with non-uniform (optimal) distribution of fibers and uniform distribution of CNTs (Design variable: V_{Fk})

In the second optimization problem, the effect of uniformly distributed CNT reinforcement of the matrix on the minimum weight design is investigated with the fibers distributed optimally across the thickness. As such the material is three-phase involving CNT/fiber combination. However, CNT reinforcement is distributed uniformly across the thickness. The optimization problem for the laminate reinforced with uniformly distributed CNTs can be stated as follows:

$$\min W_L(V_{Fk}, V_{CNTk}) \ \text{wrt} \ V_{Fk} \tag{31}$$

where V_{CNTk} is the volume content of CNTs and

$$W_L(V_{Fk}, V_{CNTk}) = \sum_{k=1}^{N} (W_{Fk} + W_{CNTk} + W_{Mk}) =$$

$$= abh \sum_{k=1}^{N} [\rho_F \, V_{Fk} + \rho_{CNT} \, V_{CNTk} + \rho_M \, (1 - V_{Fk} - V_{CNTk})] \tag{32}$$

subject to the design constraints (24) and (25) and it is noted that $V_{CNTk} = \text{constant}$ for $k = 1,2, \dots, N$. For the present problem, the design efficiency factor η, as defined by equation (26), can be expressed as

$$\eta = \frac{W_L(V_{Fk}, V_{CNTk})}{W_0(H_0)} = \frac{abh \sum_{k=1}^{N} [\rho_F \, V_{Fk} + \rho_{CNT} \, V_{CNTk} + \rho_M \, (1 - V_{Fk} - V_{CNTk})]}{abH_0 \rho_M} \tag{33}$$

4.3 Problem 3: Three-phase laminate with non-uniform (optimal) distributions of fibers and CNTs (Design variables: V_{Fk} and V_{CNTk})

In the third optimization problem, the effect of optimal distribution of CNTs as well as the optimal distribution of fibers across the thickness are investigated, that is, CNT reinforcement and fibers are distributed non-uniformly (and optimally) across the thickness with the objective of minimizing the weight. The optimization problem with both CNT and fiber distributions as the design parameters can be stated as follows:

$$\min W_L(V_{Fk}, V_{CNTk}) \ \text{ wrt } \ V_{Fk} \ \text{ and } \ V_{CNTk} \tag{34}$$

where

$$W_L(V_{Fk}, W_{CNTk}) = \sum_{k=1}^{N} (W_{Fk} + W_{CNTk} + W_{Mk}) =$$

$$= abh \textstyle\sum_{k=1}^{N} [\rho_F \, V_{Fk} + \rho_{CNT} \, V_{CNTk} + \rho_M \, (1 - V_{Fk} - V_{CNTk})] \tag{35}$$

subject to the design constraints (24) and (25) and constraints on CNT contents of the layers, specified as

$$0 \le V_{CNTk} \le V_{CNTmax} \qquad \text{for } k = 1, 2, \dots, N \tag{36}$$

$$\textstyle\sum_{k=1}^{N} W_{CNTk} \le W_{CNT} \tag{37}$$

where V_{CNTmax} is the maximum volume content of CNTs for the k^{th} layer and W_{CNT} is the maximum allowable weight of CNTs. Inequality (36) defines the constraint on CNT volume content per layer, denoted by V_{CNTmax}, and the inequality (37) defines the constraint on the total CNT weight content W_{CNT}. In the present study, the weight of the CNT per layer W_{CNTmax} is specified as 1% or 2% per layer. Inequality (37) limits the overall CNT weight. The design efficiency factor η for this case is given by equation (33).

4.4 Problem 4: Three-phase laminate with non-uniform (optimal) distributions of fibers and CNTs and optimal fiber orientations (Design variables: V_{Fk}, V_{CNTk} and θ_k)

Next, the fiber orientations are included as design parameters in addition to the distributions CNTs and fibers across the laminate thickness. As such CNTs and fibers are distributed optimally across the thickness with the layers having optimal fiber angles. The optimization problem with three design parameters can be stated as follows:

$$\min\ W_L(V_{Fk}, W_{CNTk}, \theta_k) \quad \text{wrt} \quad V_{Fk}, V_{CNTk} \text{ and } \theta_k \tag{38}$$

where $W_L(V_{Fk}, W_{CNTk}, \theta_k)$ is given by equation (35). The minimization is subject to the design constraints (24), (25), (36) and (37) and

$$-90° \leq \theta_k \leq 90° \tag{39}$$

The design efficiency factor η for this case is given by equation (33). Importance of this optimal design problem comes from the fact that reinforcing the matrix with CNTs is likely to affect the optimal fiber orientations and this effect has to be observed and quantified. This is due to the fact that the stiffness of the CNT reinforced matrix increases and this, in turn, affects the optimal values of the fiber orientations.

4.5 Problem 5: Three-phase laminate with non-uniform (optimal) distributions of fibers and CNTs and non-uniform layer thicknesses (Design variables: V_{Fk}, W_{CNTk}, θ_k and h_k)

In the final optimization problem, layer thicknesses h_k are included as design parameters. As such, distributions of CNTs and fibers, fiber angles and layer thicknesses now become design parameters. In the present case, the volume of fibers in each layer is given by $Vol_{Fk} = abh_kV_{Fk}$. The total volume of fibers is given by

$$Vol_{FTotal} = \sum_{k=1}^{N} Vol_{Fk} = ab\sum_{k=1}^{N} h_kV_{Fk} \tag{40}$$

Similarly, the weight content of CNT in each layer is given by $Weight_{CNTk} = abh_kW_{CNTk}$ and the total weight content of CNT is given by

$$W_{CNTTotal} = ab\sum_{k=1}^{N} h_kW_{CNTk} \tag{41}$$

The optimization problem now includes fiber and CNT volume fractions, fiber angles θ_k, and the layer thicknesses h_k and it can be stated as:

$$\min\ W_L\ (V_{Fk}, W_{CNTk}, \theta_k, h_k)\ \text{ wrt }\ V_{Fk},\ V_{CNTk},\ \theta_k \text{ and } h_k \tag{42}$$

where $W_L(V_{Fk}, W_{CNTk}, \theta_k, h_k)$ is given by

$$W_L(V_{Fk}, W_{CNTk}, \theta_k, h_k) = \sum_{k=1}^{N}(W_{Fk} + W_{CNTk} + W_{Mk}) =$$
$$= ab\sum_{k=1}^{N}[\rho_F\ V_{Fk}h_k + \rho_{CNT}\ V_{CNTk}h_k + \rho_M h_k(1 - V_{Fk} - V_{CNTk})] \tag{43}$$

The minimization is subject to the design constraints (24), (25), (36), (37), (39) and

$$\frac{1}{H}\sum_{k=1}^{N} h_k W_{CNTk} \leq W_{CNTmax} \tag{44}$$

$$\sum_{k=1}^{N} h_k \leq H \tag{45}$$

The design efficiency factor η for this case is given by:

$$\eta = \frac{W_L(V_{Fk}, W_{CNTk}, \theta_k, h_k)}{W_0(H_0)} =$$
$$= \frac{ab\sum_{k=1}^{N}[\rho_F\ h_k V_{Fk} + \rho_{CNT}\ h_k V_{CNTk} + \rho_M\ h_k(1 - V_{Fk} - V_{CNTk})]}{abH_0\rho_M} \tag{46}$$

5. Verification

In this section, the verification of the analytical method of solution, as well as the optimization results, are presented. For this purpose, results from published literature are compared with the results obtained by the method of solution used in the present study which is implemented using MATLAB programming code. These results are also compared to the numerical results obtained by using the commercial finite element software ANSYS. For the presented simulations, the material properties are specified as $E_{11} = 60.7$ GPa, $E_{22} = 24.8$ GPa, $G_{12} = 12.0$ GPa and $\nu_{12} = 0.23$ for E-glass/epoxy square laminates studied in [44, 45]. A non-dimensional frequency given by $\varpi = \omega_{11}\alpha^2\sqrt{\rho h/D_0}$ where

$$D_0 = \frac{E_{11}\ H_0^3}{12(1 - \nu_{12}^2)} \tag{47}$$

has been used for frequencies calculated by using ANSYS. Four-node shell elements have been used in the commercial software ANSYS based on the first-order shear deformation theory. The comparison of results shown in Table 1 indicates that results obtained by the present approach, ANSYS and references [44, 45] differ by a fraction of percentage.

Table 1: Comparison of numerical results for a 3-layered SSSS square laminate with thickness $H = 0.001\,m$, aspect ratio $b/a = 1$.

Stacking sequence	Methods	Non-dimensional frequency ω_{11}
[0°/0°/0°]	Present Method	15.115
	ANSYS	14.863
	Ref [44]	15.171
	Ref [45]	15.190
[-15°/15°/15°]	Present Method	15.491
	ANSYS	15.294
	Ref [44]	15.369
	Ref [45]	15.430
[-30°/30°/30°]	Present Method	16.215
	ANSYS	15.902
	Ref [44]	15.583
	Ref [45]	15.900
[-45°/45°/45°]	Present Method	16.566
	ANSYS	16.249
	Ref [44]	16.082
	Ref [45]	16.140

Next, verification of the results for three-phase CNT and glass fiber reinforced laminates is studied by comparing the results with the output obtained from ANSYS. The material properties for the CNTs, glass fibers and epoxy matrix are listed in Table 2 which are taken from Refs. [23, 40, 46]. For the CNTs, the following dimensions are used: $l_{CNT} = 25\ \mu\mathrm{m}$, $d_{CNT} = 1.4\ \mathrm{nm}$, $t_{CNT} = 0.34\ \mathrm{nm}$. It is noted that the same material properties are used in the numerical results sections of the present article.

Table 2: Material properties.

Material	E (GPa)	G (GPa)	ν	Density (kg/m^3)
Carbon nanotubes	640	$E/(2(1+\nu))$	0.27	1350
Matrix	3.5	$E/(2(1+\nu))$	0.35	1200
Glass fibers	72.4	$E/(2(1+\nu))$	0.20	2400

Results of this comparison are satisfactory as shown in Table 3, indicating that the formulation and the method of solution adopted in the present work is suitable for further analysis to be used in the present weight minimization problem.

Table 3: Comparison of numerical results with commercial software for a 3-ply CNT/Fiber reinforced laminate with $H = 0.001\, m$, $b/a = 1$.

Material Content	Stacking Sequence	Non-dimensional frequency ω_{11}	
		Present Study	Commercial Software
$V_F = 0.3$ $W_{CNT} = 0.01$	[0°/0°/0°]	13.760	13.172
	[-15°/15°/15°]	14.385	13.791
	[-30°/30°/30°]	15.559	14.727
	[-45°/45°/45°]	16.114	15.294
$V_F = 0.3$ $W_{CNT} = 0.05$	[0°/0°/0°]	16.535	16.426
	[-15°/15°/15°]	16.860	16.782
	[-30°/30°/30°]	17.492	17.359
	[-45°/45°/45°]	17.800	17.678
$V_F = 0.6$ $W_{CNT} = 0.01$	[0°/0°/0°]	13.411	13.250
	[-15°/15°/15°]	14.085	14.177
	[-30°/30°/30°]	15.346	15.423
	[-45°/45°/45°]	15.938	15.996

Furthermore, the optimization algorithm which is implemented in the present study is also verified. In this respect, several simulations were conducted using ANSYS for an 8-layered laminate. A uniform CNT distribution and varying fiber volume contents are considered in these simulations. For each simulation, the non-dimensional weight as well as the non-dimensional frequency are determined. Then, the proposed optimization scheme outlined in Section 4 is employed to obtain the minimum weight results.

Figure 2 and Table 4 show the results in terms of the non-dimensional weight $\eta = \frac{W_L(V_{Fk},V_{CNTk})}{W_0(H_0)}$ given by equation (33) versus the non-dimensional frequency $\Omega_O = \frac{\omega_{11}}{\omega_{IP}}$ given by equation (24). The results are obtained from the software simulations and the present optimization code. It is observed in Figure 2 that in the finite element simulations using ANSYS, as the non-dimensional frequency increases towards the limit 1.689, the non-dimensional weight gradually reduces to the value 0.65. When the proposed optimization analysis is conducted for the model with one design variable, that is, the fiber content and using the same limit 1.689 for the frequency constraint, a similar optimal minimum weight (0.642) is obtained as compared to the ANSYS simulations shown with an orange dot in Figure 2. Table 4 confirms these results with the ANSYS and present results for the minimum weight laminate being very close.

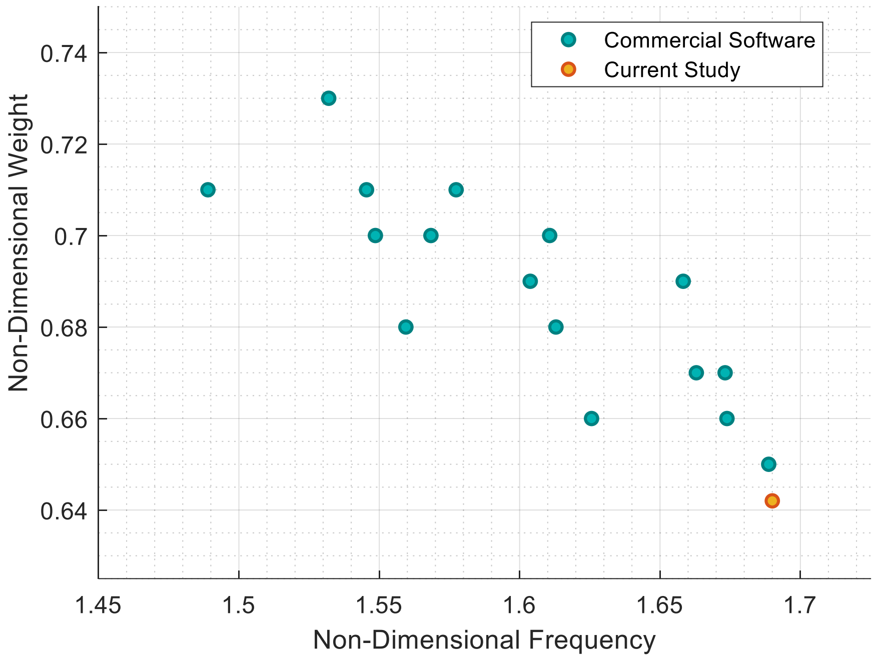


Figure 2: Verification of the weight minimization solution.

Table 4: Comparison of numerical results obtained by ANSYS software for 8-layer CNT/fiber reinforced laminate with thickness $H = 0.01\,m$, stacking sequence [45°/-45°/45°/-45°]$_{\text{anti-s}}$, $W_{CNT} = 0.00125$, and aspect ratio $b/a = 1$.

Fiber Volume Content	Non-Dimensional Weight	Non-Dimensional Frequency Ω_O
$[0.2/0/0/0]_s$	0.71	1.489
$[0.25/0/0/0.2]_s$	0.73	1.532
$[0.25/0/0/0.1]_s$	0.71	1.545

[0.2/0.1/0/0]s	0.70	1.549
[0.25/0/0/0]s	0.68	1.559
[0.25/0/0.1/0]s	0.70	1.568
[0.25/0/0.2/0]s	0.71	1.577
[0.2/0.2/0/0]s	0.69	1.604
[0.2/0.2/0.1/0]s	0.70	1.611
[0.25/0.1/0/0]s	0.68	1.613
[0.3/0/0/0]s	0.66	1.626
[0.35/0/0/0.2]s	0.69	1.658
[0.25/0.2/0/0]s	0.67	1.663
[0.35/0/0/0.1]s	0.67	1.673
[0.3/0.1/0/0]s	0.66	1.674
[0.35/0/0/0]s	0.65	1.689
Weight minimization result using the optimization process of the present study		
Fiber volume content	Non-dimensional weight	Non-dimensional frequency Ω_O
[0.3394/0/0/0]s	0.642	1.690

6. Results and discussions

6.1 Analysis

First, the results are presented to assess the weight/frequency relations and observe the effect of fiber orientations on the weight of the laminate. The material properties presented in Table 2 have been adopted for these cases. The ratio ω_{11}/ω_{IP} is used for the non-dimensional frequency as formulated in Section 4.1. For the non-dimensional weight, the ratio W_L/W_o is used where the numerator is the computed weight for the specific case and the denominator is the weight of the reference plate as defined in equation (26), that is, $W_o = abH_0\rho_M$.

It is noted that in Figures 3-11 of Sections 6.1-6.4, the fiber orientations are taken as $[\theta/-\theta/\theta/-\theta/]_{anti-s}$ to make the effect of bending-twisting coupling negligible as noted in [36]. It is observed in [36] that as the number of angles increases in a $[+/-/+/-/]_{anti-s}$ pattern, D_{16} and D_{26} terms become negligible and the orthotropic solution given by equation (6) becomes accurate. Similarly, the angles in Tables 5-10 are in the form of $[\theta/-\theta/\theta/-\theta]_{anti-s}$ making the effect of D_{16} and D_{26} terms on the solution negligible.

For these results, as well as for the subsequent results presented in Sections 6.5 and 6.6, terms γ and δ satisfy the conditions $\gamma \leq 0.2$ and $\delta \leq 0.2$ where $\gamma = D_{16}(D_{11}^3 D_{22})^{-1/4}$, $\delta = D_{26}(D_{11} D_{22}^3)^{-1/4}$. As such, the error becomes negligible for the orthotropic solution given by equation (6) as discussed in [37-39].

First the effect of the CNT and fiber volume contents on the weight of the laminate is studied. CNT weight percentages are specifed as 0%, 1% and 5% and the curves of weight plotted against the fiber angle are shown in Figures 3a, 3b and 3c for various aspect ratios. Results indicate a significant decrease in the laminate weight as CNT content increases as expected. For the aspect ratio $b/a = 1.0$, the weight becomes a minimum at $\theta = 45^{\mathrm{o}}$ as expected. Results for $b/a = 1.25$, $b/a = 1.5$, $b/a = 1.75$ and $b/a = 2.0$ are approximately $\theta = 52^{\mathrm{o}}$, $\theta = 61^{\mathrm{o}}$, $\theta = 87^{\mathrm{o}}$ and $\theta = 90^{\mathrm{o}}$ in the case of 0% CNT reinforcement (Fig. 3a), with the stacking sequence given by $[\theta/-\theta/\theta/-\theta]_{\text{anti-s}}$. The corresponding optimal fiber angles are slightly less for a CNT content of 1% (Fig. 3b). In Figure 3c, the CNT content is increased to 5% by weight and it is observed that the optimal fiber orientations now become less than the ones corresponding to CNT contents of 0% and 1% which are shown in Figures 3a and 3b. For the aspect ratio $b/a = 1.5$ and $a/b = 1.75$, the optimal angles are approximately $\theta = 56^{\mathrm{o}}$ and $\theta = 76^{\mathrm{o}}$ in the case of 5% CNT weight content. The reason for the changes in the optimal fiber angles for 0% and 5% CNTs cases is the fibers having less effect on the laminate stiffness with the addition of CNT to the matrix. As the stiffness of the matrix increases, directional stiffness provided by the reinforcing fibers become less effective leading to changes in the optimal fiber angles.

Another effect of the CNT reinforcement is the reduced range of the design efficiency factors (see equation (26)) for different fiber orientations as the CNT content increases. This effect can be observed from a comparison of Figures 3a, 3b and 3c. This is again due to the strong influence of CNT reinforcement on the weight of the laminate making the directional influence of the fibers less pronounced.

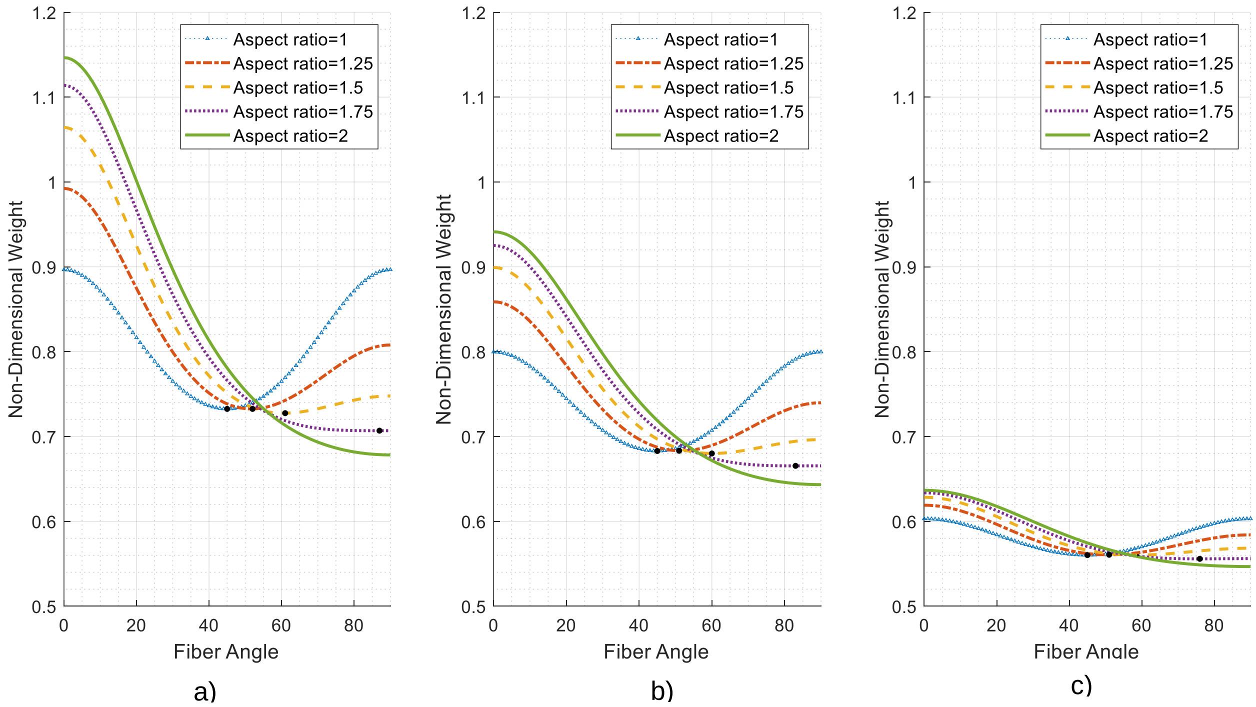


Figure 3: Design efficiency factor vs the fiber angle for a laminate with 30% fiber content and a) 0% CNT, b) 1% CNT, c) 5% CNT weight content.

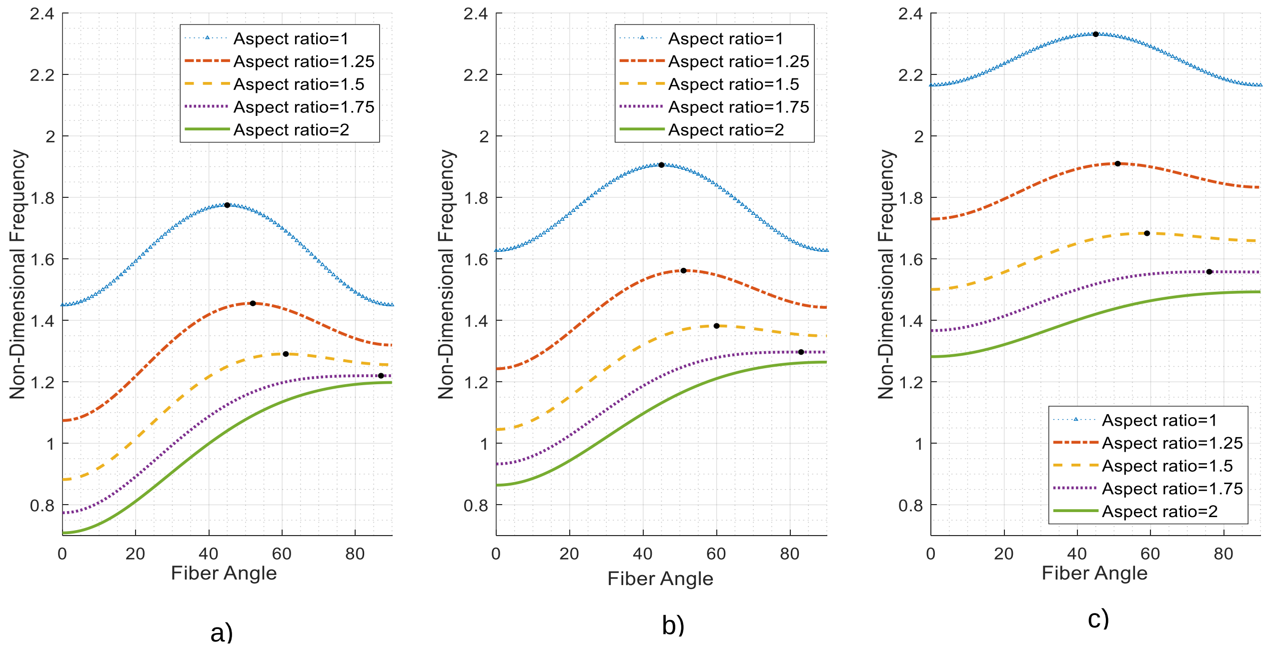


Figure 4: Non-dimensional frequency vs fiber angle for a laminate with 30% fiber content and a) 0% CNT, b) 1% CNT, c) 5% CNT weight content.

Frequency constraint normally plays a significant role in the optimal design of minimum weight problems. Figure 4 shows the curves of frequency versus fiber angle for different aspect ratios and CNT contents. Results show a higher frequency as the CNT content increases from zero to 1% and 5% as expected. The optimal fiber stacking angles are observed to be the same as those observed in Figure 3.

Table 5 summarizes the analysis results presented in Figures 3 and 4. It is noted that as CNT weight increases from 0% to 1% and 5%, the weight W_L of the laminate increases, but the non-dimensional weight (which is equal to the design efficiency factor given by equation (26)) decreases due to the increase of the weight $W_{0,}$ of the reference plate with W_0 increasing faster than the weight W_L of the optimal laminate.

Table 5: Design efficiency factors for different CNT weight contents and fiber angles.

Aspect ratio b/a	V_f per layer	W_{CNT} per layer	Fiber angle	Non-dimensional frequency	Weight W_L	Reference weight W_0	Efficiency factor
1.00	[0.3/0.3/0.3/ 0.3]s	[0/0/0/0]s	45	1.78	15.60	21.30	0.732
1.25			52	1.46	19.50	26.62	0.733
1.50			61	1.29	23.40	32.17	0.727
1.75			87	1.22	27.30	38.62	0.707
2.00			90	1.20	31.20	46.00	0.678
1.00	[0.3/0.3/0.3/ 0.3]s	[0.01/0.01/0.01/ 0.01]s	45	1.91	15.61	22.86	0.683
1.25			51	1.56	19.52	28.57	0.683
1.50			60	1.38	23.42	34.44	0.680
1.75			83	1.30	27.32	41.06	0.665
2.00			90	1.26	31.23	48.54	0.643
1.00	[0.3/0.3/0.3/ 0.3]s	[0.05/0.05/0.05/ 0.05]s	45	2.33	15.67	27.97	0.560
1.25			51	1.91	19.58	34.93	0.561
1.50			59	1.68	23.50	41.95	0.560
1.75			76	1.56	27.42	49.33	0.559
2.00			90	1.49	31.33	57.32	0.547

Next, optimization results are presented for 8-layered laminates. The formulations for the minimum weight problems are given in Section 4 and the solutions for these problems are presented next. The effects of reinforcing the laminates with CNTs, use of three-phase composites and implementing optimal designs with variable fiber and CNT contents are highlighted.

6.2. Problem 1: Two-phase laminate with non-uniform distribution of fibers (Design variables: V_{Fk}, 0% CNT reinforcement)

Figure 5 shows the results for design efficiency factor vs fiber angles for different aspect ratios with the fiber content V_{Fk} being the only design variable. It is observed that the optimal fiber angle is 45° for a square laminate as expected and increases to 90° as the aspect ratio increases. It is noted that the design efficiency (non-dimensional weight) improves as the aspect ratio increases.

Table 6 shows the minimum weights and the fiber distributions across the thickness as well as the optimal fiber angles for different aspect ratios. It is noted that the optimal fiber angles correlate with the results given in references [39, 47, 48]. It is observed that the optimal designs have a higher percentage of fiber content in the outer layers. This is due to the fact that the outer layers contribute more to the overall stiffness of the laminate compared to the inner layers as expected.

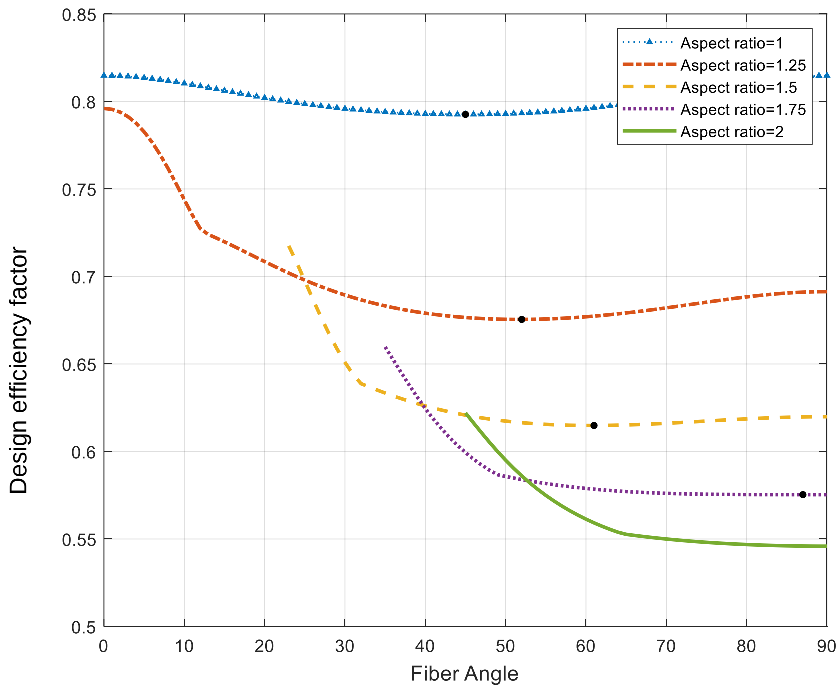


Figure 5: Design efficiency factor vs fiber orientation for two-phase laminates for different aspect ratios b/a with thickness $H = 0.01\ m$, $0 \leq V_f \leq 0.6$, frequency constraint $\omega_{11}/\omega_{IP} \geq \Omega_O$ where $\Omega_O = 1.3$.

Table 6: Optimal fiber volumes V_{fk} and the fiber orientations for two-phase laminates with thickness $H = 0.01\ m$, $V_{fk} \leq 0.6$, frequency constraint $\Omega_O = 1.3$.

Aspect ratio b/a	Optimal V_f per layer	Optimal fiber angle	Weight W_L	Reference weight W_0	Design efficiency factor
1.00	$[0.12/0/0/0]_s$	45	12.37	15.60	0.793
1.25	$[0.28/0/0/0]_s$	52	16.06	23.78	0.676
1.50	$[0.43/0/0/0]_s$	61	19.92	32.40	0.615
1.75	$[0.51/0/0/0]_s$	87	26.68	41.16	0.575
2.00	$[0.48/0/0/0]_s$	90	27.25	49.92	0.546

6.3. Problem 2: Three-phase laminate with non-uniform distribution of fibers and uniform distribution of CNTs (Design variable: V_{Fk})

Next, the effect of uniformly distributed CNT reinforcement on the minimum weight design of the laminate is investigated. Figure 6 shows the curves of the design efficiency factor versus the fiber orientation for various aspect ratios and for CNT weight contents of 1% and 2%. Comparing Figures 6a and 6b, it is observed that the efficiency factor, representing the non-dimensional weight, decreases as a result of the increase in CNT content from 1% (Figure 6a) to 2% (Figure 6b). It is noted that for an aspect ratio of 1 and CNT weight content of 2%, frequency constraint is automatically satisfied (Fig. 6b).

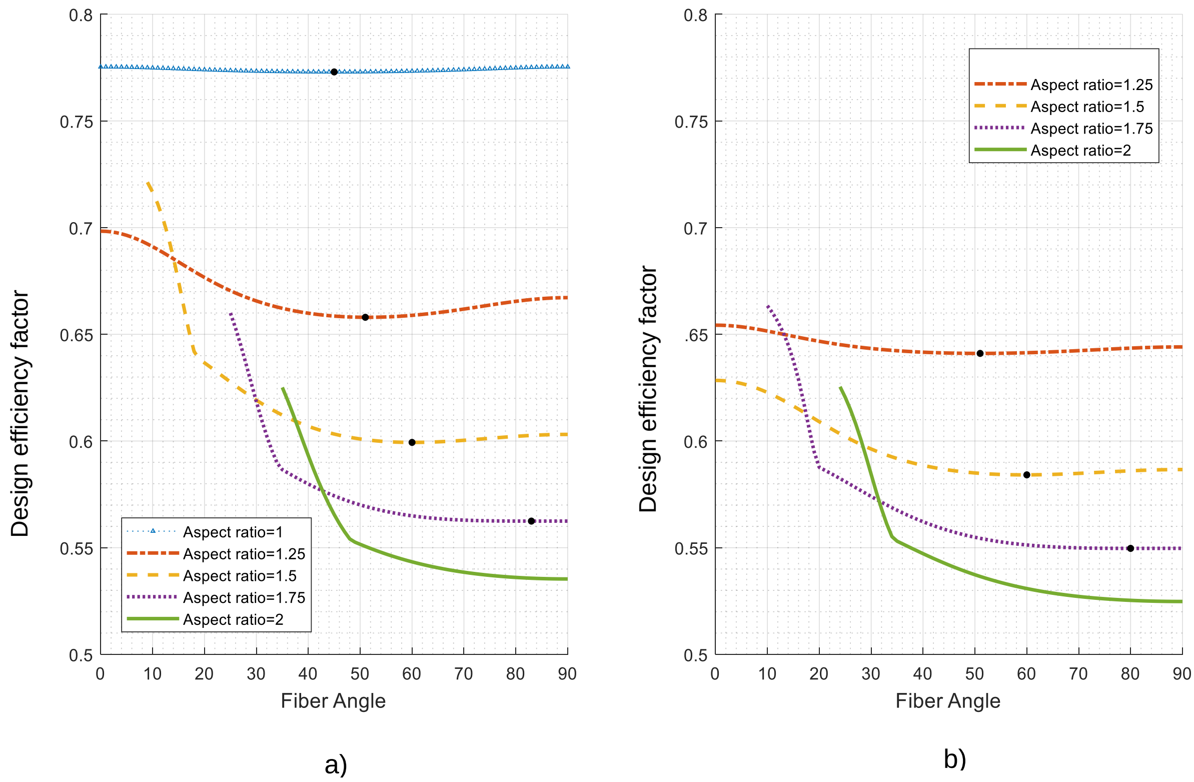


Figure 6: Design efficiency factor vs fiber orientation for three-phase laminates with thickness $H = 0.01\,m$, $0 \leq V_{fk} \leq 0.6$, frequency constraint $\Omega_O = 1.3$ for different aspect ratios, a) 1% CNT, b) 2% CNT weight.

The addition of CNTs improves the stiffness of the laminate leading to lower weights for the optimal laminate (i.e., lower design efficiency factors which is the ratio of the weight of the optimally designed laminate and the laminate with the matrix material only which is the unreinforced polymer (see equation (33)). Table 7 shows the detailed results for the three-phase laminate with 1% CNT content distributed uniformly across the thickness. By comparing the results between Table 6 (0% CNT content) and Table 7 (1% CNT content), it is observed that addition of CNT has reduced the volume content of fibers needed to satisfy the frequency constraint which, in turn, reduced the weight of the laminate. This result clearly illustrates the positive effect of using three-phase laminates to minimize weight, and in particular, using CNTs for reinforcing the matrix material.

Table 7: Optimal fiber volumes V_{fk} and the fiber orientations for three-phase laminates with thickness $H = 0.01\ m$, $V_{fk} \leq 0.6$, frequency constraint $\Omega_O = 1.3$ and $W_{CNT} = 1\%$ distributed uniformly across the thickness.

Aspect ratio b/a	Optimal V_f per layer	W_{CNT} per layer	Optimal fiber angle	Weight W_L	Reference weight W_0	Design efficiency factor
1.00	$[0.15/0/0/0]_s$	$[0.01/0.01/0.01/0.01]_s$	45	12.06	15.60	0.773
1.25	$[0.17/0/0/0]_s$	$[0.01/0.01/0.01/0.01]_s$	51	15.65	23.78	0.658
1.50	$[0.31/0/0/0]_s$	$[0.01/0.01/0.01/0.01]_s$	60	19.42	32.40	0.599
1.75	$[0.4/0/0/0]_s$	$[0.01/0.01/0.01/0.01]_s$	83	23.15	41.16	0.562
2.00	$[0.45/0/0/0]_s$	$[0.01/0.01/0.01/0.01]_s$	90	26.73	49.92	0.535

Corresponding results for laminates with 2% CNT content are given in Table 8. It is noted that increase in the CNT content leads to a further improvement of the design efficiency, i.e., lighter weight, as a comparison of Tables 7 and 8 shows, that is, the weight of the laminate is reduced further with increased CNT content as expected. For $b/a = 1$, the frequency constraint is automatically satisfied when CNT content per layer is 0.02.

Table 8: Optimal fiber volumes V_{fk} and the fiber orientations for three-phase laminates with laminate thickness $H = 0.01\ m$, $0 \leq V_{fk} \leq 0.6$, frequency constraint $\Omega_O = 1.3$ and $W_{CNT} = 2\%$.

Aspect ratio b/a	Optimal V_{fk} per layer	W_{CNT} per layer	Optimal fiber angle	Weight W_L	Reference weight W_0	Design efficiency factor
1.25	$[0.06/0/0/0]_s$	$[0.02/0.02/0.02/0.02]_s$	51	15.24	23.78	0.640
1.50	$[0.20/0/0/0]_s$	$[0.02/0.02/0.02/0.02]_s$	60	18.93	32.40	0.584
1.75	$[0.30/0/0/0]_s$	$[0.02/0.02/0.02/0.02]_s$	80	22.62	41.16	0.550
2.00	$[0.36/0/0/0]_s$	$[0.02/0.02/0.02/0.02]_s$	90	26.20	49.92	0.525

6.4. Problem 3: Three-phase laminate with non-uniform distribution of fibers and CNTs (Design variables: V_{fk} and W_{CNTk})

Next, the minimum weight problem is solved using two design variables, namely, the fiber volume contents and the CNT weight contents which are non-uniformly distributed across the thickness and should be determined optimally for each layer. As such, the minimum weight of the three-phase laminate is studied for a range of frequency constraints taking the fiber content and the CNT weight as the design variables. Figure 7 indicates that for low values of the frequencies up to the value of 1.65, fiber reinforcement is not needed and the minimum weight design is obtained by CNT reinforcement only with the frequency constraint satisfied. For higher frequency constraints ($\Omega_O \geq 1.65$), the optimal design leads to non-zero fiber reinforcement in the surface layers with the maximum CNT weight of 5% also used in the surface layers.

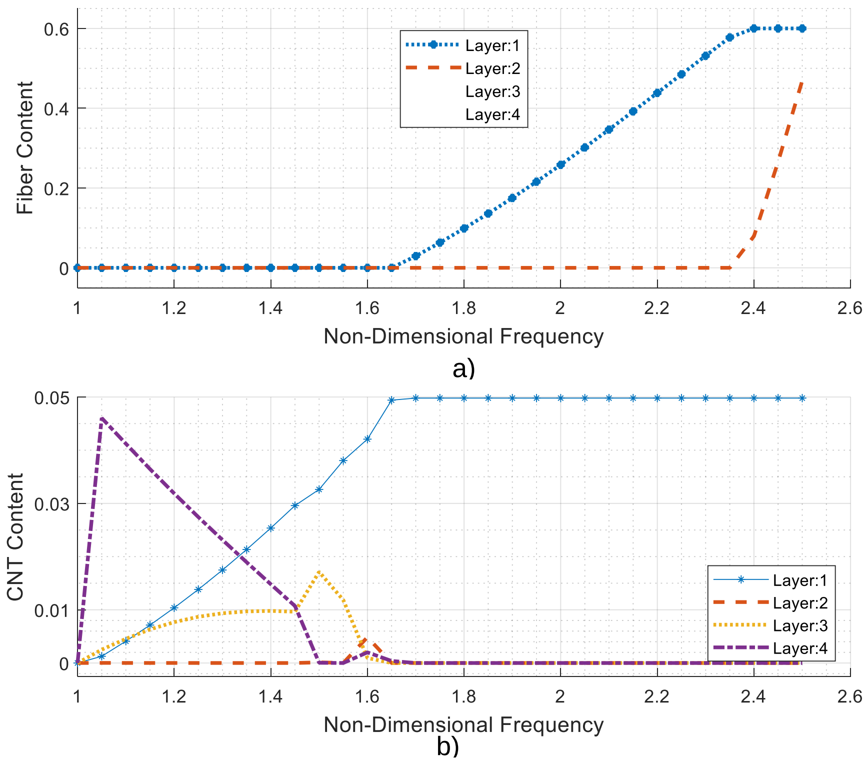


Figure 7. a) Fiber and b) CNT reinforcement per layer versus the non-dimensional frequency with thickness $H = 0.01$ m, $0 \leq V_{fk} \leq 0.6$, $0 \leq W_{CNTk} \leq 0.05$, fiber angle = 45º, aspect ratio $b/a = 1$.

To gain further insight into the effect of higher frequency constraints on design efficiency and the weight of the optimally designed laminate, three diagrams are presented in Figure 8. Figure 8a shows that the design efficiency factor (that is, non-dimensional weight η given by equation (38)) vs frequency, Figure 8b the weight of the optimally designed minimum weight laminate vs frequency, and Figure 8c the weight of the reference plate vs frequency. Figure 8a indicates that the design efficency decreases with increasing frequency as the amount of the fiber and CNT reinforcements increases. Figure 8b shows an increase of the actual weight of the optimal laminate with increasing frequency. Figure 8c shows the increase in the weight of the reference plate given by $W_0(H_0) = abH_0\rho_M$ with increasing frequency. Note that the expression for H_0 is given by equation (30). These results are expected since the higher frequencies require increased reinforcement for the optimal laminate and a higher thickness in the reference plate. It is also noted that the decrease in the efficiency factor (Figure 8a) indicates that the increase in the weight of the reference plate dominates the increase in the weight of the optimal laminate.

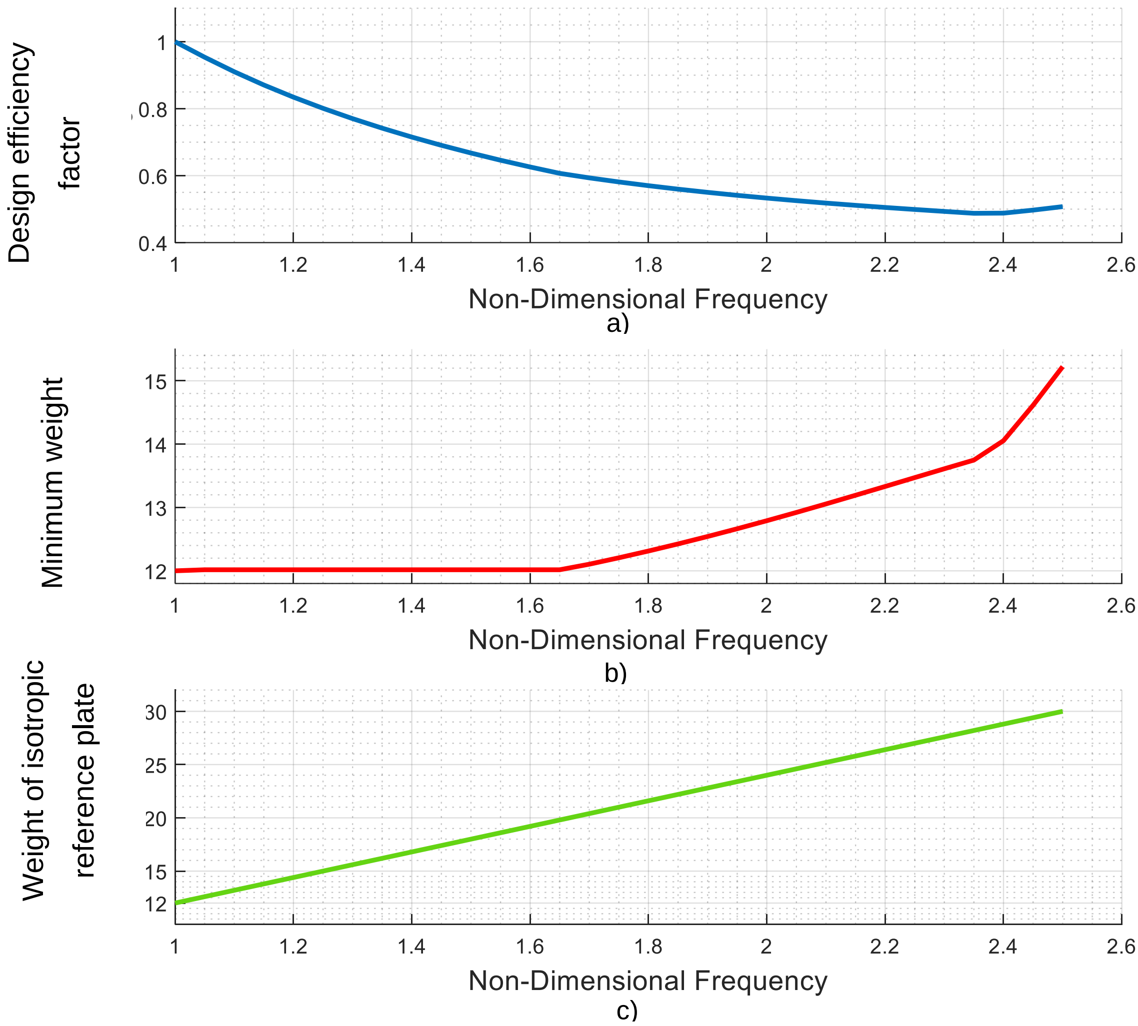
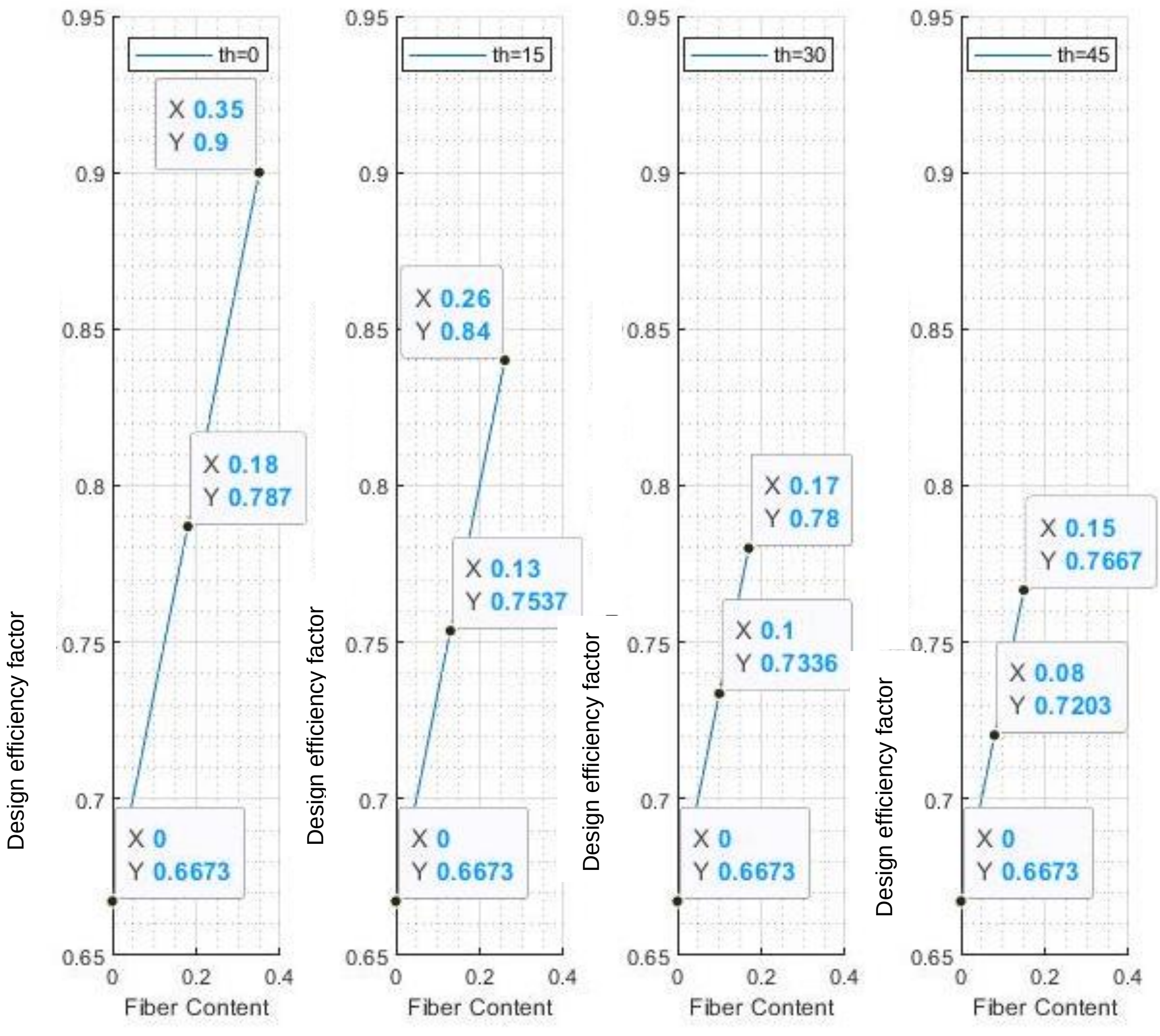


Figure 8: Graphs showing a) the non-dimensional weight of an 8-layered square laminate with $H = 0.01\ m$, $V_{fk} \leq 0.6$, $W_{CNTk} \leq 0.05$, fiber angle = 45^o, $b/a = 1$, b) the weight of the laminate, c) the weight of the reference plate versus the non-dimensional frequency.

In Figure 9, the effect of the stacking sequence on the weight of the optimal laminate is studied subject to the frequency constraint $\Omega_O \geq 1.5$ with design variable specified as CNT content. The fiber content and fiber angles are specified the same for all layers to specifically assess the effect of the CNT reinforcement only on the minimum weight. Fiber content varies along the x axis for each diagram with the provision that the fiber content applies to all the layers. In Figures 9a to 9d, fiber angles are 0º, 15º, 30º, and 45º, respectively. In Figure 9, X indicates the fiber content and Y the design efficiency factor (non-dimensional weight) given by equation (38). Figure 9 shows that, for zero fiber reinforcement, the non-dimensional minimum weight is the same in all cases. Increasing the fiber content and the fiber angle (Figure 9a to Figure 9d) results in the decrease of the weight.



a) b) c) d)

Figure 9: Design efficiency factor vs fiber volume content for 8-layered square laminate with thickness $H = 0.01\,m$, $W_{CNTk} \leq 0.05$, $b/a = 1$ and $\Omega_O \geq 1.5$ for a) 0º, b) 15º, c) 30º and d) 45º fiber angles for all layers.

Next the effect of optimal CNT distribution is investigated with the fibers distributed uniformly. Table 9 shows the results of the optimization using only the CNT content of each layer as the design variable. Four stacking sequencies are adopted, using 0º, 15º, 30º and 45º fiber angles. Results indicate that as the uniformly distributed fiber content increases, CNT content decreases in each layer to satisfy the frequency constraint. This results in an increase in the laminate weight. Thus, higher fiber content and lower CNT content result in increased weight, indicating that CNT has a greater effect on the weight minimization as compared to fibers. However, it is noted that lower CNT content leads to lower cost due to CNTs being more expensive than glass fibers. Regarding the fiber angle, Table 9 and Figure 10 verify the results shown in Figures 3 and 6 which show that the minimum weight is achieved at a fiber angle of $45°$ for a square plate.

Table 9: Optimal CNT contents and uniform fiber distributions for three-phase laminates with thickness $H = 0.01\ m$, $W_{CNTk} \le 0.05$, frequency constraint $\Omega_O = 1.5$, $b/a = 1$.

V_f per layer	W_{CNT} per layer	Fiber angle	Weight W_L	Reference weight W_0	Design efficiency factor
$[0/0/0/0]_s$	$[0.032/0/0/0]_s$	0	12.01	18.00	0.667
$[0.18/0.18/0.18/0.18]_s$	$[0.014/0/0/0]_s$	0	14.17	18.00	0.787
$[0.35/0.35/0.35/0.35]_s$	$[0.0002/0/0/0]_s$	0	16.20	18.00	0.900
$[0/0/0/0]_s$	$[0.032/0/0/0]_s$	15	12.01	18.00	0.667
$[0.13/0.13/0.13/0.13]_s$	$[0.015/0/0/0]_s$	15	13.57	18.00	0.754
$[0.26/0.26/0.26/0.26]_s$	$[0.0009/0/0/0]_s$	15	15.12	18.00	0.840
$[0/0/0/0]_s$	$[0.032/0/0/0]_s$	30	12.01	18.00	0.667
$[0.1/0.1/0.1/0.1]_s$	$[0.013/0/0/0]_s$	30	13.21	18.00	0.734
$[0.17/0.17/0.17/0.17]_s$	$[0.0013/0/0/0]_s$	30	14.04	18.00	0.780
$[0/0/0/0]_s$	$[0.032/0/0/0]_s$	45	12.01	18.00	0.667
$[0.08/0.08/0.08/0.08]_s$	$[0.015/0/0/0]_s$	45	12.97	18.00	0.720
$[0.15/0.15/0.15/0.15]_s$	$[0.0004/0/0/0]_s$	45	13.80	18.00	0.767

Next, the optimal design of the three-phase hybrid composite laminate is performed using two design variables, namely, optimal fiber and CNT contents of layers with the fiber angles kept constant. Figure 10 shows the optimal solutions for symmetric angle-ply and cross-ply laminates subject to the frequency constraint of $\Omega_O = 1.75$ for two-phase and three-phase laminates. It is observed that the laminate weight is significantly reduced in the case of three-phase laminates as compared to two-phase laminates, again indicating the weight advantage of three-phase designs and, in particular, including CNT content of layers as design variables.

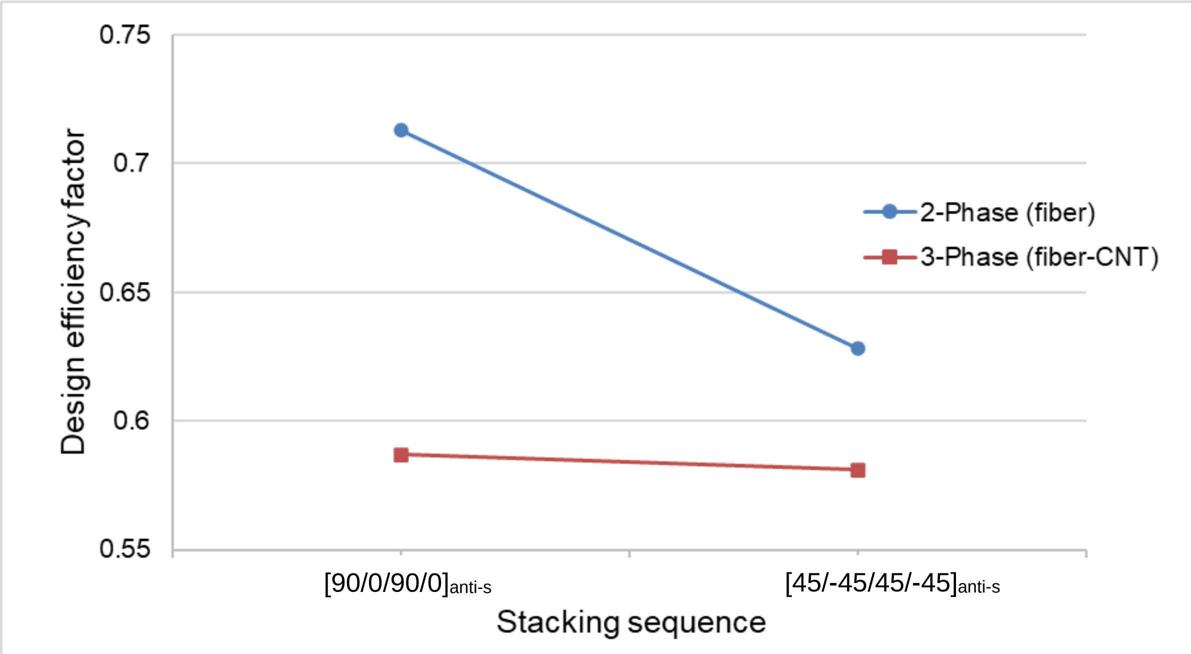


Figure 10: Design efficiency factor versus stacking sequences for two-phase and three-phase laminates with thickness $H = 0.01\,m$, $V_{fk} \leq 0.6$, $W_{CNTk} \leq 0.05$, $W_{CNTmax} = 1.25$, $\Omega_O = 1.75$, $b/a = 1$.

Results for two-phase and three-phase laminates are shown in Table 10. As expected, reinforcement contents are higher in the outer layers and the weight is lower (i.e., lower design efficiency factor) for $[45/\text{-}45/45\text{-}45]_{anti\text{-}s}$ laminate which is the optimal one for an aspect ratio of $b/a = 1$. Furthermore, the design efficiency factors of the three-phase laminates are improved by 17.6% for cross-ply laminate and 7.5% for $[45/\text{-}45/45\text{-}45]_{anti\text{-}s}$ laminate compared to two-phase laminates with fiber reinforcement only.

Table 10: Design efficiency factors of two-phase and three-phase square laminates with 8-layers and with $H = 0.01\,m$, $V_{fk} \leq 0.6$, $W_{CNTk} \leq 0.05$, $W_{CNTmax} = 1.25\%$, $\Omega_O = 1.75$ for cross-ply and angle-ply laminates with $b/a = 1$.

Two-phase laminate (Fiber/Matrix): Design variable = Fiber content per layer					
Stacking sequence	V_f per layer	W_{CNT} per layer	Weight W_L	Reference weight W_0	Design efficiency factor
$[90/0/90/90]_{anti\text{-}s}$	$[0.6/0.39/0/0]_s$	$[0/0/0/0]_s$	14.98	21.00	0.713
$[45/\text{-}45/45\text{-}45]_{anti\text{-}s}$	$[0.4/0/0/0]_s$	$[0/0/0/0]_s$	13.19	21.00	0.628

Three-phase laminate (Fiber/CNT/Matrix): Design variables = Fiber and CNT contents per layer					
Stacking sequence	V_f per layer	W_{CNT} per layer	Weight W_L	Reference weight W_0	Design efficiency factor
$[90/0/90/0]_{anti\text{-}s}$	$[0.1/0/0/0]_s$	$[0.05/0/0/0]_s$	12.33	21.00	0.587
$[45/-45/45\text{-}45]_{anti\text{-}s}$	$[0.06/0/0/0]_s$	$[0.05/0/0/0]_s$	12.21	21.00	0.581

Next, the effect of the CNT content on the weight of a square laminate is investigated. Figure 11 shows the contour plot of the non-dimensional weight (design efficiency factor) against the total CNT content and the fiber angle with optimal distributions of the fibers and CNTs across the thickness. At 0% CNT content (two-phase laminate), the dependence of the weight on the fiber angles is very high as the fiber angle changes from 0 to 90 degrees indicating that, in the absence of CNT reinforcement, fiber orientation is an effective design parameter. As the CNT content increases, the effect of the fiber angle on the weight gradually decreases and, at a CNT content of 1.25 %, this effect becomes practically negligible. Thus, increasing the total CNT content leads to a decrease in the influence of the fiber angles on the minimum weight. This effect was also observed in Figure 3.

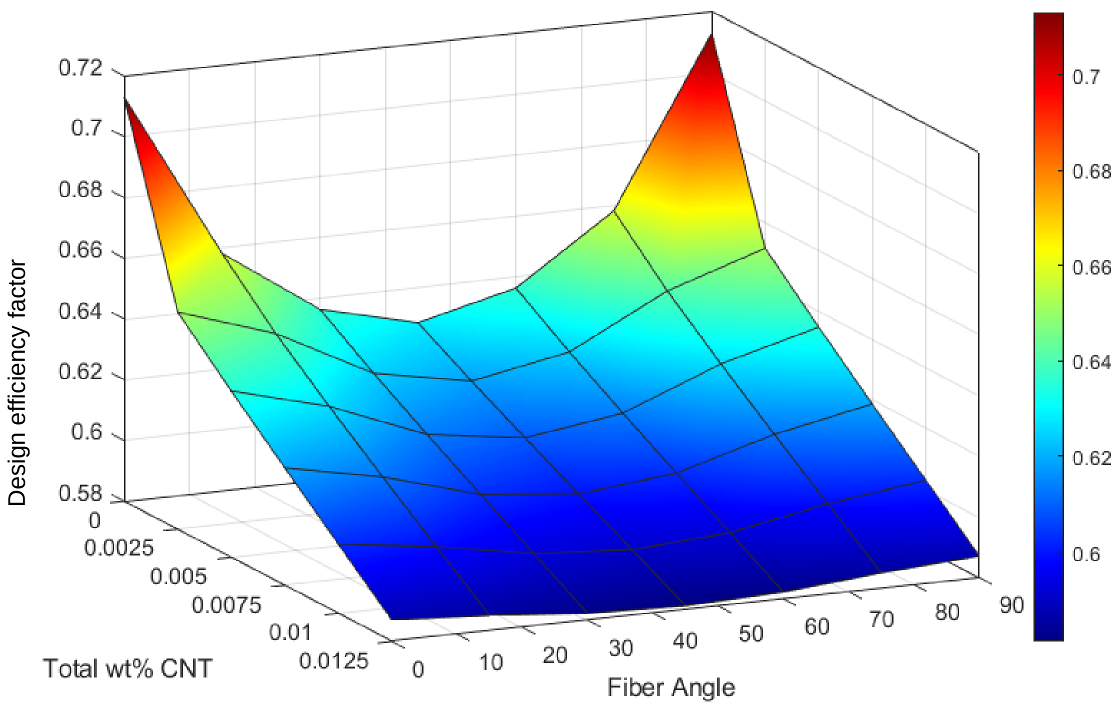


Figure 11: Contour plot for a minimum weight three-phase square laminate showing the design efficiency factor versus the total CNT content and the fiber angle with $H = 0.01\ m$, $V_{fk} \leq 0.6$, $W_{CNTk} \leq 0.05$, $\Omega_O = 1.75$, $b/a = 1$.

6.5. Problem 4: Three-phase laminate with uniform layer thicknesses and non-uniform distribution of reinforcements (Design variables: V_{fk}, W_{CNTk}, fiber angles θ_k)

Next, a third design variable, namely, the fiber angle is introduced into the weight minimization problem as design parameter in addition to the optimal fiber and CNT distributions across the thickness. In the previous problems (Problems 1 to 3), fiber angles were assigned to each layer and only the effect of the optimal distributions of the reinforcements on laminate weight was investigated. In the present problem, laminates are designed for optimal stacking sequences in addition to having optimal fiber and CNT distributions.

First, the effect of the increase in the frequency constraint on the minimum weight is investigated with the results shown in Table 11. It is observed that, for frequency constraints $\Omega_O = 1.25, 1.3 \text{ and } 1.5$, the optimal fiber contents are zero in all the layers and the frequency constraints are satisfied with the increasing CNT reinforcements only for a minimum weight design. As the frequency constraint increases further, CNT reinforcement only is no longer sufficient to satisfy the frequency constraint. Consequently, the fiber contents of layers also increase with higher frequency constraints with the surface layers having higher fiber contents as expected. Design efficiency factor decreases as the frequency constraint increases indicating that the weight of the optimal laminate improves as compared to an isotropic plate subject to the same frequency constraint.

Table 11: Minimum weight designs of three-phase square laminate with fiber angles and, fiber and CNT contents as design variables with $H = 0.01\,m$, $V_{fk} \leq 0.6$, $W_{CNTk} \leq 0.05$, $W_{CNTmax} = 1.25\%$, $-90° \leq \theta_k \leq +90°$, $b/a = 1.0$.

Frequency constraint Ω_O	Fiber angles	Fiber content by volume	CNT content by weight	Weight W_L	Reference weight W_0	Design efficiency factor
1.25	$[45/45/30/45]_s$	$[0/0/0/0]_s$	$[0.009/0.011/0.007/0.023]_s$	12.02	15.0	0.801
1.3	$[45/45/30/45]_s$	$[0/0/0/0]_s$	$[0.013/0/0.037/0]_s$	12.02	15.6	0.770
1.5	$[45/45/30/45]_s$	$[0/0/0/0]_s$	$[0.036/0/0/0.014]_s$	12.02	18.0	0.667
1.75	$[45/45/30/45]_s$	$[0.063/0/0/0]_s$	$[0.05/0/0/0]_s$	12.21	21.0	0.581
2.0	$[45/60/0/30]_s$	$[0.26/0/0/0]_s$	$[0.05/0/0/0]_s$	12.79	24.0	0.533
2.2	$[45/0/0/30]_s$	$[0.44/0/0/0]_s$	$[0.05/0/0/0]_s$	13.33	26.4	0.505
2.4	$[45/45/30/45]_s$	$[0.6/0.08/0/0]_s$	$[0.05/0/0/0]_s$	14.06	28.8	0.488
2.5	$[45/45/90/45]_s$	$[0.6/0.47/0/0]_s$	$[0.05/0/0/0]_s$	15.22	30.0	0.507

Table 12 shows the results for the optimal stacking sequences for different frequency constraints and with the fiber and CNT distributions across the thickness determined optimally. Weight minimization with three design variables improves the efficiency of the design leading to lower fiber contents which, in turn, reduces the material cost. This result follows from a comparison of the results for designs with three design variables (the present case) with one or two design variables studied in Problems 1, 2 and 3.

Table 12: Optimal fiber angles and the weight for three-phase laminates with thickness $H = 0.01\,m$, $V_{fk} \leq 0.6$, $W_{CNTk} \leq 0.05$, $W_{CNTmax} = 1.0\%$, $-90 \leq \theta_k \leq +90$, and the frequency constraint $\Omega_O = 1.75$.

Aspect ratio b/a	Optimal V_f per layer	W_{CNT} per layer	Optimal fiber angles θ_k	Weight W_L	Reference weight W_0	Design efficiency factor
1.00	$[0/0/0/0]_s$	$[0.015/0/0.025/0]_s$	$[45/45/0/45]_s$	12.01	15.60	0.770
1.25	$[0.023/0/0/0]_s$	$[0.04/0/0/0]_s$	$[45/30/30/45]_s$	15.10	23.78	0.635
1.50	$[0.17/0/0/0]_s$	$[0.04/0/0/0]_s$	$[60/45/0/45]_s$	18.76	32.40	0.579
1.75	$[0.28/0/0/0]_s$	$[0.04/0/0/0]_s$	$[90/90/0/45]_s$	22.47	41.16	0.546
2.00	$[0.34/0/0/0]_s$	$[0.04/0/0/0]_s$	$[90/0/0/45]_s$	26.07	49.92	0.522

6.6. Problem 5: Three-phase laminate with non-uniform layer thicknesses and non-uniform distribution of reinforcement materials (Design variables: V_{fk}, W_{CNTk}, fiber angles θ_k, thickness ratios h_k/H)

Next, minimum weight designs for the three-phase laminate are studied with four design variables, namely, fiber and CNT contents, fiber angles and the layer thicknesses. As such, the layer thicknesses are taken as non-uniform. Layer thickness ratio is defined as the ratio of the thickness h_k of k^{th} layer and the thickness H of the laminate. Table 13 presents the results of optimization with four design variables with increasing frequency constraints. Zero fiber content is observed for low frequency constraints and higher CNT content appears in the interior layers. For higher frequency constraints, both fiber and CNT contents increase and higher CNT reinforcement and thickness ratios are observed in the outer layers. The thicker outer layers at higher frequencies are needed to provide the necessary stiffness to satisfy the higher frequency constraints.

The benefit of using non-uniform thicknesses for weight minimization is highlighted by comparing Tables 11 (three design variables, namely fiber and CNT content, fiber angle and uniform layer thicknesses) and Table 13 (four design variables with non-uniform layer thicknesses). The design efficiency gradually improves with increasing frequency constraints in the present case with four design variables. For the first three values of the frequency constraints, the same design efficiency is obtained for problems with three and four design variables which have zero fiber content. However, for higher frequency constraints, the problem with four design variables (Table 13) shows an improvement in the design efficiency compared to the solutions with three design variables (Table 11) as expected. Also noted is that as the frequency constraint increases, the design efficiency improves, i.e., the weight of the optimal laminate decreases.

Table 13: Minimum weight design for three-phase square laminate with 8-layers subject to 4 design variables, thickness $H = 0.01$ m, $V_{fk} \leq 0.6$, $W_{CNTk} \leq 0.05$, $W_{CNTmax} = 1.0\%, -90° \leq \theta_k \leq +90°$, $0.01 \leq h_k/H \leq 0.15$, $b/a = 1$

Freq. const.	Optimal fiber angle	Optimal V_f per layer	W_{CNT} per layer	h_k/H	Weight W_L	Reference weight W_0	Design Efficiency factor
1.25	[45/30/30/45]s	[0/0/0/0]s	[0.005/0.019/0.018/0.01]s	[0.15/0.15/0.1/0.1]s	12.02	15.0	0.801
1.30	[45/30/30/45]s	[0/0/0/0]s	[0.005/0.027/0.015/0.003]s	[0.15/0.15/0.1/0.1]s	12.02	15.6	0.770
1.50	[45/30/30/45]s	[0/0/0/0]s	[0/0.04/0/0.01]s	[0.05/0.15/0.15/0.15]s	12.02	18.0	0.667
1.75	[45/0/30/30]s	[0.15/0/0/0]s	[0.05/0/0/0]s	[0.15/0.15/0.1/0.1]s	12.06	21.0	0.574
2.00	[45/45/60/45]s	[0.19/0/0/0]s	[0.05/0/0/0]s	[0.15/0.15/0.06/0.14]s	12.58	24.0	0.524
2.20	[45/90/90/30]s	[0.35/0/0/0]s	[0.05/0/0/0]s	[0.15/0.15/0.1/0.1]s	13.08	26.4	0.495
2.40	[45/45/45/30]s	[0.53/0/0/0]s	[0.05/0/0/0]s	[0.15/0.12/0.09/0.14]s	13.61	28.8	0.473
2.50	[45/45/30/45]s	[0.6/0.13/0/0]s	[0.05/0/0/0]s	[0.15/0.13/0.08/0.14]s	14.21	30.0	0.473

7. Conclusions

The minimum weight designs problems for multiscale CNT/fiber reinforced composite laminates, subject to a frequency constraint, are formulated and the results are presented in table and graph formats. In the optimal design problems, fiber and CNT reinforcements are taken as design variables as well as the fiber orientations and the ply thicknesses. Sequential Quadratic Programming (SQP) algorithm was implemented to determine the optimal values of the design parameters. The method of solution was verified by comparing the results with the numerical results obtained by using ANSYS software package. The results were given for 8-layered and simply supported laminates with the five optimal design problems formulated and solved in order of increasing complexity. Design variables were defined as fiber volume contents of layers, CNT weight contents of the layers, fiber orientations and the thicknesses of layers.

The main objective of the study is to determine the minimum weight designs of two-phase (reinforced by fibers) and the three-phase (reinforced by fibers and CNTs) composite laminates and to compare these two designs. In order to assess the effectiveness of different design parameters in minimizing the weight, a design efficiency factor was introduced. As noted, a sequence of five optimization problems was solved with the first problem involving a two-phase fiber reinforced laminate with fiber contents of layers taken as the design variables with no CNT reinforcement. In the subsequent problems, the number of design parameters was increased by including the CNT content of the layers, fiber orientations and the layer thicknesses into the design process one by one resulting in five different optimization problems. This approach was instrumental in assessing the effectiveness of each design variable in comparison to others in minimizing the weight. As expected, the introduction of the CNTs as reinforcement and the fiber orientations as design variables improved the design efficiency to a large extent.

An important result of the present study is that CNT reinforcement reduces the effectiveness of the optimal fiber orientations in minimizing the weight. This somewhat unexpected result is due to the fact that reinforcing the matrix with CNTs increases the stiffness of the matrix which, in turn, makes the fibers less effective in comparison to the overall stiffness of the CNT reinforced matrix. It was also observed in Figure 3 that the optimal fiber orientations have changed with the addition of CNTs to the matrix. This was again somewhat unexpected and was due to the increased stiffness of the CNT-reinforced polymer.

As expected, both fiber and CNT reinforcements were more effective in minimizing the weight if they are placed in the outer layers or close to the outer layers of the laminate due to the higher contribution of the outer layers to laminate stiffness. Numerical results indicated that by increasing the amount of CNTs in the design, the amount of fiber required to satisfy the frequency constraint is reduced. This, in turn, leads to a more lightweight laminate. Combining these reinforcements (fiber + CNT) with stacking sequence optimization, as was done in Problem 4, leads to further weight savings. The last problem in the study (Problem 5) involved the optimization of the layer thicknesses in addition to the other design parameters studied in Problems 1-4. Layer thickness optimization led to further improvements in reducing the laminate weight and increasing the design efficiency. Future studies on weight minimization should be extended to buckling and bending problems as well as to functionally graded nanocomposites and minimum cost designs.

Acknowledgements

This work was supported by the National Research Foundation of South Africa; and the University of Kwazulu-Natal, Durban, South Africa.

Data availability

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also forms part of an ongoing study.

References

[1] Iijima S. Helical microtubules of graphitic carbon. Nature 1991;354:56-58.

[2] Duc ND, Lee J Nguyen-Thoi T, Thang PT. Static response and free vibration of functionally graded carbon nanotube-reinforced composite rectangular plates resting on Winkler–Pasternak elastic foundations. Aerospace Science and Technology 2017;68:391-402.

[3] Garg A, Chalak HD, Belarbi, M-O, Zenkour AM, Sahoo R. Estimation of carbon nanotubes and their applications as reinforcing composite materials–An engineering review. Compos Struct 2021;272:114234.

[4] Vinyas M, Athul SJ, Harursampath D, Loja M, Thoi TN. A comprehensive review on analysis of nanocomposites: from manufacturing to properties characterization. Materials Research Express 2019;6:092002.

[5] Ervina Efzan MN, Siti Syazwani N. A review on effect of nanoreinforcement on mechanical properties of polymer nanocomposites. J Solid State Phenomena 2018;280:284-293.

[6] Kumar A, Sharma K, Dixit AR. A review on the mechanical properties of polymer composites reinforced by carbon nanotubes and graphene. Carbon Letters 2021;31: 149–165.

[7] Lal A, Markad K. Deflection and stress behaviour of multi-walled carbon nanotube reinforced laminated composite beams. Computers and Concrete 2018;22:501-514.

[8] Mehar K, Panda SK, Mahapatra TR. Large deformation bending responses of nanotube-reinforced polymer composite panel structure: Numerical and

experimental analyses. Proc IMechE Part G: J Aerospace Engineering 2019;233: 1695-1704.
[9] Moradi-Dastjerdi R, Malek-Mohammadi H. Free vibration and buckling analyses of functionally graded nanocomposite plates reinforced by carbon nanotube. Mechanics of Advanced Composite Structures 2017;4:59-73.
[10] Civalek Ö, Dastjerdi S, Akgöz B. Buckling and free vibrations of CNT-reinforced cross-ply laminated composite plates. Mechanics Based Design of Structures and Machines 2020.
[11] Patel A, Das R, Sahu SK. Experimental and numerical study on free vibration of multiwall carbon nanotube reinforced composite plates. International Journal of Structural Stability and Dynamics 2020;20:2050129.
[12] Civalek Ö. Akbaş ŞD, Akgöz B, Dastjerdi S. Forced vibration analysis of composite beams reinforced by carbon nanotubes. Nanomaterials 2021;11:571.
[13] Yas MH, Heshmati M. Dynamic analysis of functionally graded nanocomposite beams reinforced by randomly oriented carbon nanotube under the action of moving load. Appl Math Model 2012;36:1371–1394.
[14] Kazakov IA, Krasnovskii AN, Kishuk PS. The influence of randomly oriented CNTs on the elastic properties of unidirectionally aligned composites. Mechanics of Materials 2019;134,54–60.
[15] Al-Haik M, Luhrs CC, Reda Taha MM, Roy AK, Dai L, Phillips J, Doorn S. Hybrid carbon fibers/carbon nanotubes structures for next generation polymeric composites. Journal of Nanotechnology 2010;860178.
[16] Zakaria MR, Akil HM, Kudus MHA, Ullah F, Javed F, Nosbi N. Hybrid carbon fiber-carbon nanotubes reinforced polymer composites: A review. Composites Part B: Engineering 2019;176:107313.
[17] He XQ, Rafiee M, Mareishi S, Liew KM. Large amplitude vibration of fractionally damped viscoelastic CNTs/fiber/polymer multiscale composite beams. Compos Struct 2015;131:1111–1123.
[18] Al-Furjan MSH, Fereidouni M, Sedghiyan D, Habibi M, Jung DW. Three-dimensional frequency response of the CNT-Carbon-Fiber reinforced laminated circular/annular plates under initially stresses. Compos Struct 2021;257:113146.
[19] Yadav A, Amabili M, Panda SK, Dey T, Kumar R. Nonlinear damped vibrations of three-phase CNT-FRC circular cylindrical shell. Compos Struct 2021;255:112939.
[20] Sobhani E, Moradi-Dastjerdi R, Behdinan K, Masoodi AR, Ahmadi-Pari AR. Multifunctional trace of various reinforcements on vibrations of three-phase nanocomposite combined hemispherical-cylindrical shells. Compos Struct 2022;279:114798.
[21] Hemmatian H, Fereidoon A, Sadollah A, Bahreininejad A. Optimization of laminate stacking sequence for minimizing weight and cost using elitist ant system optimization. Adv Eng Software 2013;57:8–18.
[22] Montemurro M, Izzi MI, El-Yagoubi J, Fanteria D. Least-weight composite plates with unconventional stacking sequences: Design, analysis and experiments. Journal of Composite Materials 2019;53:2209-2227.
[23] Rafiee M, Nitzsche F, Labrosse M. Cross-sectional design and analysis of multiscale carbon nanotubes-reinforced composite beams and blades. International Journal of Applied Mechanics 2018;10:1850032.
[24] Kumar P, Srinivas J. Free vibration, bending and buckling of a FG-CNT reinforced composite beam: Comparative analysis with hybrid laminated composite beam. Multidiscipline Modeling in Materials and Structures 2017;13:590-611.

[25] Megahed M, Abo-Bakr RM, Mohamed SA. Optimization of hybrid natural laminated composite beams for a minimum weight and cost design. Compos Struct 2020;239:111984.
[26] Abo-Bakr RM, Abo-Bakr HM, Mohamed SA, Eltaher MA. Optimal weight for buckling of FG beam under variable axial load using Pareto optimality. Compos Struct 2021;258:113193.
[27] Abo-Bakr RM, Shanab RA, Attia MA. Multiobjective optimization for lightweight design of bi-directional functionally graded beams for maximum frequency and buckling load. Compos Struct 2021;278:114691.
[28] Shrivastava S, Mohitea PM, Yadav T, Malagaudanavar A. Multi-objective multi-laminate design and optimization of a carbon fibre composite wing torsion box using evolutionary algorithm. Compos Struct 2018;185:132–147.
[29] Kamarian S, Shakeri M, Yas MH. Natural frequency analysis and optimal design of CNT/fiber/polymer hybrid composites plates using Mori-Tanaka approach, GDQ technique, and firefly algorithm. Polymer Composites 2018;39:1433-1446.
[30] Zhou HW, Mishnaevsky Jr L, Yi, HY, Liu YQ, Hu X, Warrier A, Dai GM. Carbon fiber/carbon nanotube reinforced hierarchical composites: Effect of CNT distribution on shearing strength. Composites Part B: Engineering 2016;88 201-211.
[31] García-Macías E, Rodriguez-Tembleque L, Castro-Triguero R, Sáez A. Buckling analysis of functionally graded carbon nanotube-reinforced curved panels under axial compression and shear. Composites Part B: Engineering 2017;108:243-256.
[32] Ghasemi AR, Soleymani M. Effects of carbon nanotubes distribution on the buckling of carbon nanotubes/fiber/polymer/metal hybrid laminates cylindrical shell. Journal of Sandwich Structures & Materials 2021;23:2086-2105.
[33] Karamanli A, Aydogdu M. Vibration behaviors of two-directional carbon nanotube reinforced functionally graded composite plates. Compos Struct 2021;262:113639.
[34] Liu Q, Paavola J. Lightweight design of composite laminated structures with frequency constraint. Compos Struct 2016;156:356-360.
[35] Moleiroa F, Madeira JFA, Carrera E, Reddy JN. Design optimization of functionally graded plates under thermo-mechanical loadings to minimize stress, deformation and mass. Compos Struct 2020;245:112360.
[36] Reddy JN. Mechanics of Laminated Composite Plate and Shells, Theory and Analysis, CRC Press, 2004.
[37] Nemeth, MP. Importance of anisotropy on buckling of compression-loaded symmetric composite plates. AIAA Journal 1986;24:1831-1835.
[38] Adali S, Richter A, Verijenko VE, Summers EB. Optimal discrete ply design of hybrid laminates with angles for maximum buckling load and minimum cost. Composite Structures 1995;32:409-415.
[39] Adali S, Verijenko VE. Optimum stacking sequence design of symmetric hybrid laminates undergoing free vibrations. Compos Struct 2001;54:131-138.
[40] Gholami R, Ansari R, Gholami Y. Numerical study on the nonlinear resonant dynamics of carbon nanotube/fiber/polymer multiscale laminated composite rectangular plates with various boundary conditions. Aerospace Science and Technology 2018;78:118-129.
[41] Neto MA, Yu W, Leal RP. Generalized Timoshenko modelling of composite beam structures: sensitivity analysis and optimal design. Engineering Optimization 2008;40:891-906.
[42] Zabihollah A, Zareie S. Optimal design of adaptive laminated beam using layerwise finite element. Journal of Sensors 2011;240341.
[43] Marques F, Mota A, Loja M. Variable stiffness composites: Optimal design studies. Journal of Composites Science 2020;4:80.

[44] Wang X, Wang Y, Xu S. DSC analysis of a simply supported anisotropic rectangular plate. Compos Struct 2012;94:2576-2584.
[45] Leissa AW, Narita Y. Vibration studies for simply supported symmetrically laminated rectangular plates. Compos Struct 1989;12:113-132.
[46] Rafiee R, Eskandariyun A. Estimating Young's modulus of graphene/polymer composites using stochastic multi-scale modeling. Composites Part B: Engineering 2019;173:106842
[47] Narita Y. Layerwise optimization for the maximum fundamental frequency of laminated composite plates. Journal of Sound and Vibration 2003;263:1005-1016.
[48] Ehsani A, Rezaeepazhand J. Stacking sequence optimization of laminated composite grid plates for maximum buckling load using genetic algorithm. International Journal of Mechanical Sciences 2016; 119: 97-106.